Abstract

Design of steel structures is specified by structural codes. The European steel building code prEN Eurocode 3 is under final vote by CEN. From computational technology point of view the so-called interaction design equations are the most important phase of the codes. These equations govern the design of members as well as the bracing system of the structure. The final draft of the basic code was prepared by extended developments carried out by the so-called ECCS/TC8 working group. This expert group has improved the older formula and created a 'new' one. They suggested a new 'three-level' specification. Level 1 formula has the priority on user-friendliness, simplicity and comprehensiveness. Level 2 formula puts the priority to accuracy, consistency and continuity with other standard checks. Level 3 is the most sophisticated technique using efficient software as well as sufficient theoretical knowledge. This approach is used extensively in the calibration of the two other levels. This paper is related to the Level 2 and 3 formulas and proposes simulation based design methodology, which keeps the principal design and safety philosophy of Eurocode 3, but uses modern computational architectures.

Keywords: steel structures, beam-columns, structural Eurocodes, Monte-Carlo simulation, elasto-plastic FEA, interaction design equations.

1 Introduction

Accepting the results of ECCS/TC8 working group the final version of prEN Eurocode 3 1993-1-1, see [1], specifies tree levels for checking the resistance of beam-columns. The levels are identified by the techniques of the analysis to be used:

- Level 1: simple relationships where global factors includes several effects
- Level 2: consistent analysis which leads to more sophisticated relationships
- Level 3: non-linear numerical analysis (simulation)

From computational technology point of view the design programs based on Level 1 and 2 formulas are conservative systems. In order to justify this statement let analyse the design process. The general form of these relationships can be written as

$$\frac{N_{Sd}}{N_{Rd}} + k_1 \frac{M_{1.Sd}}{M_{1.Rd}} + k_2 \frac{M_{2.Sd}}{M_{2.Rd}} \le 1 \tag{1}$$

where N_{Sd} and M_{Sd} are respectively the design force and bending moments, N_{Rd} and M_{Rd} are respectively the design resistances, and k_i $(i=1,2)$ coefficients take into account the geometric non-linearity within the member as well as the interaction between the failing modes. However, the first item relates to the column buckling, the second item relates to the bending about major axis, if the torsional deformation is prevented, or the lateral torsional buckling, if the torsional deformation is allowed. The third item reflects the bending about the minor axis. The N_{Sd} and M_{Sd} design force and bending moments should be computed by simplified geometric nonlinear analysis, namely the so-called P-Δ approach, where the nonlinearity within the members (P-δ effect) should be neglected – the last effect is contained in the k_i coefficients (see [2] for details). The N_{Rd} and M_{Rd} are the theoretically and experimentally well-established member resistances, respectively for pure compression and pure bending. The difference between the Level 1 and Level 2 formulas is just appearances in the k coefficients. In Level 1 formula they are simple and conservative for most of the sections, while in Level 2 formula they are more sophisticated and differentiated given more accurate, general, consistence results. Furthermore, the Level 2 formula gives continuity with the other standard checks, namely the cross-section resistances for biaxial bending and compression. From computational technology point of view the similarity of the formulas is significant: N_{Sd} and M_{Sd} should be computed by simplified geometric nonlinear approach (P-Δ approach) in order to examine the resistance as an 'external' ultimate load of the member. However, the formulas are taking into account the geometric nonlinearity within the member, see more in [2]. The consequence of this design paradigm is the following:

- the analysis of the global model is restricted to a simplified method,
- using more refined analysis, the resistances will be underestimated.

