Budapest University of Technology and Economics
Department of Electric Power Engineering

PhD Theses

NOVEL HIGH FREQUENCY MODEL OF
TRANSFORMERS OF ELECTRONIC DEVICES

by
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To my mother for all of her love
PREFACE

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LIST OF SYMBOLS

$A$ cross sectional area
$A_M$ cross sectional area of core
$A_W$ cross sectional area of wire
$c$ light velocity
$C$ capacitance
$C'$ capacitance for unit length
$C_T$ turn-to-turn capacitance
$C_K$ turn-to-turn capacitance in the lump model
$C_{nC}$ capacitance to the core for one lump
$C_{nK}$ turn-to-turn capacitance for one lump
$C_{T0}$ capacitance of one turn to the core
$d$ diameter
$d_W$ diameter of wire
$D$ diameter of coil
$D_C$ diameter of core
$D_W$ medium diameter of coil
$E$ signal energy
$f$ frequency
$f_{max}$ maximum frequency
$G$ conductance
$G'$ conductance for unit length
$i$ electric current, variant
$I$ electric current, invariant
$\Delta i$ electric current deviation
$K$ reciprocal capacitance
$K'$  reciprocal capacitance for unit length

$l$  length

$l_n$  wire length of a lump

$l_W$  length of whole wire within the coil

$L$  inductance

$L'$  inductance for unit length

$L'_K$  inductance for unit length in the shunt path

$L_{HF}$  inductance in a Foster’s circuit

$L_{nK}$  self inductance of one lump in the shunt path

$L_T$  self inductance of one turn

$L_T$  inductance of one turn with core

$L_{TP}$  inductance of one turn in the shunt path

$L_n$  self inductance of a lump

$M_n$  mutual inductance between two lumps

$n$  number of turns in a lump

$N$  number of turns in a whole coil layer or coil

$R$  resistance

$R'$  resistance for unit length

$R'_K$  resistance for unit length in the shunt path

$R_{n0}$  direct current resistance of the coil within a lump

$R_{HF}$  resistance value of one lump in case of maximum frequency

$R_{nK}$  resistance of one lump in the shunt path

$R_T$  resistance value of one turn

$R_{TP}$  resistance value of one turn in the shunt path

$t$  time

$t_n$  propagation time of electromagnetic field along the coil length in one lump

$t_{nW}$  propagation time of electromagnetic field along the wire length in the coil

$t_T$  time delay for one turn

$u$  voltage, variant

$U$  voltage, invariant

$U_M$  measured voltage
$U_S$  simulated voltage  
$\Delta u$  voltage drop  
$\Delta U$  voltage difference  
$v_b$  thickness of protective pipe  
$v_V$  thickness of varnish insulation on wire  
$v_W$  velocity of electromagnetic waves along coil wire  
$x$  variable  
$z$  variable  
$Z$  impedance  
$Z_0$  wave impedance  
$Z_{0o}$  wave impedance in vacuum (air)  

$\varepsilon$  electric permittivity  
$\varepsilon_0$  electric permittivity of vacuum (air)  
$\varepsilon_r$  relative electric permittivity  
$\varepsilon_{rb}$  relative electric permittivity of material (PVC) of the protective pipe  
$\varepsilon_{rV}$  relative electric permittivity of varnish insulation on wire  
$\Phi$  magnetic flux  
$\lambda_i$  turn ratio  
$\Lambda$  magnetic conductivity  
$\mu$  magnetic permeability  
$\mu_0$  magnetic permeability of vacuum (air)  
$\mu_r$  relative magnetic permeability  
$\mu_i$  turn ratio  
$\rho$  specific resistance  
$\sigma$  specific conductance  
$\omega$  angular frequency
## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>EMP</td>
<td>Electromagnetic Pulse</td>
</tr>
<tr>
<td>ESD</td>
<td>Electrostatic Discharge / Damage</td>
</tr>
<tr>
<td>LEMP</td>
<td>Lightning Electromagnetic Pulse</td>
</tr>
<tr>
<td>SD</td>
<td>Shielding Degree</td>
</tr>
<tr>
<td>SEMP</td>
<td>Switching Electromagnetic Pulse</td>
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1. INTRODUCTION

1.1. Previous research

Impacts of over-voltages reaching transformers and transformer coils are already researched over a century. Transient over-voltages have different characteristics, like rise time, peak value, spectrum, energy, charge, etc. and different impacts on devices in turn, depending on the phenomenon causing the over-voltage: LEMP (Lightning Electromagnetic Pulse), SEMP (Switching Electromagnetic Pulse) sources of surges and bursts and electrostatic discharges (ESD).

The increasing level of electromagnetic noises on electric networks and the even higher switching frequency of power supply units built into sensible electronic devices make necessary to have more precise modelling of transformers - for low voltage and low power transformers as well -, taking more precisely into account the wave propagation phenomena along the coils. A reliable but simple high frequency model of transformer shielding is necessary to be developed for the forecast of the interaction of different over-voltages and the shielding.

Several versions of SPICE based circuit simulator software are widely used for simulating electric and electronic circuits before, during and after manufacturing. An important task of these simulation sessions is to predict the behaviour of the circuits in case of over-voltages as well. Devices like transformers of electronic equipment becoming smaller and smaller and therefore being more and more sensitive to interference, have to be modelled for transient over-voltages with high frequency content like burst and ESD (Electrostatic Discharge). In case of small transformers built into electronic devices, quick over-voltages like bursts and electrostatic discharge are of interest as well.

Bursts and ESD have rather small electric charge absorbed by the capacitance of high voltage transformers but small scale transformers are not able to absorb this small amount of charge.
They have a rise time of some nanoseconds and propagating through small transformers they can cause damages. In addition to those described above switching frequencies of supply units are near to the MHz range, so wave propagation is no more negligible either in small transformers.

For a more precise high frequency modelling of transformers, a reliable model is needed. The known models do not take into account every electromagnetic wave propagation phenomena along the coils. The current path composed by the capacitance between the neighbouring turns, not being negligible at high frequencies, is taken into account only as capacitors connected directly in series with each-other. This is the case in longitudinal and radial directions as well. In these models certain voltage values appear with no delay at every locations of the capacitance chain, i.e. along the whole length and whole radial dimension of the coil when applying the supply voltage at the input ports of the coil.

Shielding inserted between the coils of transformers have the task to conduct the electric charge of transient over-voltages to the earth avoiding so the propagation of over-voltages to the secondary coil of the transformer. However in case of fast transients with high frequency content this shielding is no more effective. Wound and cylinder type shielding has more or less inductance to the ground hindering the electric charges to reach the shielding, decreasing so the shielding efficiency. In case of over-voltages with very high frequency content no shielding is built into the transformers because their inefficiency.

A more precise simulation model for the shielding could help during the decision what type of shielding should be installed if any. The known shielding models do not take into account the capacitance of the shielding to the surrounding conductive bodies and the inductance in series with the shielding inside and outside of the transformer housing.

1.2. Objective of the research

The known high frequency transformer models are not able to simulate all aspects of the electromagnetic wave propagation along coils and transformers, neither in longitudinal nor in radial direction. In addition there is no known model for the shielding inserted between the coils of transformers taking into account the capacitance of the shielding to the surrounding
conductive bodies and the inductance in series with the shielding inside and outside of the transformer housing.

In the scope of the above shortage of the high frequency models of transformers the objectives of my research work are as follows.

(i) My purpose is to develop a novel high frequency model for one-layer, straight coils.

An inevitable part of a model of one-layer straight coils needed for high frequency examinations is the capacitance between turns along with the capacitance between the coil and the core and the housing. There are several high frequency transformer models being applicable in certain cases, resulting no contradictions to each-other concerning the basic function of the transformer. Some models have lumped parameters determined e.g. by measurements, others are of “quasi” distributed parameters and of really distributed parameters taking into account wave propagation to a certain amount.

None of the known coil models is able to take into account electromagnetic wave propagation along the path composed by the turn-to-turn capacitance, because these capacitors are connected directly in series to each-other in the model according to the classic Wagner’s theorem. In the case of these models certain voltage values appear with no delay at every locations of the capacitance chain, i.e. along the whole length of the coil when applying the supply voltage at the input ports of the coil.

For taking into account electromagnetic wave propagation along straight coils I would like to propose a length unit inductance connected in series to the length unit turn-to-turn capacitance within the distributed parameter model of one-layer straight coils proposed by Wagner. This inductance makes able the model to take into account the wave propagation along the turn-to-turn capacitance current path of the coil.

For an easier use of the model with a SPICE software I would like to develop a “quasi distributed parameter model” as well with proposed calculation methods of the parameters. This model contains several identical lumps for modelling the wave propagation but being easily applicable for the practical use.
For testing the model I would like to prove it by measurements, so a two meter long straight coil of copper wire with a diameter of 1 mm included the varnish insulation and the wire is densely wound onto a plastic protective pipe is tested. Measurements on the coil with an iron core have been realised with pulse generators and an oscilloscope to compare the results with those given by the model with the simulator software.

(ii) My purpose is to work out a novel high frequency model for multi-layer straight coils and coils on each-other.

The known high frequency coil and transformer models contain only capacitors between the turns of the neighbouring coil layers and coils, therefore these models are not able to take into account electromagnetic wave propagation along the layers of coils and between the coils, because these capacitors are connected directly in series to each-other. In the case of these models certain voltage values appear without delay at every locations of the capacitance chain, i.e. along the whole radial dimension of the coil when applying the supply voltage at one bordering layer of the coil.

For taking into account electromagnetic wave propagation along multi-layer coils and transformers i.e. coils on each-other I would like to propose a unit length inductance connected in series to the unit length layer-to-layer capacitance within the distributed parameter model of the coils and transformers. This inductance makes able the model to take into account the wave propagation along the layer-to-layer capacitance current path in radial direction as well.

For an easier use of the model with a SPICE software I propose a “quasi distributed parameter model” as well with proposed calculation methods of the parameters. Because of the several identical lumps, this model can take into account electromagnetic wave propagation, remaining meanwhile easily applicable in the practice.

To validate the model I have developed a measurement for two meter long straight coils of copper wire with a diameter of 1 mm included the varnish insulation and the wire has been densely wound onto plastic protective pipes. Measurements on the coils, with and without iron core has been realised with pulse generators and oscilloscope, to compare the results with those given by the model with the simulator software.
(iii) My purpose is to develop a novel high frequency model for the shielding between transformer coils.

Primary coils of small transformers with even less dimensions can no more absorb the rather low electric charges of bursts and electrostatic discharges, thus voltages being dangerous to electronic circuits can propagate to the secondary circuit. Shielding installed between the coils in small transformers should avoid the propagation of over-voltages to the secondary coil of the transformer. However, because of the inductance existing always in series between the shielding and the electric charge source composed by the ground, the shielding is not so effective at high frequencies belonging to fast common mode transients like bursts and electrostatic discharges as at low frequencies belonging e.g. to surges.

A rather simple simulation model can help by the decision which art of shielding should be installed if any. The known shielding models do not take into account the capacitance of the shielding to the surrounding conductive bodies and the inductance in series with the shielding inside and outside of the transformer housing.

I would like to propose a high frequency SPICE model of transformer shielding built in a circuit simulator software for the use during the dimensioning of the transformers taking also into account the ground connection aspects of the shielding.

For testing the model I have realised a measurement on a PC supply unit transformer with signal generators to test the model. I would like to introduce an inductance in series to the ground of the shielding, yielding similar results as those of the measurements. A capacitance is introduced then in parallel to this inductance, both are then split into two parts each to obtain a reliable model for shielding between transformer coils.

The research focuses on coils with a structure in general use in transformers of electronic devices and on their behaviour during the first time period after applying voltage onto the coil, thus no core losses are taken into account during the investigations. The proposed models are not valid for high voltage transformers and for other transformers with special structure.
2. BACKGROUND OF THE WORK

The dimensions of transformers built into electronic devices are becoming smaller and smaller and transient over-voltages can have more and more severe impacts on the transformers and the electronic circuits at their secondary sides. The impacts on these devices caused by electromagnetic pulses (EMP) like lightning electromagnetic pulses (LEMP), switching electromagnetic pulses (SEMP), bursts and electrostatic discharges (ESD) are intensively researched nowadays as well [1], [2].

There are numerous high frequency coil models and transformer models known. The difference between them depends on the phenomenon in the focus of modelling and on the objective of the model. These models are applicable in certain cases, resulting no contradictions to each-other concerning the basic function of the transformer. In the following, several models composing the basis for development, other theories and methods are listed contributing to the results of the research.

2.1. Known high frequency models of coils and transformers

Depending on the objective of modelling coils and transformers and of the phenomena in the focus, different kind of models are known based on more or less theoretical or practical analysis or measurements. Distributed parameter models are supported with more theory however being less useful in the daily design work. Lumped parameter models are more simple and can be used with circuit simulator software, however all the phenomena of fast transients and wave propagation can not be modelled.

Distributed parameter models of coils are based in most cases on the traditional transmission line model (Fig. 1) and are built up by a further development of the transmission line model [3], [4], [5], [6]. Elements added to the basic model take into account the specific characteristics of the coils.
Fig. 1. The traditional distributed model of transmission lines

The parameters in Fig. 1 have the following meaning: $L'$ is the inductance of the line for the unit length in H/m, $R'$ is the resistance of the line for the unit length in Ω/m, $C'$ is the parallel capacitance of the line for the unit length in F/m and $G'$ is the conductance of the line for the unit length in S/m.

The characteristic for a length of $dz$ of the line is calculated with multiplying the parameters by the length taken into account $L' dz$, $R' dz$, $C' dz$, $G' dz$. The Kirchhoff equations for the above arrangement are

$$\begin{align*}
-u + L' dz \frac{\partial i}{\partial t} + R' dz i + \left( u + \frac{\partial u}{\partial z} dz \right) &= 0, \\
-i + C' dz \frac{\partial u}{\partial t} + G' dz u + \left( i + \frac{\partial i}{\partial z} dz \right) &= 0.
\end{align*}$$

The solution of the above equations can be rather easily be achieved and an important issue is the velocity the electromagnetic wave propagates with along the line in ideal case for simplicity

$$v = \frac{1}{\sqrt{L'C'}}. \quad (2)$$

For ideal transmission lines wave impedance has similar importance too

$$Z_0 = \frac{u}{i} = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{L}{C'}}. \quad (3)$$
The distributed models proposed by me is based on this traditional theory and is the starting point for further development. To calculate wave propagation velocity and wave impedance for coils is also an objective of investigation during the research.

A significant task is to take into account the time delay of the induced voltage in the turns being apart from each-other if the electromagnetic field propagation has to be modelled as well. For example in Fig. 2 the distributed model of a Rogowski coil is shown with voltage sources in series with the original series elements in a short section of the coil [7]. Voltage $u'(x,t) \Delta x$ is induced by the currents flowing in the other sections of the coil.

Fig. 2. Distributed model of a shielded Rogowski coil

Solutions applied for delaying the voltage and current and the time delay elements in a circuit simulation software are significant parts of the models proposed by me.

