Dmitry Svirko

**DESIGN OF TRACKING CONTROLLERS FOR FLEXIBLE JOINT ROBOTS**

Ph. D thesis book

Scientific advisor
Éva Gyurkovics, Ph.D.

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Introduction

The desire for higher performance from robot manipulators has spurred research in a number of areas: mechanics, sensors and actuators, control, computer architectures and artificial intelligence, to name a few. In the area of control, the trend of the past decade has been to apply increasingly sophisticated tools from nonlinear control theory with the goal of developing controllers that achieve accurate, high speed tracking with low sensitivity to modeling errors, disturbances, payloads, sensor noise and so on. Advances in microprocessor technology have made the implementation of complicated nonlinear control algorithm feasible from a practical standpoint.

At the same time, achieving high performance by applying advanced control techniques requires an increased understanding of the dynamics of robot manipulators. To borrow from the language of linear control theory, as the bandwidth of the control system increase, dynamic effects that previously were beyond the frequency range of interest, now must be considered in the control design.

For robot manipulators this means that a dynamic model of a robot as an ideal chain of coupled rigid bodies is frequently not adequate for the design of controllers to achieve high performance. In the early 1980’s the problem of joint flexibility began to be recognized as an important factor limiting robot cycle time.

Problem Statement

This thesis is devoted to the design of a tracking control for flexible joint robots. Assuming that the motion of the actuator rotors may be considered as pure rotations with respect to an inertial frame, the model of an elastic joint robot (see Figure 1) is given by

\[ B_1(q_1) \ddot{q}_1 + C_1(q_1, \dot{q}_1) \dot{q}_1 + K(q_1 - q_2) + h(q_1) = 0, \]

\[ B_3 \ddot{q}_2 - K(q_1 - q_2) = u, \]

(1)

Here \( q_1 \) and \( q_2 \) are the \( n \times 1 \) vectors of the link and rotor relative displacements, respectively, \( B_1 \) is the link inertia matrix, \( B_3 \) is a constant diagonal matrix depending on the actuator inertias, \( h(q_1) \) takes gravitational forces into account. \( u \) represents the input generalized force from the actuators.
$K$ is the diagonal matrix of joint stiffness coefficients

$$K = \text{diag}[k_1, \ldots, k_n],$$

where $k_i$ is the elastic constant of the $i$-th joint. The centrifugal and Coriolis terms are related to the link inertia matrix by

$$C_i(q_1, \dot{q}_1) \ddot{q}_1 = \dot{B}_i(q_1) \dot{q}_1 - \frac{1}{2} \dot{\hat{q}}_1 B_i(q_1) \dot{\hat{q}}_1$$

and $\dot{B}_i - 2C_i$ is a skew-symmetric matrix for a suitable definition of $C_i$. For $B_1$ and $C_1$ there exist positive constants such that

$$B_{1m} \leq \lambda_m(B_1(q_1)) \leq \|B_1(q_1)\| \leq \lambda_M(B_1(q_1)) \leq B_{1M}, \quad \forall q_1 \in R^n,$$

$$\|C_1(q_1, \dot{q}_1)\| \leq C_{1M} \|\dot{q}_1\|, \quad \forall q_1, \dot{q}_1 \in R^n.$$  

Matrix $C_1$ has the property

$$C_i(q_1, y)z = C_i(q_1, z)y$$

where $y$ and $z$ are arbitrary $n \times 1$ vectors.

It is important to note that the control of flexible joint robots by state feedback requires the knowledge of four state variables $(q_1, \dot{q}_1, q_2, \dot{q}_2)$ for each of the joints, but the measurement of all variables is too expensive if not impossible. Most present robot designs are such that only the actuator variables are
measured. Therefore, observers that reconstruct the whole state vector by using a reduced set of measurements are needed. This problem has attracted considerable attention in the literature. They came to the upshot that on the one hand the measurement of link speed \((q_1)\) for each of the joints is not possible, on the other hand it is impossible to reconstruct the whole state vector using only the measurements of the rotor position and rotor speed \((q_2, \dot{q}_2)\). That’s why in the literature the link positions \((q_1)\) are assumed to be measurable. But the direct measurement by sensors is practically impossible too. **If we assume link positions are available, then we must show how we can get these data in case of flexible joint robots.** We have not found any result concerning this question in the literature.

Let \(q_c: [0, \infty) \rightarrow \mathbb{R}^n\) be a given continuous and bounded function with continuous and bounded derivatives up to the order 4, which is referred to as the desired reference trajectory for the link position. We choose as state variables and as desired state variables \(x^T = (q^T_1, \dot{q}^T_1, q^T_2, \dot{q}^T_2)\) and \(x_d = q_d\) respectively. An observer is a dynamic system

\[
\dot{\hat{x}} = \hat{f}(\hat{x}, y, u),
\]

such that for any initial conditions \(x(t_0) = x_0, \quad \hat{x}(t_0) = \hat{x}_0\)

\[
\lim_{t \to \infty} (\hat{x}(t) - x(t)) = 0,
\]

It is known that having an observer that asymptotically estimates the robot state variables does not guarantee the stability of the control loop when the observer is used in conjunction with a state feedback controller. **Therefore, the stability of the whole control system (controller plus observer) should be proved.**

The problem is to design a dynamic output feedback controller \(u\)

\[
\dot{\hat{x}} = f(y, \hat{x}, x_d), \quad \hat{x} \in \mathbb{R}^q, \quad u = \eta(y, \hat{x}, x_d),
\]

for which the corresponding solution of the closed loop system (1) satisfies the conditions

\[
\lim_{t \to \infty} (q_1(t) - q_d(t)) = 0, \quad \lim_{t \to \infty} (\dot{q}_1(t) - \dot{q}_d(t)) = 0, \quad \lim_{t \to \infty} (\hat{x}(t) - x(t)) = 0.
\]
Previous Results

This problem has attracted considerable attention in the literature. Beginning in the early 1980’s the problem of joint flexibility began to be recognized as an important factor limiting robot cycle time. First researchers of the flexible joint robot problem transform a nonlinear system to a linear system. The properties of this linear system allow the design of control laws, which are robust to parametric uncertainty. But robustness to parametric uncertainty is achieved only if all of the variables $q_1, \dot{q}_1, \ddot{q}_1, q_2, \dot{q}_2$ are available for feedback. Eventually this is why researchers have given up the feedback linearizing control technique for the flexible joint system.