The Level 3 approach allows using more accurate and sophisticated nonlinear analysis. These analyses should be materially nonlinear, furthermore, the model parameters should be assumed as probabilistic functions. However, Level 3 methods are computer simulations, which are experiment performed on computer models. For cross-sections with no local buckling (Class 1 and 2) numerical simulation program was developed by Papp and Szalai [3], which was based on plastic beam-column finite element model and a Monte-Carlo procedure. Others such as Hasham and Rasmussen [4] used finite strip method taking into account the local buckling effect. However, the Level 3 methods require considerable running time and theoretical knowledge. Therefore, these approaches can be adequate tools for researchers to calibrate the simple formulas Level 1 and Level 2. In this paper two approaches will be introduced. The first approach is based on 3D elastic global stability analysis and generalised design equation, which is calibrated by a numerical simulation. This CAD oriented design method can replace the ‘hand oriented’ Level 2 formula and results an automatic computer design procedure. The second approach uses the 3D global stability analysis and leads the computation of resistance back to a numerical simulation.

2 A CAD oriented Level 2 formula

It is important that the global analysis let response to the real stability behaviour of the whole structural model as well as its members. For this purpose Rajasekaran [5] developed a thin-walled beam-column finite element, which included the geometric nonlinearity and the torsional deformation as well. Using this explicit stiffness matrix formula the elastic analysis for any load combination can supply the second order design stress resultants as well as the global critical load factor, as it is seen in Figure 1.

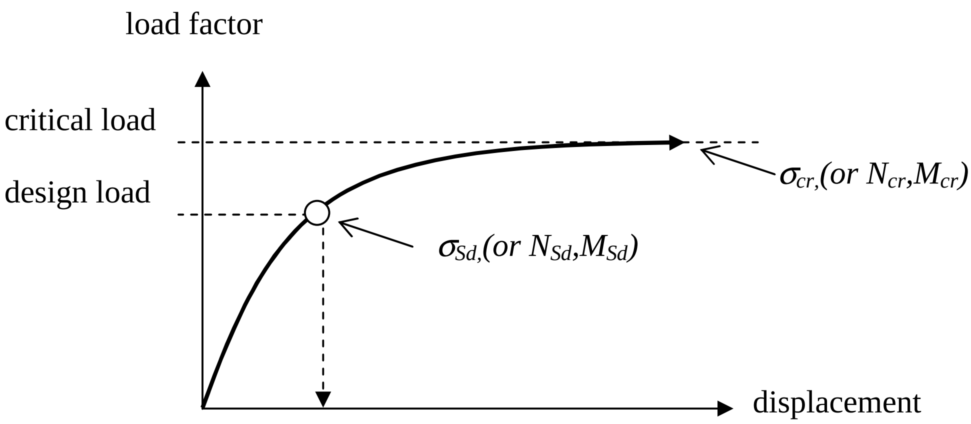
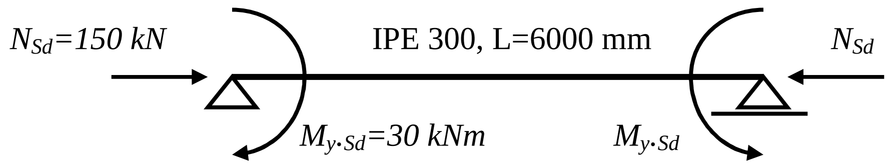


Figure 1: One parameter loading process giving design stresses as well as critical stresses.

The up-to-date design programs such as the ConSteel© published by Papp [6] apply 3D beam-column finite element analysis. Figure 2 shows the result of analysis carried out on the simple beam-column problem. The critical forces in the first row of the table were computed following the Eurocode 3 specification, which means that the N_{cr} is due to pure compression, M_{cr} is due to pure bending. Evaluating the design Eq.(1), the resistance factor is *0,987*. In the second row the critical forces were computed by 3D global stability analysis applying ConSteel© program. Using these values in design Eq.(1), the resistance factor is *1,493*. The last result shows a considerable underestimation of the resistance. The example underlines our statement at the beginning of Introduction: the Eurocode specification punishes using more accurate analysis.



elastic stability analysis	N_{cr} (kN)	M_{cr} (kNm)	design resistance Eq. (1)
Eurocode 3	343.7	83.75	**0.987**
one parameter loading*	222.6	44.52	**1.493**

* computed by FEM based on Rajasekaran's beam-column element

Figure 2: An illustrative example to show the effect of computational approaches to the elastic critical forces as well as the design resistance.