There are high frequency transformer models focusing on the resonant character of the transformer. These high frequency transformer (HTF) models belong to the class of models where the frequency dependent response at the terminals of the transformer is reproduced by means of equivalent RLC networks [8]. This implies an initial assumption of linearity being partly acceptable in some cases.

$$
\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1m} \\
Y_{21} & Y_{22} & \cdots & Y_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{m1} & Y_{m2} & \cdots & Y_{mm}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_m
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_m
\end{bmatrix}
$$

(4)
The model is applicable to a multi-winding, multi-phase transformer and is based on the frequency characteristics of the transformer admittance matrix between its terminals over the frequency range of interest. The elements of the nodal admittance matrix in equations (4), are approximated in the frequency domain consisting of real as well as complex conjugate poles and zeros (Fig. 3).

![Fig. 3. Theoretical multi-winding, multi-phase model](image)

At the considered three-phase terminals matrix \([Y_{ij}]\) is a 3x3 sub-matrix and \(m\) is the number of the groups. The fitting technique used to approximate the admittance functions of the transformer is based on a least squares curve fitting process performed with the aid of MATLAB. These numerical approximations are realised in the form of an RLC network (Fig. 4).

![Fig. 4. Structure of an RLC module](image)

The RLC module in Fig. 4 reflects the known frequency characteristics of the admittance functions of the transformer:
- inductive behaviour at low frequencies which includes frequency dependent effects due to skin effect in the windings and iron core eddy current losses. These are simulated by the \(R_iL_i\) branches in the middle of the model shown in Fig. 4.
- Series and parallel resonance phenomena from mid to high frequencies caused by winding to winding and winding to ground stray capacitances. These are simulated by the $R_iL_iC_i$ branches on the right of the model shown in Fig. 4.

- Predominantly capacitive behaviour at high frequencies represented by the single $R_1C_1$ branch.

![Diagram of the ideal line model of a high voltage transformer](image)

Fig. 5. Ideal line model of a high voltage transformer

Extensive research have been made also at the University of Karlsruhe in this field [9]. The developed lumped parameter model of a high voltage transformer is shown in Fig. 5. This model is the result of a modal analysis of the transmission and admittance function of the transformer as a four pole. The model can contain more than three blocks according to the result of the analysis.

The model in Fig. 5 is a lumped parameter model developed for circuit simulator software and contain the so called “Transmission Line” time delay element being a feature of a SPICE software [10], [11]. This circuit element is characterised by a wave impedance $Z_i$ and a time value $t_i$. Value of $Z_i$ have significant impact on the model behaviour, thus its adequate calculation is an important issue as well. The above models have been developed for power
transformers and need measurements for the accurate calculation of the parameters. None of these models can take into account all electromagnetic wave propagation effects due to their basic lumped structure.

Some other models have also lumped parameters determined e.g. by measurements [12], others are of “quasi” distributed parameters [13], [14] and of really distributed parameters [15] taking into account wave propagation to a certain amount [16], [17]. There are models based on finite element methods, e.g. using the 2-D axisymmetrical finite element analysis for calculating voltage distributions [18]. The recent researches for obtaining most possible exact transient voltage and current values being inevitable in case of current transmitters [19], [20], however in case of over-voltages reaching supply transformers a more simple models can give appropriate results as well.

None of the above models take into account every aspects of electromagnetic wave propagation along the coils and transformers. Some of them models the time needed for the wave propagation along the wire of the coils in the transformer like those in Fig. 2 and Fig. 5. The model in Fig. 2 shows a theoretical solution for the problem and that in Fig. 5 is a practical one using the “Transmission Line” time delay element for SPICE. However none of the models is able to take into account wave propagation along the shunt path. The current path composed by the turn-to-turn capacitance is either not taken into account or only by capacitors in series to each-other.

2.2. Wagner’s theorem

An inevitable part of the high frequency models of coils and transformers is the capacitance between turns, the capacitance between the coil and the core and housing. Most of the models taking into account the turn-to-turn capacitance are based on the well-known Wagner’s theorem [21], [22], [23]. Wagner’s distributed parameter model is based on the known transmission line model as well.
Wagner’s theorem is applicable for straight, one-layer, ideal coils with core. Fig. 6 shows Wagner’s model for an ideal coil with core. $L'$ means the inductance for the unit length of the main current path in H/m, $K'$ the reciprocal of the turn-to-turn capacitance for the unit length in 1/Fm and $C'$ is the capacitance between the coil and the core in F/m.

Parameter $K'$ is connected in parallel to $L'$ hindering so the model to take into account time delays of voltage values appearing at the locations of the coil being apart from the point of applying the voltage. According to this theorem a voltage wave reaching the coil causes a $U_x$ initial voltage distribution along the coil at every locations of the capacitor chain without any delay (Fig 7)

$$U_x = U \frac{\text{sh} \alpha \left(1 - \frac{x}{l}\right)}{\text{sh} \alpha l}, \quad (5)$$

with

$$\alpha = \frac{C'}{\sqrt{K'}}. \quad (6)$$

In the above equations $U_x$ means the initial voltage on the coil at a distance of x from the grounded end of the coil and $U$ the whole applied voltage. $l$ is the total length of the coil and $\alpha$ is a parameter in F.

In Fig. 7 on the right side the initial voltage distribution is shown at the moment of applying the voltage surge ($U_{\text{surge}}$) onto one (upper) end of the coil compared with the voltage distribution belonging to the rated voltage during normal operation ($U_{\text{rated}}$). In equation (6) the higher the value of $\alpha$ is, the more uneven is the initial voltage distribution along the coil.
Fig. 7. Initial voltage distribution along the coil caused by a surge

An uneven voltage distribution causes high electric stresses in the insulation of the turns at the end of the coil. Neither the other turns of the coil are protected against high turn-to-turn voltages because the voltage distribution varies for a period until the steady state will establish.

Finding the result for a more even initial voltage distribution in case of a surge attack of a transformer coil has been a priority task in the field of power transformer design [23], [24]. High voltage power transformers are expensive machines with expensive and sensitive insulation. In case of large power transformers rapid transients like bursts and electrostatic discharges are not of interest because of their low charges and energy. Power transformers are tested mainly for surges with lower frequencies but higher charges and energy, because they can cause damages to these machines as well.

The voltage distribution can be made more close to linear with decreasing the value of $\alpha$ in equation (6), i.e. decreasing the capacitance to the earth ($C'$) and increasing the capacitance between the turns ($K'$). An even initial voltage distribution belongs to $\alpha = 0$. 
During the transient period the voltage oscillates along the coil before reaching its ultimate distribution (Fig. 8). Every coil has a so called limit frequency

$$\omega_l = \frac{1}{\sqrt{L'K''}}$$

and no waves with frequencies above this limit frequency \( \omega_l \) can penetrate into the coil and thus propagate along the coil in turn. A solution for achieving more even voltage distribution is to use interleaved disc type coils where the turns are arranged in disks so the turns in voltage sequence are not arranged close to each-other but in a greater distance.

In Fig. 8 the dependence of the initial voltage distribution along the coil on \( \alpha \) is shown, the higher the value of \( \alpha \) is the more uneven is the initial voltage distribution. Fig. 9 shows the envelop curves of the oscillating voltages along the coil.

Fig. 8. Initial voltage distribution along the coil depending on the value of \( \alpha \)
The envelop curve in Fig. 9 is actually a mirrored curve of the initial voltage curve to the line corresponding to the even voltage distribution. The straight line between the envelop curves belongs to the even voltage distribution during normal operation.

As a conclusion, Wagner’s theorem takes into account the turn-to-turn capacitances of coils, thus composing a basis for high frequency distributed parameter coil models being applicable in many practical cases. It is the basis for the enhanced model proposed by me as well.

However, Wagner’s model is not able to take into account wave propagation along the shunt path composed by the turn-to-turn capacitance because of the capacitors in series to each other. At the moment of applying the voltage to the input ports of the coil the voltage value appears at every locations of the coil. This is impossible in the reality, a certain time in needed for the wave to propagate from one end to the other along the coil.
2.3. Multi-layer coil models

The deficiency of the high frequency models in case of the stray capacitance paths persists if wave propagation has to be taken into account in radial direction between coil layers or coils on each other, i.e. in transformers. There are high frequency models for high voltage transformers with interleaved disc type coils taking into account the turn-to-turn capacitance in every directions. Only capacitors connected in series with each other are taken into account in radial direction as well [9], [13], [14], [15].

Fig. 10 shows the commonly used high frequency model for multi-layer coils. The parameters \( L' \), \( C' \) and \( K' \) are the same as in the one-layer model and \( K''_L \) means the layer-to-layer capacitance for the unit length in \( 1/F_m \). In the figure only two neighbouring layers are shown. Voltage appears along the capacitance chain at every locations in radial direction as well at the time of applying the voltage onto the coil without any delay. Neither in radial direction can wave propagation taken into account with this model.

This modelling attitude appears when high frequency modelling of high voltage transformers with interleaved disc type coils (Fig. 11) [9]. The numbers in the rectangles representing the coil conductors in Fig. 11 show the sequence of the turns within the coils. The turns directly connected to each other are placed rather far from each other to obtain most possible even initial voltage distribution in case of surges as described before.
Fig. 11. Traditional high frequency model for ideal multi-layer coils

Only turn-to-turn and layer-to-layer capacitance is taken into account in this case as well. In the model in Fig. 11 every single turn of the coil is taken into consideration with its capacitance to all of the neighbouring turns being a possible way for calculations in case of power transformers with rather low number of turns.

For some problems simplified models of transformers with coils arranged in disks are also used (Fig. 12) [13], [14]. The parameters $K_i$ in the figure are actually disk-to-disk capacitances derived from turn-to-turn capacitances and $C_i$ are the capacitances between disks containing several turns and between the earth. This is actually a special case of lump reduction (see Chapter 2.4) and the model is actually a capacitor network.

Fig. 12. Simplified transient model for disk wound transformers

The model in Fig. 12 is that of a multi-layer coil reduced to a one-layer coil. For the modelling purposes only capacitors are taken into account. In case of investigations for surges
this model yields adequate results. Voltage distribution along the model shown by Fig. 12 can be calculated by matrices based on the equilibrium of electric charges (Fig. 13)

\[ \sum Q_i = 0 \]  \hspace{1cm} (8)

corresponding to the input and output currents of the nodes in the circuit. This task does not result in a long CPU time, but matrices have to be considered. In case of a simple SPICE model only the circuit is to be drawn up, all calculations are then made by the programme.

As a conclusion we can state, that none of the known models take into account the time delay needed for the electromagnetic wave to propagate from one coil layer to the other layer, because only capacitors in series compose the shunt current path of the stray capacitances between the turns in the neighbouring layers. An other disadvantage of these models is that they consist loop of capacitances. This is not allowed by circuit simulator softwares because these loops and circuits are not regular.

2.4. Lump reduction and the series Foster’s circuit

A distributed parameter model can give perfect solution in case of fast transients but can not be realised in a circuit simulator software. The only really distributed parameter element in a SPICE software is the so called “Lossy Transmission Line” [10] [11]. In my investigation I have chosen for the use of ideal transmission lines because of their simplicity and less number of parameters. This transmission line serves only for time delay and the lossy character of the
real coil is modelled by concentrated resistances. Using lossy transmission lines would not give the advantage of using a completely distributed parameter model, because it can simulate only a transmission line. In case of modelling coils it can be used only for time delay purposes like the normal transmission line. Thus a lumped parameter model is to be used in a circuit simulator software with several identical lumps connected in series to each other to maintain the ability of the circuit to model wave propagation. Thus a “quasi distributed model” is to be used which is actually a lumped parameter model but made of several identical lumps.

When modelling a coil the most precise results would be given by a circuit containing a model for each turn of the coil. In case of high voltage power transmission transformers this way can also be realised, with the help of a computer the desired results can be quickly obtained [9]. However coils of small transformers can have hundreds or thousands of turns so this way is not suitable for the practice. The solution can be given by the help of the so called turn reduction applied in case of high voltage transformers as well, namely modelling several neighbouring turns in one lumped model [14], [23], [24], [25].

The principle of turn reduction is demonstrated in Fig. 14. With adequate calculation less number of lumps can be used in the model maintaining meanwhile the necessary advantages of several lumps.

A deficiency of all the known models is that they are not adequate to take into account the wave propagation along this shunt current path composed by the turn-to-turn capacitance. As a consequence of using these models, voltage values appear along the capacitance chain at every locations at the time of applying the voltage onto the coil without any delay. For this reason I would like to propose a model built up by several current paths connected parallel to
each-other. The main current path means the path along the whole length of the coil wire belonging to the main function of the coil or of the transformer.

![Diagram of a coil model](image)

**Fig. 15. Model of one lump of a coil for the main current path**

Fig. 15 shows a lump between the neighbouring lumps of a high frequency coil model taking into account only the main current path of the coil corresponding to the main function of the coil. If \( n \) turns are covered by a lump as a result of the turn reduction within a coil with \( N \) turns, then there are \( N/n \) lumps within the model. \( R_{n0} \) is the direct current resistance of the coil within a lump,

\[
R_{n0} = \rho \frac{l_n}{A_w} = \rho \frac{N}{d_w^2} \frac{N}{4},
\]

where \( l_n \) is the wire length of a lump, \( l_w \) of the whole wire within the coil, \( A_w \) is the cross sectional area of the wire and \( d_w \) is its diameter, \( \rho \) its specific resistance. This value of \( R_{n0} \) is also \( N/n \) times less than the direct current resistance of the whole coil.

