Joint Modeling of Terrestrial Rain Attenuation and Land Mobile Satellite Multipath Fading Time Series with General N-state Markov Model

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Abstract: - This contribution provides an N-state Markov Chain model is, which is used to generate an attenuation time series on terrestrial microwave (38 GHz) and land mobile satellite links. The model is applicable for estimating the CCDFs (Complement Cumulative Distribution Function) of attenuation. The parameters of the Markov model is derived from fade slope statistics of measured attenuation. The conditional probability density function of fade slope is estimated with a Gaussian distribution function.

Key-Words: - Attenuation modeling, N-state Markov Chain model, Fade Slope modeling, CCDF of attenuation estimation, Land Mobile Satellite Link

1 Introduction

Wave propagation at high carrier frequencies (above 10 GHz) is highly influenced by precipitation especially by rain. To investigate wave propagation phenomena a rain attenuation and weather data measurement system was established. Currently in Hungary a number of point-to-point millimeter wave link operating in the frequency bands of 13, 15, 23 and 38 GHz. The received IF signal powers are collected with the meteorological data together using weather stations at different locations for data collection. In our previous work an N-state Markov Chain model is used to generate a rain attenuation time series [1],[2]. The model is applicable for estimating the CCDFs (complement cumulative distribution function) of attenuation. The model parameters were derived from fade slope statistics of attenuation, which was determined from a Gaussian fade slope model. In this contribution this model is validated on other terrestrial microwave links operating in the same frequency band and the model is applied on a land mobile satellite link as well.

2 The Considered N-state Markov Chain Model

In the considered N-state Markov Chain model there are many states according to the attenuation levels [1],[2],[3]. Because each state represents an attenuation level with 0.05 dB resolution, this model can generate time series, which will be quantized with resolution 0.05 dB. The number of states in the model is dependent for the maximum attenuation, which will occur in the time series to generate. The schematic representation of the model is depicted in Fig.1 [3]. In Fig.1 the number of states is N, the state probabilities z_i gives the probability of A_i attenuation level, and the state transition probabilities $\underline{p_{ij}}$ can arranged into the transition probability matrix $\overline{\overline{P}}$.

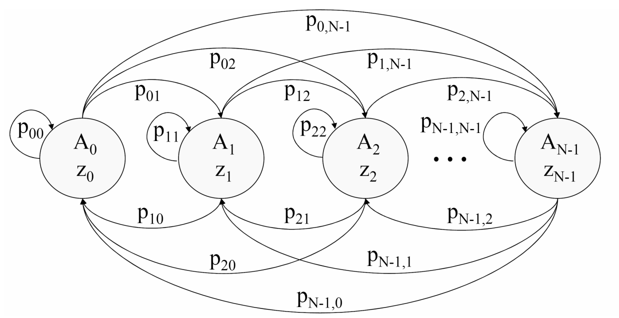

Fig.1. The schematic representation of the N-state Markov Chain model with state probabilities and state transition probabilities

3 Calculating Of Model Parameters Using Fade Slope Statistics

The state transition probability parameters of the model can be determined from the fade slope statistics of attenuation measurement $A(t)$ of the examined terrestrial microwave link. Fade slope is a relevant second-order statistics for planning purposes. Fade slope (ς) indicates the rate of change of attenuation, so gives the gradient (in dB/s) of the fading at a given fade threshold (1) [1].

Determining the $P(\varsigma|A_i)$ (CPDF, conditional probability density function) of fade slope for every A_i attenuation levels as conditions, corresponding to the i[th] state, the p_{ij} probability of transition in the next second from this state to state A_j corresponds to the $P(\varsigma=(A_j-A_i)/2\,|A_i)$ value. The details of this calculation method is described in [1].

$$\varsigma_t^{[dB/s]}=\frac{A(t+1)-A(t-1)}{2} \qquad (1)$$

In Fig.2 the $P(\varsigma|A_i)$ of fade slope is depicted for attenuation level 5 dB calculated on different terrestrial microwave links, which parameters are listed in Table 1.

Table 1. Parameters of the investigated terrestrial microwave links

Site Name	Location	Frequency [GHz]	Polarization	Length [km]
HU11	Budapest	38	H	1.5
HU36	Pécs	38	V	1.69
HU45	Miskolc	38	V	1.52
HU52	Győr	38	V	2.97

Please notice, that the CPDFs of fade slope are very similar, so they more depend on the attenuation level than the parameters of a given link.

