RESTORATION OF NONLINEARLY DISTORTED OPTICAL SOUNDTRACKS USING REGULARIZED INVERSE CHARACTERISTICS

Ph.D. Theses

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1. Introduction

At the professional sound-films the sound is usually recorded optically. Optical sound-recording has several advantages: the sound can be handled together with the image during recording, copying or cutting; these methods require the same devices both for image and sound. Another advantage is that during sound reproduction the sound reproduction device will not touch the surface of the film, hence will not reduce the life expectancy of the film-roll. This is the reason why the optical sound-recording became so popular in the 1920’s and 1930’s and why it is used also today.

However, optical sound recording has a disadvantage as well. The recorded sound is nonlinearly distorted, which comes from the fact that the density characteristic of the photosensitive materials used for sound-recording – the relation between the "blackness" of the film after development and the given light exposure – is strongly nonlinear and this nonlinear behavior appears also in the recorded sound.

Optical sound recording has two different methods. One is the so-called variable intensity recording and the other one is the variable area recording. At the beginning of sound-film technology the variable intensity method was preferred. During this method the value of the current sound sample is recorded as the blackness of the current part of the sound-band on the film. Therefore at this method the nonlinear distortion appears directly as the nonlinear distortion of the density characteristic. Since the amount of blackness depends only on the value of the current sound-sample and does not depend from the previous or later samples, this kind of distortion can be modeled as a memoryless distortion.

The variable area method become popular from about the 1940’s. At this method in the ideal case the sound-band of the film contains only pure black and pure white parts. The value of the current sound sample is determined by the ratio of the black and white areas. In the non-ideal case the nonlinear distortion also appears here. In practice the transition between the white and black area is not a sudden jump because of the non-ideal projection of the
light beam controlled by the sound and the light diffusion in the film material. In this case nonlinear distortion will appear and this distortion at a certain sample will be influenced by the previous and next samples. Therefore this distortion can be represented as a nonlinear distortion with memory.

During sound reproduction the nonlinear distortion is quite disturbing. The distorted sound is tiresome for the audience because they have to make stronger efforts to concentrate to the artistic part of the film. Sometimes the intelligibility of the film is also reduced. However, the nonlinear distortion can be compensated.

Formerly the only way to reduce the nonlinear distortion was the proper copying of the distorted sound to another film roll with special light and developing parameters. With the special parameters, during copying, the technician used a strongly distorted part of the new film roll that was able to partly compensate of the original distortion. However this solution requires several experiments, since the exact distortion of the sound is not known. Also a disadvantage of this method is that the distortion can only be partly compensated, because none of the parts of the density characteristic is the perfect inverse of another part. Another disadvantage is that after copying the noise level of the sound will be high that comes from the granularity of the photosensitive material, which causes small fluctuations in the light beam during sound-reproduction, therefore it will appear in the sound as noise.

These problems can be avoided, if the sound is compensated by digital sound-processing techniques. Today there are several sound-restoration methods already, which are able to remove clicks, cracklings, broad-band noise or pitch defects, however there are relatively small emphasize on the elimination of non-linear defects. It is the topic of current research interests in DSP for audio. The aim of my research was to clarify the reasons of nonlinear distortions in the case of optical soundtracks and propose methods based on digital signal processing to reduce the distortion and avoid the appearance of artefacts in the restored sound.
2. Research methods

The quality of old films degrades more and more as time goes by. The reason is the aging of the carrier of film. There are thousands of old films in the archives, which are waiting for restoration, therefore there is only a little time for each film restoration. The variable intensity based films are older and there is the higher need for rescue. This is the reason why I have chosen the sound restoration of variable intensity based films.

The nonlinear distortion of variable intensity based optical recording can be represented as a memoryless nonlinear distortion:

\[ y(t) = N(x(t)), \quad (1) \]

where \( x(t) \) is the original, undistorted signal at time \( t \), \( N() \) is the nonlinearity of the system and \( y(t) \) is the nonlinearly distorted signal in time \( t \).

