Capacity Analysis of Partially Overlapping NOMA System with Two Users

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***Abstract*—In recent years, the Internet of Things and other massive machine-type communication applications have demanded increased spectral efficiency for future radio multiple access schemes. One of the key enabling technologies is power domain non-orthogonal multiple access (NOMA) with successive interference cancellation (SIC), which works by superposing multiple users in the same frequency band, and at the receiver side successively decoding the signals. The key advantage here, compared to a traditional orthogonal multiple access (OMA) scheme like frequency division multiple access (FDMA), is that each user can utilize the full available bandwidth. The SIC method however has limitations. It requires precise estimation of the channel conditions of each user by the receiver. It works best when there is a large enough difference between the receiving signal strengths of the different superposed user signals. At the marginal case, when all the signals has the same signal-to-noise ratio (SNR) there is no advantage compared to FDMA. In this paper, we propose a method that lies between the full spectral overlap of the power domain NOMA and the no overlap of the FDMA schemes, allowing the user's signal to partially overlap by stretching the frequency band occupied by one user to utilize wider bandwidth, allowing the neighboring signal to partially overlap. The overlapping part of the neighboring signal causes interference, which is treated as part of the noise, reducing the capacity, but the wider bandwidth will increase the capacity. In this contribution, we investigate the balance between the gain and loss of this proposed scheme for the two user case where both users have similar SNR conditions, and we show that depending on the actual conditions in many cases significant capacity increase can be reached.**

***Index Terms*—NOMA, non-orthogonal, multiple access, FDMA**

I. Introduction

In orthogonal multiple access (OMA) communication, we separate the communication channels of the multiple users that tries to communicate at the same time in a way that no interference occurs between the signals. This separation can occur in different domains, three common cases are separating in the frequency domain (frequency division multiple access, FDMA), in the code domain (code division multiple access, CDMA), or in the time domain (time division multiple access, TDMA) [1]. In this document, we concentrate on the frequency domain multiple access. This means that the available bandwidth for the communication is divided between the users in a way that no overlap occurs between the neighboring users frequency band, this way no interference occurs. That limits the available bandwidth for each user, we can trade some capacity of one user for letting less bandwidth for some others.

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In the case of non-orthogonal multiple access (NOMA), each partner uses the full bandwidth of the communication channel with superposition coding, and the signals from all users are added. For example, in uplink channel, all the users are transmitting at the same time in the same frequency band. The signal of the user with the best channel condition has the highest signal level, all the other users' signal are considered as interference (part of noise) while decoding the strongest signal. After successfully decoding the first signal, the receiver can recover the first signal by remodulation using the successfully decoded symbols, and it can be subtracted from the received signal. This way, the first signal can be eliminated from the superposition. For decoding the next strongest signal, only the remaining weaker signals are interfering. The decoding follows by successive eliminating all the stronger signals. This method is called successive interference cancellation [2]. In this contribution we consider only the two user case.

There are cases when the NOMA communication does not work better than the OMA method. If there is no significant difference between the channel conditions of the users, the SIC method has no real advantage. In the theoretical case when two users has exactly the same signal-to-noise ratio (SNR), the achievable capacity region in the OMA and NOMA case is the same [2], [3]. But even if there is a slight difference between two users' SNR, SIC requires the knowledge of the channel conditions. Imperfect channel estimation results in imperfect cancellation and degraded decoding performance. There are many papers involving better channel estimation in different circumstances [4], [5].

In the OMA case there is no overlap of the frequency bands at all, in the NOMA case there is a full overlap. We can consider a case in between the two opposites: having a partial overlap between neighboring channels. We suppose that the signals of the two neighboring channels are uncorrelated, we treat the interference from the overlap as part of the noise. Having only partial overlap, the receiver cannot decode the symbols of the other user, so this interference cannot be canceled as it is done in the case of the original NOMA. By widening the available bandwidth for some user, the capacity of the channel can be increased, but the overlapping of the two signals increases the noise level, which will reduce the capacity. In this contribution, we can show that in some cases, the channel capacity can be increased by this kind of algorithm.

