ANALYTICAL MODELLING OF TWIST DRILL BITS

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Abstract. Twist drill bits became one of the most important tool of the manufacturing industry for the last one and a half century. Its geometry and manufacturing instructions have been well developed and are available ever since. None the less the exact mathematical modelling with differential geometry has been scarce. Its characteristic shape consists of well definable geometric objects of helicoidal and conical surfaces. In the paper the exact mathematical model of twist drill bits are discussed with the use of homogeneous matrix transformations.

Keywords: twist drill bits, working back angle, homogeneous transformation

1. ANALYTICAL MODELL OF TWIST RILL BIT

To analyse the twist drill bit analytically firstly a geometrical model have to be made in a suitable environment[1]. To form the model of the drill first an arbitrary cross section of the half drill shaft has to be defined. This can be made because the drill is centre symmetric. The cross section will be perpendicular to the Z axis of the drill (in the X.Y plane) at a general section containing only the flutes since for the analysis the shank section is not important. For the task a piecewise function[2] is best suited that describes the half of the section of the drill because the section is centre symmetric. The section is described by the following function with "u" parameter of arc length (Eq. (1)):

$$x := \begin{cases} u & u < v \\ v & u < v+h \\ 2v+h-u & u < v+h+t \\ R\cos\left(-\varepsilon - \dfrac{u-v-h-t}{R}\right) & u < v+h+t+L \\ \rho\sin\left(\dfrac{u-v-h-t-L}{\rho}\right) & u < v+h+t+L+l \\ 0 & \mathit{otherwise} \end{cases} \qquad y := \begin{cases} r & u < v \\ v+r-u & u < v+h \\ r-h & u < v+h+t \\ R\sin\left(-\varepsilon - \dfrac{u-v-h-t}{R}\right) & u < v+h+t+L \\ -r-\rho-\rho\cos\left(\dfrac{u-v-h-t-L}{\rho}\right) & u < v+h+t+L+l \\ r & \mathit{otherwise} \end{cases} \qquad (1)$$

[1] For this environment MapleV 10 symbolic mathematical system is chosen.
[2] **piecewise**: with respect to a number of discrete intervals, sets, or pieces, the *piecewise* continuous functions is one of the special function of the Maple software

Where: d is the nominal diameter of the drill shaft, r, is the radius of the core, h is the lip relief width and t is its radial height, R is the body diameter ϱ. is the radius of the semicircular flute ε is an auxiliary angle for calculating the arc length L for the section, l is the arc length of the flute, S is the overall diameter of the shaft section., P is the thread gradient of the π/3 helix angled flute.

The values for the parameters were calculated by using the following formulas (Eq.(2)..(10)):

$$v := \sqrt{\left(\frac{d}{2}\right)^2 - r^2} \tag{2}$$

$$R := \sqrt{(v-t)^2 + (r-h)^2} \tag{3}$$

$$\varepsilon := \arctan\left(\frac{h-r}{v-t}\right) \tag{4}$$

$$L := R\left(\frac{\pi}{2} - \varepsilon\right) \tag{5}$$

$$\rho := \frac{R-r}{2} \tag{6}$$

$$l := \rho\,\pi \tag{7}$$

$$S := v + h + t + L + \rho\,\pi \tag{8}$$

$$V := v + h + t \tag{9}$$

$$P := d\,\pi\,\tan(\theta) \tag{10}$$

For these parameters the following values were used to construct the parametric function of the drill cross section:

$$d := 80;\ r := 6;\ h := 2;\ t := 3$$

With these values the section of the drill can be assembled (See Fig.1):