The 1980’s is the period of analytical investigation of the design of composite control (variants of composite control: integral manifold control, adaptive control) based on a reformulation of the dynamic model of the flexible joint robots as a singularly perturbed system. But this technique has not been developing since the 90’s. On the one hand, the application of composite control have reliable result for designing a state feedback controller, but on the other hand, there is no result for the design of dynamic output feedback controller with the above technique. The robotics literature of the last decade does not contain any advance of the composite control design.

In the early 1990’s the back-stepping method was applied for the control design. This method is based on a reformulation of the dynamic model of the flexible joint robots as the cascade connection of four systems, in which the output of each system is connected to the input of the successive system. It gives the possibility to design a dynamic partial state feedback controller based on the measured states and on a nonlinear observer of the unmeasured variables.

The best result of this control algorithms approach was a semiglobal (in the sense that the region of convergence may be arbitrarily enlarged) dynamic output feedback controller which requires only link positions available from measurements and guarantees the asymptotic tracking of any bounded trajectory with bounded time derivatives up to order four. The dynamic part of the controller consists of a nonlinear observer for the unmeasured variables. The drawbacks of this controller is the following: it has been designed directly in terms of the estimated variables, therefore the controller parameters depend on the observer parameters and numerical values of controller are too large to apply it in practice. The speed of convergence is not guaranteed, so the speed of convergence may be too slow. And finally the authors assumed that link positions are measurable, but any method to obtain link positions data has not been presented (the direct measurement by sensors is practically impossible).
New Scientific Results

The robot has to be equipped with motor position sensors on each motor shaft and three acceleration sensors on the end effector. Since most robots are equipped with sensors on the motor shaft, the measurement of motor positions seems to be a reasonable assumption. Three acceleration sensors disposed on the robot end effector give ability to obtain link positions. So the positions of all links and motor rotors are available and the \( n \times 1 \) output vectors \( y_1 \) and \( y_2 \) are given by

\[ y_1 = q_1, \quad y_2 = q_2. \]

Thesis 1: The method to obtain link positions data by using three acceleration sensors disposed on the robot end effectors is presented.

To obtain link positions we apply one of the well-known methods of inverse geometrical problem, which requires to know the position and the orientation of the end effector for current time moment. Using three acceleration sensors disposed on the robot end effectors we obtain the positions of three points. The position and the orientation of the end effector for the initial time moment are known. We calculate the transformation of the end effector (using positions of the three points for initial and current time moments) and then obtain position and orientation of the end effector for current time moment. The transformation of the end effector is solved by minimizing a least squares error criterion. Further, we assume that the link positions are available. The method is illustrated by an example.

Published in [1].

Thesis 2: An observer for flexible joint robots has been provided. The proposed observer requires that the link and motor positions are available. It estimates the velocity of each link and motor rotor, and it has semiglobally exponentially stable error dynamics.

The region of convergence may be arbitrarily enlarged by increasing some observer gains (i.e. semiglobal exponential stability is proven). The speed of convergence is guaranteed. The observer is needed to design the control. The results are illustrated by simulation examples.

Published in [5].
Thesis 3: A partial state feedback controller for flexible joint robots has been proposed applying the back-stepping technique. The controller requires that the link and motor positions are available and guarantees that the closed-loop system is semiglobally exponentially stable for any given sufficiently smooth bounded reference trajectory.

The region of convergence may be arbitrarily enlarged by increasing some observer and controller gains (i.e. semiglobal exponential stability is proven). The speed of convergence is guaranteed. The dynamic part of the proposed controller consists of an observer (see the previous thesis). The results are illustrated by simulation examples.

Published in [4], [6].

The above controller and observer are designed simultaneously, therefore the controller parameters may be too large. Lately, several results have been published about the applicability of the separation principle for the stabilization of certain classes of nonlinear time-invariant systems about an equilibrium state. These results have motivated us to construct a dynamic partial state feedback controller based on the separation principle for the tracking problem, which – in contrast to the above mentioned – can only be described by a time-variant nonlinear system.

Thesis 4: A controller for flexible joint robots based on separation principle has been proposed. The validity of the separation principle for this special class of output feedback stabilization problem has been proved. The controller requires that the link and motor positions are available and guarantees that the closed-loop system is semiglobally exponentially stable for any given sufficiently smooth bounded reference trajectory.

The advantage of this approach is in the possibility of designing the controller independently of the observer. A globally stabilizing state-feedback controller is designed. Semiglobally stable nonlinear observer is constructed like as in [5]. Then the parameters of the observer have to be fitted accordingly to guarantee that closed loop system is semiglobally exponentially stable. Simulation examples illustrated the effectiveness of the proposed approach: numerical values of controller are small enough to put it into practice. The region of convergence may be arbitrarily enlarged by increasing some observer gains (i.e. semiglobal exponential stability is proven). The speed of convergence is guaranteed.

Published in [2], [3].
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Author’s Publications

2003

2002

2001
**Selected Publications on Topics Related to Dissertation**


