ANGLE DEPENDENT N-STATE MARKOV MODEL FOR RAIN ATTENUATION TIME SERIES GENERATION

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Abstract

In our previous works an N-state Markov Chain model was used to generate a rain attenuation time series [1,2,3,4]. The model is applicable for predicting the first and second order statistics of attenuation on a proposed microwave link in the planning phase. The model parameters were derived from fade slope statistics of measured attenuation. The novelty of this work is investigating the correlation angle dependency of the generated time series.

1 Introduction

The demand for broadband services claims to apply high carrier frequency in radio communication systems. Besides the obvious benefits of the high carrier frequency there is a significant disadvantage, the considerable attenuation caused by precipitation, especially by rain. For accurate planning of the proposed microwave links the ability of estimating statistics of the expectable rain attenuation is highly important.

In our previous works an N-state Markov Chain model [7] was used to generate a rain attenuation time series. The model is applicable for predicting the first and second order statistics of attenuation on a proposed microwave link in the planning phase. The model parameters were derived from fade slope statistics of measured attenuation.

The novel approach of this work is investigating the correlation angle dependency of the generated time series. In this phase of work two high frequency microwave links in star topology are considered, what enables to investigate the angle dependent correlation between attenuation time series on different links. In this work the Markov model parameters are determined from the attenuation measurement on one of the links. Then the correlation angle dependency between the measured data is investigated, and the results are applied on the generated time series.

The paper is organized as follows. Section 2 describes the attenuation measurement, Section 3 deals with the required data processing method. Section 4 shows the N-state Markov model, while model parameterization is described in Section 5. In Section 6 the angle dependency of measured attenuation time series is investigated. The time series generation method and results are provided by Section 7. Conclusion and expected future works are given in Section 8.

2 Attenuation measurement

In this work measured rain attenuation time series on high frequency microwave links were applied around measuring node Szeged, which is a part of our countrywide measurement system. The rain attenuation measurements were performed in 2006 with sampling frequency of 1 Hz. The parameters of the applied links are listed in Table 1, while the schematic representation of the topology is depicted in Figure 1, where EOV-X and EOV-Y means the axis of the uniform Hungarian geographical frame of reference. The measurement links are in star topology as can be seen in Figure 1.

Link	Site Name	Freq. [GHz]	Pol.	Length [km]	Azimuth [deg]
HU61	Kereszttöltés	38	V	3.27	264.25
HU62	Kiskundorozsma	23	V	7.79	195.54

Table 1. Parameters of links around measuring node Szeged

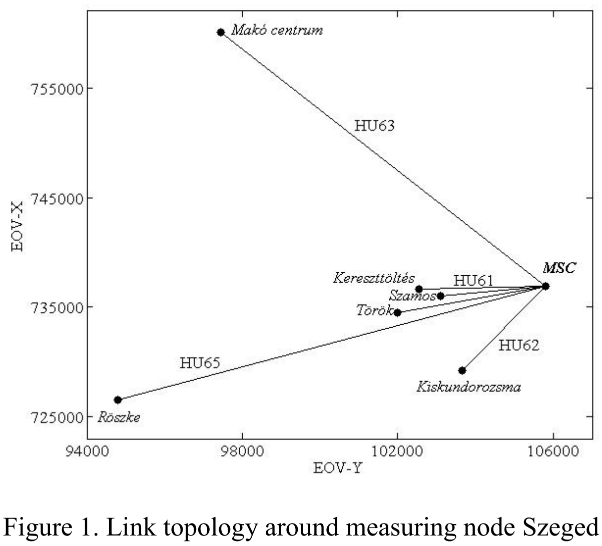

Figure 1. Link topology around measuring node Szeged

3 Data processing

The measurements were performed on microwave links with different parameters (Table 1). This varying of path length and operating frequency causes different rain attenuation for the same rain intensity. In order to easily compare the generated time series this variegation must be eliminated. Therefore all of the measured rain attenuation data had to be transformed to a reference. This reference is called hypothetical link, which is operating in 23 GHz carrier frequency with vertical polarization and with length of 1 km. The transformation was performed with Equation (1) - Equation (2) based on the based on the ITU-R P.530 recommendation [4].

