Performance Improvement of Differential Chaos Shift Keying Modulation Scheme

PhD Thesis

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1

Introduction

1.1 MOTIVATIONS

Chaotic circuits and systems are deterministic dynamical systems which are operating in the unstable region from the conventional engineering point of view. The operation in unstable region is a necessary but not a sufficient condition to be satisfied if a chaotic behavior has to be achieved. In spite of the fact that the chaotic signals are generated by deterministic circuitry, they look like a random signal in the time domain, and their spectrum is very similar to that of a band-limited white noise. However, the chaotic signals are bounded and they can be generated in any frequency band and at any power level with very simple circuitry. They can be generated by either conventional circuits (e.g. chaotic analog phase-locked loop) or circuits developed for this purpose (e.g. Chua’s circuit).

In the last one hundred years, the behavior of chaotic systems has been analyzed mainly from theoretical point of view [Poi90, Poi92, Can83, Lya93, Poi05, Bir27, Bir32, Lya47]. In electrical engineering, the intensive study of chaotic phenomenon was started at the end of 1980s [MCT84]. Since them, an appropriate mathematical terminology to describe the chaotic phenomenon, and tools for the analysis of chaotic systems and circuits have been developed and the physical explanation of the chaotic behavior has been found. A lot of case-studies have been performed to describe the behavior of chaotic systems and circuits.

Since the discovery of the chaotic phenomenon, a lot of research efforts have been done to find engineering applications for this new phenomenon. Many promising possible applications have been identified; among which one is the application of chaotic signals in telecommunications engineering.

There are many radio data communications applications where the conventional narrowband systems fail to operate. For example, if the telecommunications have to be established in a multipath environment (see mobile radio communications, indoor radio or wireless local area network) or in an industrial environment where huge spikes disturb the radio channel, or if the radio channel is not time-invariant then the conventional narrowband radio systems cannot be used. The frequency hopping and direct sequence spread spectrum techniques offer a solution to these problems. However, those
techniques require a complex circuitry, the synchronization of spreading codes has to be achieved and maintained at the receiver and due to the complex system configuration these systems have a high power consumption.

The chaotic communications, a brand new approach, offers an alternative solution to these problems. In chaotic communications, the digital information to be transmitted is mapped directly to chaotic, i.e., to inherently wideband signals. Then these wideband chaotic signals are transmitted via the telecommunications channel. Since chaotic signals are wideband signals, the chaotic telecommunications systems are not sensitive to the multipath propagation and due to the low spectral density of transmitted signal they do not cause interference in the narrowband systems operating in the same frequency band. Due to its simple circuitry, the chaotic telecommunications technique offers a cheap alternative to the wideband communications.

Over the past ten years many different modulation schemes have been proposed for chaotic communications. Of these, the Differential Chaos Shift Keying (DCSK) [KVSA96] and FM-DCSK [KKK97b] techniques, offer the best system performance. However, even the noise performance of these schemes is a few dB behind that of the conventional ones in additive white Gaussian noise channels. Hence, any improvement in the noise performance of DCSK/FM-DCSK has a great importance.

The goal of this thesis is to find suitable noise performance improvement methods for these systems and to give design rules for the transmitter part of the FM-DCSK system\(^1\). Since the theory and applications of chaotic signals is new area in electrical engineering, a short survey of chaotic signals and their application in telecommunications is given in this introductory chapter before discussing the performance improvement methods.

### 1.2 Chaotic Signals and Systems

Since Newton, one of the basic idea of the science has been that, due to the laws of cause and effect, for a dynamical system if the initial conditions and its system governing equation are known, its evolution can be predicted for all time, i.e., the behavior of the dynamical systems is completely predictable. However, it has been realized about one hundred years ago that the evolution of many nonlinear dynamical systems do not follow simple, regular, and predictable trajectories. These systems evolves in a random-like, but well-defined, fashion. As long as the process involved is nonlinear, even a simple deterministic model may develop such complex behavior. In general sense, this behavior is known as chaos.

Although the notion of chaotic behavior in nonlinear dynamical systems has been discovered in mathematics more than one hundred years ago, in the field of engineering this phenomenon was misinterpreted as noise or it was described as “strange phenomenon” which should be avoided.

By contrast, today scientists from many disciplines have recognized the benefits of chaotic systems and try to exploit their particular properties. Chaos has been reported in many scientific discipline such as: biology, chemistry, engineering, meteorology, physics, social sciences, etc. Today, the challenge from “how to avoid” the chaotic behavior has been changed to “how to control” or to “how to exploit” it.

\(^1\)The design rules for an FM-DCSK receiver has been elaborated by Gábor Kis in the framework of an Open LTR Esprit Project #31103 called INSPECT and financed by the European Commission. His results are presented in [Kis03].
This thesis is focused on a potential application area of chaotic signal, namely: on digital communications using chaotic carriers.