2.1 Generalised Design Equation

The problem of Level 1 and 2 formulas is encapsulated in the design philosophy: the beam-column problem is considered as an interaction between the column and the beam behaviour. According to a more progressive design concept: the beam-column problem is the basic problem, the column and beam problems are the extreme cases of the basic problem. As a consequence of the last concept, the Level 2 design formula should be defined on stress level, where the effects of the axial force and the bending moments can be summarised. However, following the above concept, a heuristic Generalised Design Equation (GDE) can be written as

$$\frac{\sigma_{N.Sd}^{c.max} + \sigma_{My.Sd}^{c.max}}{\chi_{bc}} + \sigma_{Mz.Sd}^{c.max} + \sigma_{B.Sd}^{c.max} \leq K_{bc} f_{yd} \tag{2}$$

where $\sigma^{c.max}$ denotes the components of the longitudinal design normal stress in the most compressed cross-section point of the member using full 3D second order elastic analysis and elastic cross-section properties. In Eq.(2) $_B$ denotes the warping effect due to torsion, χ_{bc} denotes the generalised buckling reduction factor, f_{yd} is the design strength (including appropriate partial safety factor) and K_{bc} is the generalised plasticity factor of the cross-section. Eq.(2) reflects the following design concept:

- the critical forces can be computed from the design load combinations applying one-parameter loading process (concept of coherent critical forces),

- the flexural and lateral torsional buckling are coherent failing modes, in other words: they are the characteristic cases of the beam-column buckling mode,
- the effects of stress resultants on the member buckling can be summarised on longitudinal normal stress level.

2.2 Generalised buckling reduction factor

The χ_{bc} generalised buckling reduction factor may be computed applying the Rondal-Maquoi formula established in Eurocode 3 (1992-1-1) where the generalised slenderness is

$$\bar{\lambda}_{bc} = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{3}$$

and where σ_{cr} is the critical normal stress in the most compressed cross-section point due to the critical load given by full 3D global elastic stability analysis. The application of the exact linear stability analysis in practical design of steel plane frames was concluded - among others - by Baptista et. al. [6]. The Eq.(2) is just the generalisation of their design concept for 3D beam-column behaviour. However, the Eurocode design formulas for the two characteristic buckling modes (flexural and the lateral torsional) are based on strong theoretical and experimental background. These knowledge base are encapsulated in the α and α_{LT} imperfection factors. The generalised imperfection factor for beam-columns can be interpolated as

$$\alpha_{bc} = \alpha + r(\alpha_{LT} - \alpha) \tag{4}$$

where r varies from 0 to 1 providing a linear interpolation between the two characteristic buckling modes

$$r = \frac{\sigma_{My.cr}}{\sigma_{My.cr} + \sigma_{N.cr}} \tag{5}$$

where $\sigma_{N.cr}$ and $\sigma_{My.cr}$ denote the coherent components of the elastic critical normal stress due to axial force and bending moment.

