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Combinatorial Algorithms in VLSI Routing

Summary of PhD Dissertation

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1 Introduction

The design of very large scale integrated (VLSI) circuits is one of the broadest areas in which the methods of combinatorial optimization can be applied. There are plenty of results in this topic from the last few decades. However, the list of NP-complete problems arising in this field is also very long and there is an abundance of heuristics, many of them with a very good performance, to handle these.

The term VLSI usually covers not only a single problem but a range of substantially different problems that arise during the design of circuits. However, in this summary we focus on *detailed routing*, one of the last phases of the design process.

Assume that the devices of an electric equipment to be designed have already been placed on the circuit board. In the detailed routing problem our task is to interconnect certain given subsets (or *nets*) of the pins (or *terminals*) of these devices by wires. Wires belonging to different nets must never get closer to each other than a given distance. To this end, the wires are usually embedded in a rectangular grid. However, this grid is not planar (this would make the problem unsolvable in most cases), it consists of a number of planar layers, each of them parallel with the circuit board. Wires can leave a layer for a consecutive one at any gridpoint. To sum it up from a graph-theoretical viewpoint, the detailed routing problem consists of finding vertex-disjoint Steiner-trees (trees with a given terminal vertex set) in a 3-dimensional rectangular grid.

In this summary we consider three of the numerous subproblems of detailed routing: *switchbox routing*, *channel routing* and *single active layer routing*.

2 Switchbox Routing

Assume that the terminals of the integrated circuit are situated on the four boundaries of a rectangle. In the switchbox routing problem we aim at routing all nets (that is, interconnecting their terminals by wires) using the inner part of the rectangle. Before we give a formal definition, we mention two further technological requirements: the 'corners' of the board must not be used and wires can access the terminals on any layer. The following definitions are formulated in a way to incorporate these requirements.

Definition 1 *Let the vertex set of a graph be the set* $\{0, \dots, n+1\} \times \{0, \dots, w+1\} \times \{1, \dots, k\}$. *Let two vertices be adjacent if and only if they differ in exactly one coordinate and by exactly one. Delete the 'corner' vertices from the graph, that is, the vertices* $(0,0,l)$, $(n+1,0,l)$, $(0,w+1,l)$ *and* $(n+1,w+1,l)$, *where* $l = 1, \dots, k$. *Now in the remaining graph contract all the subsets of vertices* $\{(i,j,l) : l = 1, \dots, k\}$ *into a single vertex, where*

- $i = 0$ *or* $i = n+1$ *and* $j = 1, \dots, w$*; or*
- $j = 0$ *or* $j = w+1$ *and* $i = 1, \dots, n$.

Call the thus obtained graph G_k k-layer rectangular grid graph. w *and* n *are the* width *and the* length *of the grid, respectively. Denote the vertex obtained from the contraction of the set* $\{(i,j,l) : l = 1, \dots, k\}$ *by* $t_{i,j}$. *The vertices* $t_{i,j}$ *are called* terminals. *The sets of all non-terminal vertices with a common* z*-coordinate are called* layers.

Figure 1 shows the 2-layer grid graph with $w = n = 3$.

Definition 2 *A* net *is a set of terminals. A* switchbox routing problem *is a set* $\mathcal{N} = \{N_1, \dots, N_t\}$ *of pairwise disjoint nets.*

Definition 3 *A* k-layer solution *(or* routing, *or* layout*) of a switchbox routing problem* $\mathcal{N} = \{N_1, \dots, N_t\}$ *is a set* $\mathcal{H} = \{H_1, \dots, H_t\}$ *of pairwise vertex-disjoint, connected subgraphs of the* k*-layer grid* G_k*, such that* $N_i \subset V(H_i)$*, that is,* H_i *connects the terminals of* N_i. *The subgraphs* H_i *are called* wires.

As we have already mentioned, the wires are usually chosen to be minimal, thus they are Steiner-trees. The following definition is motivated by the fact that for certain technologies it is advantageous not to have long parallel wire segments on two consecutive layers. Therefore there are many results that provide routings in the Manhattan model.