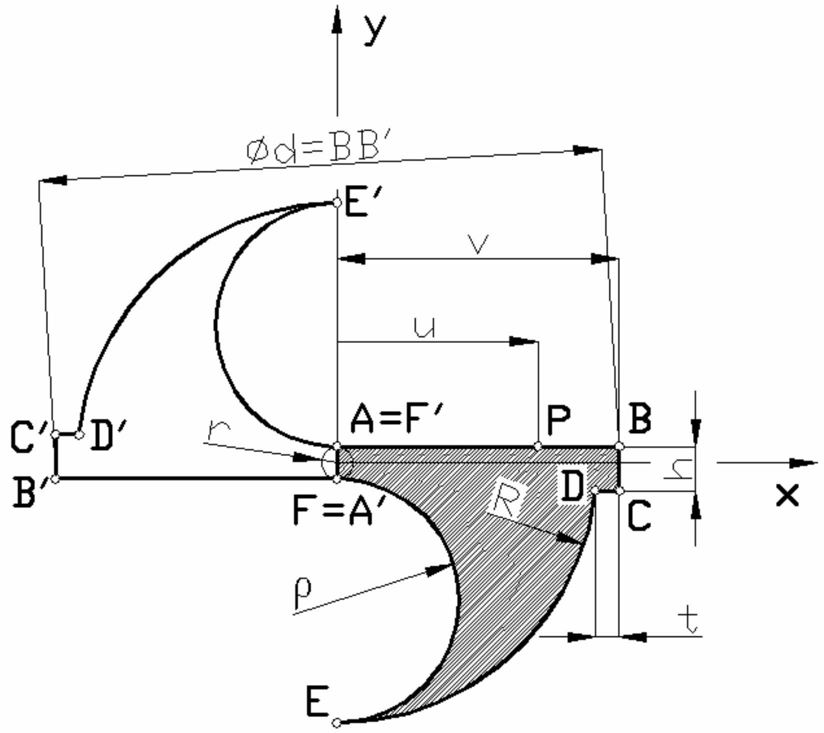

Fig. 1. Cross Section of twist drill body, parameterised by the arc length

The x(u), y(u) coordinates of the piecewise contour arc length function can be pictured as the first two coordinates of a four component homogeneous vector function $k_1=k_1(u)$:

$$k_1 := [x, y, 0, 1] \tag{11}$$

On this formula the homogeneous transformations can then be easily applied making the operations and transformations very simple.

To make the helical part of the drill the section has to undergo such a transformation (M1) that it rotates around the z axis with a defined angel (ϕ) while it also translates along the z axis with a definite distance (P is the thread lead by θ helix angle of the flute). The formula of the homogeneous transformation matrix that performs this task is given by:

$$M_1 := \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 & 0 \\ \sin(\phi) & \cos(\phi) & 0 & 0 \\ 0 & 0 & 1 & \frac{P\,\phi}{2\,\pi} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

By multiplying the previously defined k_1 with M_1 from the left hand side the desired helical section is gained, thus gaining the vector function w_1 in the function of the parameter "u" yield:

$$w_1 = M_1\, k_1 \tag{13}$$

The cutting edge of the drill that is formed by the intersection of the two centre symmetric helical part and the outer superficies of two intersecting cones flank surfaces (see Fig. 2). The sharpening of the edge of the bit is illustrated on Fig. 3.

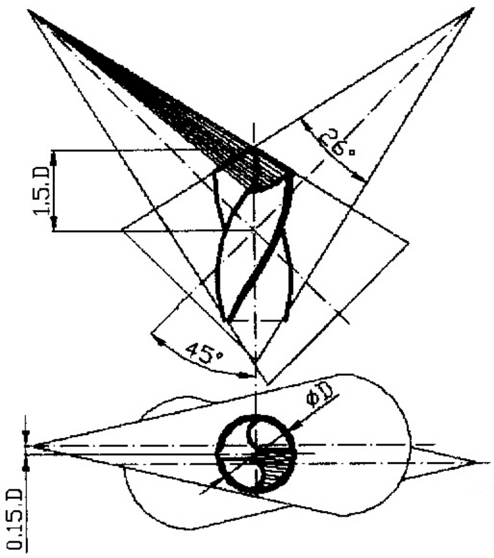

Fig. 2. **The geometric deduction of the cutting edge of the twist drill**

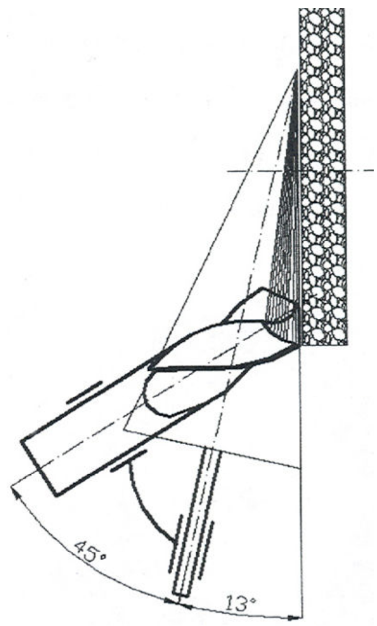

Fig. 3. **The sharpening of the cutting edge of the twist drill bit according to conical flank surfaces.**

