RAIN ATTENUATION TIME SERIES SYNTHESIS WITH SIMULATED RAIN CELL MOVEMENT

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Abstract

This paper proposes a stochastic modeling method to simulate the rain attenuation process in order to generate time series for high frequency terrestrial radio links. It is also the subject of this research to express the local attenuation differences on converging radio links belonging to the same node. The simulation scene (some tenth square kilometers) is a cellular mobile radio system which consists of a backbone node and some millimeter band radio links forming a star network around the node. The model parameters are extracted from the long-term measurement of different local meteorological parameters of an active radio network. The essentials of the model is the simulation of rain cell translation across the scene by a two dimensional non-symmetrical random walk (discrete time and discrete state homogenous Markov model). A second Markov model is applied to embed in the model the rain cell translation speed. The structure of the rain cells is approximated with an elliptical model to express the point rainfall rate in order to calculate the path attenuation at discrete time steps during the simulation. The local precipitation field allows the evaluation of the rainfall impact on the considered radio links in terms of single-link attenuation distributions. The model is also applicable to simulate the annual attenuation statistics which will be compared with the real measurement statistics for evaluation purposes.

Introduction

After the brief introduction of the simulation scene, the random walk Markov model and its parameterization are described [1, 2]. It is followed by the introduction of the rain cell translation speed model. Afterwards the elliptical rain cell model will be reviewed which is applied to express the point rainfall rate along the radio link affected by the rain cell [3, 4, 5]. In order to simulate the long term attenuation statistics of the radio links, several rain cells has to be moved over the simulation area. The peak rain rate of the individual cells is weighted by the measured annual rainfall rate statistics, allowing a realistic approximation of the rain cell distribution. Finally a simulation of multiple rain cell movement will be shown and the attenuation statistics relative to the radio links are calculated and compared with the measured ones.

The simulation scene

The simulation scene consists of star-structured high frequency terrestrial radio links in a GSM network, named HU11-HU13, whose geometry is depicted in Figure 1. and their characteristics are detailed in Table 1.

Table 1. Characteristics of the star-structured radio links

Link Code	Frequency band [GHz]	Polarization	Length [Km]	Azimuth [deg]
HU11	38	horizontal	1.5	238
HU12	38	vertical	2.98	272
HU13	23	horizontal	2.4	10

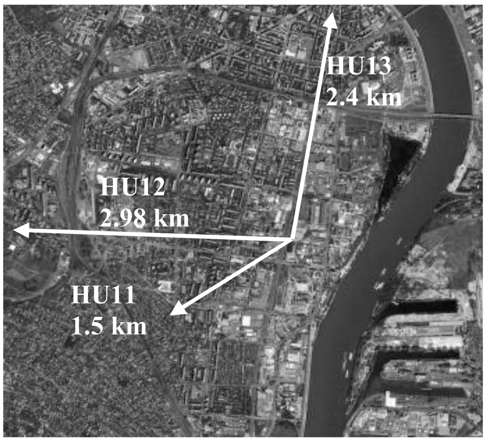

Figure 1. Link topology around the backbone node
(Location Budapest, city area)

Modeling the rain cell translation

To simulate the rain attenuation and generate attenuation time series we model the meteorological environment of the radio link area by rain cell movement simulation. To achieve our goal the translation model of rain cells simulates the direction and speed of their movement. At first the moving direction model will be introduced, followed by the description of the translation speed simulation.

In order to model the translation direction of the rain cells on the scene, a two dimensional non-symmetrical random walk is applied [2]. A four-state discrete time and state, homogenous Markov chain is envisaged (Figure 2.), for which each state represents one of the four main directions of the cell movement (up/down/left/right). The transition matrix $\overline{\overline{WD}}$ of the Markov model contains the transition probabilities wd_{ij} $(1 \leq i,\ j \leq 4)$ of the rain cell step between the four different directions.