$$A_h(t_n)=\frac{k_h \cdot L_h}{1+L_h/d_0}\cdot\left(\frac{A_m(t_n)\cdot\left(1+\frac{L_m}{d_0}\right)}{k_m\cdot L_m}\right)^{\frac{\alpha_h}{\alpha_m}} \quad (1)$$

$$d_0=35\cdot e^{(-0.015\cdot R_{0.01})} \quad (2)$$

In Equation (1) and in Equation (2) $A_h(t_n)$ and $A_m(t_n)$ are time discrete attenuation on the hypothetical and measurement links in the n^{th} time instant, k_h, α_h, k_m and α_m are polarization and operating frequency dependent variables described in [6], for the hypothetical and the measurement links, respectively. The length of measurement links and of the hypothetical link are L_m and L_h, while d_0 is the path reduction factor. The geographical location dependent rain intensity (R) is higher than or equal to $R_{0.01}$ in 0.01 percent of the year. In order to transmit data, $R_{0.01}$ must be known from [5] or from own measurement. For determining $R_{0.01}$ in different geographical locations where the considered microwave links are deployed, we utilized our own several yearly rain attenuation statistics comes from our countrywide measurements. The $R_{0.01}$ related to the hypothetical link is determined considering one of the location of our measuring nodes.

After transforming the scintillation had to be removed from the data series with a moving average filtering with 60 seconds averaging window length. In the following these processed data series were applied.

4 The N-state Markov Model

In the considered time discrete N-state Markov Chain model there are many states according to the rain attenuation levels [1,7]. Each state represents a rain attenuation level with 0.05 dB resolution (ΔA), so the model is discrete in states as well. This resolution can be chosen finer, but according to our investigations 0.05 dB is appropriate for our goals.

The schematic representation of the model is depicted in Figure 2, where the number of states is N, the minimum and maximum attenuation levels are A_0 and A_{N-1} respectively. As it can be seen in Figure 2 the applied Markov model is irreducible.

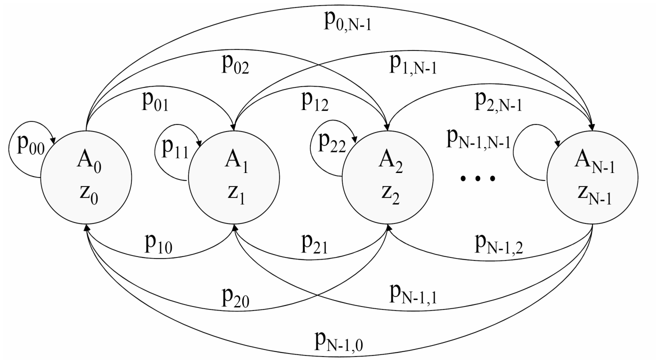

Figure 2. The schematic representation of the N-state Markov Chain model with state probabilities and state transition probabilities [3]

The state probabilities z_i gives the probability of A_i attenuation level and can be arranged into the state probability vector $\overline{z}$, while the state transition probabilities p_{ij} can be arranged into the transition probability matrix $\overline{\overline{P}}$ as given by Equation (3) and Equation (4).

$$\overline{z}=[z_0, z_1, z_2, \ldots, z_{N-1}] \quad (3)$$

$$\overline{\overline{P}}=\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1,N-1} \\ p_{21} & p_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ p_{N-1,1} & \cdots & p_{N-1,N-2} & p_{N-1,N-1} \end{pmatrix} \quad (4)$$

From the transition probability matrix of the general N-state Markov chain model $(\overline{\overline{P}})$, the CCDF $(P(A \geq A_i))$ of generated rain attenuation time series can be calculated as the steady state probability distribution of the Markov chain according to Equation (5).

$$P(A \geqslant A_i)=\sum_{j=i}^{N-1} z_j, \quad \overline{z}=\overline{\overline{P}}^T\cdot\overline{z} \quad (5)$$

5 Model parametrization

The model parameters are determined from the one year measured and processed data of HU61, therefore the model can generate time series whose statistics will be similar to statistics of measured data on HU61. The N-State Markov model has a very large transition probability matrix, so only the calculation method is presented instead of the exact elements. The probability parameters of the N-State Markov model can be calculated with fade slope statistics. Fade slope (ς) is a relevant second order statistics for planning purposes of e.g. appropriate fade mitigation techniques, gives the gradient (in dB/s) of the fading at a given A_i. The simulation time unit (STU) gives the time interval in seconds between two measured rain attenuation values. Considering the time discrete measured attenuation data and STU, the fade slope can be calculated with Equation (6) and Equation (7).