The straight line of the cone in the X-Z plane is defined by the following equation:

$$Z := H + X \tan\left(\frac{\pi}{2} + \alpha\right) \tag{14}$$

where: Z is the equation of the cone in the function of x, h is the height of the cone, and α is the half angle of the cone and $q := .15\ d;\ H := 4.5\ d$.

The homogeneous coordinates of surface line of the cone is:

$$k_2 := [X, 0, Z, 1] \tag{15}$$

Line (15) is first rotated around the X axes with the angle ψ. In the second step the cone is translated with the distance of

$$-h.\tan(\alpha).\cos(\frac{\pi}{4}) + R \tag{16}$$

in the X direction and

$$q = 0.15\ d \tag{17}$$

in the Y direction, and finally

$$-h.\tan(\alpha).\sin(\frac{\pi}{4}) \tag{18}$$

in the direction of the Z axes.

In the third step the translated cone is rotated around the Y-axis with the angle π/4 and lastly it is rotated around the Z-axis with the angle α to form the precise cutting edge. The following matrices perform the desired transformations:

M_2 is the transformation matrix of the rotation around the X-axis. M_3 -performs the necessary translations in the directions of X, Y and Z axes and M_4 -the rotation around the Y axis. Eq. (19):

$$M_2 := \begin{bmatrix} 1 & 0 & 0 & -h\tan(\alpha)\cos\left(\frac{\pi}{4}\right) + R \\ 0 & \cos(\psi) & -\sin(\psi) & q \\ 0 & \sin(\psi) & \cos(\psi) & -h\tan(\alpha)\sin\left(\frac{\pi}{4}\right) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

M_4-performs the rotation around the Y axis and M_5-the rotation around the z axis Eq. (20):

$$M_4 := \begin{bmatrix} \cos\left(\frac{\pi}{4}\right) & 0 & \sin\left(\frac{\pi}{4}\right) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\left(\frac{\pi}{4}\right) & 0 & \cos\left(\frac{\pi}{4}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_5 := \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

The cutting cone at its proper place can be acquired by multiplying the cone from the left hand side by the transformation matrices in the prescribed order:

$$w_2 = M_5\, M_4\, M_2\, K_2 \tag{21}$$

Illustrating the helical part and the conical cut of the drill with 3D Maple plot, Fig. 4:

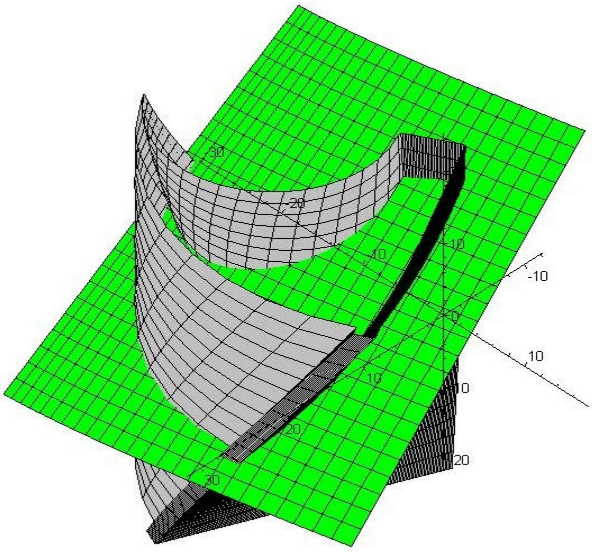

Fig. 4. Cutting the helical part of the drill shaft with conical surface

2. INTERSECTION LINES OF SURFACES OF TWIST DRILL

The intersection of the two geometrical entities can be calculated where the two surfaces have equal points. Using the equation the points can be calculated by the following system of equations where the difference of points of the two surfaces are zero:

$$w_{1_i} - w_{2_i} = 0 \tag{22}$$

To complete the model the last process is to mirror the whole part on the z axis and combine the two parts. From this process the analytical curve of the cutting edge of the drill can be obtained. This can be seen on Fig. 5:

The homogenous vector equation of the body surface of drill is defined by

$$w_1 = [\, x(u,\varphi)\,,\; y(u,\varphi),\;\; z(u,\varphi),\;\; 1\,] \tag{23}$$

and the right flank cone surface is given by Eq. (9) in the form of

$$w_2 = [\, \xi(X_1,\psi_1)\,,\; \eta(X_1,\psi_1)\,,\; \zeta(X_1,\psi_1)\,,\; 1\,] \tag{24}$$

the point co-ordinates of the intersection lines are obtained to an arbitrary value of the arc length parameter u = U by solving the following system of equations:

$$x(U,\varphi) - \xi(X_1,\psi_1) = 0 \tag{25}$$

$$y(U,\varphi) - \eta(X_1,\psi_1) = 0 \tag{26}$$

$$z(U,\varphi) - \zeta(X_1,\psi_1) = 0 \tag{27}$$

by numerical approximation of φ, X_1 and ψ_1 values. The contours of the right and left side flank surfaces can be seen on Fig. 5.

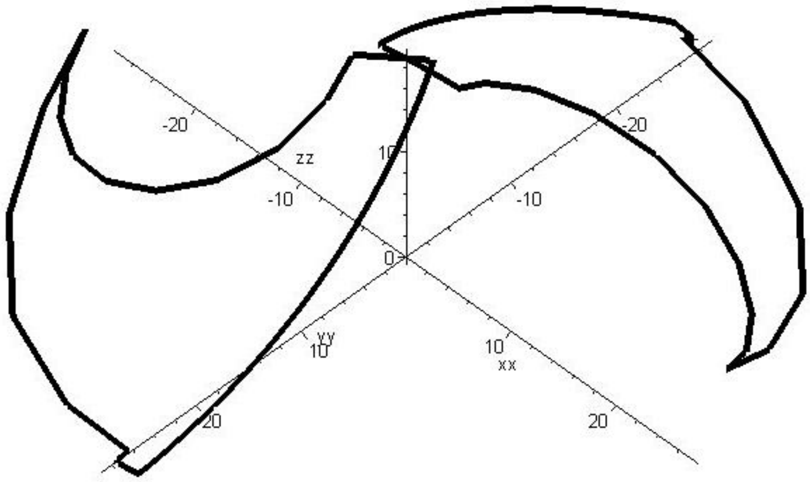

Fig. 5. Analytical curve of the cutting edge of the drill, without the chisel edge.

The equation of the left flank cone surface is formalised by:

$$w_3 = [\ -\xi(X_2,\psi_2)\ ,\ -\ \eta(X_2,\psi_2)\ ,\ \ \zeta(X_2,\psi_2)\ ,\ \ 1\ ] \tag{28}$$

The chisel edge can be deduced from the intersection of the two conical flank surfaces. The chisel edge can be calculated by making the points of the two cones equal; this will be the function of the intersecting line. The point co-ordinates of the chisel edge are obtained to arbitrary values of parameter $\mathbf{X} = X_1$ from solving the following equation system

$$\xi(\mathbf{X},\psi_1) + \xi(X_2,\psi_2) = 0 \tag{29}$$

$$\eta(\mathbf{X},\psi_1) + \eta(X_2,\psi_2) = 0 \tag{30}$$

$$\zeta(\mathbf{X},\psi_1) - \zeta(X_2,\psi_2) = 0 \tag{31}$$

by numerical approximation of ψ_1, X_2 and ψ_2 values. The intersection line can be seen of Fig. 6.

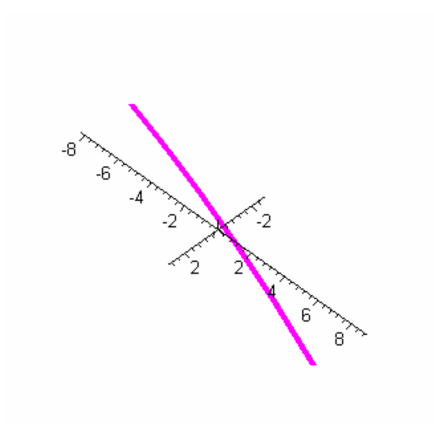

Fig. 6. The curve of the chisel edge

By exporting the points in DXF format from Maple V10 to a CAD system the 3D model of the drill can be created. The CAD model of the drill created by this method can be seen on Fig. 7.