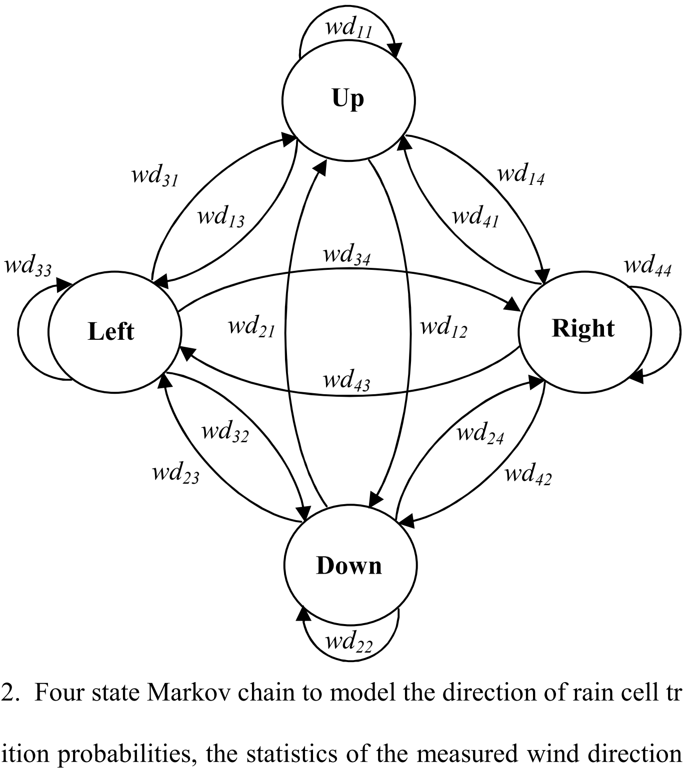

Figure 2. Four state Markov chain to model the direction of rain cell translation

In order to determine the transition probabilities, the statistics of the measured wind direction have been considered (location: Budapest, year: 2004, sampling rate: 1 sample/min). Figure 3. shows the histogram of the measured wind direction angle (in degrees, measured clockwise from North) for the above mentioned period.

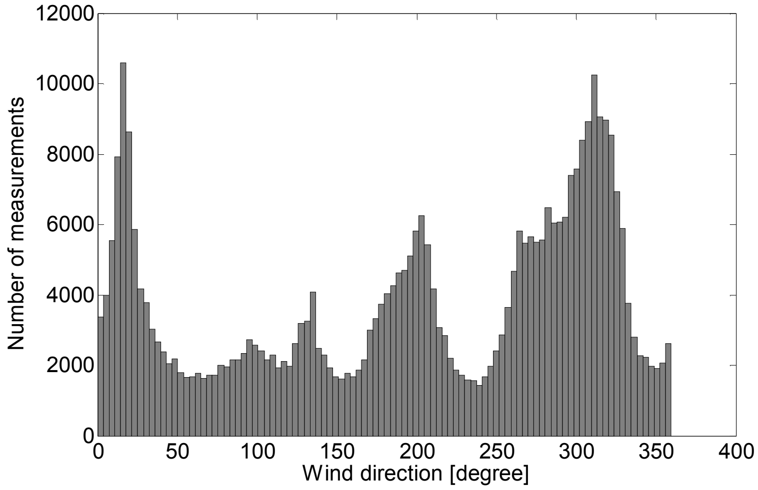

Figure 3. Measured wind direction statistics. The angle is given in degrees; measured clockwise from North.

At first, all measured wind directions have been uniformly grouped into four main classes which have been afterwards assigned to the Markov chain states (see Table 2.).

Table 2: Quantization of the wind direction

Angle range [degree]	Direction	Markov chain state
$315° < \varphi < 360°$ and $0° \le \varphi < 45°$	Up	1
$135° < \varphi < 225°$	Down	2
$225° \le \varphi \le 315°$	Left	3
$45° \le \varphi \le 135°$	Right	4

The transition probabilities wd_{ij} can be calculated with (1), where N is the total number of samples, $D(k)$ and $D(k+1)$ are denoting one of the relative frequencies the four main observable directions:

$$wd_{ij} = \frac{\sum_{k=1}^{N}(D(k+1)=j \mid D(k)=i)}{\sum_{k=1}^{N} D(k)=i}, \ 1 \le i,j \le 4 \tag{1}$$

The application of (1) to the measured wind directions leads to the determination of $\overline{\overline{WD}}$ as it follows:

$$\overline{\overline{\mathrm{WD}}} = \begin{bmatrix} 0.8697 & 0.0009 & 0.0921 & 0.0373 \\ 0.0016 & 0.9087 & 0.0380 & 0.0517 \\ 0.0994 & 0.0263 & 0.8725 & 0.0017 \\ 0.0876 & 0.0824 & 0.0049 & 0.8251 \end{bmatrix} \tag{2}$$