1.2.1 Autonomous Dynamical Systems

Dynamical systems can be classified in autonomous and non-autonomous dynamical systems.

The governing equation of a non-autonomous dynamical systems depends on the time, while the governing system equation of an autonomous dynamical systems is independent of the time. The non-autonomous dynamical systems will not be considered here, because:

- any non-autonomous system can be transformed to an equivalent higher order autonomous system, and
- the chaotic systems used in chaotic communications schemes are autonomous dynamical systems.

Considering the evolution in time, the dynamical systems can be grouped in two category:

- continuous-time dynamical systems, and
- discrete-time dynamical systems.

The evolution of the state of the continuous-time dynamical systems is described by a system of ordinary differential equations (ODEs) called state equations.

On the other hand the state evolution of a discrete-time dynamical system is described by difference equations.

1.2.1.1 Autonomous Continuous-Time Dynamical Systems

A continuous-time autonomous dynamical system is defined by a system of state equations (ODEs) of the form:

\[ \dot{x} = f(x) \]  \hspace{1cm} (1.1)

where \( x(t) \in \mathbb{R}^n \) is the state, while \( \dot{x} \) denotes the derivative of \( x(t) \) with respect to time and \( f: \mathbb{R}^n \rightarrow \mathbb{R}^n \) is called the vector field. The dynamical system (1.1) is linear if the vector field \( f(x) \) is linear with respect to \( x \).

Observe that in the autonomous dynamical systems the vector field depends only on the state, i.e., it is independent of time.

Let us denote the solution of (1.1) going through \( x_0 = x(t_0) \) with \( \{\phi_t(x_0) \in \mathbb{R}^n | t \in \mathbb{R}^+ \} \). The solution \( \phi_t(x_0) \) is called the trajectory through \( x_0 \).

If the mapping \( \phi_t \) satisfies, \( \phi_{t_1+t_2} = \phi_{t_1} \circ \phi_{t_2} \) and \( \phi_0(x_0) = x \), then it is called a flow of (1.1).

1.2.1.2 Autonomous Discrete-Time Dynamical Systems

Another important class of the autonomous dynamical systems is the discrete-time systems, defined by a system of difference equations of the form:

\[ x_{k+1} = f(x_k), \quad k = 0, 1, 2, \ldots, \infty \]  \hspace{1cm} (1.2)
where $x_k \in \mathbb{R}^n$ is the state and $f$ maps the current state $x_k$ into the next state $x_{k+1}$.

The sequence of points starting from an initial condition $x_0$, denoted by $\{\phi_k(x_0) \in \mathbb{R}^n | k \in \mathbb{N}\}$, is called an orbit through $(x_0)$.

### 1.2.2 Steady-State Behaviors of Autonomous Dynamical Systems

#### 1.2.2.1 Limit Sets

In dynamical systems we have wandering and recurrent points. Wandering points correspond to a transient behavior of the system, while recurrent points belongs to the asymptotic behavior of the system.

A state $x$ of a dynamical system is called recurrent if, by waiting long enough, the trajectory through $x$ repeatedly returns arbitrarily close to $x$. Hence, the asymptotic behavior of a dynamical system corresponds to orbits of recurrent states.

A trajectory of a dynamical system starting from $x_0$ goes through a set of wandering points and as it evolves in time it may settle, in a set of points called a limit set.

A limit set $L$ is called attracting if nearby trajectories converge towards an $L$ as $t \to \infty$.

An attracting set $A$ that contains at least one orbit that comes arbitrarily close to every point in $A$ is called an attractor.

In an asymptotically stable linear system the limit set is independent of the initial conditions and is unique. By contrast, a nonlinear system may possess several different limit sets and therefore may exhibit a variety of steady-state behaviors, depending on the initial conditions.

The set of all points in the state space that converge to a particular limit set $L$ is called the basin of attraction of $L$.

Since non-attracting limit sets cannot be observed in physical systems in steady-state, the asymptotic or steady-state behavior of an electronic circuit corresponds to a motion on an attracting limit set.

#### 1.2.2.2 Equilibrium Point

The simplest steady-state behavior of a dynamical system is an equilibrium point. An equilibrium point $x_{eq}$ of an autonomous continuous-time dynamical system is the constant solution of (1.1), i.e., $x_{eq} = \phi_t(x_{eq})$ for all $t$. Thus a trajectory starting from an equilibrium point remains indefinitely at that point.

In state space, the limit set of an equilibrium point consists of a single recurrent point. In this sense we may say that the attracting limit set of an equilibrium point is a zero dimensional object.

For discrete-time autonomous systems the equivalent of equilibrium point is called fixed point and satisfies the relation:

$$f(x_{fp}) = x_{fp}.$$

---

2The equilibrium point is often called stationary point in the literature.
1.2.2.3 Periodic Steady-State

A state \( x_p \) is called periodic solution of (1.1) if exists \( T > 0 \) such that \( \phi_t(x_p) = \phi_{T+t}(x_p) \), and \( x_p \) is not an equilibrium point. The orbit \( \phi_t(x_p) \) is called a cycle.