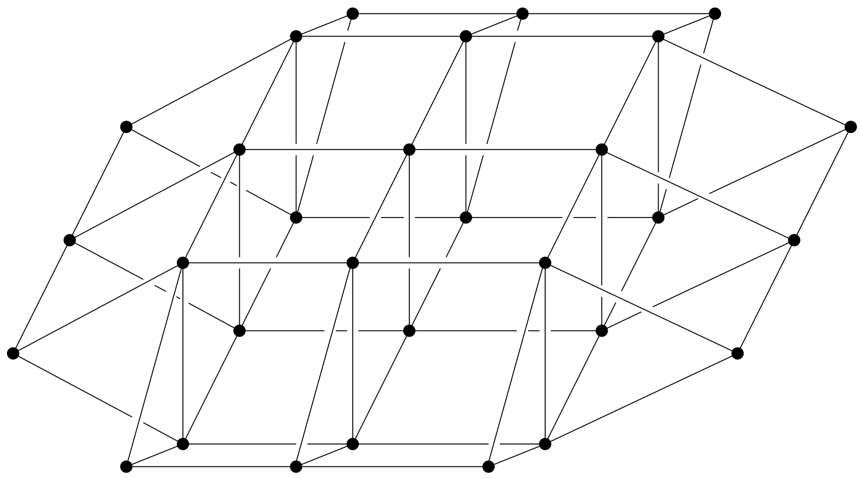

Figure 1

Definition 4 *A solution of a switchbox routing problem is said to belong to the* Manhattan model *if consecutive layers contain wire segments of different directions only. That is, layers with horizontal (east-west) and vertical (north-south) wire segments alternate. If no such restriction is imposed on a solution then it is said to be in the* unconstrained model*.*

If a switchbox routing problem is given, it is natural to choose the number of layers as the objective function to be minimized. This, however, is an NP-hard problem, even very special cases are known to be NP-hard. Furthermore, it was observed by Hambrusch [5], that there is no fixed number of layers that would suffice for all specifications. This is proved by a simple argument based on cuts: if a vertical line e cutting the grid into two separates many nets then the number of layers must be large enough to allow each wire belonging to these nets to intersect e. More precisely, the following statement is proved in [5]: if $m = \max(\frac{n}{w}, \frac{w}{n})$ denotes the ratio of the two sides of the switchbox then the number of layers needed can be as large as $\lceil m \rceil + 1$ in the worst case. A similar argument shows that if we restrict ourselves to the Manhattan model then at least $\max(4, 2\lceil m \rceil + 1)$ layers are needed in the worst case. The following proposition gives an improvement on the above lower bound in the unconstrained case for small values of m.

Proposition 1 [8] *For arbitrary values of n and w there exists a switchbox routing problem of length n and width w whose solution requires at least 4 layers in the unconstrained model.*

It is also verified in [4] that the lower bound can be improved to 6 in the Manhattan model for a certain 2×2 problem. Although no better lower bound than 4 is known in the $n \times n$ case, the following theorem is not far from being optimal.

Theorem 2 [9] *Every $n \times n$ switchbox can be solved on 6 layers in the Manhattan model.*

Turning to the general $n \times w$ case, the above bounds show that the minimum number of layers needed in the worst case can be lower bounded as a function of the ratio $m = \max(\frac{n}{w}, \frac{w}{n})$. On the other hand, E. Boros, A. Recski and F. Wettl [3] also gave an upper bound on the necessary number of layers as a function of m: they proved that $\max(18, 2m+14)$ layers suffice for every switchbox routing instance in the unconstrained model. The following theorem gives an improvement on this result.

Theorem 3 [9] *Every switchbox can be solved on $2\lceil m \rceil + 4$ layers in the Manhattan model.*

The following proposition highlights the fact that the upper bounds of the above theorems can be attained by a very effective algorithm.

Proposition 4 [9] *There exists a linear time algorithm for solving any switchbox routing problem in the Manhattan model such that the number of layers is bounded by the values given in Theorems 2 and 3.*

We have mentioned that the necessary number of layers for solving a switchbox routing problem in the Manhattan model is at least $\max(4, 2\lceil m \rceil + 1)$ in the worst case. Although Theorem 3 presents an upper bound of $2\lceil m \rceil + 4$, the algorithm attaining this bound is not an approximation algorithm since the lower bound refers to the worst case only. However, the following theorem claims that an approximation algorithm exists.

