On Object Manipulation Methods using Finger Relocation *

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Abstract

This paper presents a new smooth object manipulation method which is based on finger relocation. It relies on a special stratified manipulation concept. This method performs the stratified motion planning algorithm, not using the exact kinematic model but a fictitious system. The process provides a simpler control algorithm because it substitutes the generally hard symbolic computation problem with a simple (almost pure numerical) computations. Finally, the paper introduces the simulation results for the new method.

1 Introduction

Manipulation planning, in general sense, tries to find an appropriate trajectory for fingers which impose a desired object motion from a given initial configuration to the final grasp satisfying some constraint conditions. Fundamentals of the object manipulation can be found in [1]. Some recent interesting treatments such as simulated annealling and flat approach can be found in [2], [3]. A relevant peculiarity in the object manipulation tasks is the types of contact. Depending on the model and the applied forces, we can have fixed (constant), sliding or rolling contact. This paper considers piecewise constant contact points which means that neither sliding nor rolling but finger relocations are allowed. The manipulation with rolling contact points is investigated in some recent works, for instance in [3].

Implicitly, the most motion planning (MP) methods assume that the model of the system is smooth (i.e. all partial derivatives of the vector fields of the system, of any order, exist and are continuous). However, the manipulation problem where different constraints appear cyclically in the nonlinear system needs a careful attention from the point of view of control theory and planning. One can observe this phenomenon, if the manipulation process contain finger relocations. The stratified control introduced by Goodwine [4, 5] offers an approach for a unified discussion for the system whose (smooth) equations may change in a small region of the configuration space. The key element of the method is to divide the configuration space into smooth submanifolds (strata). In each stratum, different smooth nonlinear systems are valid. The method tries to reduce these system into a smooth common system which will be called *bottom stratified extended system.*

The paper introduces a new stratified object manipulation method. It builds on the framework of stratified control with added extensions [6, 7]. The method works on a special fictitious system reducing difficult symbolic computation which was proposed in [5]. The advantages of the proposed method arises from the special parametrization of the fictitious system providing the stratified controllability property and simple vector fields.

The paper is organized as follows. Section 2 outlines the manipulation problem. The basic theory of stratified control is based upon the smooth MP but it also uses some additional extensions. The background of the theory is summarized in section 3. The stratified MP on a fictitious system will be discussed in section 4. Finally, the section 5 presents simulation results and highlights the differences between the proposed manipulation method and the earlier approach in [5].

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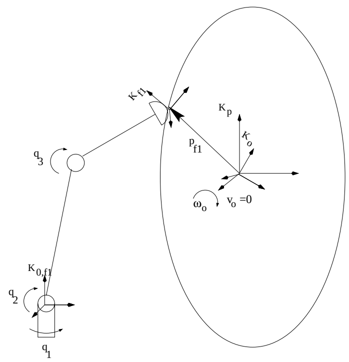

Fig. 1: The connection between a finger and the object.

2 The Manipulation Problem

This section intends to give an outline about modelling of the manipulation system. We also introduce some neccessary notations which are used in the sequel. Let us consider a smooth object [5] parametrized by the equation:

$$c(u,v) = \begin{pmatrix} (1+\frac{u}{\pi})\cos u \cos v \\ (1+\frac{u}{\pi})\cos u \sin v \\ \frac{3}{2}\sin u \end{pmatrix}, \qquad \begin{matrix} u \in (-\frac{\pi}{2}, \frac{\pi}{2}) \\ v \in (-\pi, \pi) \end{matrix} \tag{1}$$

where u and v are the parameters of the surface. The coordinates x, y and z are given with respect the object frame. In the following we will assume that the origin of the object frame is placed inside of the object. Here the robot hand equipped with four fingers where each finger has three degrees of freedom. The connection between a finger and the object is illustrated in the Fig. 1. The frame K_p denotes the palm frame which is the inertial frame in the manipulation system. For the sake of simplicity and in this paper, we assume that the origin of the object frame K_o coincides with the origin of the palm frame K_p for all time. This implies that during the whole manipulation task the object motion represents only by pure rotation. Although we consider only rotation during the simulation, the algorithm proposed in the Section 4 is able to handle the general problem where linear velocity is also allowed. Let the vector ω_o denote the angular velocity of the object frame relative to the palm frame, as seen from the palm frame. Similarly, let v_o denote the linear velocity of the object frame relative to the palm frame, as seen from the palm frame. The frames K_{f_i}, $i = 1 \ldots 4$ are fixed to the finger tips. During the whole discussion, picewise constant contact points are supposed (see [1]).