A limit cycle \( L \) of periodic solution is an isolated periodic orbit of a dynamical system. In state space, the limit cycle is a closed curve. If the initial conditions are on the limit cycle then the state of the dynamical system evolves along the limit cycle. In this sense the limit cycle of periodic solution is a one dimensional object. The limit cycle trajectory visits every point on the curve \( L \) with a period \( T \).

In discrete-time autonomous system a set of \( K \) points \( \{x_1, x_2, \ldots, x_K\} \) is called periodic solution or period-\( K \) orbit if \( x_i = f^K(x_i) \) is satisfied, where \( f^K \) denotes the \( K \)th iterate of the map \( f \).

1.2.2.4 Quasi-periodic Steady-State

A quasi-periodic signal is one that may be expressed as a countable sum of periodic signals with frequencies whose ratios give non-rational numbers.

In state space, the quasi-periodic limit set is a \( K \)-torus, i.e., a two dimensional object.

1.2.2.5 Chaotic Steady-State

At least a third-order continuous-time nonlinear system or a first-order discrete time nonlinear system with non-invertible map is required to get a chaotic behavior.

There is no widely accepted, completely satisfactory and rigorous mathematical definition of chaos. In the literature we can find several different, but closely related definitions of chaos. The most commonly used definitions are:

"Chaos is a bounded steady-state behavior that is not an equilibrium point, not periodic, and not quasi-periodic." [PC89, Ken93a, Ken93b] and

"Chaos is a low-dimensional recurrent motion in a deterministic dynamical system which is characterized by at least one positive Lyapunov exponent \(^3\)." [Ott93]

If we analyze a chaotic systems in the state-space, or in the time- and frequency-domain they exhibits simultaneously both “randomness” and “order”.

In the state-space chaos is characterized by repeated stretching and folding of bundles of trajectories. The attracting limit set of a chaotic system is called a strange attractor.

While an equilibrium point, a limit cycle, and a \( K \)-torus are objects having integer dimension, the limit set associated with a chaotic steady-state has a more complicated structure. For example, in the case of a continuous-time system governed by a third order ODE it may be something more than a curve but less than a surface [Moo92]. In particular, a chaotic attractor is typically characterized by a non-integer dimension [Man82]. An object that has a non-integer dimension is called a fractal [Man77].

Two trajectories started from almost identical initial conditions diverge and soon become uncorrelated. Realizing that all initial conditions of a system can never be known with an infinite precision and that chaotic systems tend to amplify the small differences in

---

\(^3\)The Lyapunov exponent measures the average rate of separation of trajectories along the flow.
initial conditions as time evolves, we can conclude that, despite of Newton’s Principia, determinism do not necessarily imply predictability. Similarly, two almost identical dynamical systems starting from exactly the same initial conditions will diverge in time. This property of the chaotic systems is called *sensitive dependence on initial conditions* and gives rise to long-term unpredictability.

*In the time domain*, the chaotic signals are random-like but well-defined signals which can be predicted only in the short-term.

*In the frequency domain*, due to the non-periodicity characterizing the evolution in the time domain, chaotic signals has broad-band “noise-like” spectrum.

### 1.2.3 Chaos in Built Electronic Circuits

Chaotic behavior of many electronic circuits have been reported. These circuits are either conventional circuits operating in an unstable region such as:

- Colpitts oscillator [Ken94],
- sampling phase-locked loop [KV93, KV94]
- autonomous analog phase-locked loop (APLL) [KV95]
- switched-capacitor circuits [RVHR87],

or circuits designed exclusively to generate chaotic signals:

- Chua’s circuit [CKM86, Ken93a],
- discrete-time piecewise linear chaos generators: skew tent map [CRSM00], “Bernoulli shift” generators [DRMRV93, DRRV99].

Among these, the best two candidates for chaotic communications are the APLL and the Bernoulli shift chaotic signal generator. The chaotic behavior of these circuits will be presented in the remaining part of this section.

#### 1.2.3.1 Autonomous Analog Phase-Locked Loop

The phase-locked loops exhibits rich dynamical behavior. The possibility of chaotic signal generation by a third-order autonomous APLL was shown by computer simulation in [KV95]. Later the existence of Shil’nikov orbit which proves chaotic behavior was shown in [WEK96]. The chaotic behavior of the forth order APLL was reported in [JN98]. To the best of our knowledge these are the best chaos generators which can be used in a radio frequency (RF) FM-DCSK modulator\(^4\).

Because, the goal of this introductory chapter is to describe the starting point of the new results presented in this thesis, the summary of the results published in [KHEK97] will be repeated here for convenience. However, the chaotic behavior of the, more complex, forth order APLL published in [JK98] will be omitted here.