This paper is organized as follows. First, we lay down the basic definitions for calculating the achievable communication

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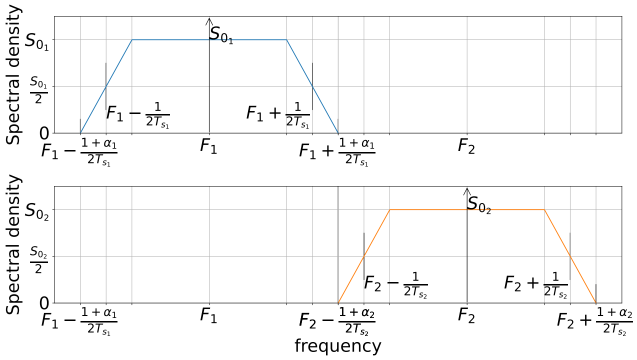

Figure 1: Two adjacent channels without overlap

rate in a channel. Then we examine the effect of partially overlapping frequency bands. Then we evaluate the effect of the increased noise level due to the partial overlap. Finally, we calculate the overall effect of widening the bandwidth of a communication channel for a particular practically important case. We finish with summarizing the conclusions of this study.

II. Basic definitions

The achievable bit rate of a communication channel with bandwidth W and power P is [6], [7]:

$$R = W \cdot \log_2 \left(1 + \frac{P}{N_0 \cdot W}\right) = W \cdot \log_2 (1 + SNR) \quad (1)$$

where N_0 is the noise power spectral density. The noise power in the band is $N = N_0 \cdot W$ and SNR is the signal-to-noise ratio ($SNR = \frac{P}{N} = \frac{P}{N_0 \cdot W}$).

In the case of NOMA communication with interference cancellation, one of the user's signal is decoded while the other user's signal is considered as interference and part of the noise power. For the second user's signal decoding, we can cancel that interference of the previously decoded signal. So the achievable rates for a two simultaneously communicating users are:

$$R_1 = W \cdot \log_2 \left(1 + \frac{p_1 |h_1|^2}{W \cdot N_0 + p_2 |h_2|^2}\right) \quad (2)$$

$$R_2 = W \cdot \log_2 \left(1 + \frac{p_2 |h_2|^2}{W \cdot N_0}\right) \quad (3)$$

where we denote the transmit power of the two users by $p_1$and p_2, respectively, the (complex) channel characteristic of the two users' channel are h_1 and h_2, and the common bandwidth by W. Since both user occupies the full bandwidth, while decoding the first (stronger) user's signal, the whole received power of the other user's signal is considered as noise.

In the case of classical OMA communication where adjacent channels do not overlap, (1) describes the channel capacity. In figure 1 we denote the center frequencies of the signals with F_1 and F_2, respectively, and the signaling periods are denoted by T_{s_1} and T_{s_2}. We are considering an optimal coherent receiver [8]. The effective bandwidth, which is the frequency distance between the symmetry point of the signal's spectral

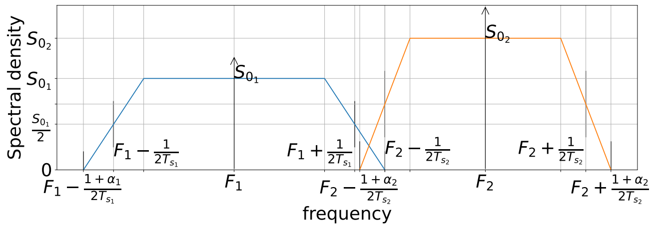

Figure 2: Two adjacent channels with partial overlap

density, can be considered as $W_1 = \frac{1}{T_{s_1}}$ and $W_2 = \frac{1}{T_{s_2}}$. The actual bandwidth that is occupied by the channel is wider than that due to the finite cut off of the transmit and receive filters, denoted by the roll-off parameter α_1 and α_2. The actual occupied bandwidth is $W_{1_{tot}} = \frac{1+\alpha_1}{T_{s_1}}$ and $W_{2_{tot}} = \frac{1+\alpha_2}{T_{s_2}}$.