3.1 Calculating the model parameters from a Gaussian fade slope model

We carried out, that we can estimate the CPDF of fade slope for different attenuation levels with a Gaussian distribution function (2) [2].

$$P(\varsigma|A)=\frac{1}{\sqrt{2\pi}\sigma_\varsigma}\cdot e^{-\frac{1}{2}\cdot\left(\frac{\varsigma}{\sigma_\varsigma}\right)^2}, \qquad (2)$$

where A is the attenuation in dB, ς is the fade slope in dB/s. Because of the characteristics of the fade slope, the expected value of the normal distribution is zero. The standard deviation parameter (σ_ς), which depends on the attenuation level can be estimated from the standard deviation of $P(\varsigma|A_i)$ of fade slope measured on either investigated microwave links.

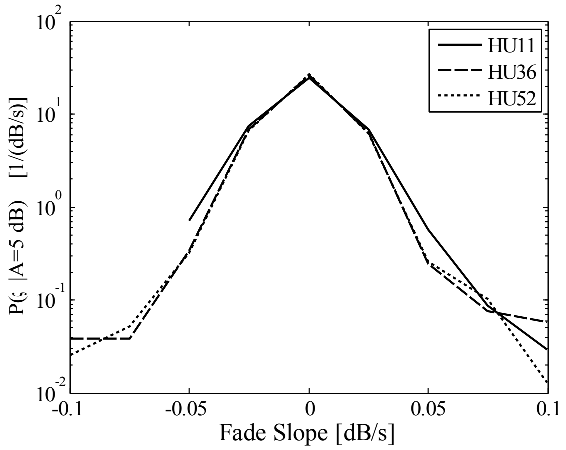

Fig.2. CPDFs of fade slope for attenuation level 5 dB calculated on different terrestrial microwave links

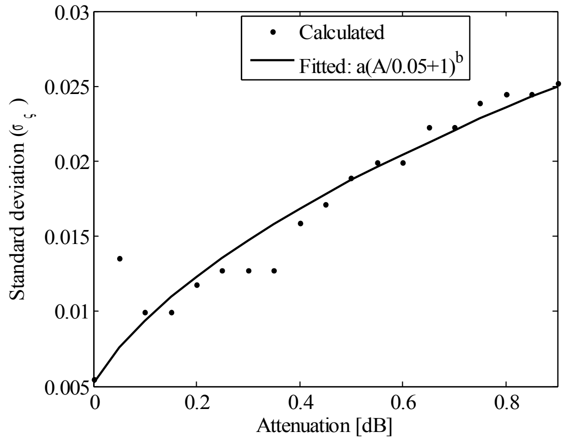

Fig.3. The calculated and the fitted attenuation dependent standard deviation of the conditional probability density of fade slope for attenuation levels lower than 1 dB

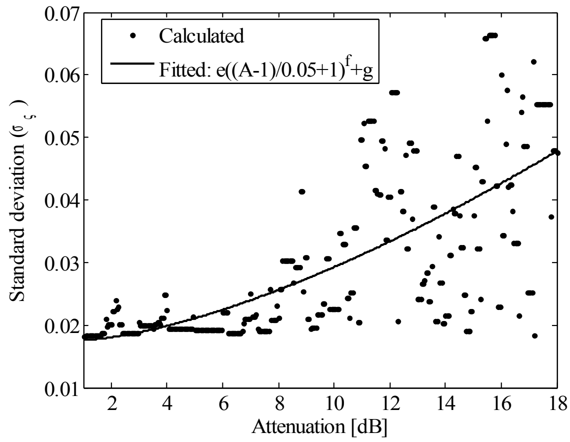

Fig.4. The calculated and the fitted attenuation dependent standard deviation of the conditional probability density of fade slope for attenuation levels equal or higher than 1 dB

In our work we used the $P(\varsigma|A_i)$ of fade slope measured on HU11. This choice depends only on us, because of the similarity between the CPDFs calculated on different microwave link, but at the same attenuation level (Fig.2).