The unit of fade slope is dB/STU, t_n is the n^{th} time instant. In our case STU equals to 1 second because of the 1 Hz sampling frequency.

$$\varsigma^{[dB/STU]} = \frac{A(t_{n+1}) - A(t_{n-1})}{2}\Bigg|_{A(t_n)=A_i} \qquad (6)$$

$$t_n = n \cdot STU\,, \quad n \in N \qquad (7)$$

Determining the $P(\varsigma|A_i)$ (Conditional Probability Density Function (CPDF) of fade slope) with the Gaussian fade slope model for every A_i attenuation levels as conditions, corresponding to the i^{th} state, the p_{ij} transition probability (from state A_i to state A_j) corresponds to the $P(\varsigma=(A_j - A_i)/2|A_i)$ value. In Figure 3 in the right a typical CPDF of fade $P(\varsigma_j=(A_{i+j} - A_i)/2|A_i)$ slope at A_i attenuation level $(P(\varsigma|A=A_i))$ is presented. Two states of the N-State Markov Model with transition probabilities according to the CPDF of fade slope are also depicted in Figure 3 in the left, where $\varsigma_j=(A_{i+j}-A_i)/2$. As it is presented in the figure as well, the $p_{i,i+j}$ probability corresponds to the value. If the CPDF of fade slope is a continuous function, we get the exact value of the transition probability with an integral around the proper fade slope value. In our discrete case a sum is used instead of the integral.

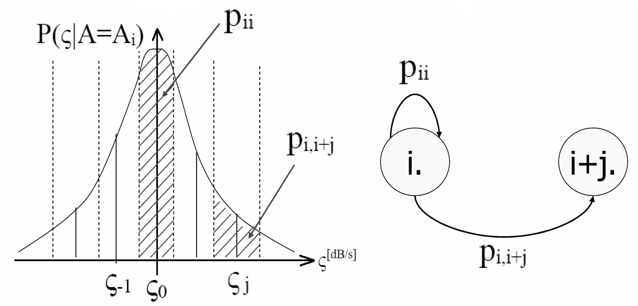

Figure 3. Determination of the state transition probabilities from the CPDF of fade slope (on the left) with two signed states of the Markov Chain [3]

6 Angle dependency

In this section the angle dependency of the measured time series on links HU61 and HU62 is investigated. In Figure 4 the measured and processed attenuation time functions are depicted during a rain event on HU61 and HU62. The event length is approximately 5000 seconds.

Please observe that the maximum attenuation is lower on HU62 than on HU62 and it occurs later in case of HU62. This difference is only caused by the different angle of microwave link (Figure 1) because the other factors are eliminated by data transformation. Let D_i sign the difference between the measured and processed attenuation value on HU61 $(A_{HU61,i})$ and on HU62 $(A_{HU61,i})$ during the rain attenuation event, at the i^{th} time moment as shown by Equation(8).

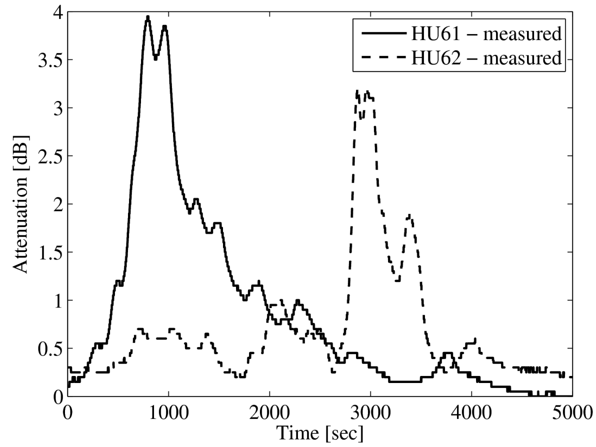

Figure 4. Measured and processed rain attenuation time function during a rain event on links HU61 and HU62

$$D_i = A_{HU61,i} - A_{HU62,i} \qquad (8)$$

These differences contain information about the correlation between the two measured data series. The probability distribution of D_i is depicted in Figure 5. The distribution is not symmetric to zero and the probability of negative difference is about 0.2. This means than statistically attenuation value on HU61 is higher than on HU62 with high probability.