The signal to noise ratio for these signals are:

$$SNR = \frac{P_i}{N_0 W_i} = \frac{P_i T_{s_i}}{N_0}$$

III. Basic model: Partially overlapping channels

Our goal is to increase the achievable communication rate according to (1). In our model, we consider the transmit power of the user (P) as a given constraint and keep it constant. The base noise density (N_0) is also an environmental constraint outside of our influence. So we are increasing the bandwidth by stretching the signal to a wider frequency band, allowing it to overlap with the neighboring channel. In our model, we don't consider changing any other parameter of the communication (like the shape of the spectral density function or the α roll-off parameter). The wider bandwidth (W') allows us a shorter signaling period (T'_s):

$$W'_1 = \frac{1}{T'_{s_1}} = n \cdot W_1 = \frac{n}{T_{s_1}} \quad (4)$$

$$T_{s_1} > T'_{s_1} \quad (5)$$

where n is a stretching parameter.

Since the transmitted power of the user does not change, the power density level of the signal S_{0_1} decreases:

$$P_1 = S_{0_1} \frac{1}{T_{s_1}} = S'_{0_1} \frac{1}{T'_{s_1}} \quad (6)$$

$$S'_{0_1} = S_{0_1} \frac{T'_{s_1}}{T_{s_1}} \quad (7)$$

so $S'_{0_1} < S_{0_1}$.

The widening of the bandwidth means that the signal will partially overlap with the neighbor channel, as can be seen in figure 2.

The partial overlap with the neighboring channel causes interference in the decoding of the signal. In our model, we consider the uncorrelated partially overlapping signal as a Gaussian noise with some equivalent extra noise power density N'_0:

$$R = W \cdot \log_2 \left(1 + \frac{P}{(N_0 + N'_0) \cdot W}\right) \quad (8)$$

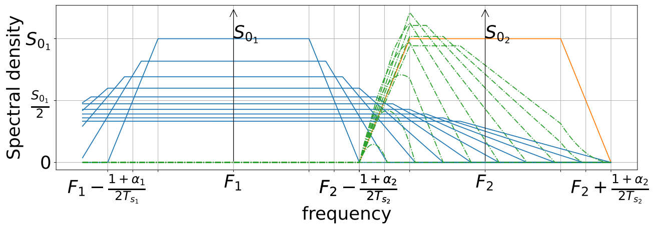

Figure 3: The first channel is stretched between a factor of 1 and 3. The dotted line is the product of the two spectral density function.

According to [9] the N_0' equivalent extra noise power density can be calculated as:

$$N_0' = \frac{\int_{-\infty}^{\infty} S_1'(f - F_1) S_2(f - F_2) df}{\int_{-\infty}^{\infty} S_1'(f - F_1) df} \tag{9}$$

where the denominator is the total power of the signal: $P_1 = \int_{-\infty}^{\infty} S_1'(f - F_1) df$, and in the numerator, we can get rid of the improper integral by utilizing the fact that the range where $S_1(f)$ and $S_2(f)$ are non-zero is finite. Without loosing generality, we can assume that $F_1 < F_2$, so:

$$N_0' = \frac{\int_{F_1 - \frac{1+\alpha_1}{2T_{s1}}}^{F_2 + \frac{1+\alpha_2}{2T_{s2}}} S_1'(f - F_1) S_2(f - F_2) df}{P_1} \tag{10}$$

In the classical case where the two spectral density do not overlap, one or the other term of the product in the numerator is zero, so $N_0' = 0$.

When we allow some overlap, this extra noise power density is added to the already present noise floor:

$$N_0 + N_0' = N_0 \left(1 + \frac{N_0'}{N_0}\right) \tag{11}$$

In figure 3, the bandwidth of signal 1 is stretched between a factor of 1 (no stretch) and 3 (see (4)). The product of the two spectral density function (the integrand in (9)) is also displayed as a dotted line. Note that it is non-zero only where the two functions overlap, and the shape of the product function depends on which part of the density functions overlap.

IV. Evaluation of the Increase of the Equivalent Noise Power Density

To evaluate the effect of extending the bandwidth of the channel to overlap with the neighbor, we need to evaluate the extra noise N_0' with respect of the base noise level N_0 and the signal power P.