The transition matrix $\overline{\overline{WD}}$ defines a two-dimensional random walk on an infinite plane. Each translation step of a rain cell on this plane is performed by the transition probabilities defined by $\overline{\overline{WD}}$.
The rain cell position after n steps can be calculated from $Z^{(0)}$, defined as the initial probability vector, and the transition matrix $\overline{\overline{WD}}$, according to the following equation:

$$Z^{(n)} = Z^{(0)}\overline{\overline{WD}}^{\,n} \tag{3}$$

The highest probability element in the $Z^{(n)}$ vector is the instantaneous translation direction at step n.

As the wd_{ij} probabilities are determined from the measured wind direction statistics, this Markov chain is applicable to simulate the translation of a rain cell, starting from an arbitrary state and a random position on the scene, until the cell quits the simulation area.

To take into account the rain cell translation velocity, accordingly the cell movement size between two simulation steps a second Markov chain model is applied. This model is based on the wind speed measurement at the simulation area (location: Budapest, year: 2004, sampling rate: 1 sample/min). Its statistics is presented in Figure 4. in the form of histogram.

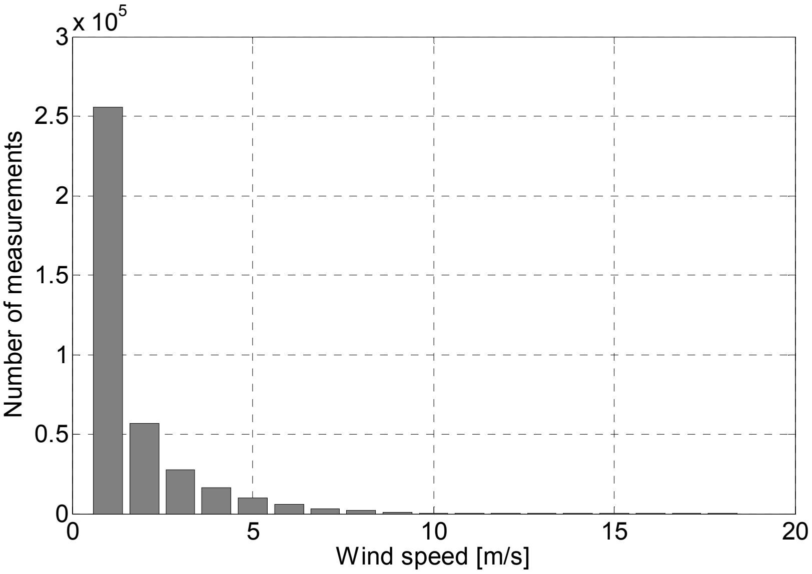

Figure 4. Measured wind speed statistics.

After a quantization of the measurements to the nearest integer value a finite state Markov chain (Figure 5.) can be assigned to the wind speed process, where each state of the chain represents a discrete wind speed value (see Figure 4.). The values are between 0 and 17 m/s, according to the measurement data.

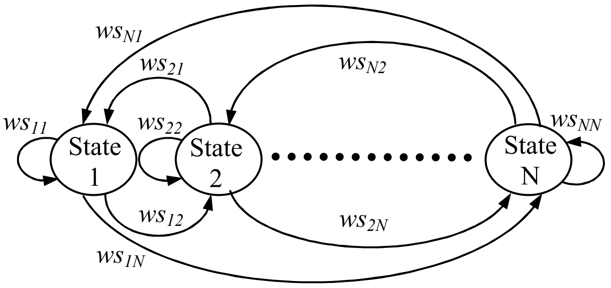

Figure 5. Markov model of the wind speed

The transition probabilities between the states can be determined from the measured wind speed statistics in a similar manner than in the case of the wind direction, taking into account that the number of states N is depending on the quantized wind speed measurement statistics (see Figure 4.), which is in our case $N=18$.

$$ws_{ij} = \frac{\sum_{k=1}^{N} (S(k+1) = j \mid S(k) = i)}{\sum_{k=1}^{N} S(k) = i}, \quad 1 \le i,\ j \le 18 \tag{4}$$