As the circuit elements in the main path of the current are changing with the frequency, so \( R_{HF} \) and \( L_{HF} \) are introduced to meet this requirement. This high frequency resistance model complies with the simplified series Foster’s circuit [25]. The basic principle of the series Foster’s circuit can be seen in Fig. 16. In the circuit \( R_0 \) belongs to the direct current resistance of the element and the other lumps are calculated so that at low frequencies the inductive reactance values of \( L_i \) are negligible compared to \( R_0 \). With increasing frequency more and more further \( R_i \) values will be effective modelling so the frequency dependence of the resistance.
Fig. 16. Model of one lump of a coil for the main current path

In the model in Fig. 15 $R_{HF}$ has the resistance value of one lump in case of maximum frequency occurring during fast transients. If the first peak of the voltage has been reached within a certain time period after the voltage pulse arrives the coil, this value can considered as a quarter period of $T$ corresponding to the $f_{\text{max}}$ maximum frequency of the voltage wave and the minimum skin depth by the measured coil can be considered as

$$\delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}},$$

assuming $\mu_0$ for the magnetic permeability of the non-magnetic wire and $\sigma$ is the specific conductivity of it and $\omega$ is the angular frequency for $f_{\text{max}}$, so $R_{HF}$ can be calculated as,

$$R_{HF} = \rho \frac{l_n}{4\frac{1}{d_w^2}-(d_w-2\delta)^2} - R_{n0},$$

where $R_{n0}$ calculated in (9) is connected always in series with $R_{HF}$ so it must be substracted from $R_{HF}$. Parameter $L_{HF}$ is introduced to short circuit $R_{HF}$ in case of low frequencies and to compose a much greater impedance at high frequencies. So the resistance of the lump can vary between two decades depending on the frequency. The value of $L_{HF}$ must be chosen so, that its reactance is negligible compared to $R_{HF}$, i.e. at least two decades lower than that of $R_{HF}$ at low frequencies. On the other hand inductive reactance of $L_{HF}$ must be much higher than $R_{HF}$ at high frequencies.
Parameter $L_n$ is the self inductance of one lump and $M_n$ the mutual inductance with the next lump. When calculating $L_n$ and $M_n$ for a lump it is to be taken into account that the sum of the self inductance and mutual inductance values have to add up the value of the self inductance $L$ of the coil in stationary case. The self and mutual inductance for a lump are

$$L_n = M_n = \frac{1}{2} \frac{n}{N} L = \frac{1}{2} \frac{n}{N} N^2 \Lambda = \frac{1}{2} n N \mu_0 \mu_r A_M l.$$  \label{eq:12}

In \eqref{eq:12} $L$ is the self inductance of the whole coil, $N$ is the number of turns in the coil and $n$ is that in one lump, $A$ is the magnetic conductance of the coil, $\mu_r$ is the relative magnetic permeability of core, $A_M$ is the internal cross sectional area of coil and $l$ is the length of the coil. According to studies the value of the mutual inductance decreases very fast between turns laying far from each-other within the coils \cite{26}, \cite{27}. With optimum choice of the number of turns $n$ a good modelling of the mutual inductance can be achieved. In general the values of all series elements in the circuit are to be divided by the number of lump to obtain the value of the element in one lump and the values of all parallel elements are to be multiplied with the number of lumps. An exception is the turn-to-turn capacitance where it is to be multiplied by the lump number although it is a series element. The calculation of the turn-to-turn capacitance can be realised on several ways, there are exact methods for the calculation of a capacitance with extremely small dimensions as well \cite{28}.

According to \cite{28} the stray (parasite) parameters of interconnects in integrated circuits influence the data transfer too. The parameters are calculated on a mesh with reduced nodes using multilayer dielectric Green’s function approach to compute the quasi-TEM transmission line interconnecting parameters in multi-layered dielectric media with infinitely thin conductors in the top layer composing capacitance and partly shielding within the integrated circuit. During my investigations the above methods of turn reduction and the simplified, series Foster’s circuit are extended to build up a more detailed model.

\section{2.5. Possibilities for modelling time delays}

In case of modelling electromagnetic wave propagation along coils and transformers it is important to have adequate time delay elements in the model. Elements with the characters $t_n$
and $t_{nW}$ in the model plotted in Fig. 15 correspond to the time span needed for electromagnetic wave propagation. Value of $t_n$ is for the propagation of electromagnetic field along the coil length for one lump

$$t_n = \frac{n}{N} \frac{l}{c}. \tag{13}$$

This is the time span needed for the electromagnetic waves to propagate directly along the length of the coil in air between the coil and core, where $c$ is the velocity of light in vacuum. For the propagation of the current inside the wire caused by applying the voltage onto the coil more time $t_{nW}$ is needed because of the much greater length of the wire

$$t_{nW} = n \cdot t_T = n \cdot \frac{l_w}{N \cdot v_w} = \frac{n}{N} \sqrt{\frac{\varepsilon_r \mu_0}{c}} \cdot l_w, \tag{14}$$

where $t_T$ is the time delay for one turn. In (14) $t_{nW}$ belongs to one lump, $l_w$ wire length of the whole wire, $v_w$ is the velocity of electromagnetic waves along coil wire and $c$ is the velocity of light in vacuum.

These circuit elements are necessary because lumped parameter models do not take into account the time elapsing during electromagnetic wave propagation along the coil. Only the circuit element “Lossy Transmission Line” contains distributed parameters in a circuit simulator software [10], [11].

The above time delays can be realised through several methods with the help of a circuit simulator software, using

- transmission lines,
- lossy transmission lines [15],
- n port systems or
- all pass filters.

All pass filters give a nearly distortion free voltage curve at the model output, however they are applicable only for very short time periods [29], [30], [31], [32]. Fig. 17 shows the most simple all pass filter.
Port system elements featuring by the circuit simulator software have the parameters of the input and output resistances causing a need of reduction of the parameters really existing in the circuit. According to the measurements ports result in a distortion of the voltage curves to an inacceptable amount.

In a circuit simulator software lossy transmission line models are determined by the four parameters, \( L', R', C' \) and \( G' \) of the traditional lossy transmission lines. When ideal transmission lines are used, in excess to the time \( t \) this circuit element needs also a wave impedance \( Z_0 \) to be entered. The circuit shown in Fig. 5 contains transmission lines too with the parameters \( Z_i, t_i \). If using transmission lines, the rectangular elements of the main current path in Fig. 15 are replaced by these cylinder shaped transmission line elements requiring \( t \) and \( Z_0 \) parameters. For the main current path the \( Z_{0W} \) wave impedance of the helical line can be calculated [33] instead of that of the simple transmission line (3),

\[
Z_{0w} = Z_{n0} \left[ 1 + \frac{(n,\pi d)^2}{2 \ln \frac{D}{d}} \left[ 1 - \left( \frac{d}{D} \right)^2 \right] \right]^{-1} , \tag{15}
\]

with wave impedance of air \( Z_{n0} = 377 \, \Omega \), the outer diameter \( D = D_W + d_W \), the inner diameter \( d = D_W - d_W \), the number of turns \( n_1 \) within a length of 1 cm. The above equation is a practical mean for telecommunication experts for designing helical antennas or helical lines for e.g. delaying purposes. In this case the similarity of the structure of coils to that of helical antennas is utilised.
During the research I would like to make investigations and test the different time delay possibilities and would like to apply the “Transmission Line” in the proposed models. For the main current path of the coil I would like to model coil layout with the helical line formula for calculating the wave impedance.

2.6. Shielding between transformer coils

Transformers built into electronic devices are becoming even smaller, so their higher voltage coils are not able to absorb the electric loads propagating with fast transients like bursts and electrostatic discharges. Since several decades shielding is built into transformers between the primary and secondary coils to drain the electric charges propagating with over-voltages. This shielding is effective against e.g. surges with relatively low frequency content, the charge propagating with the pulse is drained to the earth by the grounded shielding. It is important not to short-circuit the shielding, because then it works as a turn in the transformer consuming energy from the magnetic field.

At low frequencies common mode interference currents can not propagate from one coil of the transformer to the other because of the galvanic insulation between them. However at high frequencies the parasite capacitance ($C_p$ in Fig. 18) contributes to their propagation between the coils [34], [35], [36], [37].

Fig. 18. Propagation of common mode interference current through transformer coils
This problem can be remedied with a shielding inserted between the two coils (Fig. 19). The shielding composes capacitors with the primary and the secondary coils as well ($C_{S1}$ and $C_{S2}$ in Fig. 19) [38]. This capacitance couples the interference current to the ground avoiding so the propagation to the other coil. There are also double and triple shielding applied connected to separate groundings.

In case of the high frequency ranges of fast transients like bursts and electrostatic discharges the shielding inserted between the two coils, i.e. between the primary and the secondary coils, is not so effective as at low frequencies e.g. of surges [39]. Recent researches have demonstrated that neither a shielding made of superconductive material is effective at high frequencies [40]. Shielding degree (SD) begins to sink over 30 kHz. In general no shielding is installed in the transformers of high frequency, chopping supply units for it is ineffective because of its rather high inductance to the ground. The more common shielding coil has a less shielding degree at the same frequency than that of a shielding foil.

Fig. 20 shows the inductance $L_S$ of the shielding to the ground hindering the electric charges to reach the shielding and as a result of it the shielding will be “transparent” in case of fast transients with high frequency content. This inductance depends on the internal layout but also on the outer circumstances of the transformer and the shielding.
A grounding point near to the transformer can not always be considered as an unlimited source of electric charges because of the layout of the electric installation of the room and of the building where the transformer operates. A rather simple simulation model could help by the decision which art of shielding should be installed if any. There are precise methods modelling the shielding and metal foil cylinders itself, e.g. [41], [42], [43], [44] but their use is rather complicated.

After this review of the results I would like to propose a high frequency SPICE model of transformer shielding built in a circuit simulator software for the application by the dimensioning of the transformers taking also into account the ground connection aspects of the shielding. For testing the model I would like to build a test transformer by dismantling a transformer built into a PC supply unit and winding 100 turns for primary and secondary coil each. Thus a rather quick change of the shielding between the coils could be achieved.

I would like to make several measurement sequences with the test transformer using different pulse generators, shielding types and layer numbers of the shielding. Measurements results will be than compared with those of simulation sessions.
3. A NOVEL HIGH FREQUENCY MODEL FOR ONE-LAYER, STRAIGHT COILS

The increasing level of electromagnetic noises on electric networks and the even higher switching frequency of power supply units built into sensible electronic devices make necessary to have more precise modelling of transformers, taking more precisely into account the aspects of wave propagation along the coils. An inevitable part of a high frequency coil model needed for these examinations is the so called stray capacitance, i.e. the capacitance between turns, the capacitance between the coil and the core and housing. Capacitive reactance composed by this stray capacitance is negligible at the rated frequency of the coil or of the transformer, however it is not negligible in case of the frequencies of fast transient over-voltages.

In case of small transformers also fast over-voltages like bursts and electrostatic discharges (ESD) are of interest. Bursts and ESD have small electric charge absorbed by the capacitance of high voltage transformers. They have a rise time of some nanoseconds and propagating through small transformers they can cause damages. In excess, switching frequencies of supply units are near to the MHz range, thus wave propagation is no more negligible.

There are several high frequency coil models being applicable in certain cases, resulting no contradictions to each-other concerning the basic function of the coil. None of the known coil models is able to take into account electromagnetic wave propagation along the path composed by the turn-to-turn capacitance, because these capacitors are connected directly in series to each-other according to the classic Wagner’s theorem. In the case of these models certain voltage appears without delay at every locations of the capacitance chain, i.e. along the whole length of the coil when applying the supply voltage at the input ports of the coil.

I would like to work out a novel one-layer distributed parameter coil model suitable for modelling electromagnetic wave propagation also along the shunt current path composed by the turn-to-turn capacitance and I propose a lumped parameter model as well for the application with simulation software (e.g. xSPICE) by introducing an inductance in series to
the turn-to-turn capacitance in the model circuit. For testing the developed model I would like to build a two meter long straight coil of copper wire with a diameter of 1 mm included also the varnish insulation so that the wire has been densely wound onto a plastic (PVC) protective pipe (Fig. 21). In Fig. 21 several coils are shown, both of the lower coils have been built for wave impedance measurements and the two upper coil has been tested also as a “transformer” (see also Chapter 4.). For the investigation of the one-layer coil referred in this chapter I have measured the second uppermost coil in the figure.

Measurements on the coil with an iron core have been realised at the University for Applied Sciences of Würzburg-Schweinfurt at the Department of Electrical Engineering in Schweinfurt in 2000 and 2004 in the Laboratory for Telecommunication Technology running by Professor Dr. Peter Möhringer PhD. The coil has been supplied with a pulse generator and the response has been pitched up by an oscilloscope for comparing the results with those given by the model with the simulator software. The output ports of the coil was practically open, i.e. closed by 1 MΩ.

Fig. 21. Laboratory made test coils for the measurement
Measurements have been realised in a laboratory for microwave tests with PVC floor and with furniture made of mainly non-conductive materials (Fig. 22).

Fig. 22. The measurement layout

The parameters of the coil can be seen in Fig. 23. These parameters have been applied for the determination calculation of the circuit elements of the model.

Fig. 23. Parameters of the measured coil
Parameter $l$ is the actual length of the coil ($l = 2000$ mm); $N$ is the total number of turns ($N = 2000$); $d_W$ the diameter of the coil wire ($d_W = 1$ mm); $A_W$ is its cross sectional area; $\rho$ is its specific resistance i.e. that of copper ($\rho = 0.0178 \, \Omega \, \text{mm}^2/\text{m}$); $\nu_V$ is the thickness of the varnish insulation on the wire and $\varepsilon_V$ the relative dielectric permittivity of it ($\varepsilon_V = 3.5$), while $\varepsilon_{rb}$ is the relative dielectric permittivity of the protective installation pipe of PVC ($\varepsilon_{rb} = 3.4$) and $v_b$ is its thickness. $D_W$ is the medium diameter of the coil ($D_W = 14$ mm) and $D_C$ is the core diameter, $A_M$ the cross section area of the core ($A_M = 1.33 \times 10^{-4} \, \text{m}^2$) and $\mu_r = 6.94$ is the relative magnetic permeability of the core.

3.1. Model parameters of one-layer straight coils

When applying voltage at the input ports of a coil there are four paths for the electromagnetic wave to propagate to the output ports of the coil, (a) main current path along the coil wire, (b) shunt path composed by the turn-to-turn capacitance, (c) shunt path composed by the capacitance between the turns and the core, (d) shunt path composed by the capacitance between the turns and the housing.

When modelling a coil the most precise results would be given by a circuit containing a model for each turn of the coil. In case of high voltage power transmission transformers this way can also be realised, with the help of a computer and the desired results can be quickly obtained. Coils of small transformers can however have several thousands of turns so this cannot be a suitable way for the practice. The solution can be given by the use of the method of turn reduction used also in case of high voltage transformers, namely modelling several neighbouring turns in one lumped model. Simulation has shown, that the measured two meter long coil having two thousand of turns ($N = 2000$) practically ten lumps yields good results.

3.1.1. High frequency model of the main current path

The main current path realises the main function of the coil, it belongs always to the coil model also in low frequency cases. In a high frequency model frequency dependence of the resistance and inductance of the coil has to be taken into account. Fig. 24 shows the principle
of the magnetic field related parameters and resistance of a turn in case of a one-layer coil taking into account the two neighbouring turns.

![Diagram of magnetic field related parameters of a coil's turn](image)

**Fig. 24. Magnetic field related parameters of a coil’s turn**

In Fig. 24 the self inductance and resistance of turn $i$ and its mutual inductances with turns $i + 1$, $i - 1$ are shown. Based on Fig. 15 the model of the main current path proposed for the use with circuit simulation software is shown in Fig. 25 for one lump.