For a signal with power density $S(f)$ and total power P, we can define a normalized spectral density function:

$$S^n(f) = \frac{S(f)}{P} \tag{12}$$

so that:

$$\int_{-\infty}^{\infty} S^n(f) df = 1 \tag{13}$$

Expressing the extra noise power N_0' with this normalized spectral densities:

$$\begin{aligned} N_0' &= P_2 \int_{-\infty}^{\infty} \frac{S_1'(f - F_1)}{P_1} \frac{S_2(f - F_2)}{P_2} df \\ &= P_2 \int_{-\infty}^{\infty} S_1^{n\prime}(f - F_1) S_2^n(f - F_2) df \end{aligned} \tag{14}$$

With this we can express

$$\begin{aligned} \frac{N_0'}{N_0} &= \frac{P_2}{N_0} \int_{-\infty}^{\infty} S_1^{n\prime}(f - F_1) S_2^n(f - F_2) df \\ &= \frac{P_2 T_{s_2}}{N_0} \frac{1}{T_{s_2}} \int_{-\infty}^{\infty} S_1^{n\prime}(f - F_1) S_2^n(f - F_2) df \\ &= SNR_2 \frac{1}{T_{s_2}} \int_{-\infty}^{\infty} S_1^{n\prime}(f - F_1) S_2^n(f - F_2) df \end{aligned} \tag{15}$$

Note, that the functions under the integral contain only normalized power density functions, hence it is independent of the actual power level of the signals. Denoting the value of the integral with

$$N^e = \int_{-\infty}^{\infty} S_1^{n\prime}(f - F_1) S_2^n(f - F_2) df \tag{16}$$

we can write (15) as:

$$\frac{N_0'}{N_0} = SNR_2 \frac{N^e}{T_{s_2}} \tag{17}$$

The achievable rate for the channel is:

$$\begin{aligned} R_1 &= W_1' \cdot \log_2 \left(1 + \frac{P_1}{W_1' \cdot N_0 \cdot \left(1 + \frac{N_0'}{N_0}\right)}\right) \\ &= \frac{1}{T_{s_1}'} \cdot \log_2 \left(1 + \frac{P_1 T_{s_1}'}{N_0} \frac{1}{\left(1 + \frac{N_0'}{N_0}\right)}\right) \\ &= \frac{1}{T_{s_1}'} \cdot \log_2 \left(1 + \frac{P_1 \cdot T_{s_1}' \cdot T_{s_1}}{N_0 \cdot T_{s_1}} \frac{1}{\left(1 + \frac{N_0'}{N_0}\right)}\right) \\ &= \frac{1}{T_{s_1}'} \cdot \log_2 \left(1 + SNR_1 \frac{T_{s_1}'}{T_{s_1}} \frac{1}{\left(1 + \frac{N_0'}{N_0}\right)}\right) \\ &= \frac{1}{T_{s_1}'} \cdot \log_2 \left(1 + SNR_1 \frac{T_{s_1}'}{T_{s_1}} \frac{1}{\left(1 + SNR_2 \frac{N^e}{T_{s_2}}\right)}\right) \end{aligned} \tag{18}$$

The resulting SNR value for the stretched signal can be expressed as:

$$SNR_1' = SNR_1 \frac{T_{s_1}'}{T_{s_1}} \frac{1}{\left(1 + SNR_2 \frac{N^e}{T_{s_2}}\right)} \tag{19}$$

The great achievement of (18) is that this expression does not contain any values that depends on the actual level of any of the signals or the noise floor. Only normalized power density functions and the original signal-to-noise ratios of the signals are present in this formula.

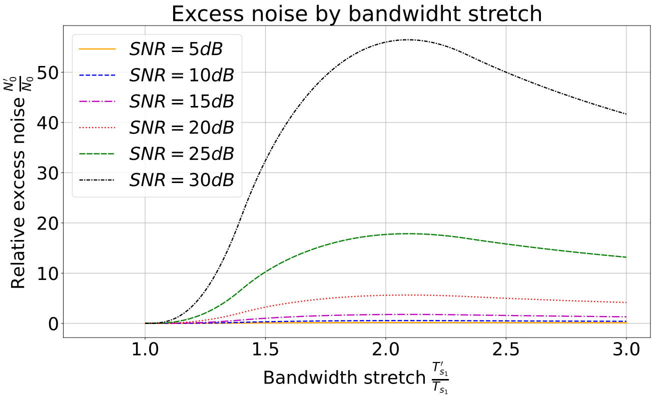

Figure 4: The excess noise relative to the noise floor ($\frac{N_0'}{N_0}$), for different SNR levels. The neighbor channel has the same SNR as our channel.