Theorem 5 [8] *There exists a linear time algorithm that approximates the minimum number of layers needed for a switchbox routing problem in the Manhattan model with an additive constant of 5.*

3 Channel Routing in the 2-layer Manhattan Model

The *channel routing problem* is the special case of the switchbox routing problem in which all the terminals of each net are situated on two opposite (say, the upper and lower) boundaries of the grid. In this case the specification of a routing problem only fixes the length n. Therefore the usual formulation of the problem is to fix the number of layers and ask for the minimum width routing.

Considering the channel routing problem on 2-layers in the Manhattan model is one of the most popular and most investigated fields in VLSI routing. Trivial examples show that some specifications are unsolvable with an arbitrary width (see Theorem 6). Therefore the usual problem setting, which is adopted in many papers, allows an arbitrary number of extra empty columns to be added to the left and right ends of the grid.

It was proved by Szymanski [11] that finding a minimum width routing is NP-hard. However, Baker, Bhatt and Leighton [2] provided an approximation algorithm. For this, they used two independent lower bounds on the minimum width. The *density* d, which is the maximum number of nets separated by a vertical line, is a naturally arising lower bound. However, the definition of the *flux* f of a routing problem, which is the other one of the above mentioned lower bounds, is much more technical; we omit the details here, we only mention that $f = O(\sqrt{t})$ (where t is the number of nets). It is proved in [2] that every channel routing problem can be solved on 2 layers in the Manhattan model with width at most $2d + O(f)$. The disadvantage of the method is that the number of extra columns that have to be added to the original problem instance can be as large as $O(f)$.

Let us assume now that the length n of the grid is fixed, that is, no extra columns are allowed to be added to the grid any more. The following theorem characterizes those problem instances that are solvable under this condition and also provides an upper bound on the minimum width.

A channel routing problem is called *bipartite* if each net contains exactly two terminals, one on the upper, and one on the lower boundary. A channel routing problem is *dense* if each terminal (on both boundaries) belongs to some net. A net is called *trivial* if it consits of two terminals which are situated in the same column.

Theorem 6 [10] *A channel routing problem is* not *solvable in the 2-layer Manhattan model (with an arbitrary width) if and only if it is bipartite, dense and has at least one non-trivial net. Moreover, if a specification is solvable then it can be solved in linear time with width at most* $\frac{3}{2}n$ *in the bipartite, and* $\frac{7}{4}n$ *in the general case.*

We also mention a somewhat surprising corollary of the above theorem.

Corollary 7 *Every channel routing problem can be solved in linear time in the 2-layer Manhattan model if it is allowed to extend the length of the channel by at most one (by introducing an extra column).*

The algorithm provided by Theorem 6 does not approximate the minimum width within a constant factor. It is natural to ask whether the above mentioned theorem of Baker, Bhatt and Leighton [2] remains to be true even if the length of the channel is declared to be fix? The following proposition answers this question in the negative.

Proposition 8 [8] *For any value of* n *there exists a channel routing problem with length* n*, density* $d = 2$ *and flux* $f \leq \lceil \sqrt{n} \rceil$ *whose minimum width routing requires* $n - 1$ *tracks.*

4 Single Active Layer Routing

We mentioned in Section 2 that the solution of a switchbox problem can require arbitrarily many layers. Nevertheless, switchbox routing can still be regarded as a 2-dimensional problem: the input consists of four sequences of terminals and the output consists of a fixed number of planar layers.

Due to the quick improvement of routing technology, research has recently turned towards 'real' 3-dimensional routing. There are plenty of deep results in this area from the last two decades.

In what follows, we restrict ourselves to the *single active layer routing problem*, where the terminals to be interconnected are situated on a rectangular planar grid of size $n \times w$ and the routing should be realized in a cubic grid of height h above the original grid that contains the terminals.

One can easily see even in small instances like 4×1 or 2×2 that a routing is usually impossible unless either the length n or the width w may be extended by introducing extra rows or columns between rows and columns of the original grid.