3 Smooth and Stratified MP

The paper deals with a manipulation using finger relocation. Since the concept is based on the stratified control, this section gives a brief overview of stratified control approach which was proposed by Goodwine [4] and [5]. This theory intensively relies on the smooth MP due to Lafferriere and Sussmann [8] so we begin the discussion with this method. Consider a smooth nonlinear system with m inputs which has no drift, i.e. $\Sigma\ : \dot{x} = u_1 f_1(x) + \ldots + u_m f_m(x)$, $x \in \mathsf{R}^n$. Assume that the vector fields f_i are real analytic and the system Σ is controllable.

Definition 1. Nilpotent Lie algebra with order k is defined by Lie algebra L where all the Lie brackets $[v_1, [v_2, \ldots, [v_k, v_{k+1}] \ldots]]$ equal to zero.

Definition 2. The system Σ is said to be *nilpotent* if its controllability Lie algebra $L(f)$ is nilpotent.

The strategy of SMP. The proposal in [8] is to extend the system Σ to $\Sigma_e :\ \dot{x} = v_1 f_1(x) + \ldots + v_m f_m(x) + v_{m+1} f_{m+1}(x) + \ldots + v_r f_r(x)$ where vector fields f_{m+1}, $\ldots, f_r$ are defined by higher order Lie brackets of the f_i selected so that $span\{f_1(x), \ldots, f_r(x)\} = \mathsf{R}^n$. The strategy consists of two main steps. At first, find a control v that steers the extended system Σ_e from p to q. Since the vector fields of Σ_e span the whole configuration space, the simplest case for smooth system is the straight-line segment in the configuration space. In the second step, a control u for original system Σ will be computed that substitutes the extended control v. This step consists of additional steps. At the first time, one has to obtain the P. Hall basis (which is a set of Lie brackets where "the order of right element brackets" is equal or greater than the order of the left element) and the order of system nilpotency. Then, we can solve the formal differential equation $\Sigma_{fe}\ :\quad \dot{S}(t) = S(t)(v_1(t)f_1 + \ldots + v_m(t)f_m + v_{m+1}(t)f_{m+1} + \ldots + v_r(t)f_r)$ on a special nilpotent Lie group in the form $S = e^{\tilde{h}_1 B_1} \ldots e^{\tilde{h}_{s-1} B_{s-1}} e^{\tilde{h}_s B_s}$ where B_i are the P. Hall basis, $\tilde{h_i}$ are the forward P. Hall coordinates. Finally, we obtain the controller u from the P. Hall coordinates.

Remark 1. The solution of the smooth MP problem is a sequence of flows along the vector fields of the system. It means that at a given time point, only one of the inputs is not zero, the other ones are constant zero. The active input is equal to constant 1 or -1 in its active time interval since the algorithm adjusts only the length of the time intervals belonging to the actual flows. ◁

The sketched theory does not work on nonlinear system which has discontinuous equations of motion

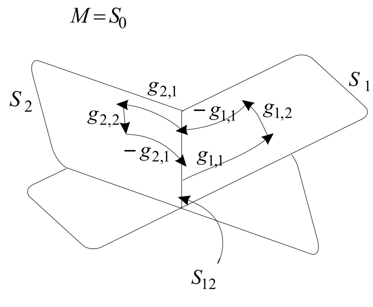

Fig. 2: Flow sequences in stratified configuration space.

because the equations depend on the actual state vector. However, the stratified control based on [4] overcomes the difficulty. In the following, this method will be outlined.