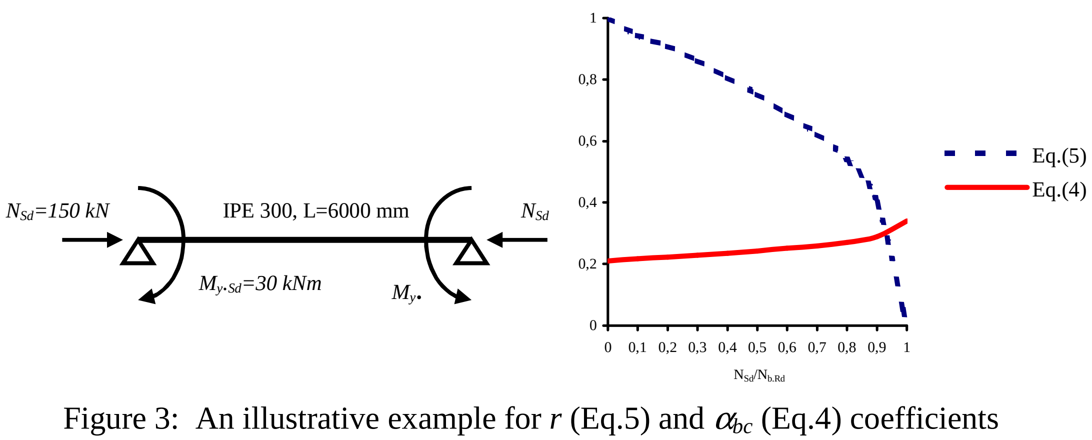


Figure 3: An illustrative example for r (Eq.5) and α_{bc} (Eq.4) coefficients

Figure 3 shows a simple member with IPE 160 cross-section where the r and the α_{bc} are dotted in terms of ratio of the design normal force N_{Sd} and the corresponding buckling resistance $N_{b.Rd}$. The non-linearity of r throws light on the convexity of the interaction between the elastic buckling modes of beam-columns, mainly about the mostly compressed and slightly bent range. The convexity depends on both the profile and the slenderness of the member. However, the design formula Eq.(2) using the reduction factor Eq.(3-5) gives back the flexural and the lateral torsional buckling formulas established by Eurocode 3.

2.3 Generalised plasticity factor K_{bc}

The tension part of all the Classes and the compressed part of Class 1 and 2 profiles may have full or partial plasticity that results higher resistance than that computed from the elastic stress distribution. When the cross-section properties are used in the buckling formula, the development and effect of plasticity depend on the actual slenderness – at higher slenderness the buckling starts in elastic range, consequently elastic section properties should be used. However, the generalised K_{bc} plasticity factor should depend firstly on the complex stress distribution in the plate segments of the actual cross-section, secondly on the slenderness and the axial force. Boissonnade et. al. [8] suggested to use a calibrated coefficient that allows the behaviour of the member to tend in a continuous way from plasticity to pure elasticity when length and axial force increasing. With some modification the generalised plasticity factor may be written as

$$K_{bc} = 1 + (K - 1)(1 - \bar{\lambda}_{bc}^{2}) \geq 1.0 \tag{6}$$

where K factor is the ratio of the plastic cross-section resistance to the elastic cross-section resistance, it is the function of the cross-section topology and the actual stress resultants. To ensure the continuity between the cross-section resistance and the global stability resistance the K should be determined from the corresponding cross-section design equation using the following iterative procedure:

step 1: compute the appropriate design stress resultants N_{Sd}, $M_{y.Sd}$, $M_{z.Sd}$ and $M_{B.Sd}$,

step 2: compute the uniform p loading factor when the factored design stress resultants $p*N_{Sd}$, $p*M_{y.Sd}$, $p*M_{z.Sd}$ and $p*M_{B.Sd}$ just satisfy the appropriate cross-section design equation,

step 3: compute the K factor as

$$K = p \frac{\dfrac{N_{Sd}}{A} + \dfrac{M_{y.Sd}}{W_{y.el}} + \dfrac{M_{z.Sd}}{W_{z.el}} + \dfrac{B}{W_{\omega.el}}}{f_{yd}} \tag{12}$$

For example it is easy to check that Eq.(6) will give $K = W_{y.pl}/W_{y.el}$ for hot rolled Class 1 H sections bent about their major axis. Figure 3 shows the K factor for the