In the above formula $S(k)$ and $S(k+1)$ are the observed relative frequencies of the quantized wind speed measurements.
The combination of the 2D random walk model and the wind speed model, each parallel step of these Markov chain models is simulating a realistic rain cell movement both in direction and length. Considering that each transition of the Markov model

represents the duration of 1 minute (corresponding to the original sampling rate of the measured wind directions and speeds), every step determines the exact direction and size of rain cell translation. The n^{th} simulation step of the model results d_n rain cell translation in meters, if ws_n is the actual state in the wind speed Markov chain:

$$d_n = ws_n / 60\,[m] \tag{5}$$

Figure 6.Figure shows a realization of a rain cell movement on the scene for the duration of 48 hours. The figure axes are indicating the position of the rain cell relative to the starting point (x = -12.5 km, y = 12.5 km).
The final cell positions are x = 20.38 km, y = -9.04 km.

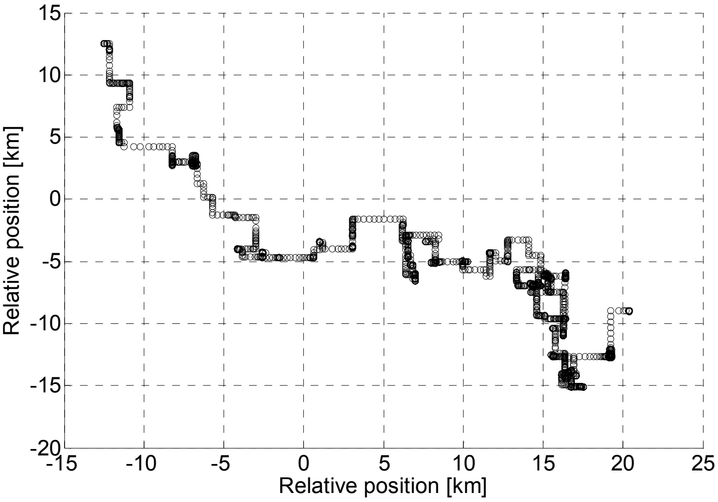

Figure 6. Realization of a rain cell movement on the scene according to the Markov chain model (duration of 48 hours)

It can be observed in the histogram of the measured wind direction (Figure 3.Figure) that the prevailing wind direction is around 300°: this trend is also clearly visible in the rain cell motion modeled by means of the transition matrix $\overline{\overline{WD}}$ and $\overline{\overline{WS}}$.

Modeling the rain cells and calculating their signal attenuation

In the previous section the method of the rain cell movement modeling has been described, which is applicable to predict the position of the rain cell center during the simulation period. To calculate the influence of the rainy area to the propagation, the extent of the rain cell and the point rainfall rate has to be taken into account. In [3, 4, 5] a model of the horizontal structure of the rain cell is described which is known as the EXCELL (Exponential Cell) model. This model is employed here to evaluate the impact of the local rainfall field on the radio link network.

The elliptical rain cell model gives the rain rate within the rain cell's horizontal plane according to the next equation:

$$R(x,y) = R_E \exp\left[-\left(\frac{x^2}{a_E^2}+\frac{y^2}{b_E^2}\right)^{1/2}\right] \tag{6}$$

This model applies an elliptical horizontal shape, the parameters are the following: R_E is the peak rain rate, a_E and b_E are the distances along the axes from the rain cell center for which the rain rate decreases by a factor $1/e$ with respect to R_E. The validity of the EXCELL model is restricted to the rain rates $R \geq R_{min}$ where R_{min} is the minimum rain rate what can be more or less arbitrarily chosen around 1 mm/h.

The EXCELL model allows determining the point rainfall rate for each time step causing variable signal attenuation along the path. Let us define the position of the backbone node in the microwave network as the $(0, 0)$ point of an orthogonal coordinate system and the initial position of the rain cell center by a couple of coordinates (r_{x0}, r_{y0}). Applying the Markov chains to simulate the translation of the rain cell, by (3) and (5) is possible to calculate the exact rain cell position after n time steps, identified by the couple of coordinates (r_{xn}, r_{yn}).