Definition 3. A set $\aleph \subset \mathsf{R}^n$ defined by union of smooth manifolds (i.e. strata) is said to be *regularly stratified set.*

Definition 4. The system is *stratified* if its configuration space is defined by regularly stratified sets.

In each stratum, a smooth nonlinear system is defined by vector fields. The main problem occurs when one wants to move the system from one stratum to another one.

Consider an object manipulation problem with "two fingered" hand. Let $M \equiv S_0$ be the whole configuration space where is no constraint. Let the stratum $S_i \subset M$ be a codimension one submanifold where only one finger is in contact with the object. Roughly speaking, this stratum corresponds to dimension $n-1$ manifold in the configuration space. Let $S_{ij} = S_i \cap S_j$ where both the ith finger and jth finger are in contact with the object (Fig. 2). Recursively, $S_I = S_{i_1 i_2 \dots i_k} = S_{i_1} \cap S_{i_2} \cap \cdots S_{i_k}$ where $I = i_1 i_2 \dots i_k$ is a multi-index [9]. The stratum with lowest dimension is said to be the *bottom stratum.* A stratum is called *lower stratum* if its dimension is lower than the dimension of the other one. The *higher stratum* is defined vice versa.

Theorem 1. (Goodwine) Let $T_{x_0}M$ be the tangent space of M in x_0 and let $\bar{\triangle}_{S_j} \mid_{x_0}$ denote the involutive closures of a distribution which is spanned by the vector fields of a stratum S_j in x_0. If there exists a nested sequence of strata $x_0 \in S_p \subset S_{p-1} \subset \cdots \subset S_1 \subset S_0$, such that the involutive closures of distributions (of strata) fulfill $\sum_{j=0}^{p} \bar{\triangle}_{S_j} \mid_{x_0} = T_{x_0}M$ then the system is locally stratified controllable from x_0.

Definition 5. A vector field is said to be a *moving off* vector field if the existence of contact between the finger and the object depend on it. In other words, the moving off vector fields switches between strata.

Definition 6. A vector field is said to be a *moving on* vector field if it does not leave the actual stratum.

Proposition 1. If the moving on vector fields commute with moving off vector fields (i.e. by definition, their Lie brackets are zero) then the flow sequence can be rearranged and reduced to bottom stratum. ♣

Corollary 1. The switching between higher and lower strata is possible if the vector fields which separate a finger from the object are decoupled from all vector fields defined on the substratum and higher strata (in other words, their Lie brackets are zero).

It is clear from *Theorem 1* that the stratified motion planning algorithm requires a fictitious system on the bottom stratum S_p (as a common space). In order to obtain such a system, one has to create the union set of vector fields in all strata and define with them a system called *bottom stratified system.* It has to contain all the moving on vector fields that commute with the moving off vector fields (i.e. the vector fields which disconnect a finger from the object). To apply the above mentioned smooth MP, in a general case, one need also the *bottom stratified extended system* whose vector fields consist of the Lie brackets among all vector fields of bottom stratified system.

Example 1. (Bottom Stratified System)
$\dot{x} = g_{0,1}u^{0,1} + \cdots + g_{0,n_0}u^{0,n_0} + g_{1,1} \mid_{S_0} u^{1,1} + \cdots + g_{1,n_1} \mid_{S_0} u^{1,n_1} + g_{I,1} \mid_{S_0} u^{I,1} + \cdots + g_{I,n_I} \mid_{S_0} u^{I,n_I}$ where the notation $\mid_{S_0}$ refers to the vector fields which take a part in bottom stratified system, however, they are defined originally not in this stratum.

The algorithm of MP on the bottom stratified extended system also solves indirectly (operating only with moving on vector fields) the stratified MP problem. The total stratified solution needs to insert suitable flows along the moving off vector fields into the flow sequence of moving on vector fields.