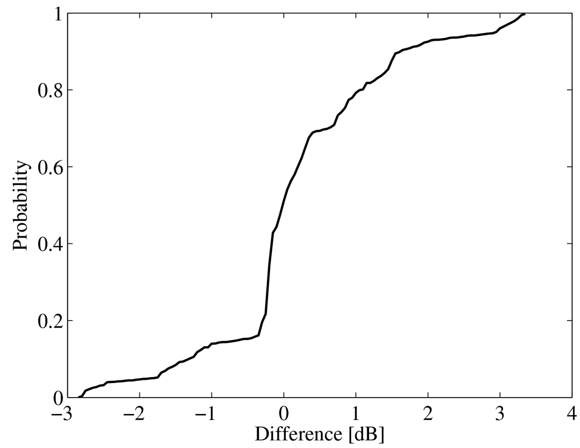

Figure 5. Probability distribution of difference between measured rain attenuation on HU61 and on HU62 during the rain event

7 Time series generation

Our goal was to generate attenuation time series, which could be occurred on HU62. But our N-state Markov model is parametrized from HU61 data. To consider the correlation between link he following method was applied. The measured attenuation time series on HU61 during a rain event (Figure 4) is considered as it would be generated by the Markov model. This assumption is correct because it is already proved, that the N-state Markov model is able to generate time series with very similar statistics to the measured data [2,3]. At a *t=0* start time moment, for example when the measured rain attenuation event on HU61 $(A_{HU61}(t=0))$ has maximum value, a $(D(t=0))$ difference value is taken into account between data value on HU61 and on HU62. This difference value can be obtained by drawing by lot based on difference distribution depicted on Figure 5, or it can be exactly calculated from the measured and processed rain attenuation event on HU61 and on HU62 (Figure 4). The start state of the N-state Markov model can be calculated from the $A_{g,HU62}(t=0)$ start attenuation value on HU62, where *g* notes that $A_{g,HU62}(t=0)$ belong to the generated time series on HU62. Its value is obtained from the rain attenuation on HU61 at start time moment and the difference as shown by Equation (9).

$$A_{g,HU62}(t=0)=A_{HU61}(t=0)-D(t=0) \tag{9}$$

Then let the Markov model generate time series from this start state with duration of so called attach time, which is signed with τ_A. After this attach time duration a new difference value is calculated with which the actual state of Markov model can be corrected. This method is going on until the time series generation ends. With this method the correlation between measured data on HU61 and on HU62 is periodically considered via the *D* difference value. The period is the τ_A attach time, when the Markov model generating data on HU61 (in our case this generated time series is the same as the measured data) is attached to Markov model generating data on HU62. Depending on the attach time different time series can be generated on HU62.
According to the difference calculation method two different cases are investigated.

- Case A: The *D* difference is draw by lot from the difference distribution.
- Case B: The *D* difference calculated exactly from measured data on HU61 and HU62.

7.1 Case A

In this case the *D* difference value is draw by lot based on its distribution in every τ_A time moment and the current state of Markov model is corrected applying *D*. For different attach time values the generated time series are depicted in Figure 6 - Figure 8. It can be observed that the time series generation on HU62 works well. The attach time of 100 seconds is to short therefore the generate time series follows the measured data on HU61 very well, because the Markov model hasn't enough time to go over higher states.

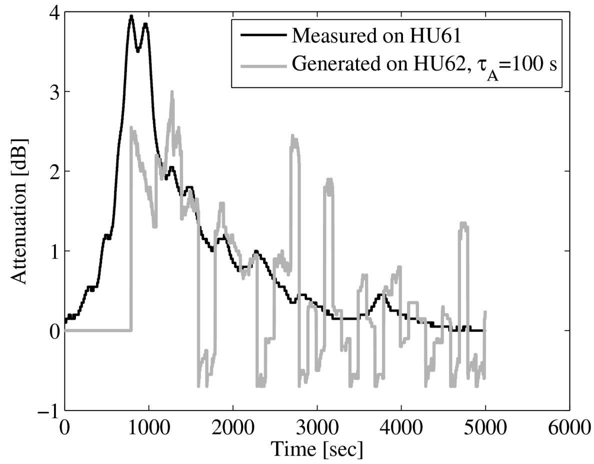

Figure 6. Measured rain attenuation event on HU61 and generated time series on HU62. Case A, τ_A =100 s.