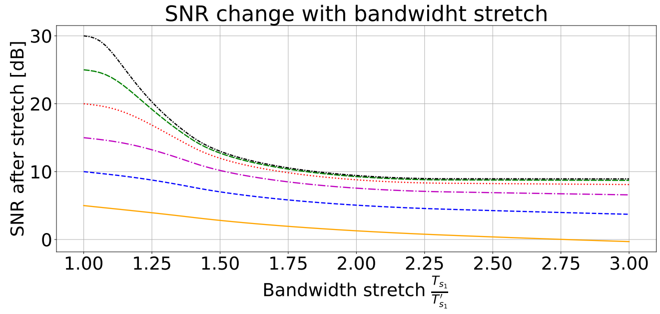

Figure 5: The resulting signal to noise ratio as a result of the bandwidth stretch. The initial SNR for both channels are 5, 10, 15, 20, 25, 30 dB

V. Analysis of the Physical Dimensions

In order to understand the equations on the previous section deeper, we analyze the dimensions of the quantities we got.

The dimension of the signaling periods (T_s) is time, measured in seconds $[s]$. The bandwidths in (4) (W) are measured in $[Hz]$ or $\left[\frac{1}{s}\right]$. The power spectral density (S_0 and $S(f)$) is measured in $\left[\frac{W}{Hz}\right]$. The integral of the power spectral density function over the bandwidth is the total power of the signal, measured in $[W]$:

$$P = \int_{-\infty}^{\infty} S(f)df \tag{20}$$

The Noise power density (N_0) and the extra noise density N_0' in (9) also measured in $\left[\frac{W}{Hz}\right]$. The normalized spectral density in (12) has a dimension of $\left[\frac{1}{Hz}\right]$. Integrating that over the bandwidth gives a dimensionless quantity. The relative extra noise density $\frac{N_0'}{N_0}$ in (15) is dimensionless. The signal to noise ratios (SNR) are also dimensionless. The quantity N^e in (16) has a dimension of $\left[\frac{1}{Hz}\right]$ (or $[s]$), so the quantity in (17) is also dimensionless.

VI. Evaluation of the System Capacity

In this section, we are evaluating the excess noise, the change in the SNR, and the achievable bandwidth for a particular case when we have a neighboring channel with the same characteristic as our signal channel. That is the neighboring channel has the same bandwidth and signal-to-noise ratio as our channel. This case is a practically important case when users with similar conditions has to coexists nearby to each other. As for the shape of the spectral density function, we are considering a simple linear roll-off, as can be seen in figure 1 and figure 3.

We are handling the interference caused by the overlapping neighboring signal as if it was part of the noise background, this is expressed in (8). The base idea behind this, according to [9], is that since the interfering signal is uncorrelated to our signal and we are approximating it as a Gaussian distribution signal. With this assumptions both the base noise and the interference signal is Gaussian so can be added in power domain. The main difference between the interference and the base noise is that the interference occupies only a (possible small) part of the signal bandwidth, so in (8) the quantity N_0' is an equivalent excess noise power density, which, when multiplied by the bandwidth W, gives us the same power as the interfering signal power. We can look at it as if the interfering power would have been smeared over the whole bandwidth.