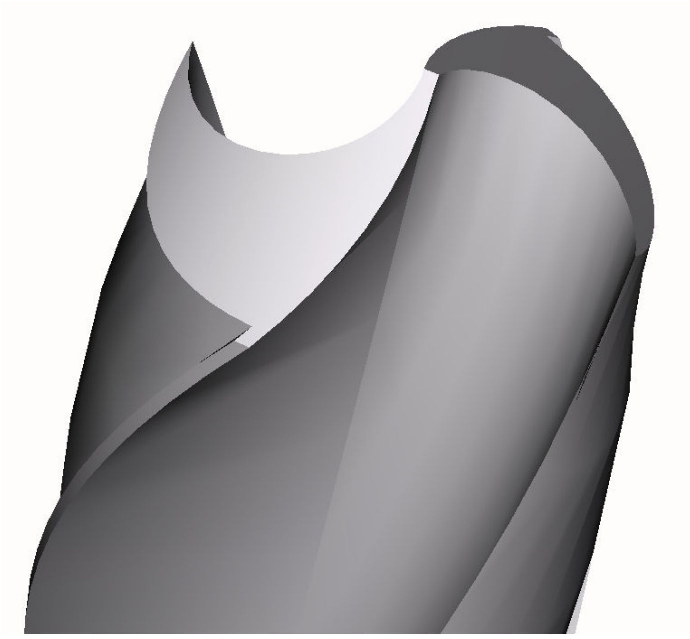

Fig. 7. CAD model of the drill from the analytical points

3. ANALYSIS OF ACTIVE CLEARANCE ANGLE

A clearance angle is necessary for the cutting edge to work freely without unnecessary rubbing. Cutting behaviours of the twist drill can now be done analytically using the previously described geometric model. The aspect that will be looked at in the final part of the paper is the changing of the dynamic rack angel of the drill in the function of the radial parameter of the cutting edge. To begin the analysis the drill bit is placed in a reference frame in a way that the axis of the bit is collinear to the z-axis of the coordinate system.

Analysis of cutting parameters of the drill can now be done analytically using the previously described twist drill model. The aspect that will be looked at in the final part of the paper is the changing of the dynamic back angel of the drill in the function of the radial parameter of the cutting edge.

To begin the analysis the drill bit is placed in a reference frame in a way that the axis of the bit is collinear to the z axis of the coordinate system.

The motion of the drill consists of to parts, the main cutting motion a rotation around its axis (Z axis) and a feed motion parallel to the z axis. In a coordinate way the two motions can be described the following way Eq.(32), and Eq. (33:

$$\omega = [\,0,\,0,\,\omega] \tag{32}$$

$$f = [\,0,\,0,\,f\,] \tag{33}$$

Where ω is the angular velocity in [1⁄s], and *Feed* is the feed rate in [m⁄s].

The cutting speed can be calculated using these data and the geometry of the cutting edge of the drill.

The angular cutting speed can be calculated by the vector product of the angular velocity and the radial vector points of the cutting edge.

$$\underline{v}_1 = \underline{\omega} \times \underline{r}_i \tag{34}$$

where $\underline{v}_1$ is the radial cutting speed and $\underline{r}_i$ are the space vector points of the cutting edge.