![Diagram of the model of one lump of a coil for the main current path](image)

**Fig. 25. The model of one lump of a coil for the main current path**

The following calculation of the model parameters are made according to [45] and [46]. If $n$ turns are covered by a lump as a result of the turn reduction, then there are $N/n$ lumps within the model. $R_{n0}$ is the direct current resistance of the coil within a lump based on (9),

$$R_{n0} = \rho \frac{l_n}{A_w} = \rho \frac{n l_w}{N A_w} = \frac{n l_w}{d_w^2 \pi} = 0.226 \, \Omega,$$

$$n = 226.0^2 \, \pi$$
where \( l_n \) is the wire length of a lump and is one tenth of the whole wire length \( l_W = 90 \text{ m} \) if the number of turns within a lump is \( n = 200 \) and the whole number of the coil’s turns is \( N = 2000 \), \( A_W \) is the cross section area of the wire and \( d_W = 1 \text{ mm} \) is its diameter, \( \rho = 0.0178 \Omega \text{ mm}^2/\text{m} \) its specific resistance. This value of \( R_{n0} \) is also one tenth of the direct current resistance of the tested coil (parameter values see [45]).

As the circuit elements in the main path of the current are changing with the frequency, so \( R_{HF} \) and \( L_{HF} \) are introduced according to the simplified series Foster’s circuit. \( R_{HF} \) has the resistance value of one lump in case of maximum frequency occurring during fast transients. During the measurements the rise time of the pulses was set to 2 ns on the pulse generator, namely to the minimum adjustable value. When the generator is loaded with the coil the first peak has been reached within 5 ns after applying the pulse. Taking this value as a quarter period of \( T \) the maximum frequency can be assumed to be \( f_{\text{max}} = 50 \text{ MHz} \), so the minimum skin depth by the measured coil based on (10) is

\[
\delta = \sqrt[4]{\frac{2}{\mu_0 \sigma \omega \rho}} = 9.51 \mu\text{m},
\]

assuming \( \mu_0 \) for the magnetic permeability of the copper wire and \( \sigma = 1/\rho = 5.6 \cdot 10^7 \text{ S/m} \) is the specific conductance of the wire and \( \omega \) is the angular frequency for 50 MHz, so \( R_{HF} \) can be calculated according to (11)

\[
R_{HF} = \rho \cdot \frac{l_n}{d_W^2 - (d_W - 2\delta)^2} \frac{2}{\pi} - R_{n0} = 5.8 \Omega,
\]

where \( R_{n0} \) calculated above is always in series with \( R_{HF} \) so it must be substracted from \( R_{HF} \). Parameter \( L_{HF} \) is introduced to short circuit \( R_{HF} \) in case of low frequencies and to compose a much greater impedance at high frequencies. So the resistance of the lump can vary between two decades depending on the frequency. The value of \( L_{HF} \) must be chosen so, that its reactance can be negligible to \( R_{HF} \), i.e. at least two decades lower than that of \( R_{HF} \) at low frequencies. On the other hand inductive reactance of \( L_{HF} \) must be much higher than \( R_{HF} \) at high frequencies. So let
If this inductance value gives at least hundred times lower reactance value than $R_{HF}$ at 50 Hz, then it meets the other requirement

$$X_{50\text{MHz}} = \omega_{50\text{MHz}}L_{HF} = 580 \ \mu\Omega \leq \frac{R_{HF}}{100} = 58 \ m\Omega ,$$

the requirement is met. There is also a third requirement for $L_{HF}$ namely it must be negligible compared to the other series inductance in the lump and if not then it must be substracted from it. For stationary cases it is enough to take one self inductance $L$ into account for the whole coil. In case of transients, however other currents flow in each turn. Using turn reduction the same current flows in a lump having $L_n$ self inductance for its own current.

The more lumps used the more precise simulation results can be obtained. With a high number of lumps however the mutual inductance with several other lumps must be taken into account with varying values and this would complicate the use of the model. The least possible lump number is then proposed to select for having inductive coupling only with the two neighbouring lumps with coupling factors of 1 and there are no coupling with the lumps laying far from each-other.

When calculating $L_n$ and $M_n$ for a lump it is to be taken into consideration that the sum of the self inductance and mutual inductance values have to add up the value of the self inductance $L$ of the coil in stationary case based on (12). In (12) $L$ is the self inductance of the whole coil, $N = 2000$ is the number of turns in the coil and $n = 200$ is that in one lump, $A$ is the magnetic conductance of the coil, $\mu_r = 6.94$ is the relative magnetic permeability of core, $A_M = 1.33 \cdot 10^{-4} \ m^2$ the internal cross section area of coil and $l = 2 \ m$ is the length of the coil. In the case of the measured test coil $L_n$ and $M_n$ from (12)

$$L_n = M_n = \frac{1}{2} nN\mu_0\mu_r \frac{A_M}{l} = 220 \ \mu H$$
with core and $L_n = M_n = 16.68 \, \mu H$ without core. There are $N/n$ lumps in the circuit model, so adding up $N/n$ times both of the above values result the original $L$ value of the coil

$$L = \frac{N}{n} \left( L_n + M_n \right) = \frac{N}{n} 2L_n = 2.2 \, mH \tag{23}$$

is equal to the measured value reinforcing the applicability of the model. Every circuit simulator software can take into account the frequency dependence of the magnetic conductance of cores and the impacts of eddy currents.

Elements with the characters $t_n$ and $t_{nW}$ in the model plotted in Fig. 25 model the time span needed for electromagnetic wave propagation. Value of $t_n$ is for propagation of magnetic field along the coil length according to (13)

$$t_n = \frac{n}{N} \frac{l}{c} = 0.67 \, ns \tag{24}$$

This is the time necessary for the electromagnetic waves to propagate directly along the length of the coil in air between the coil and core. This value belongs to one lump for the test piece. For the whole coil length of 2 m containing 10 lumps the time span is 6.7 ns. For the propagation of the current inside the wire caused by applying the voltage onto the coil a higher time value $t_{nW}$ is needed because of the much greater length of the wire based on (14). For the test piece

$$t_{nW} = \frac{n}{N} \frac{\sqrt{\varepsilon \mu}}{c} N D_p \pi = 29 \, ns \tag{25}$$

These circuit elements are necessary because lumped parameter models do not take into account the time elapsing during electromagnetic wave propagation along the coil as it have been already describe in Chapter 2. The above time delays can be realised through several methods with the help of a circuit simulator software, using transmission lines, lossy transmission lines or all pass filters.
In my research to simulate the coil, transmission lines have been applied. In excess to the time value this circuit element needs also a \( Z_0 \) wave impedance to be entered. For the main current path the \( Z_{0nW} \) wave impedance of the helical line can be calculated based on (15) and according to the geometrical parameters: the outer diameter \( D = D_W + d_W = 15 \text{ mm} \), the inner diameter \( d = D_W - d_W = 13 \text{ mm} \), the number of turns within a length of 1 cm \( n_1 = 10 \).

\[
Z_{0nW} = Z_{n0} \sqrt{1 + \frac{(n_1 \pi d)^2}{2 \ln \frac{D}{d} \left(1 - \left(\frac{d}{D}\right)^2\right)}} = 619 \text{ } \Omega .
\]  

(26)

3.1.2. High frequency model of the path composed by the turn-to-turn capacitance

A pair of turns laying close to each-other composes a capacitance with a reactance being small enough at high frequencies that considerable current flows through them and composing a shunt current path to the coil’s wire length. Additionally a turn composes a capacitor with other neighbouring conductive bodies like core as well.

![Fig. 26. A model of one turn of a coil with core](image)

In Fig. 26 a possible model of one turn of a one-layer coil with core is shown with the parameters of the main path on one hand: resistance of the turn \( R_T \), the stray inductance of it
$L_T$, the inductance with the core. Parameters of the parallel path of the turn-to-turn capacitance $C_T$ on the other hand with $R_{TP}$, resistance of the turn-to-turn capacitance and $L_{TP}$, inductance of it, the $C_{TO}$ capacitance to the core, its $R_{CTO}$ series resistance and the $R_{TP0}$ resistance of the insulation [47]. However, the parameters of the shunt current path are calculated for the case of turn reduction as well. The development of a lump of the “quasi distributed parameter” model for its shunt path is discussed below. For the theoretical support of the “quasi distributed parameter” model as the final purpose of the research at first I would like to propose a distributed parameter model for one-layer, straight coils.

3.1.2.1. Distributed parameter model of the shunt path

Former high frequency coil models took only the turn-to-turn capacitance into account for this shunt current path composed by the turn-to-turn capacitance on the basis of Wagner’s theorem (see Chapter 2.2). Fig. 6 shows Wagner’s model for an ideal coil with core. As a consequence of this model, voltage appears along the capacitance chain at every locations at the time of applying the voltage onto the coil without any delay. This model is therefore not adequate to take into account the wave propagation along this current path. In the reality also an inductance $L'_{K}$ can be found in series with the unit length turn-to-turn capacitance $K'$. Current through the turn-to-turn capacitance is composed partly of conductive current within the wire along its diameter and partly of displacement current between the turns in varnish and in other insulating materials if exist (air in case of the measured coil). So this current flows within the wall of a cylinder composed by the turns of the coil. A part of this current flows through the capacitance to the core.

![Diagram](image)

Fig. 27. High frequency model of ideal coils with an inductance in series with the turn-to-turn capacitance
Between the coil cylinder and the core there is the magnetic flux of this current defining an $L'_K$ inductance and as a matter of course an $R'_K$ resistance must also be taken into account. Fig. 27 shows the high frequency model of ideal coils taking into account the proposed inductance in series with the turn-to-turn capacitance.

Fig. 28 shows the proposed distributed parameter circuit for a lossy coil taking also into account the $R'$ resistance of the main path in $\Omega$/m, $R'_K$ of the turn-to-turn shunt path and the $G'$ conductance between coil and core in S/m.

![Diagram of a coil model](image)

**Fig. 28. Comprehensive high frequency distributed parameter model of lossy coils**

The model on Fig. 28 is based on the traditional transmission line model and is the developed extension of Wagner’s model. In addition to Wagner’s model I propose the parameter $L'_K$ inserted in series with $K'$ making so able the model to take into account the electromagnetic wave propagation along the path composed by the turn-to-turn capacitance.

Equations describing this distribution parameter model can be formulated with the help of Fig. 29 showing the voltages and currents for a segment of differential length of the coil. The difference to the transmission line is the existence of the current path of the turn-to-turn capacitance $K'$ and the difference to the Wagner’s model is the existence of $L'_K$ in series with the reciprocal capacitance $K'$ defined by the magnetic flux between the coil cylinder and the core generated by the current of the turn-to-turn capacitance.
Kirchhoff’s equations describing the above four pole model are

I. \[-u + L’ dz \frac{\partial i_m}{\partial t} + R’ dz i_m + \left( u + \frac{\partial u}{\partial z} dz \right) = 0 \tag{29a} \]

II. \[-u + L’ K dz \frac{\partial i_s}{\partial t} + u_K + R’ K dz i_s + \left( u + \frac{\partial u}{\partial z} dz \right) = 0 \tag{29b} \]

III. \[-i_m - i_s + C’ dz \frac{\partial u}{\partial t} + G’ u + \left( i_m + i_s + \frac{\partial (i_m+i_s)}{\partial z} dz \right) = 0 \tag{29c} \]

since

\[ u_K = K’ dz \int_i i_d t \tag{29d} \]

then

\[ \frac{\partial u_K}{\partial t} = K’ dz i_s \tag{29e} \]

and after rearranging them it yields

I. \[ \frac{\partial u}{\partial z} = -L’ \frac{\partial i_m}{\partial t} - R’ i_m \tag{30a} \]

II. \[ \frac{\partial u}{\partial z \partial t} = -L’ K \frac{\partial^2 i_s}{\partial t^2} - K’ i_s - R’ K \frac{\partial i_s}{\partial t} \tag{30b} \]

III. \[ \frac{\partial (i_m+i_s)}{\partial z} = -C’ \frac{\partial u}{\partial t} - G’ u \tag{30c} \]
The above equations (30a - 30c) are valid for the comprehensive distributed parameter model for lossy, one-layer, straight coils. Deriving an analytical solution is nearly impossible and numerical handling of them is also very complicated. For the sake of simplicity I propose a “quasi distributed parameter model” actually a lumped parameter model for the practical use. It is composed by a certain number of identical lumps making the model more simple while maintaining its capability for modelling wave propagation.

Fig. 30. Effective circuit elements at the moment of applying the voltage

Some useful results can be achieved with the help of the above distributed model. At the moment of applying the voltage onto the coil every turn-to-turn capacitance is discharged compared to the incoming over-voltage, so it can be considered as a short circuit for the first running through of the wave. An other aspect is, that inductance $L_n = L'_K \frac{n}{N}$ (~ nH) for one lump calculated after a lump reduction from $L'_K$ is several decades less then $L_n + M_n$ (~ 100 μH) connected parallel to $L_{nk}$ (12), so at that time they can be considered as breaks in the path. Fig. 30 shows the circumstances at this moment when $R_{nk} = R'_K \frac{n}{N}$ l and $G_n = G'_K \frac{n}{N}$ are also neglected. As a consequence of these, for the first propagation of the wave the coil can be considered like a transmission line with a propagation velocity calculated according to (2)

$$v = \frac{1}{\sqrt{L'_K C'}}. \quad (31)$$

Considering the copper coil as ideal coil, this results only small differences compared to the measurements. This velocity value equals to that defined by the medium between the coil and
core in coaxial cables. So this path contains also the \( t_n = 0.67 \) ns time value described above. In the measured data a part of the voltage appeared at the output ports of the coil within the above time delay. The measured time delay is about 8% longer than calculated because of the layout of the varnish insulation have not been taken into account precisely. This time delay has to be modelled in the shunt current path of the SPICE model. When applying transmission line for time delay wave impedance has also to be determined. In this case the current layout is not a helical line but a coaxial cable, because the current of the shunt path flows within a wall of a cylinder, so the wave impedance is

\[
Z_{\omega} = \sqrt{\frac{\mu_r}{\varepsilon_r}} 60 \ln \frac{D_W - d_W}{D_C} = 15.7 \, \Omega, \quad (32)
\]

where the coil has an inner radius of \( r_2 = 6.5 \) mm and the core the radius of \( r_1 = 5 \) mm (from \( D_W \) and \( d_W \)) and the other parameters are defined above [48].

3.1.2.2. Lumped parameter model of the turn-to-turn capacitance path

For the use in a circuit simulation software I propose a “quasi distributed parameter model” reducing several turns into lumps and the model is composed by several identical lumps. Simulation circuit model for the path composed by the turn-to-turn capacitance are shown on Fig. 31.

![Fig. 31. Proposed model for the current path of the turn-to-turn capacitance](image)

For the calculation of the capacitance between two neighbouring turns several methods can be used. For the test piece the simple rule is applicable, that two varnish insulated round wires with a diameter of maximum 1 mm being close to each other have 1 pF capacity per each cm of their length, so the value of one turn-to-turn capacitance within the measured coil
The value given by (33) belongs to one turn-to-turn capacitance. That for a lump will be
calculated as
\[
C_{nk} = \frac{C_k}{n} = \frac{4.4 \cdot 10^{-12}}{200} = 0.022 \ pF .
\]  
(34)

Resistance for one lump in the coil cylinder, i.e. the resistance in series with the turn-to-turn
 capacitance is
\[
R_{nk} = \rho \frac{n l}{d_w \pi (D_w - d_w) / 2} = 174 \ \mu \Omega ,
\]  
(35)

and the inductance for one lump in the coil cylinder can be calculated like that of a coaxial
cables,
\[
L_{nk} = \frac{\mu_0}{2\pi} \frac{n l}{N} \ln \frac{D_w + d_w}{D_w - d_w} = 3.2 \ nH .
\]  
(36)

The above parameters define the shunt current path, thus both longitudinal paths being
parallel to each-other are ready for use, however there are transversal current path as well
composed by capacitors and conductors to the core and/or to the housing having resistance
parameters as well.