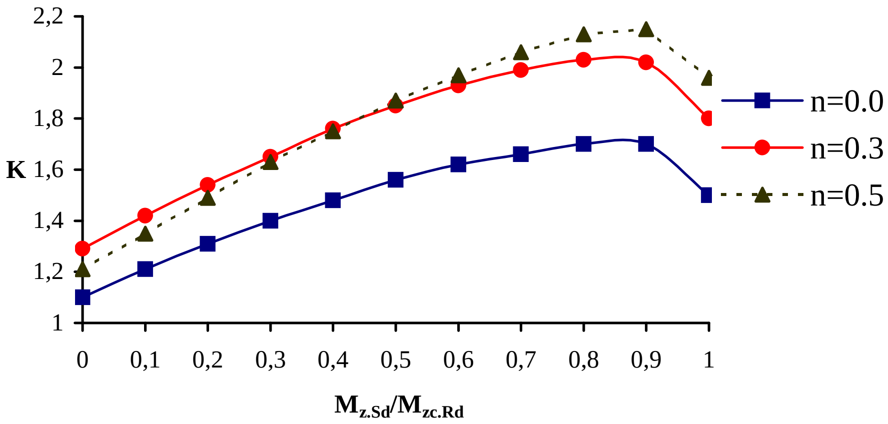


Figure 3: Generalised plasticity factor *K* for IPE 300 profile according to design equation for biaxial bending and compression ($n=N_{Sd}/N_{pl.Rd}$) applying Eq.5.35 of Eurocode 3 (1992-1-1).

compressed and biaxially bended IPE 300 (S 235) profile using the design Eq.(5.35) of Eurocode 3 (1992-1-1) as

$$\left[\frac{p*M_{y.Sd}}{M_{Ny.Rd}}\right]^2+\left[\frac{p*M_{z.Sd}}{M_{Nz.Rd}}\right]^{max(5n,1)}=1 \qquad (13)$$

where the plastic resistance moments are reduced by the effect of the actual normal and shear forces according to Eq.(5.25-26) of Eurocode 3 (1992-1-1).

2.4 Calibration by numerical simulation

The verification of Eq.(2) needs extended and time consumable work. The column-buckling problem has strongly established background based on tests and numerical simulations. The beam-buckling problem has also test background. The beam-column buckling problem has pure background - therefore the numerical simulation seems to be the adequate tool for the verification. The simulation model may be calibrated by test results. In the framework of the present research Eq.(2) was verified for hot rolled Class 1 and Class 2 I sections using thin-walled beam-column finite element model and a Monte Carlo method. The details of the verification were published by Szalai and Papp [2]. The concept of this verification was the following:

step 1: use the distribution functions of the model parameters established by Strating and Vos [9] for IPE 160 profile,
step 2: repeat the simulation of column buckling curve simulated by Strating and Vos [9],
step 3: establish the *design distribution functions* of model parameters,

step 4: compare the simulated curves to the column and lateral torsional buckling curves of Eurocode 3 (1992-1-1),

step 5: use the calibrated simulation model applying the design distribution functions to simulate beam-column buckling problems and compare the simulated curves to the Eq.(2).

Figure 4 shows the typical results of the above verification.