Compensation of nonlinear distortions can be divided into three groups:

- If we can modify the structure of the system, we can re-design it in order to reduce the nonlinear distortion.

- If we can’t modify the structure, but we have access to the input, we can pre-distort the original input signal to compensate the distortion.

\[ \hat{x} = N(P(x)), \quad (2) \]

where \( P() \) is the pre-distorter nonlinearity and \( \hat{x} \) the estimate about the original signal.

- If we have neither access to the structure, nor to the input, we can post-process the output signal to compensate the distortion:

\[ \hat{x} = K(y) = K(N(x)), \quad (3) \]

where \( K() \) is the compensation characteristic.
Since in the case of old movies we have only the film-roll that contains the distorted and noisy sound, the first two possibilities cannot be used. The solution could be only a kind of post-compensation.

If the nonlinear function is invertible, the post compensation can be made perfectly in ideal case. However, in practice the original signal cannot be restored exactly, because the restoration is limited by the noises, which appear during recording and sound-reproduction. If we take the noise into account that appears at the output then eq. (3) will be modified to

\[
\hat{x} = K(y + n) = K(N(x) + n),
\]

(4)

where \( n \) is the output noise that is assumed to have zero mean and be independent from the input signal.

In this case one problem is that the restoration could be ill-posed in the case of strong nonlinear distortions, which means that small disturbances at \( y \) will cause high deviations in the restored signal.

The ill-posedness can be shown, if we examine the equation of the compensation for small changes of \( n \):

\[
\hat{x}\big|_{x=x_0} = K(N(x_0) + n) \approx K(N(x_0)) + \frac{dK(x)}{dx}\big|_{x=x_0} \cdot n.
\]

(5)

It can be seen that if the inverse nonlinear characteristic has high derivative at a given \( x_0 \) point then the noise will be strongly amplified, which makes the result of the restoration completely useless. We could get better results with a special characteristic that produces exact results, if the derivative of the exact inverse is small and damps the noise, when the derivative of the original, exact inverse would amplify the noise too much.

Another, a bit different problem from the previous one is that the output noise will also be distorted during the restoration process. This distortion can make the measurement of certain parameters inaccurate (e.g. the amplitude of the restored signal or the estimate value of it). We can determine the expected value of the compensated signal as

\[
E\{\hat{x}\} = \int_{-\infty}^{\infty} f_{\hat{x}}(\hat{x}) \hat{x} \, d\hat{x} = \int_{-\infty}^{\infty} f_n(n) \cdot K(N(x) + n) \, dn,
\]

(6)
where \( E\{a\} \) means the expected value of the probability variable \( a \) and \( f_a(a) \) is the probability density function of \( a \). It means that the expected value of the restored data at a given time point will not be equal with the expected value of the original data in that time point. Due to the noise, the expected value of \( \hat{x} \) will not be \( K(E\{y\}) \), but \( K(E\{y\}) \) where \( K() \) is the correlation of the noise probability function and the compensation characteristic:

\[
R_{f_n(n), K(n)}(z) = \int_{-\infty}^{\infty} f_n(n) \cdot K(z + n) \, dn. \tag{7}
\]

To avoid this, and to make an unbiased estimate, \( \hat{x} \) from \( o \), we have to make such a \( K(o) \) compensation characteristic, where

\[
\int_{-\infty}^{\infty} p_n(o - y) K(o) \, do = \hat{x} = N^{-1}(y), \tag{8}
\]

which means that the correlation of \( K(o) \) and \( p_n(n) \) have to be the inverse of \( N() \).