3.1.3. Comprehensive model of the coil

In excess to the parameters calculated the capacitance between coil and core has to be
determined
\[
C_{cc} = \frac{2\varepsilon_0 \pi \cdot n l / N}{\ln \nu_c + \ln \nu_b} = 41 nF ,
\]  
(37)
where \( v_v = 1.01 \) is the ratio of thickness of varnish on wire to coil radius and \( \varepsilon_r = 3.5 \) is its relative dielectric constant and \( v_b = 1.1 \) is the ratio of thickness of the bobbin between the coil and the core to coil radius and \( \varepsilon_r = 3.5 \) is the relative dielectric constant of the bobbin.

Fig. 32. One lump of the high frequency coil model

In Fig. 32 \( R_{nc} \) is the core resistance calculated similar to that of the coil wire for one lump. This circuit is a theoretical version of the developed model actually introduced for simulation with the circuit simulation software. All the elements applied in the theoretical model can be directly chosen in a SPICE software except the time delay element.

3.1.3.1. Investigation of time delay elements

For simulation the Berkeley Spice based software TINA has been used. During simulation sessions different time delay elements have been used:
- all pass filters,
- delay ports,
- ideal transmission lines,
- lossy transmission lines.

All pass filters have given perfect, distortion free delay but only for periods of some nanoseconds. In Fig. 33 a test circuit applied by me is shown with the simplest all pass filters. However increasing the time delay by increasing the values of the filter elements, distortion
appears and will be even greater. As a remedy of this problem several all pass filter elements with low element values - 2 nH and 0.8 pF - can be connected in series to each other however with that the CPU time increases and the benefits of model simplicity will be lost. For the required time delay nearly one hundred filter elements should be introduced in one lump of the model, thus drafting the model needs too much time. For these reasons I have not decided for applying all pass filters.

An other possibility with a SPICE software is to utilise delay ports. In Fig. 34 a lump of the actual coil model is shown with delay ports with the reference $U_1$ at the points of the model needing delays. The simulated results were much poorer with these ports, because the distortion was intolerably high in every tested cases of input and output resistances needed by SPICE in case of these ports.

Fig. 33. Test circuit with all pass filters

Fig. 34. One model lump with delay ports
For the actual simulation the schematic in Fig. 35 has been utilised. I have decided for using ideal “Transmission line” with the parameters time delay and wave impedance. The best results have been yielded by these transmission lines. However applying transmission lines for realising the time delays some of the circuit elements are involved by them.

![Schematic diagram](image)

Fig. 35. The schematic model for simulation

Utilising transmission lines as time delay elements longer time periods (> 10 ns) can be achieved however the resulted voltage curves are very sensitive to the wave impedance $Z_0$. Little differences cause rather great distortions in the voltage curves.

### 3.1.3.2. Description of the SPICE model

Fig. 35 shows only one lump of the used model. Actually ten lumps have been used according to the turn reduction optimalisation (see Chapter 3.3). In the circuit the element with the reference of VG1 simulated the over-voltage pulse respectively the signal of the pulse generators used during the measurements. The elements with the reference of VM1 and VM2 are the voltage measuring points in the circuit plotted by the software as the results of the transient simulation.

The elements with the reference of TL1 are the transmission lines for time delaying purposes. The software does not show the parameters $t$ and $Z_0$ of these elements. The three transmission lines in the parallel paths under each-other have the same parameters - short time period and
wave impedance of the coaxial cable - and that placed only in the main current path (the second transmission line in this path) has the longer time parameter and the wave impedance of the helical line i.e. of the coil wire.

3.2. Comparison of measurement and simulation results

For testing the model I have wound a two meter long coil with the dimension shown in Fig. 23. This 2 m length could be easily achieved and serves for more reliable measurement results on the oscilloscope display.

Measurement layout can be seen in Fig. 36. The coil was laid on a wooden table on wooden stands with a height of 20 cm (Fig. 22). The coil has been fed by several types of pulse generators, the following figures are plotted with 5 V pulses generated by a pulse generator type HP 8007B Pulse Generator with an output resistance of 50 Ω at the input ports of the coil. The voltage has been measured by an oscilloscope here and at the output ports of the coil as well. Type Tektronix TDS 540 four channel digitising oscilloscope with a sampling frequency of 1 GS/s and with channel input parameters of 1 MΩ and 10 pF has been used.

Measurements have been realised with and without iron core being ungrounded and grounded. The oscilloscope has been layed in the middle and was connected to the coil ports through 1 m long BNC cables laying parallel to the coil. Except grounded core the nearest grounded metal bodies were the shields of the measuring cables of the oscilloscope at a distance of 30 cm with the same length.

Before the measurements, tests have been accomplished to determine which amount of voltage is transported from one end of the coil to the other via electromagnetic radiation between the connection wires as antennas at both ends. The measurements have been repeated with other generators and oscilloscope as well. The different results do not alter significantly from each-other.
The comparison of the measured and simulated results can be seen on the following figures. Fig. 37 shows the pulses supplied by the pulse generator measured (1) and simulated (2) on the input ports of the coil. Simulations have been performed with the circuit simulator software TINA based on Berkeley Spice as well. If the core is ungrounded, a part of the voltage arrives after about 7 ns to the output ports of the coil (Fig. 38), because the iron wire with a diameter of 10 mm composes a conductive shunt path for the electromagnetic waves. In this case ferromagnetic character of the iron is irrelevant; placing an aluminium core into the coil a larger part of the voltage arrives at the same time there.

Fig. 37. Measured (1 - solid line) and simulated (2 - dashed line) voltage curves at the input ports of the coil connected to the pulse generator
If the core is grounded no voltage appears until about 300 ns on the output ports of the coil (Fig. 39), electric charges in the core caused by the capacitive coupling between coil and core flow to the ground.

Fig. 38. Measured (1 - solid line) and simulated (2 - dashed line) voltage curves at the output ports of the coil in case of ungrounded core

Simulated curves in Fig. 38 and 39 are similar to that measured. Fig. 40 shows two simulated curves: curve 1 simulated with Wagner’s traditional model, i.e. without the transmission line elements in the capacitance path and the core path. Curve 2 simulated with the proposed new
model being the same as in Fig. 38. Within the time period in focus Wagner’s model yields an oscillating voltage curve being far from the one measured.

![Graph](image1)

Fig. 40. Voltage curves at the output ports simulated with Wagner’s model (1 - solid line) and with the new model (2 - dashed line)

Measurement and simulation have shown that the turn-to-turn capacitance path of coils with a high number of turns and low capacitance values cause only a voltage of some percent of the applied voltage after $t_n = 7$ ns oscillating with a rather high frequency, about 33 MHz (Fig. 41).

![Graph](image2)

Fig. 41. Measured voltages at the input (1) and output ports (2) of the coil with low turn-to-turn capacitance
In case of high voltage transformers with a low number of turns but with high turn-to-turn capacitance values can result higher voltages after elapsing $t_n$ and can even bring the whole voltage through within this time span (Fig. 42).

![Graph showing simulated voltages at the input (1) and output ports (2) of a coil with high turn-to-turn capacitance.](image)

Fig. 42. Simulated voltages at the input (1) and output ports (2) of a coil with high turn-to-turn capacitance

Voltage curve yielded by the new model proposed by me shows differences compared to the measured one. In Fig. 38 simulated curve lays above the measured curve at the very beginning of the time period and at the end of it and lays below it in between. Reasons can be the energy irradiated in the reality not taken into account in the new model and the wave reflection phenomena dealt differently by the new model.

3.3. An aspect for determining the number of lumps

In the proposed model time delays due to wave propagation has been modelled by the simple “Transmission Line” and as a result of this one lump or a few lumps can not model the effects of wave propagation perfectly. A quasi distributed parameter model has therefore to be built up with a certain number of identical lumps. Simplicity of modelling requires the least possible lump number, but the less number of lumps composing the model results in less fidelity of the real phenomena, the higher number of lumps results in a more complex model.
and the longer CPU time. The fast periodic phenomenon observed by the measurements can help to find the optimum lump number. Simulation sessions have been made with different lump numbers. The value of a series impedance element $Z_S$ is then calculated with

$$Z_S = \frac{Z}{n}$$  \hspace{1cm} (38)

where $Z_S$ is the value for one lump, $Z$ is the value for the whole coil and $n$ is the number of lumps. The only parallel element, $C_{\text{par}}$ and the delay time values are calculated the same way according to Fig. 32. During these simulation sessions attention has been focused only onto the very first time span between applying the voltage on the input ports and its arrival at the output ports of the coil. Output voltages obtained by one and two lumps are plotted in Fig. 43. Curve 1 has no break points and curve 2 has two breakpoints within the examined time span, but none of them is similar to that measured (Fig. 41, curve 2), there are no periodic phenomena, so this lump numbers do not model the reality.

![Fig. 43. Simulated voltages at the output ports of a coil with one lump (1 - solid line) and two lumps (2 - dashed line)](image-url)
The basic frequency increases with the number of lumps drawn and at a lump number of ten it is already about twice as high than that measured, 33 MHz. The voltage simulated with ten lumps is plotted with solid line (1) in Fig 44. The harmonic content is higher as well than measured, which is nearly sinusoidal (Fig 41). In Fig. 44 the curve plotted with dashed line shows the output voltage simulated with twenty lumps. Basic frequency is nearly the same in this case too and neither harmonic content decreases but it increases.

Fig. 44. Simulated voltages at the output ports of a coil with ten lumps (1 - solid line) and twenty lumps (2 - dashed line)

Simulation has shown that a periodic phenomenon can be obtained with a lump number of ten, however the frequency is then about twice as high and harmonic content is also higher than measured. It has been observed too that simulation of this initial high frequency periodic phenomenon is very sensitive to the values of the elements, a little deviation causes a great differences in the shape of the voltage curves and resonance can occur as well. A further increase of the lump number above ten does not yield more beneficial curves, basic frequency and harmonic content further increase.
3.4. Error analysis

The objective of this research is to find an appropriate model for the current path of coils composed by the turn-to-turn capacitance. This current path supports the voltage to propagate along the coil within a much shorter time period than propagating along the whole length of the coil wire. Thus most interesting results are those obtained between 6 ns and 300 ns by the measurements and simulations (Fig. 38). In the followings an error analysis of the simulation results can be found - what an extent does simulation results differ from those measured - taking the measured curve as a reference.

![Graph showing measured and simulated voltages](image)

Fig. 45. Measured (1) and simulated (2) voltages at the output ports with sampling times

In Fig. 45 on the basis of Fig. 38 the principle of the error analysis can be seen: Measured (curve 1 - solid line) and simulated voltage values (curve 2 - dashed line) are taken from the curves at a number of sampling time values being, 16.7 ns apart from each-other, and listed in Table 1. In the fifth column of Table 1 the absolute error values

$$\Delta U = U_M - U_S$$

(39)

are listed, while in the sixth column their relative values

$$\Delta U(\%) = \frac{U_M - U_S}{U_M} \cdot 100(\%)$$

(40)
can be found and column 7 lists the quadratic error between the measured and simulated values

$$\Delta U^2(\%) = \left( \frac{U_M - U_S}{U_M} \right)^2 \cdot 100 \, (\%) .$$

(41)

Table 1: Listing of the error analysis results

<table>
<thead>
<tr>
<th>#</th>
<th>( t ) (ns)</th>
<th>( U_M ) (V)</th>
<th>( U_S ) (V)</th>
<th>( \Delta U ) (V)</th>
<th>( \Delta U ) (%)</th>
<th>( \Delta U^2 ) (%)</th>
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<td>-333.30</td>
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<td>0.55</td>
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<td>0.90</td>
<td>0.35</td>
<td>28.00</td>
<td>7.80*</td>
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<td>0.58</td>
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<td>0.68</td>
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<td>0.50</td>
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</table>
Considering the values in column 5 a short periodic phenomenon can be observed at the beginning of the simulated pulse being completely absent in the measured curve. These rather small periodic voltages are the least ones obtained during simulation sessions when modifying the parameter values.

An other phenomenon is that at the beginning the simulated curve lays below the measured one and this deficit decreases until the time value of 200 ns, then the simulated curve lays above the measured one. This phenomenon is resulted mainly by the fact that the simulated voltage signal has the same characteristic. The reason of this phenomenon is to be investigated further. The requirements raising the simulated signal between 7 ns and 300 ns and to lower the overshot after 300 ns have been found to be contradicting.

Fig. 46 shows the plot of the relative error. The curve begins and ends with negative values corresponding to the phenomenon described above. The figure does not show the value at \( t_2 \) corresponding to the periodic phase and being an extreme value. Between the two negative section the error decreases from a relative high value of 45.8 % to zero.

![Fig. 46. Plotting of the relative error](image)

Finally the signal energy is calculated for the measured and simulated transient voltage signals with

\[
E = \int_{-\infty}^{\infty} U^2(t) dt ,
\] (42)
i.e. for the above discrete values

\[ E = \sum_{i}^{25} U_i^2 \Delta t_i = 15.7 \cdot 10^{-9} \cdot \sum_{i}^{25} U_i^2. \] (43)

The signal energy for the measured voltage curve \( E_M = 2.084 \cdot 10^{-6} \) \( \text{V}^2 \text{s} \) and that of the simulated curve \( E_S = 1.980 \cdot 10^{-6} \) \( \text{V}^2 \text{s} \). Simulation gives a slightly smaller energy value than measured. Its difference is 5% from the measured.

Taking into consideration only the voltage values until 300 ns, the difference is even greater, thus the energy transported by the very first wave propagation along the shunt path is less than measured.

Possible causes of simulation errors:
- higher simulated voltage values at 7 ns and 300 ns can be caused by the fact that simulation do not take into consideration the energy emission occurring during fast transients in the reality;
- an other reason of the errors can be the different handling of the wave reflection phenomena by the model. Every transmission line circuit element causes reflections.