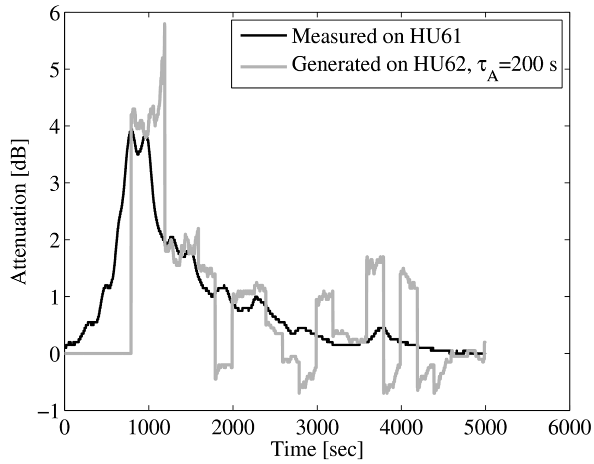

Figure 7. Measured rain attenuation event on HU61 and generated time series on HU62. Case A, τ_A =200 s.

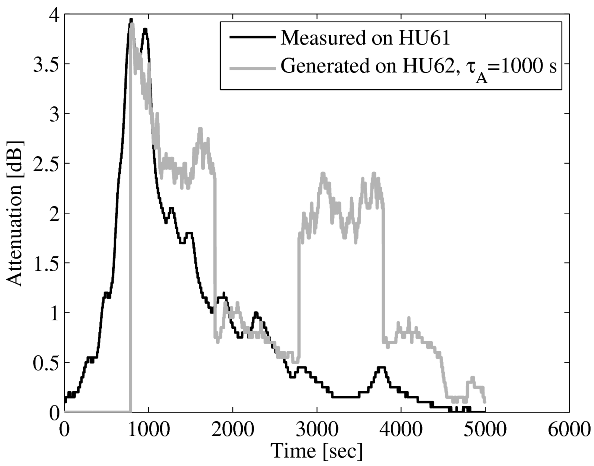

Figure 8. Measured rain attenuation event on HU61 and generated time series on HU62. Case A, τ_A =1000 s.

This behaviour is the same for attach time of 200 seconds. But in case of 1000 seconds attach time the model has enough time to generate much more different time series than the measured data on HU61.
Moreover this generated time series is similar to the measured time series on HU62 (Figure 4). The maximum attenuation occurs later and it has lower value than in case of HU61. So choosing the attach time to 1000 seconds a similar generated time series can be obtained to the measured data on HU62 which has approximately 5000 seconds duration. This result is remarkable because the only the difference distribution was applied and the Markov model parameters were determined from the measured data on HU61.
Additionally it must be mentioned that both the generated time series both the measured data on HU62 are only realizations of the stochastic process, so different rain attenuation event could be obviously generated.

7.2 Case B

In this case the D difference value is calculated exactly from the measured attenuation time series on HU61 and HU62 during the rain event. For different attach time values the generated time series are depicted in Figure 9 - Figure 11.
As it is expected in this case the generated time series in HU62 is much more similar to the measured time series in HU62. In this case the time series generation on HU62 works very well. If the attach time is 100 or 200 seconds the model generates very similar time series to the measured data, because in relatively short time period the model state is corrected with the exactly calculated difference between measured data on links.
On the other hand attach time value of 1000 seconds is too long in this case so the model generates too long time series before correction with the new calculated difference value (Figure 11). Nevertheless the most important property of the generated time series on HU62 is true in this case as well.