In figure 4, we can see the calculated value of the excess noise power relative to the base noise level, that is, $\frac{N_0'}{N_0}$ from (11) calculated according to (17) by evaluating the integral in (16) numerically. On the horizontal axis, we can find the bandwidth stretch factor $\frac{T_{s_1}'}{T_{s_1}}$. The case $\frac{T_{s_1}'}{T_{s_1}} = 1$ means no stretch, hence no overlap, so in this case, there is no excess noise: $\frac{N_0'}{N_0} = 1$. On the other end, $\frac{T_{s_1}'}{T_{s_1}} = 3$ means that the signal bandwidth is stretched three times, occupying both to the left and to the right the same amount of bandwidth as the base bandwidth. In our case, that means that the neighboring channel is fully overlapping with our signal (see figure 3). On the vertical axis we can see the relative excess noise $\frac{N_0'}{N_0}$. The value 0 means no excess noise at all, the value 1 means the excess noise is equal to the base noise level, that is, the total noise level is double. The expression in (17) is evaluated for different starting SNR conditions. We can see that the excess noise is more and more prominent relative to the base noise level as we consider higher starting SNR condition. That is because the better SNR condition means higher signal level if we consider the base noise power density (N_0) the same, which results in higher interfering signal power.

For lower stretch values, the relative excess noise rapidly increases as we raise the stretch factor, as more part of the signals are overlapping. There is a maximum of the excess noise, more stretching causes a decrease in the N_0' value. This is because the interfering signal power is smeared over a wider and wider bandwidth, while the overlapping part does not increases at the same rate (see the product value represented by the dotted line in figure 3). This does not mean that there is less excess noise power present for higher stretch factor, only the spectral density equivalent is lowering.

The resulting new SNR of the stretched signal according to (19) can be seen in figure 5 for the same stretching range

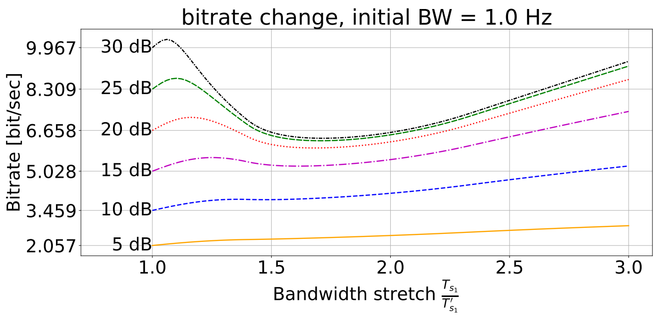

Figure 6: The resulting bit rate as a result of the bandwidth stretch.

and the same starting SNR values. Here we can see that the SNR value can only decrease as the overlap increases. The decrease, expressed in dB, is more prominent for the better starting SNR values (20-30 dB), but not negligible for even 5 dB starting SNR, and monotonically decreases with increasing stretch.

Our goal is to increase the capacity of the channel measured in bit/second. The theoretical limit (sometimes called the Shannon limit) is expressed in (8) and (18). There are two factors in these expressions that changes with the stretching of the bandwidth of the signal. One is the SNR value decreasing due to the interfering signal. The other factor is the bandwidth W_1' increasing due to the stretch. There is a balance between these two effects. In figure 6, we can see the effect of these two factors. We can see that there is a range of stretching for which the resulting capacity is increasing for all the considered starting SNR conditions. For high starting SNR values (20-30 dB), we can gain capacity with a little bit overlap with the neighbor, but we loose capacity with bigger overlaps. But when the starting channel condition is lower (below about 15 dB SNR) we can only gain capacity for all stretch factors.

VII. Conclusions

In this paper, we have examined the possibility to increase the capacity of a communication channel in a multi user environment by widening the occupied bandwidth, allowing it to partially overlap with the neighboring communication channels. This proposal can be seen as an in-between case between the fully orthogonal resource allocation in traditional FDMA schemes and the fully overlapping frequency band allocation of the power domain NOMA schemes. We have seen that there is a trade-off between the capacity gain due to the wider frequency band and the capacity loss due to the increased noise due to the interference caused by the neighboring channel. Our calculation results show that there are practically important cases where we can clearly gain capacity with the proposed method. The capacity gain is highly dependent on the actual spectral shape of the communication channels, the relative signal strength of the neighboring channels, and the starting SNR conditions.

Further study can examine of the effect of other spectral shapes, like root raised cosine channel filter characteristic. We did our study for the two communicating user case only, further study can examine the effect of having also a neighboring channel on the other side of our frequency band. It would also worth studying the effect if the neighboring channel also implements the same widening scheme.

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