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\(^4\)The FM-DCSK modulation will be presented in Sec. 1.4.2. The basic idea in a nutshell is the following: the FM-DCSK modulator requires a bandpass RF signal that carries a chaotic signal by FM. The APLL generates not only a lowpass chaotic signal, which is the VCO control voltage, but also a bandpass FM modulated RF signal at the VCO output. Hence, if APLL is used as the carrier generator, the application of auxiliary FM modulator, and/or additional mixer, filter and amplifier circuits can be avoided.
The nonlinear baseband model of the APLL [Lin72, Kol99] is shown in Fig. 1.1, where $F(s)$ denotes the transfer function of the loop-filter, $K_p$ and $K_v$ are the gains of the phase detector and the VCO, respectively.

$$F(s) = A_{u0} \frac{(1 + s\tau_1)(1 + s\tau_2)}{(1 + s\tau_{p1})(1 + s\tau_{p2})} (1.3)$$

where $A_{u0}$ is the DC gain of the loop-filter. In the APLL discussed here, the poles and zeros of the loop filter are $\tau_{p1} = \tau_{p2} = 1.65$ ms and $\tau_{z1} = \tau_{z2} = 165$ µs, respectively [KHEK97].

Adopting $x_1 = d^2\theta_e/dt^2$, $x_2 = d\theta_e/dt$ and $x_3 = \theta_e$ as the state-space variables, the ODE governing the system behavior is obtained as

$$\dot{x}_1 = \frac{\Omega_0}{\tau_{p1}\tau_{p2}} - \frac{\tau_{p1} + \tau_{p2} + \tau_{z1}\tau_{z2}G_0 \cos(x_3)}{\tau_{p1}\tau_{p2}} x_1 - \frac{1 + G_0(\tau_{z1} + \tau_{z2}) \cos(x_3)}{\tau_{p1}\tau_{p2}} x_2$$

$$+ \frac{\tau_{z1}\tau_{z2}G_0}{\tau_{p1}\tau_{p2}} x_2^2 \sin(x_3) - \frac{G_0}{\tau_{p1}\tau_{p2}} \sin(x_3)$$

$$\dot{x}_2 = x_1$$

$$\dot{x}_3 = x_2$$

where $\theta_e$ is the phase error defined as $\theta_e = \theta_i - \theta_o$, $G_0 = K_pA_{u0}K_v$ is the DC closed-loop gain and $\Omega_0$ gives the frequency detuning between the reference and VCO free-running frequencies.

Hence, the control parameters of the APLL are $G_0$ and $\Omega_0$. We will consider here only the case when the frequency detuning $\Omega_0$ is zero.

The introduction of phase error makes the development of a simple APLL baseband model possible [Lin72]. However, the phase error is not bounded, that is the “strange attractor” cannot be plotted in a finite state space.

The PD output voltage $v_{PD}(t)$ and VCO control voltage $v_C(t)$ can be expressed as

$$v_{PD} = K_p \sin \theta_e$$

$$v_C = (\Omega_0 - \dot{\theta}_e)/K_v.$$  

These voltages are always bounded. Their further advantage is that they can be measured in a built APLL.

Figure 1.2 shows the 2-D view of the attractors in a third-order autonomous APLL for different control parameters. Observe, that the APLL has periodic windows which are separated by chaotic regions as a function of control parameter $G_0$. 

The phase detector (PD), implemented by an analog multiplier, compares the input phase $\theta_i$ to the output phase $\theta_o$ and produces the PD output voltage $v_{PD}$. The sum-frequency component of the PD output is suppressed by a low-pass filter included in the PD. The instantaneous VCO frequency is proportional to the VCO control voltage $v_C$. We will assume that the only source of nonlinearity is the phase detector.
Figure 1.2 2D view of attractors for $\Omega_0 = 0$ rad/s, $\tau_p = \tau_{p2} = 1.65$ ms and $\tau_{z1} = \tau_{z2} = 165$ $\mu$s. The control parameter is the closed-loop gain: (a) $G_0 = 71.7$ dB, (b) $G_0 = 81.3$ dB, (c) $G_0 = 88.6$ dB, (d) $G_0 = 90.8$ dB, (e) $G_0 = 94$ dB and (f) $G_0 = 97.3$ dB.
1.2.3.2 Bernoulli Shift Chaos Generator

The second candidate for chaotic signal generation in chaotic communications is the Bernoulli shift mapping [Ott93]. It is described by a difference equation

\[
  x_{k+1} = \begin{cases} 
    2x_k + 1 & x_k < 0 \\
    2x_k - 1 & x_k \geq 0.
  \end{cases}
\]  

(1.6)

Figure 1.3 shows the block diagram of chaos generator which has been implemented by the Centro Nacional de Microelectrónica (CNM), Spain. The main futures of this chaos generator are [DRRV99]:

- clock frequency: 10–20 MHz,
- resolution: 10 bits,
- realized in AMS 0.35 μm 2p-3m CMOS technology,
- power supply: 2.7 V.