4 Stratified Manipulation on Fictitious System

In order to apply the stratified MP, one has to obtain the equation of motions in each strata. If we follow the conventional contact frame description [5], we arrive at symbolic expression which generate hard symbolic computations. An added problem arises from the derivation of the equations of motion which are obtained in the drifted form $\dot{x} = F(x, u)u$. This form is not suitable in the context of the stratified MP because the model is to be in the driftless form $\dot{x} = F(x)u$. In order to avoid this problem, the following stratified system model is proposed. The idea is that we use a finger tip position described by a vector $(u_i \ v_i \ z_i)^T$ where u_i and v_i determine the orthogonal projection of the finger tip position on the object expressed by the explicit parameters u and v of the object surface. The z_i denotes the distance between the ith finger tip and the object (Fig. 3).

Definition 7. [Fictitious stratified system Σ_f] Let the strata of system Σ_f be defined by the following equations of motion:

S_{1234}: (Bottom stratum where all the finger contact the object without sliding or rolling)

$$\Sigma_{f,S_{1234}} \ : \quad \begin{pmatrix} v_o \\ \omega_o \end{pmatrix} = I_6 \begin{pmatrix} v_o^d \\ \omega_o^d \end{pmatrix} \tag{2}$$

$\dot{v}_i = \dot{u}_i = z_i = 0$, $i = 1, \ldots, 4$

$S_{j_1 j_2 j_3}$: (A higher stratum where the multi-index $j_1 j_2 j_3$ dentotes the fingers which are in contact with the object)

$$\Sigma_{f,S_{j_1 j_2 j_3}} \ : \quad \begin{pmatrix} v_o \\ \omega_o \\ \dot{u}_i \\ \dot{v}_i \\ \dot{z}_i \end{pmatrix} = \begin{bmatrix} I_6 & 0 \\ 0 & I_3 \end{bmatrix} \begin{pmatrix} v_o^d \\ \omega_o^d \\ \dot{u}_i^d \\ \dot{v}_i^d \\ \dot{z}_i^d \end{pmatrix} \tag{3}$$

$\dot{v}_{j_k} = \dot{u}_{j_k} = z_{j_k} = 0, \quad j_1 j_2 j_3 \in I_4,\ j_k \neq i,\ k = 1 \ldots 3$

The vectors

$$u^1 = \begin{pmatrix} v_o^d \\ \omega_o^d \end{pmatrix} \qquad u^2 = \begin{pmatrix} v_o^d \\ \omega_o^d \\ \dot{u}_i^d \\ \dot{v}_i^d \\ \dot{z}_i^d \end{pmatrix} \tag{4}$$

are the inputs of the system in the bottom and the higher strata. The superscript d refers to the fact that the derivatives of the state can be influenced by the desired values of the velocties as the inputs of the fictitious system Σ_f. The u_i, v_i in (3) are the projection of the unconstrained finger tip onto the object.

Proposition 2. The stratified fictitious system Σ_f makes the switching between two strata possible.

Proof: At first, we show that the vector fields related to the variables z_i^d (e.g. last column in the matrix in (3)) are moving off vector fields and all the other vector fields are moving on vector fields. For this, one can see that the inputs z_i^d act only on the state variables z_i. However, the actual equations of motion (and the actual stratum) depend on the (not neccessary one) i for which $z_i = 0$. In other words, z_i^d determines the current stratum and at the same time, does not influence the state vector in the bottom stratum. In accordance with the definition of the moving off vector fields (see *Definition 5*), the vector fields related to the inputs z_i^d are moving off vector fields. Furthermore, let e_i denote the unit vector

$$e_i = (0, \ldots, 0, \underbrace{1}_{i^{\text{th}}\ pos.}, 0, \ldots 0)^T. \tag{5}$$

Each vector field in the definition of the system Σ_f is equal e_i. It is an elementary result from the differential geometry that the Lie brackets $[e_i, e_j] = 0$. It holds for arbitrary two vector fields, so obviously, it holds also for the Lie brackets which is defined between any moving on and any moving off vector field. Based on *Corollary 1*, one can switch between arbitrary two strata in the system Σ_f. ♣

Corollary 2. The stratified control algorithm can be applied for the fictitious system Σ_f.