The resultant cutting speed is the sum of the feed rate and the angular cutting speed Eq (35).

$$\underline{v}_2 = \underline{v}_1 + \underline{f} \tag{35}$$

For further calculations the unit vector of the cutting speed have to be determined. This is done by dividing the each component of the cutting speed vector by its length:

$$v_3 = \frac{v_2}{|v_2|} \tag{36}$$

From the two parameter analytical surface of the cone, the normal vectors of every point of the surface can be calculated by:

The normal vectors of the surface of the back edge can be determined by:

$$n_1 = \frac{\partial w_2}{\partial X} \times \frac{\partial w_2}{\partial \psi} \tag{37}$$

here: $\frac{\partial w_2}{\partial X}$ and $\frac{\partial w_2}{\partial \Psi}$ are the partial derivatives of the surface according to its two parameters. Taking the unit vector of the surface normals by dividing its components by their length we get:

$$n_2 = \frac{n_1}{|n_1|} \tag{38}$$

Using the scalar product of the two vectors (unit cutting speed and unit surface normals)

$$\cos(\beta) = n_2 \cdot v_3 \tag{39}$$

The active clearance angle α of the drill bit can be calculated at every point of the cutting edge:

$$\beta_i := \frac{\left(\frac{\pi}{2} + \arctan\left(\frac{\sqrt{1 - wq^2}}{wq}\right)\right) 180}{\pi} \tag{40}$$

Using these values (d = 80 [mm], h = 3 [mm], t = 2 [mm], f = 0.2 [mm/revolution]) the changing of the active clearance angle of the cutting edge along can be illustrated in the function of the radius **Fig. 8**:

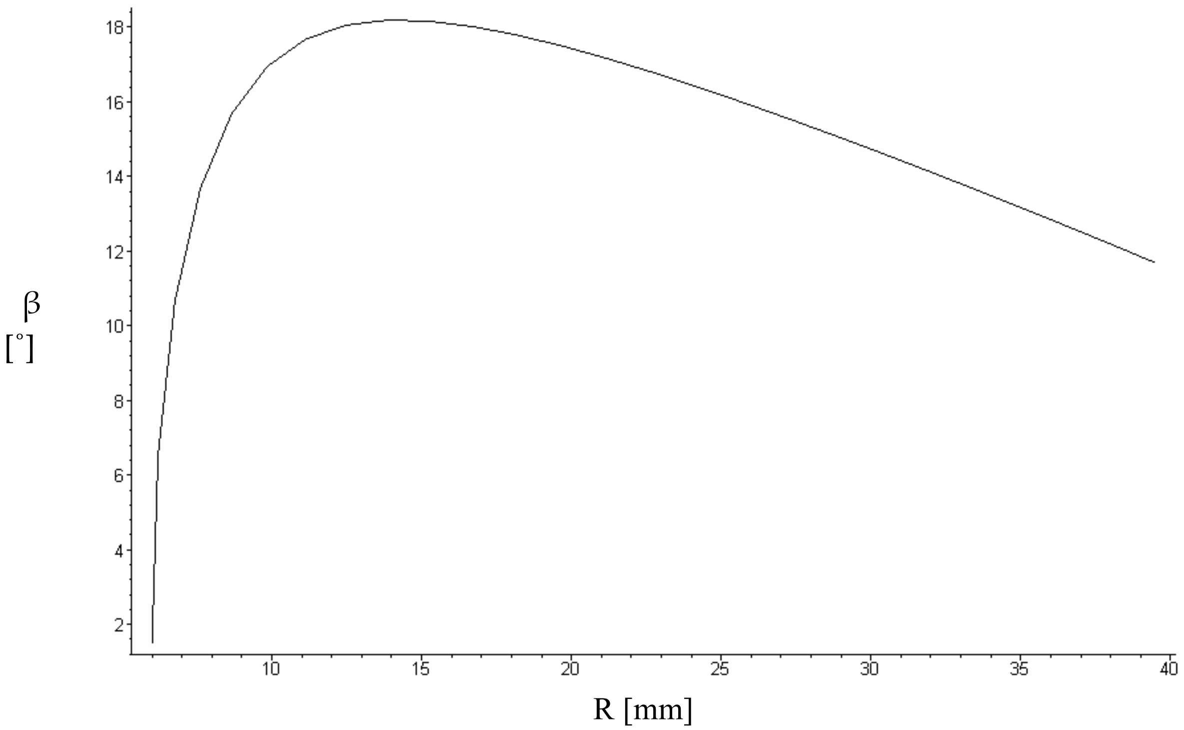

Fig. 8. The working clearance in the function of the radius

From the figure it can be seen that the working clearance angle of the bit is more complex then it was first thought. The curve starts at the final point of the chisel edge and ends at the end of the cutting edge. It has a steep slope at the start and after reaching a maximum value it gradually decreases to an end value.