3.5. New scientific result

Thesis 1:

*I have developed a novel high frequency distributed parameter model and a lumped parameter model for one-layer, straight coils. These models are able to take into account the electromagnetic wave propagation along the current path of the coil composed by the turn-to-turn capacitance as a result of an inductance inserted in series to this capacitance. Former models can not take this phenomenon into account, as they model this current path only by a capacitance chain, [45], [46].*
a) I propose a novel high frequency distributed parameter model for one-layer straight coils on the basis of Wagner’s model introducing an inductance of unit length in series with the reciprocal turn-to-turn capacitance of unit length. This distributed parameter circuit can model electromagnetic wave propagation along the current path of the coil composed by the turn-to-turn capacitance unlike the former models, because they model this path only by a capacitance chain, on which the voltage appears on its whole length with no delay. Calculation of this inductance is based on that of coaxial cables depending on the dimension and materials of the coil.

b) I propose a novel high frequency lumped parameter model for one-layer straight coils for the use with circuit simulation software introducing an inductance in series with the turn-to-turn capacitance. With a model composed by an appropriate number of the developed identical lumps electromagnetic wave propagation along the current path of the coil composed by the turn-to-turn capacitance can be modelled.
4. A NOVEL HIGH FREQUENCY MODEL FOR MULTI-LAYER, STRAIGHT COILS AND FOR COILS ON EACH-OTHER

The known high frequency models of coils and transformers take into account the capacitance between turns of different coil layers. These models are not able to take into account electromagnetic wave propagation between coil layers and coils, i.e. in radial direction, because these capacitors are connected directly in series to each-other. In the case of these models voltage appears without delay at every locations of the capacitance chain, i.e. along the whole radial dimension of the coil when applying the supply voltage at the input ports of the coil.

I would like to propose a novel multi-layer distributed parameter coil model suitable for modelling electromagnetic wave propagation between coil layers and coils and a lumped parameter model as well for the use with simulation software (e.g. xSPICE) by introducing an inductance in series to the layer-to-layer capacitance.

For testing the developed model two meter long straight coils have been measured of copper wire with a diameter of 1 mm included also the varnish insulation so that the wire was densely wound onto a plastic (PVC) protective pipe (Fig. 21). Measurements on the coils with and without an iron core have been realised with pulse generators applying voltage on the input ports of the coil and with an oscilloscope to compare the results with those given by the model with the simulator software. The output ports of the coils were practically open, i.e. closed by 1 MΩ.

The parameters of the coils can be seen in Fig. 47. These parameters are used for the calculation of the model’s circuit elements. Both of the coils have been made of the same wire, so both of them have the same number of turns, namely \( N = 2000 \). The material and thickness of the supporting pipes are also identical.
Measurements and simulation show, that after applying voltage on the input ports of the primary coil within the same time span nearly the same voltage part appears at the output ports of both the primary and secondary coils.

Simulation sessions have been made with this transformer model on transformers with low and high turn-to-turn capacitance, i.e. on small transformers and on high voltage power transformers. Simulations have been completed with the Berkeley Spice based circuit simulator software TINA. Simulation sessions have also been made with a model proposed for coils with several layers and with a thickness being comparable to the coil length. Propagation time was investigated in radial direction in the coil. Most of the parameters indicated in Fig. 47 are identical with those in Fig. 23; the only new dimension is $D_{W2}$ the medium diameter of the secondary coil as well.

### 4.1. Model parameters of the coils

In Fig. 35 the proposed lumped parameter model of the one-layer coil can be seen for the use with a SPICE software. Calculation of the parameters for the two test coils are based on [49]. Both of the two measured coils have the same structure. Based on equations (9) and (16) the
The value of the direct current resistance within a lump for the primary coil is $R_{n01} = 0.226 \, \Omega$, and that of the coil with the bigger diameter, i.e. of the secondary coil is $R_{n02} = 0.452 \, \Omega$.

Parameters $R_{HF}$ and $L_{HF}$ model the frequency dependence according to the series Foster model. Based on (10), (11) and (18), (19) their values for the primary coil are $R_{HF1} = 5.8 \, \Omega$, $L_{HF1} = 1.85 \, \mu H$ and for the secondary coil $R_{HF2} = 11.6 \, \Omega$, $L_{HF1} = 3.7 \, \mu H$ for the highest frequency occurring by the measurements, i.e. for $f_{max} = 50 \, MHz$ when $\delta = 9.51 \, \mu m$ is the minimum skin depth by this frequency.

In the case of the primary coil $L_{n1} = M_{n1} = 220 \, \mu H$ are calculated with (22). The sum of these values for all lumps add up the self inductance $L = 220 \, mH$ of the whole coil having also been measured. The values for the secondary coil $L_{n2} = M_{n2} = 220 \, \mu H$ are the same as those for the primary because the core cross section $A_M = 1.33 \cdot 10^{-4} \, m^2$, relative permeability of core $\mu_r = 6.94$ and the coil lengths $l = 2 \, m$ are the same.

The value of the time delay element $t_n = 0.67 \, ns$ is for propagation of magnetic field along the coil length. This is the time needed for the electromagnetic waves to propagate directly along the length of the coil in air between the coil and core, so it is the same for both coils. This value belongs to one lump for the test piece. For the whole coil length of 2 m containing 10 lumps the time span is 6.7 ns in case of both coils. The value for the primary coil is $t_{nW1} = 29 \, ns$ and for the secondary coil is $t_{nW2} = 53 \, ns$ calculated with equation (25). These values differ from each-other because of the different wire length of the two coils caused by the different diameter of them. For the main current path is wave impedance $Z_{0nW}$ of the helical line can be calculated. Its value for the primary coil $Z_{0nW1} = 619 \, \Omega$ and for the secondary coil $Z_{0nW2} = 700 \, \Omega$ based on (26).

Turn-to-turn capacitance for the primary coil is $C_{K1} = 4.4 \, pF$ and for the secondary $C_{K2} = 6 \, pF$. These values belong to one turn-to-turn capacitance each. That for a lump of the primary $C_{nK1} = 0.022 \, pF$ and of the secondary coil $C_{nK2} = 0.03 \, pF$ (38).

Parameter $L_{nk}$ is the inductance value in series with $C_{nk}$ for the current of this capacitance flowing partly as conductive current along the diameter of the wire and partly as displacement current flowing between the turns. The inductance for one lump in the coil cylinder can be
calculated as a coaxial cable. Its values for the primary and secondary coils are \( L_{nK1} = 3.2 \) nH, \( L_{nK2} = 4.2 \) nH (42). \( R_{nK} \) is the resistance for one lump in the coil cylinder, i.e. the resistance in series with the turn-to-turn capacitance for the primary coil \( R_{nK1} = 174 \) μΩ and for the secondary \( R_{nK2} = 126 \) μΩ (35). This path contains also the \( t_n = 0.67 \) ns time value described above. In this case the current layout for the wave impedance is not a helical line but a coaxial cable, so the wave impedance for the primary and secondary coils are \( Z_{0n1} = 15.7 \) Ω, \( Z_{0n2} = 27.6 \) Ω. The capacitance between the coil and the core for one lump is \( C_{nC} = 41 \) nF, the resistance of the coil is \( R_{nC} = 200 \) mΩ.

### 4.2. Distributed and lumped parameter models of multi-layer coils and transformers

The formerly worked out high frequency models of multi-layer coils and transformers consist only the layer-to-layer and coil-to-coil capacitance (Fig. 10). On these capacitance chains certain voltages appear at the time of applying the voltage without any delay. In case of thin, long coils this is acceptable. However, when the thickness of the coil belongs to the same magnitude as that of its length and wave propagation time is to be taken into account also in radial direction, these models are no more applicable.

![Fig. 48. Turn-to-turn parameters of the measured physical transformer model](image)

In Fig. 48 a model of the neighbouring turns of the measured transformer model built up from two coils separated by a PVC tube is shown with the parameters of the capacitance path
composed by $C_{KT1}$ and $C_{KT2}$, Resistance of the turns $R_{KT1}$, $R_{KT2}$, the stray inductance of it $L_{KT1}$, $L_{KT2}$. In the model only the capacitance $C_{T12}$ between the turns of the neighbouring layers are taken into account. Their calculation method can be found later. For the theoretical support of the “quasi distributed parameter” model as the final purpose of the research at first I propose a distributed parameter model for multi-layer coils.

### 4.2.1. Distributed parameter model for multi-layer coils

Similar to one-layer coils an inductance of unit length connected in series to the layer-to-layer capacitance gives the solution for the problem. In Fig. 49 the proposed high frequency, distributed parameter model can be seen for multi-layer coils and transformers.

The figure shows two neighbouring layers, the difference in the values of their parameters is not shown by superscripts. The introduced new element proposed by me is $L''_L$ in series with the layer-to-layer capacitance $K''_L$.

![Fig. 49. Proposed distributed model for multi-layer coils](image-url)

Compared to the one-layer coil new elements of the circuit are $K''_L$ and $L''_L$ where $K''_L$ is the reciprocal of the layer-to-layer capacitance of unit length in radial direction in $1/Fm$ and $L''_L$.
is the inductance of unit length in radial direction in H/m. With the help of the proposed \( L'' \), it is possible to model wave propagation in radial direction as well. One dash means length dependence, i.e. \( z \) dependence, two dashes mean \( r \) dependence (radial dependence).

In this case the values of all the parameters depend on \( r \). The physical structure of the coil is not the same in \( z \) and in \( r \) direction because in \( z \) direction it has a continuous distribution while in \( r \) direction not. The layer-to-layer capacitance is actually bounded to two layers having a given distance from each-other. However for the sake of the uniform handling of the problem in case of the lumped parameter model I suppose that the distribution is continuous in \( r \) direction as well. Detailed calculation methods for these distributed parameters are not shown, because the main goal is to develop a lumped parameter model respectively a “quasi distributed parameter model”.

### 4.2.2. Lumped parameter model for multi-layer coils

In Fig. 50 one lump in \( z \) direction of the developed “quasi distributed” parameter model for multi-layer coils, can be seen, it is applied in a circuit simulator software. The number of the lumps has been selected so, that the \( M_n \) mutual inductance only between the neighbouring lumps gives the same result as in the reality for an acceptable extent. According to studies the mutual inductance decreases very fast between turns laying far from each-other within the coils.

If \( n \) turns are covered by a lump as a result of the reduction of turns, then there are \( N/n \) lumps within the model. Turn reduction can also be made in \( r \) direction. It is recommended to do so in case of many layers but was unnecessary in case of the measured two coils.

The parameter \( M_{n120} \) is the mutual inductance between the lumps being on each-other of the two coils, \( M_{n1} \) and \( M_{n2} \) are the mutual inductances between two neighbouring lumps of the same coil each. Finally \( M_{n12} \) and \( M_{n21} \) are the mutual inductance values between two neighbouring lumps of the two coils. The values are

\[
M_{n120} = M_{n12} = M_{n21} = \frac{1}{2} \frac{n}{N} L = \frac{n}{N} N^2 \Lambda = \frac{1}{2} n N \mu_0 \mu_r A_{\mu} l. \quad (44)
\]
where $L$ is the self inductance of the whole coil, $N = 2000$ is the number of turns in the coil and $n = 200$ is that in one lump, $A$ is the magnetic conductance of the coil, $\mu_r = 6.94$ the relative magnetic permeability of core, $A_M = 1.33 \cdot 10^{-4}$ m$^2$ is the internal cross section area of coil and $l = 2$ m is the length of the coil.

![Fig. 50. One lump in the proposed high frequency multi-layer coil and transformer model](image)

Parameters $M_{n120} = M_{n12} = M_{n21} = L_{n1} = M_{n1} = L_{n2} = M_{n2} = 220$ $\mu$H are the same because of the same lump and core parameters. The sum of $M_{n120}$, $M_{n12}$ and $M_{n21}$ results in the mutual inductance $M$ between the two coils. Every circuit simulator software can take into account the frequency dependence of the magnetic conductance of cores and the impacts of eddy currents.

The capacitance between the two coils per lump for the measurements is $C_{n12} = 48$ nF (40). The parameter $L_l$ belongs to a part of the same magnetic path as that of $L_{nk2}$. After flowing through $L_l$ current flows further along the current path composed by the turn-to-turn capacitors. $L_l$ quasi lengthens the path of $L_{nk2}$ by a length being equal to the radius difference of the two neighbouring layers. For the measured coil pair $L_l$ can be calculated as
In equation (45) $D_{W1}$ is the diameter of the primary coil at the middle of the wire, $D_{W2}$ is that of the secondary coil and $d_W$ is the wire diameter. The parameter $t_l$ is the time span taking by the voltage wave to propagate from one layer to the next in radial direction. For the measured coils it results in

$$t_l = \frac{D_{W2} - D_{W1}}{2} \cdot \frac{\sqrt{\varepsilon_r \mu_r}}{c} = 18 \text{ ps}.$$  

(46)

This time period is valid for the measured coils in Fig. 21.

![Proposed SPICE model for the turn-to-turn and coil-to-coil capacitance paths with lossy transmission lines](image)

Fig. 51. Proposed SPICE model for the turn-to-turn and coil-to-coil capacitance paths with lossy transmission lines
Fig. 5.2 Three lumps of the proposed SPICE model
In Fig. 51 a SPICE model with Lossy Transmission Lines is proposed for “thick” coils. The figure shows only the turn-to-turn and coil-to-coil capacitance paths. $C_{ij}$ and $C_{jk}$ are the layer-to-layer capacitance values for the $i$-th, $j$-th and $k$-th layers. Capacitance between the secondary coil and the housing, i.e. in case of the measured transformer between the measuring cable shielding is $C_{nH} = 2 \text{nF}$ (34) and the resistance in series with it is $R_{nH} = 800 \text{m}\Omega$ (35). Both of these current paths have the same $t_n$ time delay value in case of the measured transformer.

The model in Fig. 50 is the theoretical version of the circuit and that in Fig. 51 is a possibility to model the stray capacitance paths. Two lumps of the actual model simulated with the software TINA can be seen in Fig. 52. The circuit in Fig. 52 developed for the software TINA has actually 10 lumps like that used for the simulation of one layer because this lump number has been found as optimum during the investigations (see Chapter 3.3).

4.3. Comparison of experiment and simulation results

Experiments have been made in 2004 and 2005 at the University of Applied Sciences of Würzburg-Schweinfurt at the Department for Electrical Engineering in Schweinfurt, Germany at the laboratory for Telecommunication Technology running by Professor Dr. Peter Möhringer, Ph.D.

Measurement layout can be seen in Fig. 53. The coils have been laid on a wooden table on wooden stands with a height of 20 cm. One of the coils has been fed by 5 V pulses generated by a pulse generator type HP 8007B Pulse Generator with an output resistance of 50 $\Omega$ at the input ports of the coil. The voltage has been measured by an oscilloscope here and at the output ports of the coils as well. Type Tektronix TDS 540 four channel digitising oscilloscope with a sampling frequency of 1 GS/s and with channel input parameters of 1 M$\Omega$ and 10 pF has been applied. Measurements have been evaluated with and without iron core being ungrounded and grounded. The oscilloscope has been inserted in the middle and has been connected to the coil ports through 1 m long BNC cables laying parallel to the coils. Except
grounded core the nearest grounded metal bodies were shielding of the measuring cables of the oscilloscope at a distance of 30 cm with the same length.

![Diagram of the test layout](image)

**Fig. 53. The test layout**

The comparison of the measured and simulated results can be seen in the following figures. Fig. 37 shows the pulses supplied by the pulse generator measured (curve 1) and simulated (curve 2) on the input ports of one of the coils. Simulated curve is lower than that measured because of the rough calculation of the parameters.