Actually, the stratified control algorithm does not require complicate symbolic computation because the vector fields are very simple, they do not contain symbolic variable. The bottom stratified fictitious system which is needed for the stratified motion planning has the form

$$\begin{pmatrix} v_o \\ \omega_o \\ \dot{u}_1 \\ \dot{v}_1 \\ \vdots \\ \dot{u}_4 \\ \dot{v}_4 \end{pmatrix} = \begin{bmatrix} I_6 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 \\ 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & 0 & I_2 & 0 \\ 0 & 0 & 0 & 0 & I_2 \end{bmatrix} \begin{pmatrix} v_o^d \\ \omega_o^d \\ \dot{u}_1^d \\ \dot{v}_1^d \\ \vdots \\ \dot{u}_4^d \\ \dot{v}_4^d \end{pmatrix} \tag{6}$$

It is easy to see that the manipulation task can be defined on this system. We ommitted the discussion of moving off vector fields because a finger relocation can be planned independently from them, if we know the initial and final points in the object. The dimension of the system (6) equals to $3 + 3 + 4 \times 2 = 14$. One can also see that the creation of *extended* bottom stratified fictitious system is unneccessary because the vector fields of the bottom stratified fictitious system span its entire state space. Observe that all the vector fields in (6) are constant which is different from the earlier stratified manipulation approach applied in [5] where $F(x)$ in the system description $\dot{x} = F(x)u$ is tipically given in complicate symbolic form. Actually, this difference is the main advantage of the proposed approach since the symbolic computations introduced

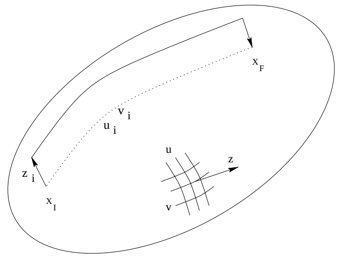

Fig. 3: Trajectory by finger relocation and its orthogonal projection onto the object

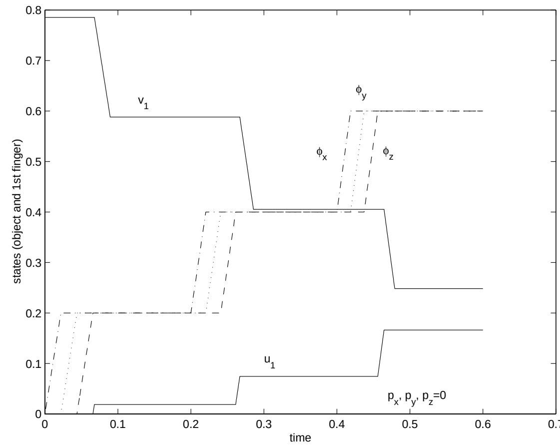

Fig. 4: The states related to the object orientation and the 1st finger tip position ($p_x = 0$, $p_y = 0$, $p_z = 0$)

in Section 3 are reduced almost to pure numerical computations whose cost is not so expensive as shown in [5]. Beyond this, one can easily show any important property (e.g. controllability) on the proposed fictitious system with considerably less work than in the earlier stratified approaches since all kind of computation with the constant vector fields are elementary in the differential geometry. The new approach needs some additional (but not complicate) computations which transform the variables from the fictitious sytem into the real system and vica versa. The algorithm can be described as follows.

Algorithm 1. ([Fictitious stratified manipulation])

Step 1. Planning the motion for the object and the finger tips.

Step 2. Create the fictitious system Σ_f.

Step 3. Create the bottom stratified fictitious system.

Step 4. Convert the prescribed reference point into the state space of the bottom stratified fictitious system. It is based upon geometric computation.

Step 5. Stratified motion planning on the bottom stratified fictitious system using the time scaling as proposed in [6].