As obtained in Fig.3 and in Fig.4 the calculated attenuation dependent standard deviation of the conditional probability density of fade slope can be estimated by a power function. A well fitting power function was found for attenuation levels lower than 1 dB, and an other was found for higher attenuation levels (3).

$$\sigma_\varsigma = \begin{Bmatrix} a\cdot\left(\dfrac{A}{0.05}+1\right)^b & , A<1\,dB \\ e\cdot\left(\dfrac{A-1}{0.05}+1\right)^f+g & , A\geqslant 1\,dB \end{Bmatrix} \quad (3)$$

where a, b, e, f, and g are experimental parameters; they values are listed in Table 2.

Table 2. The experimental parameters of the Gaussian fade slope model

a	$5.242\cdot10^{-3}$
b	$5.307\cdot10^{-1}$
e	$4.802\cdot10^{-6}$
f	1.5
g	$1.758\cdot10^{-2}$

. This Gaussian fade slope model fits to the CPDF of fade slope calculated from the HU11 measurement well for lower attenuation level as obtained in Fig.5.

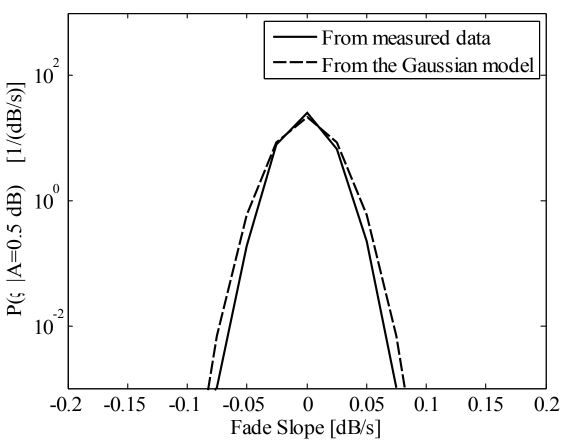

Fig.5. CPDF of fade slope for attenuation level 0.5 dB calculated from the HU11 link measurement and from the Gaussian model

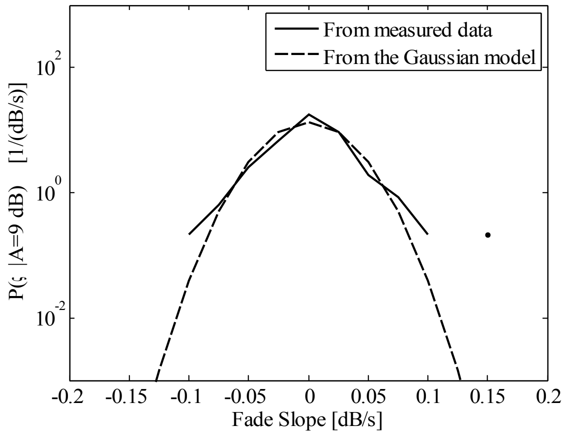

Fig.6. CPDF of fade slope for attenuation level 9 dB calculated from the HU11 link measurement and from the Gaussian model

At higher attenuation levels the Gaussian model fits also well (Fig.6), but because of the few available measured data at higher attenuation levels, the fitted Gaussian functions may not describe the measured fade slope well.

4 Steady State CCDF Of The N-state Markov Chain Model

From the transition probability matrix of the general N-state Markov Chain model $(\overline{\overline{P}})$, the CCDF $(P(A\geq A_i))$ of generated attenuation time series can be calculated with (4), where $\overline{z}$ is the steady state probability vector [4].

$$P(A\geqslant A_i)=\sum_{j=i}^{N-1} z_j \quad , \quad \overline{z}=\overline{\overline{P}}^T\cdot\overline{z} \quad (4)$$

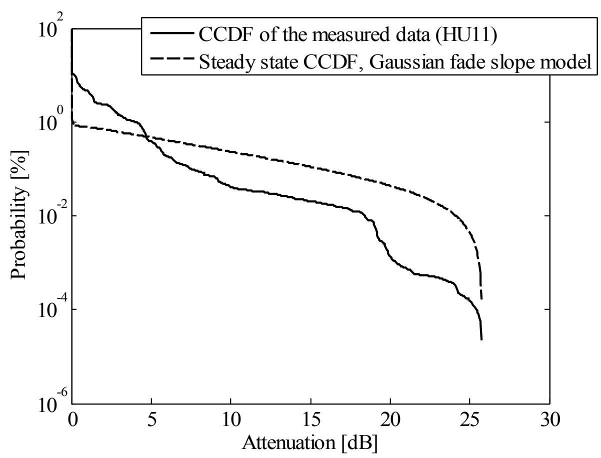

Fig.7. The CCDFs of the mesuared data (HU11) and of the generated time series. The p_{ij} parameters are derived from the Gaussian model

Please notice in Fig.7, that the CCDF of attenuation measured on HU11 link can be estimated well, if the p_{ij} parameters of the Markov model are derived from the Gaussian fade slope model described above.