![Graph of measured and simulated voltages](image)

**Fig. 54. Measured (1 - solid line) and simulated (2 - dashed line) voltage curves at the output ports of the secondary coil in case of ungrounded iron core**
When the metal core is ungrounded, a part of the voltage arrives after about 7 ns to the output ports of the secondary coil (Fig. 54), because the iron wire core with a diameter of 10 mm composes also a conductive current path for the electromagnetic waves for the secondary coil too and current flows through the coil-to-coil capacitance. In this case ferromagnetic character of the iron is irrelevant, if placing an aluminium core into the coil a larger part of the voltage arrives at the same time there. Time values are identical with those measured, however simulated curve is lower because of the same causes as in case of the pulse.

If the iron core is grounded no voltage appears until about 300 ns on the output ports of the coil (Fig. 55), electric charges in the core caused by the capacitive coupling between coil and core flow to the ground.

![Graph](image)

**Fig. 55.** Measured (1 - solid line) and simulated (2 - dashed line) voltage curves at the output ports of the secondary coil in case of grounded iron core and low turn-to-turn capacitance in both coils

Fig. 56 shows the simulated voltage curves at the output ports of the primary (1) and of the secondary (2) coil in case of ungrounded core. There is a very small difference between the two curves. Because of the much higher wire length of the secondary coil the secondary voltage increases later (after 530 ns) than in case of the one-layer coil. Similar results are obtained when the secondary coil is supplied, then its voltage curve is higher than that of the primary coil. Both measurement and simulation supports the close coupling through electric field between the coils in this frequency range.
4.4. Simulation results for low and high turn-to-turn capacitance values

In Fig. 57 the simulated voltage curves can be seen at the output ports of the primary (curve 1) and of the secondary (curve 2) coil in case of grounded core with high primary turn-to-turn capacitance and low secondary turn-to-turn capacitance. In this case the voltages at the output ports begin to increase very soon, i.e. after 7 ns again.
A very high turn-to-turn capacitance supports the arrival of the voltage at the output ports. In case of the lump capacitance values of 0.022 pF or 0.03 pF of the measured coils only a small (0.1 - 0.2 V) oscillating voltage is present at the output ports in case of grounded core before the electromagnetic wave arrives there along the main current paths (300 ns).

![Graph showing simulated voltage curves at the output ports of the primary (solid line) and of the secondary (dashed line) coil in case of grounded core with extreme high primary and high secondary turn-to-turn capacitance.](image)

**Fig. 58.** Simulated voltage curves at the output ports of the primary (solid line) and of the secondary (dashed line) coil in case of grounded core with extreme high primary and high secondary turn-to-turn capacitance

In high voltage transformers turn-to-turn capacitance of a simulation lump can be up to 6 - 7 orders of magnitude higher than that of the measured coils. One of the reasons of this can be, that the turns are arranged in discs resulting so a capacitance with several hundreds or thousands of cm$^2$ and an other reason is that the turn reduction involves a less number of turns resulting so a higher lump capacitance.

Simulation has shown, that if any of the coils has high turn-to-turn capacitance, it supports the fast voltage propagation along both of the coils due to the coil-to-coil capacitance. During the simulation 4 nF was used as “high” turn-to-turn capacitance value. With low primary and high secondary turn-to-turn capacitance both curves are lower than in the opposite case. The highest voltage curves are obtained with high turn-to-turn capacitance of both coils. In Fig. 58 an extreme high turn-to-turn capacitance of 100 nF has been applied for the primary coil and a very fast increase in output voltages has been resulted on both the primary (curve 1) and secondary (curve 2) coils.
4.5. Simulation results with the proposed multi-layer coil model

A virtual four-layer coil has been tested with the proposed model and the four voltage curves obtained are shown in Fig. 59. Simulation has been developed under high turn-to-turn capacitance (4 nF) in all of the layers and with the outer most layer (curve 1) supplied. The only difference between the layers is in the values of the wave impedance including also the self inductance values of each layer related to its turn-to-turn capacitance path ($L_{nk}$). In Fig. 59 the slight delay of the voltage curves can be seen related to each other caused by the longer current paths of the different layers and a certain damping can be noticed too.

![Fig. 59. Simulated voltage curves at the output ports of four coil layers with delay circuit elements](image)

Voltage supplied at the input ports of a primary coil initiates electromagnetic waves propagating along the wire on several paths also of a secondary coil inserted onto or into the primary coil. A part of the voltage arrives to the output ports within time spans determined by the length of the coils wire and the length of the coils. Due to the coil-to-coil capacitance certain voltage appears on the secondary coil having no contact with the primary coil.

With the appropriate modelling of this transformer-like layout, similar results can be obtained to those supplied by measurements. Ungrounded conductive cores can support fast voltage wave propagation along both of the coils. In case of coils with high turn-to-turn capacitance this path can have similar impacts. Extreme high turn-to-turn capacitance can support the
propagation of even the whole voltage along both coils within a time span determined by the coil length. Electromagnetic wave propagation along coils also in radial direction between coil layers can be taken into account in SPICE models with inductance and delay element in series with the layer-to-layer or coil-to-coil capacitance.

4.6. Error analysis

In this investigation the measured and simulated voltage curves at the output ports of the secondary coil are of interest (Fig. 54) in case of ungrounded core. Curve 1 (solid line) corresponds to the measured results and curve 2 (dashed line) to the simulated one. Now the investigated time interval from 0 to 500 ns is longer than that in one layer case. In the followings an error analysis of the simulation results can be found taking the measured curve as a reference. The difference between simulated and measured results are evaluated.

In Fig. 60 the principle of the error analysis can be seen: measured and simulated voltage values are taken from the curves at a number of sampling time values being, 16.7 ns apart from each-other, and listed in Table 2. Equations for the calculations are the same as in the one layer case (39 - 43).

![Fig. 60. Measured and simulated voltage curves at the output ports of the secondary coil in case of ungrounded core with sampling times](image-url)
Table 2: Listing of the error analysis results

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<td>-1.1</td>
<td>0.0</td>
</tr>
<tr>
<td>26</td>
<td>416.7</td>
<td>4.75</td>
<td>5.10</td>
<td>-0.35</td>
<td>-7.4</td>
<td>0.5</td>
</tr>
<tr>
<td>27</td>
<td>433.3</td>
<td>4.90</td>
<td>5.20</td>
<td>-0.30</td>
<td>-6.0</td>
<td>0.4</td>
</tr>
<tr>
<td>28</td>
<td>450.0</td>
<td>5.00</td>
<td>5.40</td>
<td>-0.40</td>
<td>-8.0</td>
<td>0.6</td>
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<tr>
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<td>466.7</td>
<td>5.10</td>
<td>5.70</td>
<td>-0.60</td>
<td>-11.8</td>
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<tr>
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<td>-1.00</td>
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<tr>
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<td>5.20</td>
<td>6.50</td>
<td>-1.30</td>
<td>-25.0</td>
<td>6.3</td>
</tr>
</tbody>
</table>
The basic differences are coincident with those at the one-layer coil: there is a short periodic phenomenon at the beginning of the simulated pulse being completely absent in the measured curve and the simulated curve lays below the measured one and this difference decreases until the time value of 460 ns, then the simulated curve lays above it. In this case the simulated curve lays even lower. While in the one-layer case the simulated voltage curve lays nearly to the same amount as the supply voltage curve, in this case the difference is larger.

Fig. 61 shows the plotting of the relative error. The curve begins and ends with negative values as in the other case. The figure does not show the value at \( t_3 \) corresponding to the periodic phase and being an extreme value. In this case the error is higher and remains at values of around 50 - 60 % and it decreases only after 350 ns. In case of the multi-layer coil the error is higher than that at the one-layer coil. This fact refers to the need of increasing the layer-to-layer capacitance, but its increase raises the overshoot after the fast transient phase. A further raffinery of the circuit is necessary.

The signal energy for the measured voltage curve \( E_{\text{Mm}} = 6.836 \times 10^{-6} \text{ V}^2\text{s} \) and that of the simulated curve \( E_{\text{Sm}} = 5.963 \times 10^{-6} \text{ V}^2\text{s} \). Simulation gives a slightly smaller energy value according to the less voltage values in general. A further research is necessary finding the reason why the error increases in case of radial propagation.
4.7. New scientific result

Thesis 2:

I have developed a novel high frequency distributed parameter model and a lumped parameter model for multi-layer coils and coils on each-other, i.e. for transformers of electronic devices. These models are able to take into account the electromagnetic wave propagation between coil layers and coils, i.e. in radial direction along the current path composed by the layer-to-layer capacitance as a result of an inductance inserted in series to this capacitance. Former models can not take this phenomenon into account, as they model this path only by a capacitance chain between the coil layers, [49].

a) I propose a novel high frequency distributed parameter model for multi-layer coils and coils on each-other, i.e. for transformers, introducing an inductance of unit length in series with the reciprocal layer-to-layer capacitance of unit length. Unlike the former models taking this path into account only by capacitance chain, on which the voltage appears without delay, this distributed parameter circuit can model electromagnetic wave propagation in radial direction, i.e. along the current path of the coil composed by the layer-to-layer capacitance. Calculation of this inductance between two layers for each layer pair is based on that of coaxial cables depending on the dimension and materials of the outer coil layer.

b) I propose a novel high frequency lumped parameter model for multi-layer coils and coils on each-other, i.e. for transformers for the use with circuit simulation software introducing an inductance in series with the layer-to-layer capacitance. With a model composed by an adequate number of the developed identical lumps electromagnetic wave propagation in radial direction, i.e. along the current path of the coil composed by the layer-to-layer capacitance can be modelled as well.

The above model is valid for transformers with a structure in general use in electronic devices, containing several turns densely wound near to each-other and layers being densely on each-other.
5. A NOVEL HIGH FREQUENCY MODEL FOR TRANSFORMER SHIELDING

Shielding inserted between the coils of transformers of electronic devices have the task to conduct the electric charge of transient over-voltages to the ground avoiding so the propagation of over-voltages to the secondary coil of the transformer. However in case of fast common mode transients with high frequency content like bursts and electrostatic discharges this shielding is no more so effective as at low frequencies belonging e.g. to surges. Wound or cylinder type shielding has more or less inductance to the ground hindering the electric charges in reaching the shielding, decreasing the shielding efficiency in turn. In general no shielding is installed in the transformers of high frequency chopping supply units for it is ineffective because of its rather high inductance to the ground.

The more common shielding coil has a less shielding degree (SD) at the same frequency than that of a shielding foil. Recent researches have demonstrated that neither a shielding made of superconductive material is effective at high frequencies. Shielding degree begins to decrease over 30 kHz. According to measurements electric shielding degree (SD\textsubscript{V}) decreases with the frequency, with the locations being closer to the edges of the shielding and with the less layers of the shielding. A rather simple simulation model can help by the decision which art of shielding should be installed if any. There are precise methods for modelling the shielding and metal foil cylinders itself, but their use is rather complicated. The known shielding models do not take into account the capacitance of the shielding to the surrounding conductive bodies and the inductance in series with the shielding inside and outside of the transformer housing.

I would like to propose a high frequency SPICE model of transformer shielding built for a circuit simulator software. I develop measurements on a PC supply unit transformer with signal generators to test the model as well. I dismount the transformer and wind out its coils and thread new coils with the same number of turns and different shielding between the coils. Measurements have been realised with surge, burst and other signal generators to determine the shielding degree. The signals have been coupled as a common mode interference.
Measurements have been performed at the University for Applied Sciences of Würzburg-Schweinfurt at the Department of Electrical Engineering in Schweinfurt, Germany in 2000 and 2004 in the Laboratory for Telecommunication Technology running by Prof. Dr. Peter Möhringer PhD [50]. The measurements have been repeated in the Laboratory for High Voltage Technology at the Technical University of Budapest in 2005 with the help of Dr. István Kiss, Ph.D.

Results of the simulation sessions with the developed SPICE model have given similar results to those of the measurements. I have introduced an inductance in series to the ground of the shielding model, then a capacitance in parallel to this inductance, both have been then split into two parts each to obtain a reliable model for shielding between transformer coils. Considering the main character of the degree curve the model have yielded similar results as those of the measurements. Because of the different potentionalities of the two laboratories a significant difference is shown by the results of the measurements conducted in Schweinfurt and in Budapest. With an adequate calculation of the model parameters for both cases the proposed model have mirrored the different circumstances with its results.

5.1. Experimental procedure

The test transformer has been made by using the bobbin and the ferrite core of a transformer dismantled from a PC supply unit. Fifty turns have been wound for the primary and secondary coils each. This low number of turns allowed a quick replacement of the shielding inserted between the coils.

The measurements have been completed with the help of a Haefely Trench Surge Generator, a HP 8007B Pulse Generator and a Schaffner NSG 222A Interference Simulator. Voltage signals on the primary and secondary coils have been measured by a four channel digitising oscilloscope type Tektronix TDS 540 with a sampling frequency of 1 GS/s and with the channel input parameters of 1 MΩ and 10 pF. The measurement layout can be seen on Fig. 62. At first the measurements have been developed in Schweinfurt in the Laboratory for Microwave Technology with PVC floor and furniture made of mainly non-conductive materials.
Measurements have demonstrated that a thin aluminium shielding results in a shielding degree $SD_V = 26$ dB in case of surge with a rise time of $2 \mu$s, a shielding degree of 16 in case of burst with a rise time of $40$ ns and a degree of 6 in case of a rectangle pulse with a rise time of $10$ ns. Calculations were made with

$$SD_V = 20 \log\left(\frac{\hat{U}_2}{\hat{U}_1}\right),$$  \hspace{2cm} (47)$$

where $\hat{U}_1$ is the peak value of the voltage at the primary and $\hat{U}_2$ is the peak value of the voltage at the secondary coil. Shielding degree decreases with the frequency. Laboratory conditions contributed to this result. The ground as an unlimited source of electric charges is rather “far” from the transformer shielding in this laboratory of mainly non-conductive materials. A rather high series inductance is effective in this environment belonging to the power cord of the oscilloscope and the signal generator and between the socket and the ground.

### 5.2. Development of the simulation model

Fig. 63 shows the initial simulation circuit of my proposition for the transformer shielding. Parameter $C_S$ models the capacitance between a coil and the shielding. Simulation has shown that a grounded shielding has a perfect shielding effect without the $L_S$ inductance between the
ground and the shielding at every frequencies, so this does not model the reality. With increasing $L_S$ the shielding degree decreases and at a value of $L_S = 200 \, \text{nH}$ the same voltage peak appears at the secondary coil as that on the primary. Parameters $R_i$ and $L_i$ are from the dimensions of the test transformer calculated values being negligible to the reactances of $C_S$ and $L_S$.