Definition 5 *The vertices of a given (planar) grid of size $w \times n$ (consisting of w rows and n columns) are called* terminals. *A* net *N is a set of terminals. A single active layer routing problem (or* SALRP *for short) is a set $\mathcal{N} = \{N_1, N_2, \ldots, N_t\}$ of pairwise disjoint nets. n and w are the* length *and the* width *of the routing problem, respectively.*

Definition 6 *By a* spacing of s_n in direction n *we are going to mean that we introduce $s_n - 1$ pieces of extra columns between every two consecutive columns (and also to the right hand side of the rightmost column) of the original grid. This way the length of the grid is extended to $n' = s_n \cdot n$. A* spacing of s_w in direction w *is defined analogously.*

Definition 7 *A* solution with a given spacing s_w and s_n *of a routing problem $\mathcal{N} = \{N_1, N_2, \ldots, N_t\}$ is a set $\mathcal{H} = \{H_1, H_2, \ldots, H_t\}$ of pairwise vertex-disjoint, connected subgraphs in the cubic grid of size $(w \cdot s_w) \times (n \cdot s_n) \times h$ (above the original planar grid containing the terminals) such that $N_i \subset V(H_i)$, that is, H_i connects the terminals of N_i. The subgraphs H_i are again called* wires. *h is called the* height *of the routing.*

If a SALRP problem is given together with the values s_w and s_n, we aim at minimizing the height h of a solution. A very similar argument as that in

[5] proves that the minimum height can be as large as $\frac{n}{2s_w}$ in the worst case [6]. In other words, if s_w is thought of as being fixed, $h = \Omega(n)$ holds for the minimum height in the worst case.

On the other hand, an upper bound of $h \leq \frac{wn}{2}$ is provided by a very simple argument, but only if $s_w, s_n \geq 2$ holds [6]. This statement can also be interpreted in the following way: if the value of w is fixed then there is a routing of height $h = O(n)$, provided that $s_w, s_n \geq 2$. The following theorem (which is far from being obvious) claims that essentially the same statement is true even if $s_n = 1$.

Theorem 9 [6] *If $s_w \geq 8$ then for any fixed value of w and for any n a SALRP problem can always be solved in time $t = O(n)$ and with height $h = O(n)$ such that the length n is preserved or increased by at most one.*

Let us assume $s_w, s_n \geq 2$ from now on. We mentioned above that a solution of heigh $h = O(wn)$ can be found in a trivial way. However, Aggarwal, Kleinberg and Williamson [1] proved that if each net consists of two terminals only then the nets of an $n \times n$ SALRP can be partitioned into $O(n \log^2 n)$ classes such that each class of nets can be routed on a copy of the grid (of size $n \times n$). This theorem easily implies that a solution with height $h = O(n \log^2 n)$ exists if $s_w, s_n \geq 2$ (and each net contains 2 terminals). The following theorem improves on this by providing a linear bound.

Theorem 10 [7] *If each net consists of two terminals only then a SALRP can be solved*

- *with $s_n = \lceil \frac{w}{2n} \rceil + 1$, $s_w = 2$ and height $h = 3n$;*
- *with $s_w = s_n = 2$ and height $h = 3\max(n, w)$.*

If w is much larger than n then the statement of Theorem 10 offers two options: either the height is relatively small ($h = 3n$) and the spacing is large, or the height is large ($h = 3w$) and then the spacing is only 2. The following theorem claims that a trade-off between these two extremities can be found.

Theorem 11 [7] *If each net consists of two terminals only then a SALRP can be solved with* $s_n = \lceil \frac{w}{4n} \rceil + 1$, $s_w = 2$ *and height* $h = \lfloor \frac{9}{2} n \rfloor$.

Let us consider the general SALRP now, where the nets may have an arbitrary number of terminals. The following results claim that the above linear bounds remain to be true with an increased constant.

Theorem 12 [7] *Any SALRP can be solved*

- *with* $s_n = \lceil \frac{w}{2n} \rceil + 1$, $s_w = 2$ *and height* $h = 6n$;
- *with* $s_w = s_n = 2$ *and height* $h = 6 \max(n, w)$.

Similarly as it was the case with Theorem 11, the statement of Theorem 12 can again be modified to achieve a smaller spacing in return for an increase in the height.

Theorem 13 [7] *Any SALRP can be solved with* $s_n = \lceil \frac{w}{4n} \rceil + 1$, $s_w = 2$ *and height* $h = 9n$.

Finally, the following proposition highlights the fact that each one of the above presented bounds can be attained by a polynomial algorithm.

Proposition 14 [7] *The bounds provided by Theorems 10, 11, 12, 13 can be attained by algorithms that run in* $O(t \cdot (w + n))$ *time (where* t *is the number of nets). That is, if* $A = w \cdot n$ *denotes the size of the input, the running time of the algorithms is* $O(A^{\frac{3}{2}})$, *provided that* $w = \Theta(n)$ *is assumed.*

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