5 Validating The Model

The transition matrix of the applied N-state Markov Chain model was derived from our Gaussian fade slope model, whose experimental coefficients were determined from the HU11 link measurement.

In this section the model is applied on three different microwaves links to estimate its annual CCDF of attenuation. These links are operating in the same frequency band, but with different length and locations in Hungary (Table 1).

On the investigated terrestrial microwave links the maximum value of the occurred attenuation are different.

By the transition matrix determination applying the Gaussian fade slope model the (σ_ς) parameter must be calculated with (3) until the maximal attenuation value of the appropriate microwave link, whose CCDF is being estimated.

The annual CCDF of attenuation and the estimated CCDF of attenuation belong to the investigated links are depicted in Fig.8-10.

Please notice, that the maximal attenuation supposed by the model equals to occurred maximal attenuation on the given link.

The logarithmic RMSE (root mean square error), which was calculated by (5) and (6), between the calculated and the estimated CCDFs are listed in Table 3.

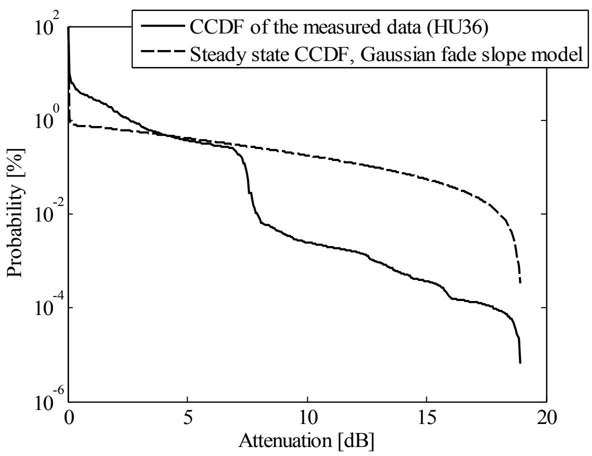

Fig.8. The CCDFs of the mesuared data (HU36) and of the generated time series. The p_{ij} parameters are derived from the Gaussian model

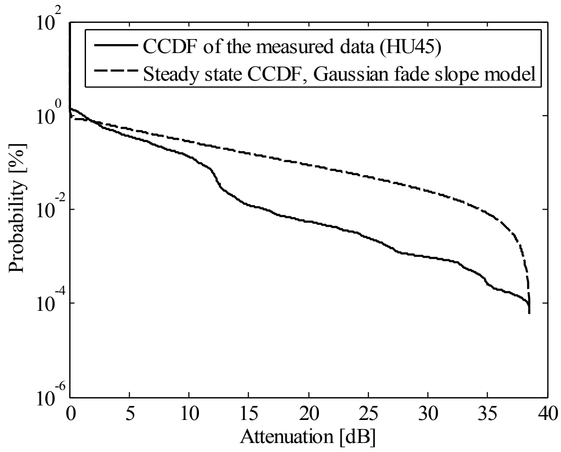

Fig.9. The CCDFs of the mesuared data (HU45) and of the generated time series. The p_{ij} parameters are derived from the Gaussian model