While the APLL offers the simplest solution for an FM-DCSK transmitter if it has to be implemented directly in the RF, this generator offers slight advantages if the FM-DCSK transmitter has to be implemented in the base-band using digital circuits.

The return map of the Bernoulli shift (1.6), i.e., \( x_{k+1} \) in function of \( x_k \) is shown in Fig. 1.4(a). Figure 1.4(b) shows that two orbits starting from two nearly identical initial conditions, their difference is only \( 10^{-8} \), becomes uncorrelated after a few iterations.

Figure 1.4 “Bernoulli shift” return map (a) and two orbits starting from “slightly” different initial conditions (b). The two orbits on figure (b) are denoted by “×” and “◦” marks.
1.2.4 Conclusions

To get chaotic behavior, at least a third-order nonlinear dynamical system has to be considered in the continuous-time domain, or a first-order discrete-time systems with non-invertible map has to be used. The main properties of a chaotic dynamical system can be summarized as follows:

- **Sensitivity to initial conditions**: two trajectories started from almost identical initial conditions diverge and soon become uncorrelated;
- **Sensitivity to parameter mismatch**: two almost identical systems started the same initial conditions diverge and soon become uncorrelated;
- **Positive Lyapunov exponent(s)**, where the Lyapunov exponent(s) [GHJM97] give the average rate of separation of trajectories along the flow;
- **Long-term unpredictability**: this is a direct consequence of the sensitivity to initial conditions. Recall, that in any real system the initial conditions cannot be specified or measured with infinite precision;
- **“Random” evolution**: chaos may be defined as bounded steady-state behavior which is not an equilibrium, periodic or quasi-periodic solution;
- **“Noise-like” spectrum**: because the chaotic signals are neither periodic nor quasi-periodic signals, they have a continuous noise-like spectrum. However, the spectrum of chaotic signal generators which have been derived from sinusoidal oscillators generally has a very strong periodic component;
- **Attractors with fractal dimension**: the dimension of a chaotic attractor in the state space is a non-integer number.

Many electronic circuits can be used to generate chaotic signals. Among these, for the FM-DCSK modulation scheme the two most promising one has been survived in this section.

1.3 DIGITAL COMMUNICATIONS

In digital communications, the digital information has to be transmitted over a band-pass analog channel. Because digital information cannot be directly transmitted over an analog channel, first it has to be mapped to analog sample functions having finite duration. Based on the noisy and distorted received sample functions, the receiver must recover, more precisely estimate, the digital information transmitted.

This basic structure of digital telecommunications can be recognized in both conventional and chaotic modulation schemes. However, the application of chaotic signals as information carriers requires a few brand new solutions.

In the first period, the application of chaotic signals in telecommunications was proposed and studied by physicists and mathematicians [PC90, COS93, Has98]. These scientists where experts in nonlinear dynamics, but not familiar with the theory and practice of conventional digital communications, consequently, the modulation methods

\footnote{A system with more that one positive Lyapunov exponent is called hyper-chaotic system.}
and transceiver schemes had been developed in a heuristic manner. To avoid this mistake, to make digital communications understandable for physicists and mathematicians and to elaborate a unified framework for the study, analysis and comparison of conventional and chaotic telecommunications systems, first the basic ideas of conventional digital telecommunications will be surveyed here.

We will describe the major components of a conventional digital telecommunications system and recall that the primary source of errors is the analog channel. We will explain why a realistic channel model must include at least Additive White Gaussian Noise (AWGN) source and band-limiting. We will review the notion of Bit Error Rate (BER) as a way of comparing digital modulation schemes.

In Section 1.3.5, we will survey the basic concepts of digital modulations. While in Section 1.3.6, the theory of correlation based coherent and noncoherent receivers will be summarized.

Under poor propagation conditions, where carrier synchronization cannot be maintained, coherent detection cannot be used. In such circumstances, a noncoherent receiver offers a solution. Section 1.3.6.2 gives a comparison of coherent and noncoherent reception.

### 1.3.1 Basic Structure of Digital Communications System

Communication system theory is concerned with the transmission of information from a source to a receiver through a channel [Hay94, Pro83].

The goal of a digital communications system, shown schematically in Fig. 1.5, is to convey information from a digital information source (such as a computer, digitized speech or video, etc.) to a receiver as effectively as possible. This is accomplished by mapping the digital information to a sequence of symbols which vary some properties of an analog electromagnetic wave called the carrier. This process is called **modulation**. Modulation is always necessary because all practical telecommunications channels are band-pass analog channels which cannot transmit digital signals directly.

At the receiver, the signal to be received is selected by a channel filter, demodulated, interpreted, and the information is recovered.

![Diagram of digital communications system](image)

*Figure 1.5* Digital communications system showing source and channel coding, modulation, and channel.