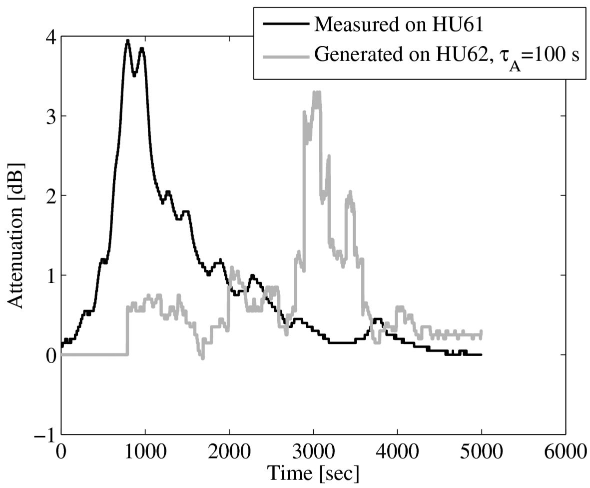

Figure 9. Measured rain attenuation event on HU61 and generated time series on HU62. Case B, τ_A =100 s.

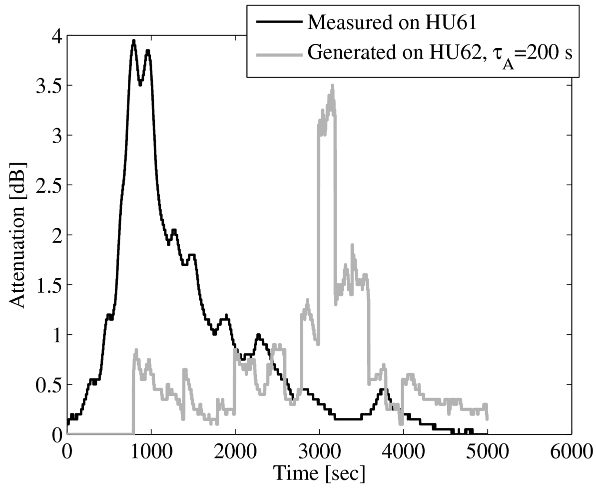

Figure 10. Measured rain attenuation event on HU61 and generated time series on HU62. Case B, τ_A =200 s.

The maximum attenuation occurs later and it has lower value than in case of HU61. So as it is expected in case of exact calculation of difference value the higher the attach time the worse the generated time series, however, attach time value of 1000 seconds could be barely good for generating a time series with approximately 5000 seconds duration.

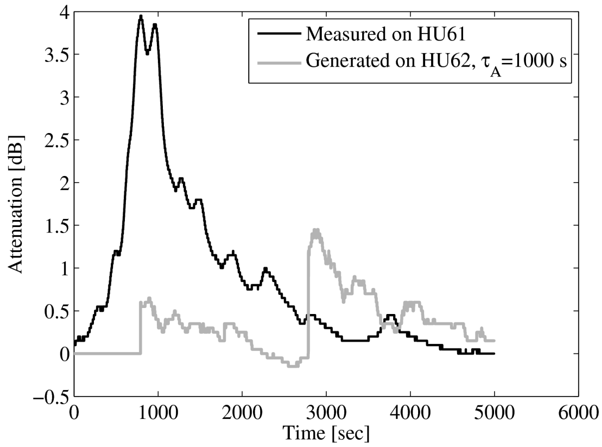

Figure 11. Measured rain attenuation event on HU61 and generated time series on HU62. Case B, τ_A =1000 s.

8 Conclusion

In this contribution we focused on investigating the angular dependence of rain attenuation on high frequency microwave links in star topology. The investigation is based on the same measured rain attenuation event on two different microwave links. We were interested only in the deviation caused by the different angle in the star topology therefore a data transformation was applied to eliminate the effects of other differences in link parameters. The most notable deviance between the two measured event is that the maximum attenuation is different and it occurs different time moment.

The novelty of this work is presenting how to consider this effect of different angle by generating attenuation time series with our N-state Markov Chain model, which can provide very accurate realization of the physical fading process. The basic idea is to calculate the attenuation difference between the two measured data during the rain attenuation event. This differences contains the information about the correlation between the two measured data series. By the time series generation the current Markov model state was corrected after every attach time duration with a difference value.
Two different methods were applied. First case the difference distribution was calculated from the measurement and the by the time series generation the applied difference value was draw by lot. In this case with an attach time value of 1000 seconds similar generated time series can be obtained to the measured data, which has 5000 seconds duration. The other method was apply the exact difference between the measured attenuation on the links instead of drawing by lot. As it is expected in this case the time series generation worked very well.
As future work angle dependent correlation between transition probability matrices of two N-state Markov model parametrized from two different measurements should be determined in order to get a really angle dependent N-state Markov model.

9 Acknowledgements

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10 References

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