Once the rain cell center at step n has been determined, the rainfall rate values $R_n(x_n, y_n)$ at location (x_n, y_n) on the simulation area can be calculated using (6). The attenuation A $[dB/km]$ can be expressed by applying ITU-R recommendations [6] and [7]:

$$A = kR^{\alpha} \tag{7}$$

where *k* and α are conversion coefficients dependent on the radio wave frequency and polarization [7].
When a rain cell interacts with a link, the specific exponential profile of that cell has to be taken into account for the calculation of the total signal attenuation A_l experienced by the link. To this aim, equation (8) should be calculated numerically [8].

$$A_l = \int_0^l kR_n(x_n, y_n)^{\alpha} dl \tag{8}$$

In (8), *l* is the link length and $R_n(x_n, y_n)$ represents the rainfall rate values interacting with the link at position (x_n, y_n) at time step *n*. Figure 7.Figure shows an example when a rain cell affects the HU13 link.

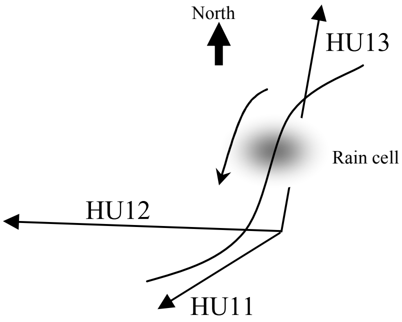

Figure 7. Rain cell transition over the link area

Simulating long term rain attenuation statistics for converging links

In the previous sections a method has been described to simulate a rain cell movement and to calculate the instantaneous path attenuation caused by the rain field. This gives the opportunity to synthesize rain attenuation time series, as depicted, as an example in Figure 8. In this example one rain cell translated over the simulation area with the next parameters: size of the simulation area *50*50 km*, nodes is centered, R_E*=50 mm/h*, a_E*=1 km*, b_E*=2 km,* duration was *2 days*. The resolution of the synthesized time series is determined by the Markov model, which is in our case one sample/minute. The simulation procedure has a significant benefit, that it is capable to produce different attenuation values for the various links of the node according to the different link parameters: frequency, polarization, length, link position and direction.

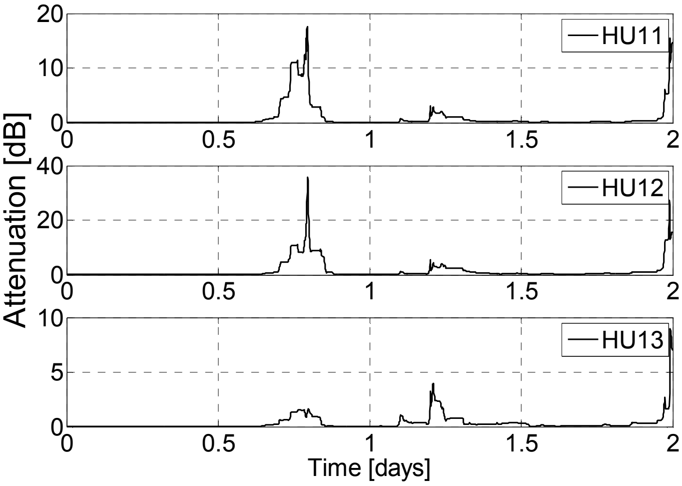

Figure 8. Simulated rain attenuation time series; translation of one rain cell over the link area

The selection of the rain cell starting position is a key task to achieve the best simulation result. The main idea what has been applied during this work that the dominant wind direction determines the most probable entry point of a new rain cell to the simulation area. Applying this assumption, the initial position of a new rain cell has been chosen at the middle of the upper left quarter of the simulation area (see Figure 3. as reference for wind directions). To simulate long term (annual) rain attenuation time series and calculate different statistics, e.g. complementary cumulative distribution function (CCDF), multiple rain cells should be translated over the simulation area. A realistic simulation can be foreseen if the duration of the simulated period is one year and the translated rain cell properties (peak rain rate and dimensions) are corresponding to the long term measured local statistics. To achieve this goal the measured rainfall rate distribution will be applied in order to weight the peak rainfall rate values of the generated rain cells. In Figure 9. the CCDF of the annual rainfall rate is depicted.