Conversion of the digital information stream to an analog signal for transmission may be accompanied by encryption and coding to add end-to-end security, data compression, and error-correction capability.
Built-in error-correction is often required because real channels distort analog signals by a variety of linear and nonlinear mechanisms: attenuation, dispersion, intersymbol interference, intermodulation, PM/AM and AM/PM conversions, noise, interference, multipath effects, etc. A channel encoder introduces algorithmic redundancy into the transmitted symbol sequence that can be used to reduce the probability of incorrect decisions at the receiver.

Modulation is the process by which a symbol is transformed into an analog waveform that is suitable for transmission. Common digital modulation schemes include Amplitude Shift Keying (ASK), Phase Shift Keying (PSK), Frequency Shift Keying (FSK), Continuous Phase Modulation (CPM), where a one-to-one correspondence is established between the symbols and the amplitudes, phases, frequencies, phase and phase transitions, respectively, of a sinusoidal carrier.

The channel is the physical medium through which the information-carrying analog waveform passes as it travels from the transmitter to the receiver.

The transmitted signal is invariably corrupted in the channel. Hence, the receiver never receives exactly what was transmitted. The role of the demodulator at the receiver is to produce from the received corrupted analog signal an estimate of the transmitted symbol sequence. The role of the channel decoder is to reconstruct the original bit stream, i.e., the information, from the estimated symbol sequence. Because of disturbances added in real communications channels, error-free transmission is never possible.

Nonlinear dynamics has potential applications in several of the building blocks of a digital communications system: data compression, encryption, and modulation [Ken96]. Data compression and encryption are potentially reversible, error-free digital processes. By contrast, the transmission of an analog signal through a channel and its subsequent interpretation as a stream of digital data is inherently error-prone.

1.3.2 Minimum Requirements for a Channel Model

The definition of the telecommunications channel depends on the goal of the analysis performed. In the strict sense, the channel is the physical medium that carries the signal from the transmitter to the receiver. If the performance of a modulation scheme has to be evaluated, then the channel model contains everything being between the modulator output and the demodulator input. Even if the physical medium can be modeled by a constant attenuation, the following effects have to be taken into account:

- in order to get maximum power transfer, the input and output impedances of the circuits of telecommunications system are matched. This is why thermal noise modeled as Additive White Gaussian Noise\(^6\) is always present at the input to a radio-frequency (RF) receiver, and

- the bandwidth of the channel has to be limited by a so-called channel (selection) filter in order to suppress the unwanted input signals that are always present at the input of a radio receiver and that cause interference due to the nonlinearities of the receiver.

\(^6\)The definition for the Gaussian process is given in [Hay94, BP66]. The autocorrelation of white noise is a Dirac delta function multiplied by \(N_0/2\) and located at \(\tau = 0\), where \(N_0\) is the power spectral density of the noise.
The simplest channel model that can be justified when evaluating the performance of a modulation scheme is shown in Fig. 1.6. Note that the channel filter is used only to select the desired transmission frequency band at the receiver and not to model any frequency dependence of the physical transmission medium. If, in addition to noise and attenuation, other nonidealities of the physical transmission medium (such as frequency dependence, selective fading, interferences, multipath, etc.) have to be taken into account, then these effects should be included in the first block in Fig. 1.6.

![Diagram of channel model](image)

**Figure 1.6** Model of an Additive White Gaussian Noise channel including the frequency selectivity of the receiver.

In the model shown in Fig. 1.6, we have assumed that the received signal is corrupted by AWGN. In a real telecommunications system, the noise may not be exactly white or Gaussian. The reasons for assuming AWGN are that

- it makes calculations tractable,
- thermal noise, which is of this form, is dominant in many practical communications systems, and
- experience has shown that the relative performance of different modulation schemes determined using the AWGN channel model remains valid under real channel conditions, i.e., a scheme showing better performance than another for the AWGN model also performs better under real conditions [Pro83, Hay94].

### 1.3.3 Performance Measures

The primary source of errors in a digital communications system is the analog channel. The fundamental problem of digital communications is to maximize the effectiveness of transmission through this channel, i.e., to minimize the energy required to transmit one bit of information.

The performance of a digital communications system is measured in terms of the Bit Error Rate (BER), which gives the probability of bit errors in the received bit stream. In general, this depends on the coding scheme, the type of modulation scheme used, transmitter power, channel characteristics, and the demodulation scheme. The conventional graphical representation of performance in a linear channel with AWGN, depicted in Fig. 1.7, shows BER versus $E_b/N_0$, where $E_b$ is the energy per bit and $N_0$ is the power spectral density of the noise introduced in the channel.

For a given background noise level, the BER may be reduced by increasing the energy associated with each bit, either by transmitting with higher power or for a longer period per bit. The main challenge in digital communications is to achieve a specified BER with minimum energy per bit.
1.3.4 Factors Affecting the Choice of Modulation Scheme

For a given BER and background noise level, the main goal in the design of a digital communications system is to minimize the energy required for the transmission of each bit. The second goal is the efficient utilization of channel bandwidth. There are special applications, for example indoor radio, where other effects, such as multipath propagation, limit the overall system performance. These design requirements affect the choice of the modulation scheme to be used.