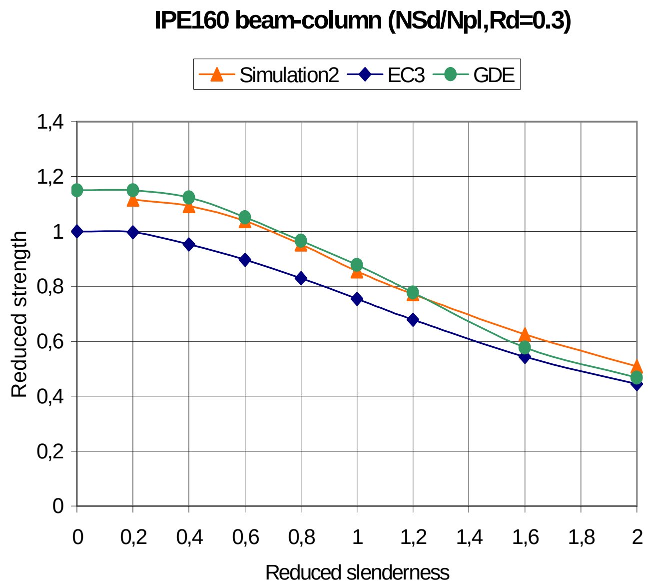


Figure 4: Simulated beam-column buckling curves for IPE 160 profile

2.5 A practical application: the ConSteel© program

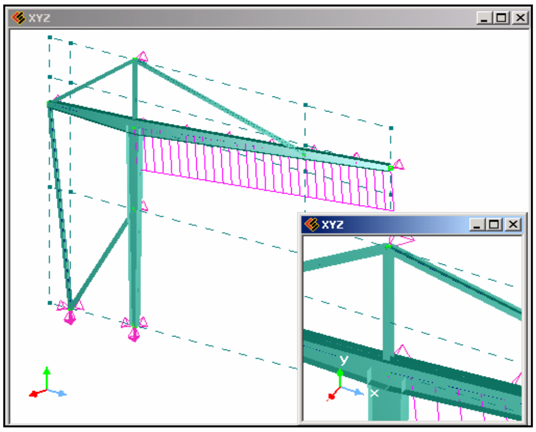
In the framework of the ConSteel© project a practical application was developed using the above Level 2 design method. The ConSteel© program was based on a 3D nonlinear thin-walled beam-column finite element method [6] and a unified object oriented cross-section definition published by Papp and *et.al.* [10]. The procedure neglected the contradiction of the Eurocode 3 formula: exact second order elastic analysis is carried out during the analysis, and the resistance of members including stability behaviour is evaluated during cross-section check. By this approach the design procedure is fully automatic. In case of structures where the buckling mode is uniform for the whole model, the design based on the first buckling mode is correct – the critical stress in Eq.(3) can be computed from the first critical load factor of the whole model (see Figure 1). In case of structures where more real buckling modes can be computed, the method may underestimate the resistance of some members, which belong to the higher buckling modes. The principle question of the design philosophy whether

- let us use more buckling modes for design of members, or
- let us use the first buckling mode and design uniformly stiff structures.

step 1

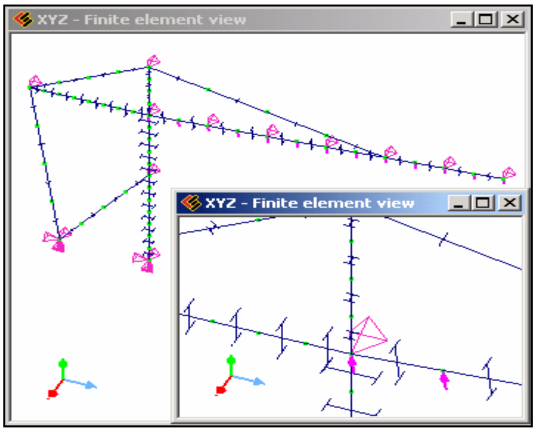
Modelling the designed structure, or a part of it, which can be designed as an individual. Modelling is carried out on CAD oriented 3D user interface. Main properties of the present model:

- tapered column and beam
- thin-walled webs
- hot-rolled members



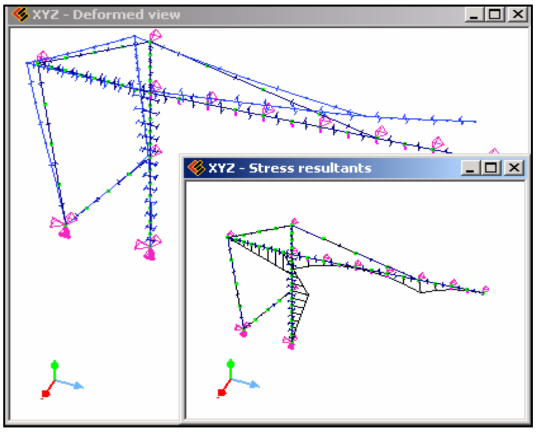
step 2

Automatic decomposition of the CAD model into appropriate thin-walled beam-column finite element model.



step 3

Automatic and exact second order elastic analysis by 3D thin-walled beam-column finite element method to compute the design stress and the critical stress database.



step 4

Automatic evaluation of the Generalised Design Equation to the whole structural model. Colour-mapped visualisation of the comprehensive result, and table based detailing of the design parameters.