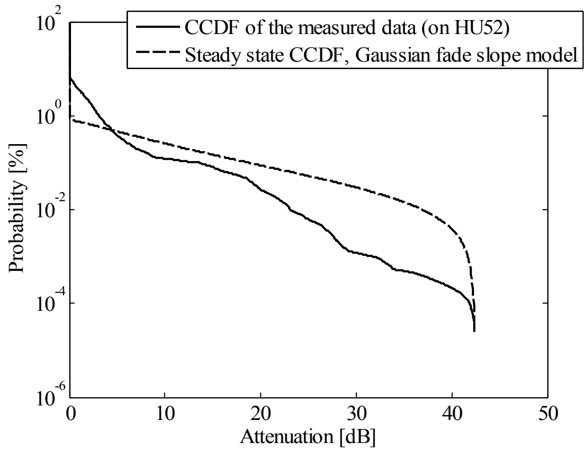

Fig.10. The CCDFs of the mesuared data (HU52) and of the generated time series. The p_{ij} parameters are derived from the Gaussian model

$$\varepsilon_i = \log\left(P_e\left(A \geqslant A_i\right)\right) - \log\left(P_c\left(A \geqslant A_i\right)\right) \quad (5)$$

$$RMSE = \sqrt{\left(\sum_{i=1}^{M} \varepsilon_i^2\right) / M}, \quad (6)$$

where ε_i is the error, $P_e\left(A \geqslant A_i\right)$ and $P_c\left(A \geqslant A_i\right)$ are the values of the estimated and the calculated CCDFs at A_i attenuation level and M is the number of the examined attenuation levels.

As obtained from Fig.8 the estimated CCDF belongs to link HU36 fits to the calculated CCDF less. On the other hand the CCDF estimation for link HU52 is quite well (Fig.10), in this case the logarithmic RMSE is less than in case of HU11 (Table 3); however, the transition probability matrix of the N-state Markov Chain model is derived from the HU11 measurement.

Table 3. The logarithmic RMSE between the estimated and the calculated CCDFs for the four investigated terrestrial microwave links

Site Name	Logarithmic RMSE
HU11	0.9506
HU36	1.5546
HU45	1.0523
HU52	0.9205

6 Applying The Model On A Satellite Link

Now, the described N-State Markov Chain model is applied on our available land mobile satellite measured attenuation data, so the transition parameters of the Markov model is derived from this measurement. The data measurement was performed by DLR between 1984-1987 on a mobile land satellite channel [4]. During the measurement the vehicle was moving on a highway with speed of 60 km/h. The parameteres of the measurement are listed in Table 4.

Table. 4. Parameters of the measured land mobile satellite channel

Geostationary Satellite Name	MARECS
Elevation	24°
Frequency	1.54 GHz
Sampling rate	300.5 Hz
Measurement period	81.2 min

After calculating the fade slope for all investigated attenuation levels with (1) the CPDFs of fade slope are estimated by Gaussian ditsribution functions (2). The attenuation dependent standard deviation (σ_ς) can be estimated with two functions. For lower attenuation level a quite well fitting power function was found, while for higher attenuation levels a linear function was found (7).

$$\sigma_\varsigma = \begin{cases} h\cdot\left(\dfrac{A}{0.05}+1\right)^i + j & , A<1\,dB \\ k\cdot\left(\dfrac{A}{0.05}+1\right)+l & , A\geqslant 1\,dB \end{cases} \tag{7}$$

The values of the h,i,j,k and l experimental parameters are listed in Table 5. The attenuation dependent standard deviation of the Gaussian distribution function with the fitting functions are depicted in Fig. 11 for lower attenuation levels and in Fig. 12 for higher attenuation levels.

Table 5. The experimental parameters of the Gaussian fade slope model applied for a satellite link

h	−0.008691
i	−1.488
j	0.01745
k	0.0003673
l	0.006194

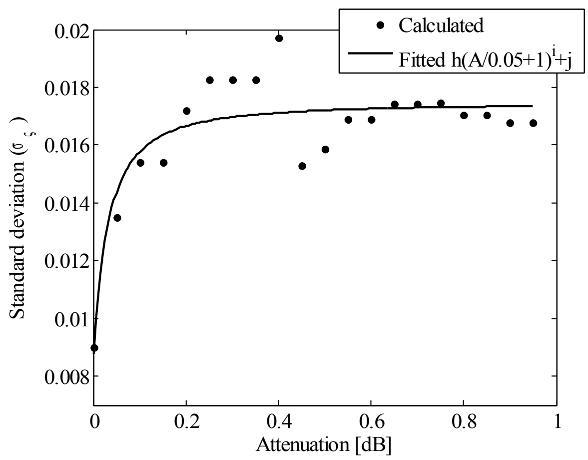

Fig. 11. The calculated and the fitted attenuation dependent standard deviation of the conditional probability density of fade slope for attenuation levels lower than 1 dB for the investigated satellite link