While modulation is a relatively straightforward process of mapping symbols to analog waveforms in a deterministic manner, demodulation, which is concerned with mapping corrupted stochastic analog signals back to symbols, is a more difficult and error-prone task. The theoretical background of modulation and demodulation will be discussed in the following two sections.

1.3.5 Modulation

Modulation is the process of mapping symbols to analog waveforms [the elements of the so-called “signal set” $s_m(t)$] in a deterministic manner.

1.3.5.1 Orthonormal Basis Functions

Let $s_m(t)$, $m = 1, 2, \ldots, M$ denote the elements of the signal set. Our goal is to minimize the number of special signals, called basis functions, that have to be known at the receiver to perform demodulation. Let us introduce $N$ real-valued orthonormal basis functions

$$g_j(t), \quad j = 1, 2, \ldots, N$$

where

$$\int_0^T g_l(t)g_j(t) \, dt = \begin{cases} 1, & \text{if } l = j \\ 0, & \text{elsewhere} \end{cases}$$
and $T$ denotes the symbol duration. Then each element of the signal set can be represented as a linear combination of $N$ basis functions

$$s_m(t) = \sum_{j=1}^{N} s_{mj} g_j(t), \quad \begin{cases} 0 \leq t \leq T \\ m = 1, 2, \ldots, M \end{cases}$$

(1.7)

where $N \leq M$. In conventional digital telecommunications systems, sinusoidal basis functions are used; the most common situation involves a quadrature pair of sinusoids.

1.3.5.2 Generation of Signal Set

The coefficient $s_{mj}$ in (1.7) may be thought of as the $j$th element of an $N$-dimensional signal vector $s_m$. The incoming bit stream is first transformed into a symbol sequence; the elements of the signal vector are then determined from the symbols. The signals $s_m(t)$ to be transmitted are generated as a weighted sum of basis functions as given by (1.7).

The generation of the elements of a signal set is shown in Fig. 1.8.

![Figure 1.8](image)

*Figure 1.8* Generation of the elements of a signal set.

1.3.6 Demodulation

The signal $s_m(t)$ is transmitted through an analog channel where it is possibly distorted and noise is added, i.e., at the receiver we have a distorted signal $\tilde{r}_m(t) = s_m(t) + \tilde{n}(t)$. Demodulation is the process by which the received signal $\tilde{r}_m(t)$ (a noisy and filtered analog signal) is mapped back to a sequence of symbols.

1.3.6.1 Recovery of the Signal Vector by Correlation

If we assume an ideal transmission channel then, because the basis functions are orthonormal, the elements of the signal vector can be recovered from the elements of the signal set, i.e., from the received signal, if every basis function is known at the receiver. In particular,

$$s_{mj} = \int_{0}^{T} s_m(t) g_j(t) \, dt, \quad \begin{cases} m = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{cases}$$

(1.8)

Thus, a demodulator can be thought of as a bank of $N$ correlators, each of which recover the weight $s_{mj}$ of basis function $g_j(t)$. Since there exists a one-to-one correspondence between signal vectors and symbols, the transmitted symbols can be recovered by post-processing the outputs of the correlators, and the demodulated bit stream can thus be regenerated.
1.3.6.2 Coherent and Noncoherent Correlation Receivers

The receiver must recognize the symbols sent via the channel in order to recover the information which has been transmitted. For the sake of simplicity, only the detection of a single isolated symbol is considered in the thesis, the effect of Intersymbol Interference (ISI), i.e., the interference between successive symbols is neglected [FSV89].

Recall that in chaotic communications, the digital information to be transmitted is mapped into wideband carriers. Consequently, the product of RF bandwidth $2B$ and bit duration is a large number. For example, for the built INSPECT data communications systems $BT = 17$. In these systems, the effect of ISI on the noise performance is extremely low [BK99].

Our goal is to minimize the average probability of symbol errors, i.e., to develop an optimum receiver configuration. For an AWGN channel and for the case when all symbols to be transmitted are equally likely, Maximum Likelihood (ML) detection has to be used in order to get an optimum receiver [LM93]. The ML detection method can be implemented by either correlation or matched filter receivers [Hay94].

In the case of matched filter receivers, the basis functions are stored at the receiver as the impulse responses of the matched filters. In chaotic communications systems (as will be shown in Section 1.4), the basis functions used to transmit the information differ from symbol to symbol, i.e., it is not possible to perform the demodulation with matched filters. Since the goal of this section is to introduce the basic ideas which are commonly used in both conventional and chaotic communications systems, the receivers based on the matched filter are not considered here.