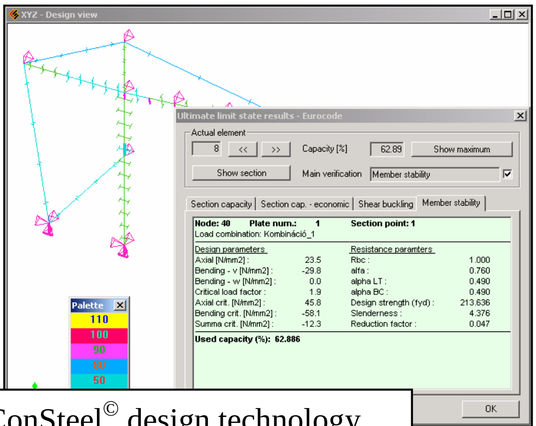


Figure 5: Main steps of the ConSteel© design technology

The ConSteel© program supports the last approach: the so-called *stability sensitivity analysis* function helps the user to design structures where the members have approximately the same critical load factor. It is not in order to make the ConSteel© concept being valid; it is to make the design being economic and quality. The design example in Figure 5 illustrates the main steps of the ConSteel© design technology.

3 Level 2 formula using parallel simulation

The numerical simulation method based on any appropriate finite element (or strip) method and appropriate design probabilistic model parameters can be applied directly to predict the design resistance of the structural members. Theoretically, the method can be extended to complex structures but there are at least two reasons why using it in the engineering practice is hardly possible:

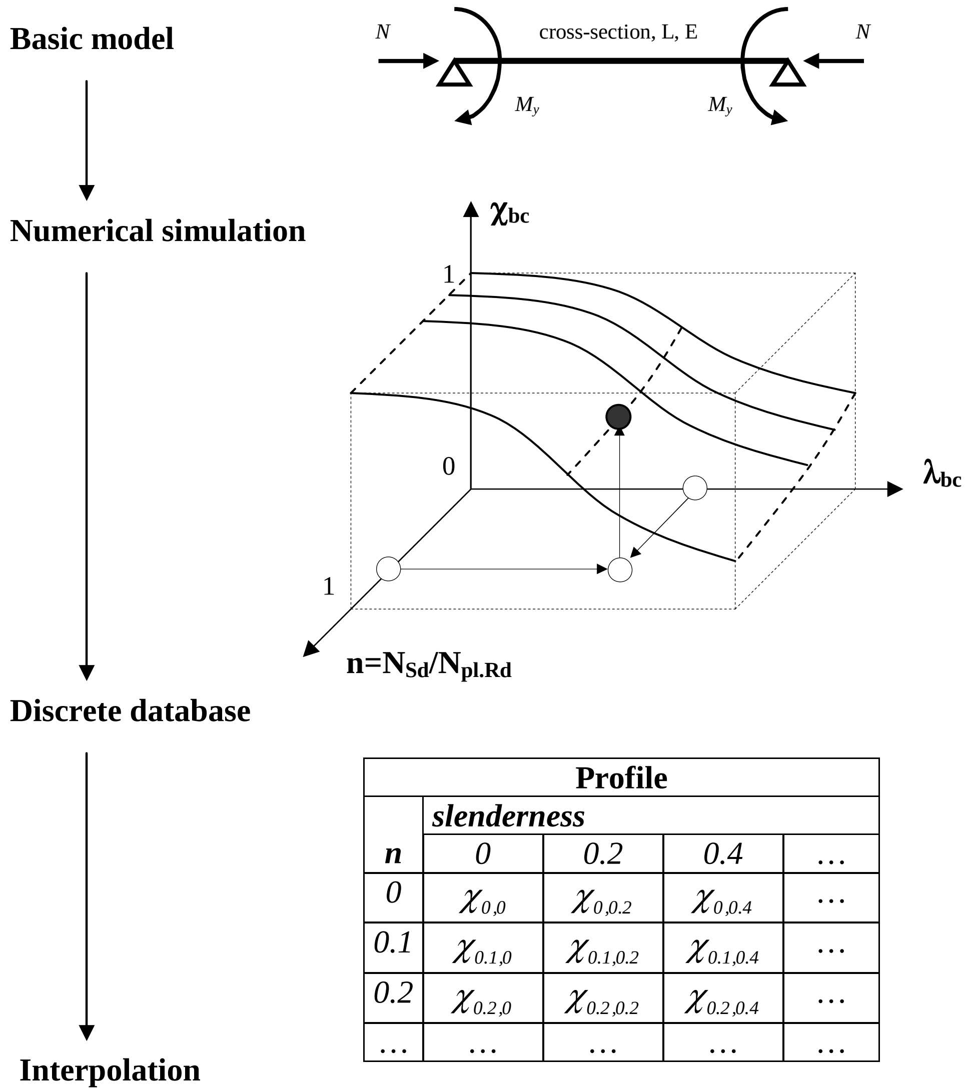
- the numerical simulation technique is time consumable, and
- in more complex cases the use of technique requires strong theoretical knowledge of theory of analysis and design.