Step 6. Performing the finger relocation by means of the insertation of moving off vector fields.

Step 7. Transform the motion trajectory from the state space of the fictitious system into the state space of the real system. This task requires to solve the inverse geometry of the hand. As a result, we obtain the joint variables for the desired manipulation.

5 Simulation Results

We discussed a MP algorithm for the smooth object manipulation problem. The simulation results are depicted in the Fig. 5 - Fig. 4. The prescribed object motion is a rotation around the axis $[\ 1\ 1\ 1\ ]^T$ and the position does not change. Introducing the notations $\omega_o = [\dot\phi_x\ \dot\phi_y\ \dot\phi_z]^T$, $v_o = [\dot p_x\ \dot p_y\ \dot p_z]^T$ for the components of the angular and the linear velocities of the object, the Fig. 4 illustrates the position and orientation of the object and the states u_1, v_1 during the simulation. At each time instant, only one state changes because all inputs influence only one state and the stratified MP allows only one active input. Since the vector fields of the fictitious system are very simple, one can easily imagine the real motion of the object. More exactly, the object motion consists of a sequence of the rotation around the axis x, y and z which are performed one after each other The process is illustrated in Fig. 5 and Fig. 6. The finger relocation are carried out along distinct direction. At beginning, the finger lifts from the object in the direction of the normal vector of the surface. Holding the distance during the finger relocation, the finger tip moves above the object as shown in Fig. 6. The projection of this motion onto the object is described by the explicit parameters of the object. A feature of this stratified approach is that the system moves along only one parameter in the configuration space, at a given time instant. Furthermore, one can also see from the structure of the fictitious system and the algorithm of the stratified control that the finger relocations are decoupled. The object is always grasped at least with 3 fingers. The finger relocations are illustrated in the Fig. 6 - Fig. 7. After the finger relocations, the manipulation is followed a new local manipulation task, in the same way as before (Fig. 5 - Fig. 6). It is also remarkable that the extension of the bottom stratified fictitious system is not needed, so the exact reaching of the final state is possible.

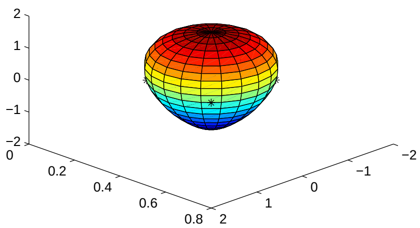

Fig. 5: Snap 1. Initial configuration of the fingers/object.

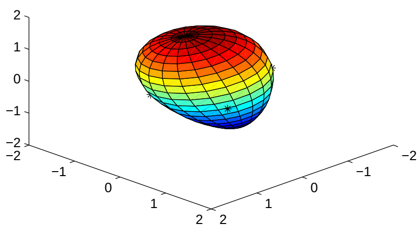

Fig. 6: Snap 2. Manipulation with four contact points.

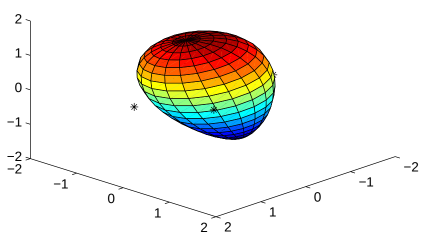

Fig. 7: Snap 3. Finger relocation while the object does not move.

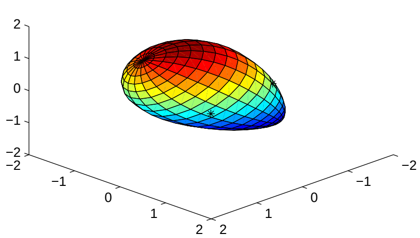

Fig. 8: Snap 4. All fingers are in contact with the object, again.

6 Conclusions

The paper proposed a manipulation method which is based on finger relocation. The method using stratified control and fictitious system at the same time shows advantages in comparison of the pure stratified motion planning, such as avoiding the hard symbolic computation, easily imaginable trajectory. All of them is based on a fictitious system equipped by special parametrization.

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