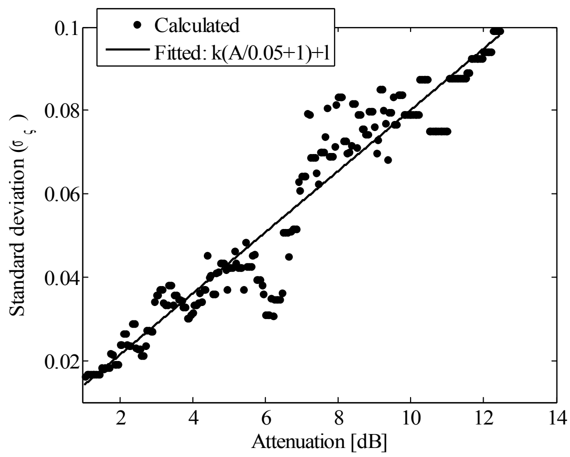

Fig. 12. The calculated and the fitted attenuation dependent standard deviation of the conditional probability density of fade slope for attenuation levels equal or higher than 1 dB for the investigated satellite link

Applying this Gaussian fade slope model with these experimental parameters for estimating the CPDF of fade slope and then calculating the transition probabilily matrix of the N-State Markov Chain model, the CCDF of the generated time series can be determined with (4). The CCDF of the generated time series and the CCDF of the measured land mobile satellite data are depicted in Fig.13.

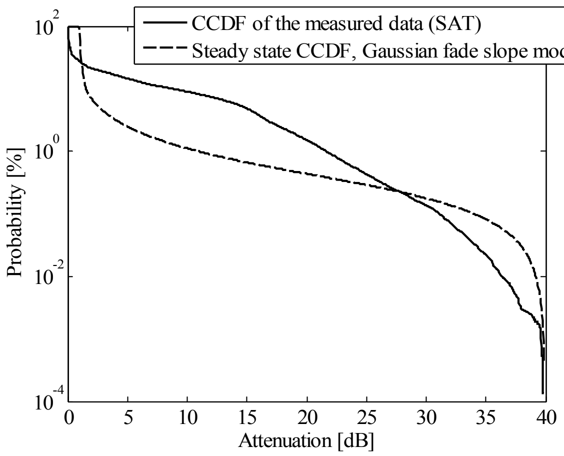

Fig. 13. The CCDFs of the mesuared data (satellite link) and of the generated time series. The p_{ij} parameters are derived from the Gaussian model

As obtained from Fig.13, the CCDF of the generated time series estimates the CCDF of the measured data well. The logarithic RMSE error, which is calculated with (5) and (6), is 0.6375. Please compare this value to the terrestrial results (Table 3), which are much more higher. So applying the N-State Markov Chain model, the CCDF of a land mobile satellite link can be estimated as well.

7 Conclusion, Future Work

In this contribution we focused on applying a general N-state Markov Chain model, which is proposed for attenuation time series generation. The model was applied on terrestrial microwave links and a land mobile satellite link as well. The transition matrix parameters were derived from the conditional probability density of fade slope, which was estimated by a Gaussian fade slope model for every attenuation level.

The attenuation dependent deviation parameter of the Gaussian fade slope model was determined from measurement performed on a terrestrial microwave link (HU11) and on a land mobile satellite link.

In the terrestrial case, as validation of the general N-state Markov Chain model, this contribution shows how well the CCDF of the generated time series estimates the CCDF of annual attenuation time series measured on an given terrestrial microwave link.

A comparison was given of the CCDF of the generated time series and of the measured data belong to three different microwave link. We found, that this CCDF estimation can be quite good, but it can be quite bad too. The main reason of this difference, that the model parameters were determined considering only one measured attenuation time series.

We expect that the CCDF of the generated time series would estimate better the CCDF of the measured attenuation data on any terrestrial microwave link, if the attenuation dependent standard deviation parameter of the applied Gaussian fade slope model was derived considering several measured attenuation time series simultaneously.

The N-State Markov Chain model was applied on a land mobile satellite link as well. The CCDF of the generated time series estimates well the CCDF of the measured land mobile satellite data. To provide an N-State Markov model, which can be applied for any land mobile satellite links, several measured attenuation time series should be considered (also in this case) by the model parameterization.

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