Doctoral dissertation
summary

Structure of complex networks

Gábor Szabó

Adviser
Prof. János Kertész

DEPARTMENT OF THEORETICAL PHYSICS
BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS

2005

Introduction

In diverse fields of scientific interest underlying network structures can be recognized, which provide a unifying concept of investigation. Examples range from biology (metabolic networks, protein nets in the cell) to sociology (movie actor relationships, coauthor networks), to informatics (Internet, the WWW). In all these examples it is easy to identify the constituents of the problem with the nodes of a graph and their relationships with links. During the last few years a great deal of information has accumulated about such structures. The *scale-free* graph description attained particular attention, for which the node degree distribution follows a decaying power law, so as small-degree nodes dominate the ensemble while highly connected nodes serve as hubs in the network.

Objective and applied methods

The general network description of the many structures as we find them in Nature is a comparatively new area of science. Although the tools to analyze them have been at hand for centuries, experimental discoveries had to pave the way to their widespread acceptance. To model them with qualitative accuracy is a relatively easy and intuitive process; this fact gave rise to the proliferation of research undertaken on them. The first steps taken in this direction had been the structural characterization of the most common of these networks. This is where I began to focus on the distance distribution between randomly chosen nodes in a scale-free tree, which had not been studied yet. An extension of this work that incorporated analytical calculations and simulation support was the vertex load estimations in which I examined the number of shortest paths going through any node. While doing this, I digressed to study dynamical processes on networks to compare the results with a true model of packet communication for which I developed an easy to use parallel simulation framework for the deployment of time consuming simulations.

A different direction of my research was that of minimum spanning trees of scale-free networks where the edge weights were chosen in an independently random manner. These are somewhat important objects in practical optimization problems and since more and more real networks turn out to have scale-free degree distribution, possessing knowledge about their relation to the original graph is certainly useful. Arguments could be given for the tail of the weight distribution on the minimum spanning trees and also for their degree of degeneracy if the weights on the network are all the same.

I turned my attention to clustering when evidence started to mount that the local clustering coefficient as a function of the degree decays as $\sim 1/k$. By using a preferential attachment model that favors the formation of triangles in the network by linking common friends together I could give an explanation for that. The method that I used in the calculations is applicable to other growing models as well.

The packet transfer scenario outlined above led me to the recognition that actual communication patterns may be used to yield information about the structure of the network itself. Namely, from the timing of the packets it could be possible to recover the path they followed while they traveled to their destination. I used extensive simulations to show that in theory it is possible to do that to any degree of confidence.

New results

1. I analyzed the probability distribution function of the shortest paths on scale-free graphs of N nodes. Using mean-field arguments and approximating the $m = 1$ case of the Barabási-Albert (BA) model by a deterministic tree, I showed the origins of the average distance scaling and demonstrated the mechanism how the distribution approaches a Gaussian in the large N limit. Using the above approximation, I derived a scaling for the load, i.e., the number of shortest paths passing through any node.

2. Real networks are often characterized by a high degree of clustering, and the local clustering coefficient oftentimes decays as an inverse function of the node degree. In a simple modification of the Barabási-Albert model that promotes the forming of local communities, I showed that the $1/k$ scaling emerges naturally, and it crosses over to a k-independent region for large degrees, also observed in certain networks. I introduced a mean-field framework to treat growing networks from a clustering perspective, which can be used to solve problems of the same spirit.

3. I showed that the minimum spanning trees on scale-free graphs are scale-free as well, in the presence of random edge weights. The probability distribution of the weights on the tree were pointed out to differ from regular lattices reflecting the typically short distances (small-world property). I also considered trees in the absence of such randomness and the ensuing massive degeneracy, which is analyzed with graph theoretical arguments.

4. As an application of the network tools, I showed how the connection structure of a loopless communication network may be discovered using only ubiquitous echo requests or as a byproduct of normal two-way transport. The key factor is the correlation effect in waiting times of successively sent messages, which is caused by background traffic on the routers.

List of publications

Publications related to this dissertation

1. G. Szabó, 'Mapping a communication tree with correlation of packets', to be published in the International Journal of Modern Physics C.

2. G. Szabó, M. Alava, and J. Kertész, 'Clustering in complex networks', *Springer's Lecture Notes in Physics "Networks: structure, dynamics, and function"* **650**, pp. 139-162 (2004).

3. G. Szabó, M. Alava, and J. Kertész, 'Geometry of minimum spanning trees on scale-free networks', Physica A **330**, 31 (2003).

4. G. Szabó, M. Alava, and J. Kertész, 'Structural transitions in scale-free networks', Phys. Rev. E **67**, 056102 (2003).

5. G. Szabó, M. Alava, and J. Kertész, 'Shortest paths and load scaling in scale-free trees', Phys. Rev. E **66**, 026101 (2002).

Further publications

1. G. Szabó, M. Alava, and J. Kertész, 'Self-organized criticality in the Kardar–Parisi–Zhang equation', Europhys. Lett. **57**, 665 (2002).

2. G. Szabó and M. Alava, 'Mapping a depinning transition to polynuclear growth', Physica A **301**, 17 (2001).