![Signal generator and transformer diagram]

**Fig. 63. Initial simulation circuit**

In case of these common mode simulations the transformer is practically unused, there is no current flowing through it. During the measurements two terminals of the primary coil were connected to each-other and to the signal generator. A simulation software does not allow this, that is why capacitors $C_S$ are applied as coupling between the coils and the shielding. With the help of a SPICE software feature a voltage source has been used as a signal generator in the model circuit with the user function signal

$$U_G = 7 \cdot e^{-10^9 t} - 7 \cdot e^{-4 \cdot 10^9 t} (V),$$

(48)

corresponding to the rise time of 2.5 ns set on the signal generator during the measurements. The actual programming lines entered in the SPICE based Tina software [11] are

```plaintext
Function Signal (t);
Begin
Signal := -7.2*exp(-t/2.5e-9)+7.2*exp(-t/1e-8);
End;
```

80
In Fig. 64 the measured and simulated voltage curves can be seen at the primary and secondary coils of the transformer. Curve (1) (thick solid line) represents the voltage measured at the primary coil with the fastest rectangle signal generator, curve (2) (thin solid line) the voltage measured at the secondary, curve (3) (dashed line) represents the voltage simulated at the primary coil and curve (4) (dotted line) that simulated at the secondary.

During the measurements the less rise time was reached with the rectangle signal generator, Fig. 64 shows the measured voltage curves in this case. The figure shows that the measured voltage curves have a wider spectrum than those simulated, for a more accurate simulation other aspects must be taken into account as well. It can be also seen that the simulated voltage curve at the secondary coil reaches its peak sooner than the measured curve.

Fig. 65 shows the voltage curves on the secondary coil simulated at different values of $L_s$ inductance. Curve (1) (thin solid line) represents the voltage simulated on the secondary coil
in case of an inductance of $L_S = 10\ \text{nH}$ between the shielding and the ground, curve (2) (dotted line) in case of $L_S = 20\ \text{nH}$, curve (3) (dashed line) for $L_S = 50\ \text{nH}$ and curve (4) (dashed line) for $L_S = 100\ \text{nH}$. With $L_S = 100\ \text{nH}$ nearly the same voltage peak appears on the secondary as on the primary coil.

In Fig. 66 the dependence of the shielding degree on the $L_S$ grounding inductance can be seen. From the figure it can be seen that the shielding degree $SD_V$ decreases with increasing inductance.

Fig. 65. Simulated voltage curves on the secondary coil. Curve (1) (thin solid line) represents the voltage simulated on the secondary coil in case of $L_S = 10\ \text{nH}$, curve (2) (dotted line) in case of $L_S = 20\ \text{nH}$, curve (3) (dashed line) for $L_S = 50\ \text{nH}$ and curve (4) (dashed line) for $L_S = 100\ \text{nH}$
Fig. 66. The dependence of the shielding degree $SD_V$ on grounding inductance $L_S$

Inductance $L_S$ in Fig. 66 contains the inductance between the shielding and the supply port of the transformer and the inductance between the supply port and the “real” far ground of inexhaustible charge source.

5.3. Impact of the capacitance to the environment

Measurements have been realised first on a wooden table without the housing of the transformer thus with a negligible capacitance to the ground in the Laboratory for Microwave Technology in Schweinfurt. When measurements have been repeated on an EMC measuring table with a grounded aluminium plate of more than a m$^2$ in the High Voltage Laboratory at the University of Budapest, shielding degree has been registered to be much higher than in case of the former measurements.

The reason of this fact is that a significant capacitance has to be taken into account between the shielding and the grounded surface like housing of the transformer or the power unit. This laboratory has a “nearer” ground than the other one. Simulation circuit with the capacitance $C_H$ to the housing is shown in Fig. 67.
Fig. 67. Simulation circuit with the capacitance $C_H$ between the shielding and the housing.

Fig. 68 shows the dependence of the developed simulated voltage on the secondary coil versus the value of the $C_H$ capacitance to the housing - respectively to the aluminium plate in the environment during the actual measurement.

Fig. 68. Simulated voltage curves on the secondary coil with the capacitance $C_H$ between the shielding and the housing. Curve 1 (dashed line) represents the voltage on the secondary coil in case of $C_H = 100$ pF, curve 2 (thick solid line) in case of $C_H = 10$ pF, curve 3 (thin solid line) for $C_H = 1$ pF.
All the curves are achieved for $L_S = 100\,\text{nH}$ resulting nearly the same voltage peak at the secondary as on the primary coil and $C_H$ varies from 1 pF to 100 pF.

Fig. 69 shows the shielding degree $SD_V$ versus grounding capacitance $C_H$ for a fixed grounding inductance of $L_S = 100\,\text{nH}$, i.e. for the worst case of $L_S$. The shape of the curve is nearly the opposite of that in Fig. 66, the nearer are $C_H$ and $L_S$ to the resonance, the better is the $SD_V$.

Fig. 69. The dependence of the shielding degree $SD_V$ on grounding capacitance $C_H$ for an grounding inductance of $L_S = 100\,\text{nH}$

Fig. 70. Simulation circuit with the capacitance $C_H$ between the outer coil and the housing
In the reality the outer coil of the transformer fully covers the shielding between the two coils thus its capacitance to the housing has to be taken into account. This model is plotted in Fig. 70. In this case $C_H$ does not reduce only the voltage peak on secondary coil but also that of on primary coil. Fig. 71 shows simulation results obtained with different $C_H$ values.

![Simulated voltage curves](image)

Fig. 71. Simulated voltage curves with the capacitance $C_H$ between the outer coil and the housing. Curve (1) (thin solid line) represents the voltage of the signal generator, curve (2) (thick solid line) represents the voltage on the primary coil and curve (3) (dashed line) on the secondary.

This impact of a capacitance between the primary coil and the grounded surfaces, i.e. that it reduces the common mode interference, is known. However a housing is not an inexhaustibly available charge source for the shielding, there is a further inductance in series to the housing with a value depending on the cable type and length supplying the electronic device. The developed simulation model taking into account also this fact can bee seen in Fig. 72.
In Fig. 72 the $L_S$ inductance in series in the grounding circuit I have split it into two parts, $L_{SH}$ is the inductance between the shielding and the housing and $L_{SE}$ is the inductance between the housing and a solid grounding point. I have split $C_H$ into two parts respectively ($C_{HH}$ and $C_{HE}$). Values of $L_{SH}$ and $C_{HH}$ can be influenced by the design, the lower the value of $L_{SH}$ and the higher the value of $C_{HH}$ the higher shielding factor can be achieved and the same is true for $L_{SE}$ and $C_{HE}$.

However, values of $L_{SE}$ and $C_{HE}$ can only be influenced by the layout of the cable and grounding circuit outside from the electronic equipment. I have found that a higher shielding factor can be achieved using short cables to a nearest possible, stable grounding point.

In my investigation I found that if $L_{SH}$ and $C_{HH}$ are set near to resonance then simulation gives nearly the same voltage curves for the different $L_{SE}$ and $C_{HE}$ values like in case of $L_S$ and $C_H$. The least inductance values could be achieved with no galvanic connection between shielding and ground and with a high capacitance between them.

I have found that in case of a high capacitance between the primary coil and e.g. the grounded housing a perfect high frequency shielding could be achieved also without any shielding between the coils. However then no shielding would be obtained for low frequency interference like surge being more dangerous to the device.
Simulation model actually used with the SPICE based circuit simulation software TINA is plotted in Fig. 73. There are some differences to the theoretical circuit, e.g. the handling of the transformer, but these differences have no influence to the results.

5.4. Measurements with a spectrum analyser

For additional testing the model, measurements have then been made with a Rohde Schwarz R&S FSH type spectrum analyser with integrated tracking generator at the University of Pécs, Pollack Mihály Faculty of Engineering in a laboratory of the Institute for Information Technology and Electrical Engineering in 2005. The measurement layout can be seen on Fig. 74 [51].

Measurements with the spectrum analyser gave similar results than those made by the signal generators this giving the $H(j\omega)$ transfer function of the transformer from 0 Hz up to 1 GHz

$$20\lg|H(j\omega)| = 20\lg\left|\frac{U_z(j\omega)}{U_i(j\omega)}\right|. \quad (49)$$
As an example Fig. 75 shows a curve plotted by the spectrum analyser for the tested transformer with shielding. The shielding has a good shielding effect up to frequency of about 2.5 MHz (-40 – -50 dB) then it will be worth and worth with the frequency.

Applying an inductance in series with the shielding the shielding degree will be even lower and connecting a capacitance parallel to it increases the degree again.
Fig. 76 shows the transfer function simulated and plotted by the software TINA between 1 MHz and 20 MHz. The basic trend of the curve is in accordance with that of measured, there is a good shielding degree between 1 and 3 MHz then a high and a low extreme value can be found and above 6 MHz the shielding degree will be even worth.

The curve on Fig. 76 is much more simple than the measured one, because the model is rather simple (Fig. 73). The peak value at about 4 MHz can also be found on the measured curve, however its value is slightly higher than the measured one. The lower extreme value above 4 MHz can be found on the measured curve as well, however the measured one is much slighter and smoother than that simulated.

Between 1 and 3 MHz there are a lot of local extreme values on the measured curve can not be seen in the simulated one. The reason of this can be the simplicity of the model circuit. A solution to model this phenomena could be a combination of the model principle developed in Karlsruhe with this model (Fig. 5).
5.5. New scientific result

Thesis 3:

I have developed a novel high frequency transformer shielding model for the shielding installed between the two - primary and secondary - coils of transformers of electronic devices. This model is able to take into account the dependence of the shielding efficiency on the internal and external characteristics of the transformer and its environment unlike former models being unable for this purpose, [50], [51].

I propose a novel high frequency lumped parameter model for the shielding between the primary and secondary coils of transformers by introducing two inductances in series with each-other between the shielding and the grounding and two capacitances parallel to these inductances. One inductance-capacitance pair corresponds to the internal and the other pair corresponds to the external layout of the transformer and its environment. Unlike the former models this circuit can model the dependence of the shielding efficiency on the internal and external characteristics of the transformer and its environment.

The internal inductance and capacitance are to be calculated according to the dimensions and material characteristics inside the transformer to the connecting ports of it. The external inductance is to be calculated taking into account the inductance of the supply cable of the transformer and the inductance of the grounding circuit of the electrical installation in the room and building where the transformer is installed. The external capacitance is to be calculated between the primary coil resp. the housing of the transformer if exists and the surrounding conductive, grounded bodies.
6. THESES

Thesis 1

I have developed a novel high frequency distributed parameter model and a lumped parameter model for one-layer, straight coils. These models are able to take into account the electromagnetic wave propagation along the shunt current path of the coil composed by the turn-to-turn capacitance as a result of an inductance inserted in series to this capacitance. Former models can not take this phenomenon into account, as they model this shunt path only by a capacitance chain, [45], [46].

a) I propose a novel high frequency distributed parameter model for one-layer straight coils on the basis of Wagner’s model introducing an inductance of unit length in series with the reciprocal turn-to-turn capacitance of unit length. This distributed parameter circuit can model electromagnetic wave propagation along the shunt path of the coil composed by the turn-to-turn capacitance unlike the former models, because they model this path only by a capacitance chain, on which the voltage appears on its whole length with no delay. Calculation of this inductance is based on that of coaxial cables depending on the dimension and materials of the coil.

b) I propose a novel high frequency lumped parameter model for one-layer straight coils for the use with circuit simulation software introducing an inductance in series with the turn-to-turn capacitance. With a model composed by an adequate number of the developed identical lumps electromagnetic wave propagation along the shunt path of the coil composed by the turn-to-turn capacitance can be modelled.
Thesis 2

I have developed a novel high frequency distributed parameter model and a lumped parameter model for multi-layer coils and coils on each-other, i.e. for transformers. These models are able to take into account the electromagnetic wave propagation between coil layers and coils, i.e. in radial direction along the current path composed by the layer-to-layer capacitance as a result of an inductance inserted in series to this capacitance. Former models can not take this phenomenon into account, as they model this path only by a capacitance chain between the coil layers, [49].

a) I propose a novel high frequency distributed parameter model for multi-layer coils and coils on each-other, i.e. for transformers, introducing an inductance of unit length in series with the reciprocal layer-to-layer capacitance of unit length. Unlike the former models taking this path into account only by capacitance chain, on which the voltage appears without delay, this distributed parameter circuit can model electromagnetic wave propagation in radial direction, i.e. along the shunt path of the coil composed by the layer-to-layer capacitance. Calculation of this inductance between two layers for each layer pair is based on that of coaxial cables depending on the dimension and materials of the outer coil layer.

b) I propose a novel high frequency lumped parameter model for multi-layer coils and coils on each-other, i.e. for transformers for the use with circuit simulation software introducing an inductance in series with the layer-to-layer capacitance. With a model composed by an adequate number of the developed identical lumps electromagnetic wave propagation in radial direction, i.e. along the current path of the coil composed by the layer-to-layer capacitance can be modelled as well.

The above model is valid for transformers with a structure in general use in electronic devices, containing several turns densely wound near to each-other and layers being densely on each-other.
Thesis 3

I have developed a novel high frequency transformer shielding model for the shielding installed between the two - primary and secondary - coils of transformers. This model is able to take into account the dependence of the shielding efficiency on the internal and external characteristics of the transformer and its environment unlike former models being unable for this purpose, [50], [51].

I propose a novel high frequency lumped parameter model for the shielding between the primary and secondary coils of transformers by introducing two inductances in series with each-other between the shielding and the grounding and two capacitances parallel to these inductances, one inductance-capacitance pair corresponding to the internal and the other pair corresponding to the external layout of the transformer and its environment. Unlike the former models this circuit can model the dependence of the shielding efficiency on the internal and external characteristics of the transformer and its environment.

The internal inductance and capacitance are to be calculated according to the dimensions and material characteristics inside the transformer to the connecting ports of it. The external inductance is to be calculated taking into account the inductance of the supply cable of the transformer and the inductance of the grounding circuit of the electrical installation in the room and building where the transformer is installed. The external capacitance is to be calculated between the primary coil resp. the housing of the transformer if exists and the surrounding conductive, grounded bodies.
7. FURTHER RESEARCH

Measurements realised on the one-layer coil give similar, robust results in case of repeating them with other signal generators and oscilloscopes. Simulation with the proposed quasi distributed parameter models are sensible to the parameters and give results with slightly other curve shapes at the investigated time period. A further research is necessary to make the model being able to give results being more identical to the measured results.

The simulated transfer function curve for the shielding model is much more simple than the measured one, because the model is rather simple as well. The second peak value differs to a rather great amount from the measured one, it is much slighter and smoother than that simulated. The reason of this fact has to be found.

Between 1 and 3 MHz there are a lot of local extreme values on the measured curve can not be found on the simulated curve. The reason of this may be the simplicity of the model circuit. A solution to model this phenomena could be a combination of the model principle developed in Karlsruhe or of other models with this model.

An other direction of the further research is to develop a distributed and a quasi distributed parameter parameter model for transformer shielding and simulate it jointly with the quasi distributed parameter parameter model of transformer coils described above.
REFERENCES