By changing the geometrical and technologic parameters of the calculation characteristic of the curve is not altered only the values of the points of the curve change. This can be explained if we take a look at Fig. **9**. We can see the surface normal vectors (blue) and the cutting speed vectors (red) change along the cutting edge. This means that the cutting speed vectors are not on the same generatrix of the deducing cone. This explains the maximum part of the curve.

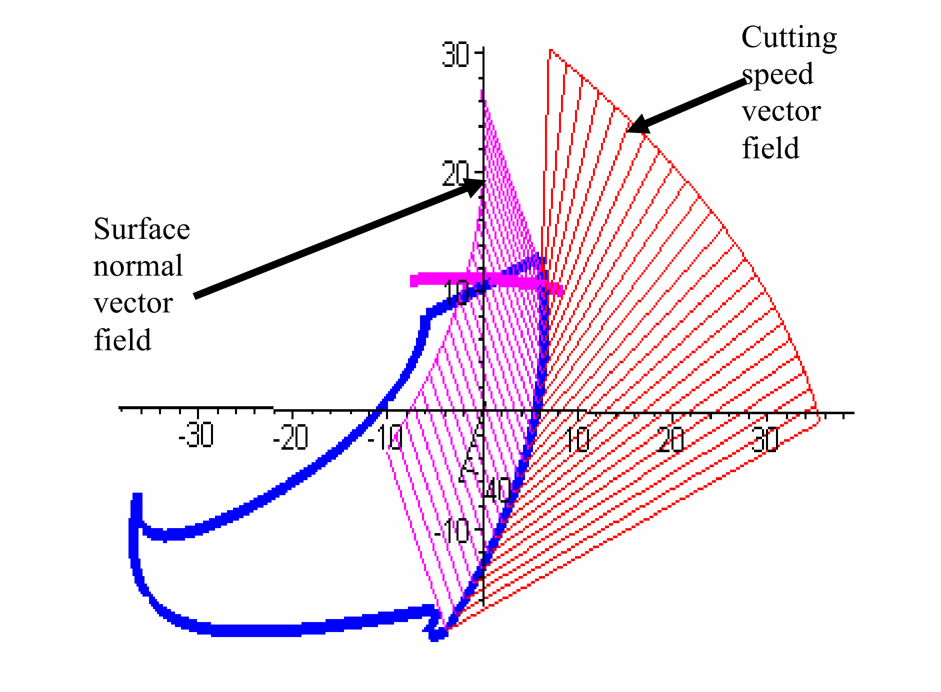

Fig. 9. Surface normal and cutting speed vector fields on the cutting edge of the drill

4. CONCLUSION

By defining an exact geometrical model of the twist drill bit various dynamic analytical analysis can be made using simulations only, where previously these data could only be gathered by difficult measurements and tests. The measurements of the working back angel could only be obtained indirectly conducting difficult and expensive tests.

The drill bit model is only a simple one, where the cross section consisting of the simplest geometrical objects such as straight lines and circle arcs. This is the classical deduction of the tool. From this model the working back angle characteristics is obtained that has the significant character shown on **Fig. 8**

Such a change in active clearance angle is disadvantageous from wear point of view where a more gradual or even better a constant clearance angle would be preferable.

In earlier publications a lot of improvements were made on the classic geometry of the bit. The flute were constructed from more complex geometry pieces than just a single circle arc [3]. From these designs a better working back angle in the function of the radius could be achieved. Although these tries were only done by trial and error.

A much better solution could be achieved by using equation (22) and defining it to be constant along the cutting edge. This will form a partial differential equation that can be possibly can be solved analytically. From this differential equation a cutting edge geometry can be calculated that then can be used for designing the drill.

The process is under development and will be published soon.

4. REFERENCES

[1] Modern Metal Cutting, Sandvik Coromant, 1994

[2] Hohenberg, F.: Konstruktive Geometrie in der Technik, Springer-Verlag Wien, 1961

[3] Alekszejev G.A., Arsinov V.A., Szmolnyikov E.A.: Forgácsoló szerszámok szerkesztése és számítása, Nehézipari Könyv és Folyóiratkiadó Vállalat, Budapest, 1952