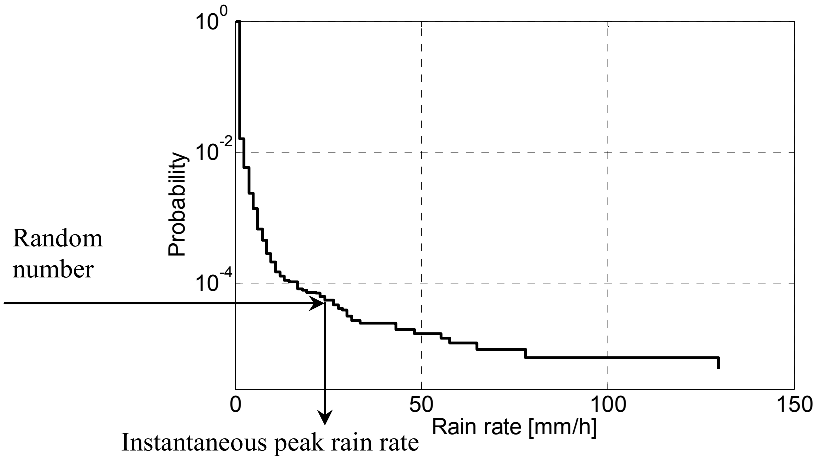

Figure 9. Annual rainfall rate CCDF; measured in 2004 and the peak rainfall rate selection method

To determine the individual peak rain rate for the actual cell an inverse transformation of the measured rainfall rate CCDF is required. A uniformly distributed random number between 0 and 1 is generated to select the actual peak rainfall rate probability. The corresponding rain intensity can be read from the vertical axis of the CCDF and this value will be applied as the peak rain rate in the EXCELL model. The process is graphed in Figure 9.

Further parameters of the rain cell are the a_E and b_E extents. Their values can be also specified according to the average rain cell dimensions based on local measurements. According to [9], during the simulation the rain cell extents are randomized between 0-2 km. The next enumeration summarizes the parameters applied for the annual rain attenuation CCDF simulation:

- Simulation area: 50x50 km^2; the location of HU11-13 links node is the center of the scene
- Duration of the simulation: 365 days
- Rain cell moving direction and speed are controlled by Markov chains
- Peak rain rate value R_M : randomly selected by inverse transformation of the measured rain rate CCDF
- Equivalent rain cell radius a_E and b_E: randomized between 0-2 km
- Rain cells initial position: middle of the upper left quarter of the simulation area
- Individual movement duration: 48 hours (an adequate time interval to cross the cell the whole scene)
- 5 annual simulation runs are averaged

The result of the simulation is depicted in Figure 10.:

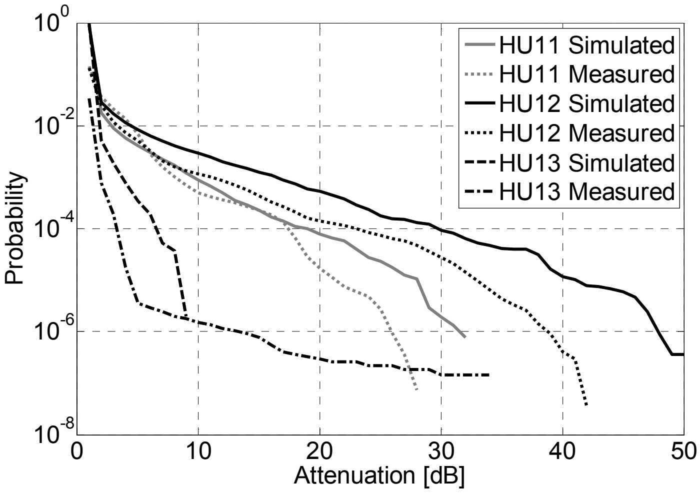

Figure 10. Simulated and measured rain attenuation CCDFs

The measured CCDF of rain attenuation is compared with the simulated CCDFs for the investigated links HU11, HU12 and HU13. It is very well observable that the model approximates with low deviation the measured statistics. The local differences due to the variant link parameters are also correctly simulated.

Conclusion

In this paper a Markov chain based stochastic model for the simulation of rain cells translation has been introduced and used to generate rain attenuation time series for converging terrestrial radio links. The method is also applicable to simulate the long term rain attenuation statistics. The cell movement model is parameterized from local wind direction and speed measurements. Several rain cells are translated over to simulation area to get the annual attenuation statistics. The impact of the individual rain cells are weighted by the measured local rainfall rate distribution. The rain cell profile has been modeled with the EXCELL method, which is applicable to express the point rainfall rate and in this way the instantaneous path attenuation can be calculated.

The simulation results have quite good correlation with the measured annual attenuation statistics relative to the individual links. The future work can be also focused on the investigation of the dynamic parameters of the synthesized time series e.g. fade and interfade duration and fade slope statistics.

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