Present nonlinear compensation methods at different fields are based on iterative methods. The disadvantage of these methods are that they require high computational power and in several cases it is not possible to determine the velocity of convergence of these algorithms, therefore adapting them to real-time applications is quite difficult. My aim was to develop a method that is fast and requires as less iteration and human interaction as possible.
3. New scientific results

I developed two new methods for compensation of memoryless distortions in noisy environment. The aim of the first method is to make an estimate about the original signal that is close to that one in least squares sense. The aim of the second method is to produce an unbiased estimate about the original signal. I examined the convergence of the algorithms and I proposed a method to determine the optimal value of the parameters of the methods.

I. Thesis: I have developed a new, non-iterative method for the post-compensation of memoryless nonlinear distortions, which is based on Tikhonov-regularization:

I/1. I shown that the derivative of the Tikhonov compensation characteristic can be calculated as follows (PhD thesis, section 6.4, pp. 50–53):

\[
\frac{dK(y)}{dy} \bigg|_{y=N(x_0)} = \frac{dN(x)}{dx} \bigg|_{x=x_0} \left( \frac{dN(x)}{dx} \right)^2 + \lambda
\]

(9)

where \( K(y) \) is the compensaton characteristic, \( N(x) \) is the original nonlinear function and \( \lambda \) is the so-called Tikhonov regularization parameter that finds a trade-off between the strong noise amplification and the high nonlinear distortion.

From eq. (9), \( K(y) \) can be computed by numerical integration. The integration constant can be estimated with the following equation:

\[
E\{N(F(y) + C)\} = E\{y\},
\]

(10)

where \( F(y) \) is the numerical integral of eq. (9) with an arbitrarily chosen integration constant and \( C \) is a modification value of the chosen integration constant.

I/2. I shown that the previous method at \( \lambda = \frac{E\{n^2\}}{E\{x^2\}} \) will provide a solution that is close to the optimal solution in least-squares sense.

If we have no other way to calculate a more accurate regularization parameter, this estimate will give good results. Another advantage
of this estimate is that it can be easily calculated (PhD thesis, section 6.4.1, pp. 53–60).

I/3. In practice we do not have information about the original nonlinear distortion. In the case of optical soundtracks we also have this problem. In this case, using certain attributes of the input signal and the nonlinear distortion, the characteristic of the nonlinearity can be determined by blind identification. If the examined part of the input signal is assumed to be periodic and the generally used parametric formula of the density characteristic is known, then a method can be given that produces good estimate about the parameters of the density characteristic. (PhD thesis, section 6.2, pp. 46–58).

In the method the input signal is modeled with a finite length Fourier-series: a fundamental sinusoid and it’s harmonics. (The length is about 100 samples in which region the signal can be treated to be periodic.) The model of the nonlinearity is the widely used γ function:

$$y = x^\gamma.$$  \hspace{1cm} (11)

The offset of the input ($O_1$), the offset of the output ($O_2$) and the output amplification are separate parameters ($G$):

$$y = G \cdot (x + O_1)^\gamma + O_2.$$  \hspace{1cm} (12)

The phase and amplitude of the sinusoids, furthermore the parameters of the nonlinearity is optimized by Monte-Carlo method so, that the difference of the noisy, distorted sound-part and the modeled signal will be minimal in least squares sense.

II. Thesis: At the compensation method described in the previous thesis-point at a given regularization parameter value, the expected value of the energy of the error signal can be calculated as

$$E\{(x - \hat{x}(\lambda))^2\} = \int_{-\infty}^{\infty} p_x(\chi) \cdot \int_{-\infty}^{\infty} p_n(\nu) \cdot (K(N(\chi) + \nu, \lambda) - \chi)^2 d\nu d\chi,$$  \hspace{1cm} (13)
where \( p_x(x) \) is the probability distribution function of the input signal, \( x \).

The accurate value of the regularization parameter can be calculated by the minimization of eq. (13) according to \( \lambda \). However the probability distribution function of the input signal is usually not known, therefore the equation usually cannot be solved in practice.