The simulation technique in case of simple beam-columns is a well-established automatic procedure, for details see [3]. In the advanced Level 2 formula the design process has the following steps:

step 1: compute the generalised slenderness using appropriate 3D global elastic stability analysis, see Eq.(3),
step 2: approximate the generalised imperfection factor using the α and α_{LT} standard design database and simple interpolation, see Eq.(4),
step 3: compute the generalised reduction factor using the Rondal-Maquoi formula,
step 4: evaluate the generalised design equation, see Eq.(2).

The weakest point of Level 1 and 2 formulas is the interaction relationship between the ultimate values of the axial force and the bending moments, especially the interpolation of the generalised imperfection factor in the above Level 2 formula. In the procedure, which uses direct simulation technique the *step 2* and *step 3* can be replaced by directly computed reduction factors that are included in the basic database of the design system. In other words, the generalised reduction factor of the simple beam-column model can be simulated in the function of the discrete values of slenderness and reduced normal force. In this case the reduction factor is given discretely on the domain of reduced slenderness and normal force, as Figure 6 shows. According to this technique the steps of the procedure are the following:

step 1: compute the normal force factor ($n = N_{Sd} / N_{pl.Rd}$) and the generalised slenderness ($\bar{\lambda}_{bc}$) using proper 3D global elastic analysis,
step 2: take the generalised reduction factor stored in the database of the system,
step 3: evaluate the resistance using the generalised design equation.



Profile				
	slenderness			
n	*0*	*0.2*	*0.4*	…
0	$\chi_{0,0}$	$\chi_{0,0.2}$	$\chi_{0,0.4}$	…
0.1	$\chi_{0.1,0}$	$\chi_{0.1,0.2}$	$\chi_{0.1,0.4}$	…
0.2	$\chi_{0.2,0}$	$\chi_{0.2,0.2}$	$\chi_{0.2,0.4}$	…
…	…	…	…	…

Figure 6: Database of generalised beam-column reduction factor

From programming technology point of view the question is that, when the reduction factor database is created. In case of standard profiles the program developers can create the database as a system component. In case of user defined cross-section the appropriate database should be created in run-time. The simulator program is the part of the design software and it can be run in the background just the cross-section has been defined. The simulation may be run parallel to the modelling. Unfortunately, usage of this procedure is theoretical when common PCs are used, because of the considerable running time. The ConSteel© design system was developed for distributed computation. Networks contain an application server with high performances in computational time. While the client is working on the model, the simulation on the user-defined cross-sections can be run on the application server. Normally, by the time the model has been designed, the reduction factor database is available.

4 Conclusion

The paper shows the conservatism of the Eurocode 3 interaction design formulas related to beam-columns. It suggest applying an advanced Level 2 formula, which keeps the well-established knowledge base of the code but it is CAD oriented and automatic. On this base advanced and high performance design software can be developed for the engineering practice.

Acknowledgement

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