I have developed an iterative algorithm, which is able to give a good estimate about the optimal value of the regularization parameter without the knowledge of \( x \) and \( p_x(x) \) (PhD thesis, section 6.4.2, pp. 60–64):

1. \( p_x(x)_0 = p_y(y) \).
2. Calculate \( \lambda \) by minimizing eq. (13).
3. Calculate \( K(y, \lambda) \) and \( \hat{x} \). From \( \hat{x} \), \( p_{\hat{x}}(\hat{x}) \) can be calculated.
4. \( p_x(x)_i = p_{\hat{x}}(\hat{x})_i \).
5. If the number of iterations is \( \geq N \) or the reduction of the deviation \( \leq \varepsilon \) then exit, else go back to step 2.

III. Thesis: I shown that in the case of noisy and distorted signals – in the knowledge of the probability distribution function of the noise and the shape of the nonlinear characteristic – a compensation function can be constructed, which restores the signal so that the estimate signal will be unbiased.

III/1. I shown that infinite number of characteristics exists, which are able to produce unbiased results, however most of these characteristics are not appropriate because they contain oscillations (PhD thesis, section 6.7, pp. 82–83).

III/2. I gave an iterative algorithm that is able to find a solution that does not have oscillations (PhD thesis, section 6.7, p. 84):

\[
\begin{align*}
K_0(y) &= N^{-1}(y), \\
K_i(y) &= K_{i-1}(y) + \alpha \cdot (N^{-1}(y) - K_{i-1}(y) \ast f_n(y)),
\end{align*}
\] (14)
where $\ast$ denotes correlation, $K_i()$ is the $i^{th}$ iteration of the compensation characteristic, $N^{-1}$ is the exact inverse of the original nonlinearity, $f_n(y)$ is the probability distribution function of noise and $\alpha$ is the parameter, which influences the convergence.

III/3. I proved that the algorithm is convergent with a proper $\alpha$ value and the value of $\alpha$ depends only on the probability density function of the noise (PhD thesis, section 6.7.2, pp. 84–85). The constraint of convergence is:

$$\alpha \leq \frac{1}{\max(|F_n(f)|)},$$

where $F_n(f)$ is the Fourier-transform of $f_n$. 

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4. Possibilities of practical applications

The film-sound that is recorded by variable intensity method is extremely sensitive to the parameters of recording and development. At high signal level of sound, or at wrong working point of the density curve, the recorded sound can be strongly distorted. This distortion can be modeled by a memoryless nonlinearity.

The widely used method for the reduction of this kind of distortion is the copying of the film-sound to a new film-roll with different exposure and development parameters. With appropriate parameters, a proper part of the density characteristic can be used, which is able to partly compensate the original distortion. However, with this method the optimal compensation cannot be achieved, moreover the optimization of the exposure and development parameters can be made only manually, which means human interaction and long processing time.

I shown with simulations that with the method describe in the first thesis, noisy and distorted signals can be restored efficiently and with good quality. I shown with film-parts provided by the Hungarian National Film Archive that the method can be used also in practice.

The algorithm is already implemented as a part of a commercial sound-restoration software. The testing of the algorithm is still in progress.
5. Scientific publications

Articles appeared in international transactions


Articles appeared in international conference-books


3. Tamás B. Bakó, Balázs Bank, Tamás Dabóczy, "Restoration of Nonlinearly Distorted Audio with the Application to Old Motion Pictures", AES 20th International Conference on Archiving, Restoration and New Methods of Recording, Budapest, Hungary, Oct 5-7, 2001, No. 88-650002, pp. 191-198. referenced

Proceedings of the IEEE Instrumentation and Measurement Technology Conference, No. 00CH37276, pp. 1099-1102. referenced


Articles appeared in Hungarian conference-books


6. Bakó Tamás, "Restoration of film-sound – a practical application of nonlinear compensation, in Hungarian (Filmhang restaurálás: a nemlineáris kompenzálás egy gyakorlati alkalmazása)", Pro Scientia Ara-
Electronic publications