Equation (1.8) shows how the signal vector can be recovered from a received signal by correlators if the $N$ basis functions $g_j(t)$ are orthonormal and are known at the receiver. Note that, in addition to the basis functions, both the symbol duration $T$ and the initial time instant of symbol transmission have to be known at the receiver. The latter data are called timing information. The idea suggested by (1.8) is exploited in the correlation receiver shown in Fig. 1.9.

In any practical telecommunications system, the received signal is corrupted by noise, i.e., the input to each of the correlators is the sum of the filtered transmitted signal
$s_m(t)$ and a sample function $n(t)$ of a zero-mean, stationary, white, Gaussian noise. The elements of the signal vector can still be estimated using correlators, although the estimates may differ from their nominal values, due to corruption in the channel.

The outputs of the correlators, called the observation vector, are the inputs to a decision circuit. The decision circuit applies the ML detection method, i.e., it chooses the signal vector from among all the possibilities that is the closest to the observation vector. Estimates of the symbols are determined from the signal vector and finally the demodulated bit sequence is recovered from the estimated symbol.

Note that in a correlation receiver all the basis functions and the timing information are required; these must be recovered from the (noisy) received signal.

**Coherent Receivers**

Receivers in which exact copies of all the basis functions are available are called coherent receivers. In practice, coherent correlation receivers are used almost exclusively to demodulate ASK, PSK, and their special cases such as Quadrature Phase Shift Keying (QPSK), $M$-ary PSK (M-PSK) and $M$-ary Quadrature Amplitude Modulation (M-QAM) signals.

**Noncoherent Receivers**

In applications where the propagation conditions are poor, the basis functions $g_j(t)$ cannot be recovered from the received signal. In these cases, the conventional solution is to use $M$-ary FSK (M-FSK, $M \geq 2$) modulation and a noncoherent receiver.

The basis functions $g_j(t)$ or the elements $s_m(t)$ of the signal set are not known in a noncoherent receiver, but one or more robust characteristics of $s_m(t)$, $m = 1, 2, \ldots, M$ can be determined. Demodulation is performed by evaluating one or more selected characteristics of the received signal.

**Relative Merits of Coherent and Noncoherent Receivers**

The real advantage of the coherent technique is not the better noise performance but the fact that huge signal sets can be generated by means of very few orthonormal basis functions [FSV89]. For example, in terrestrial digital microwave radio systems, 256 signals are typically generated using a pair of quadrature sinusoidal signals. This huge signal set results in excellent bandwidth efficiency. Moreover, the receiver must recover only one sinusoidal signal from the incoming signal.

In contrast, noncoherent techniques offer two advantages over coherent detection:

- when propagation conditions are poor, the basis functions cannot be recovered from the received signal because $\tilde{r}_m(t)$ differs too much from $s_m(t)$. In this case, a noncoherent receiver is the only possible solution.

- noncoherent receivers can, in principle, be implemented with very simple circuitry, because the basis functions do not need to be recovered.

### 1.4 DIGITAL COMMUNICATIONS USING CHAOTIC CARRIER

The aim of this section is to describe the state of the art in digital modulation schemes which use chaotic rather than periodic basis functions. Problems and opportunities
arising from the non-periodicity of chaotic signals are highlighted, solutions are proposed.

The generation of the analog signal set to be transmitted, mapping of the digital bit stream to symbols, and mapping of the symbols to the elements of signal set have been discussed in Sec. 1.3 for conventional digital communications systems. To compare conventional and chaotic communications schemes, we will use the same terminology for the discussion of chaotic modulation schemes in this section.

In conventional communications, the modulated signal consists of segments of periodic waveforms corresponding to the individual symbols. When sinusoidal basis functions are used without spread spectrum techniques, the transmitted signal is a narrowband signal. Consequently, multipath propagation (due to the reception of multiple copies of the transmitted signal traveling along different paths) can cause high attenuation or even dropout of the received narrowband signal.

A chaotic signal generator produces an inherently wideband noise like signal with robust and reproducible statistical properties [Ken93a, Ken93b, KV95]. Due to its wideband nature, a modulation scheme using chaotic basis functions is potentially more resistant to multipath propagation than one based on sinusoids.

In a conventional communications system, the analog sample function of duration $T$ which represents a symbol is a linear combination of sinusoidal basis functions and the symbol duration $T$ is an integer multiple of the period of the basis function. In a chaotic digital communications system, shown schematically in Fig. 1.10, the analog sample function of duration $T$ which represents a symbol is a chaotic basis function.

![Figure 1.10](image-url) Block diagram of a chaotic communications scheme.

The decision as to which symbol was transmitted is made by estimating some property of the received sample function [PWO95, IW97]. That property might be the energy of the chaotic sample function or the correlation measured between different parts of the transmitted signal, for example.

Since chaotic waveforms are not periodic, each sample function of duration $T$ is different even if the same symbol is transmitted repeatedly. This has the advantage that each transmitted symbol is represented by a unique analog sample function, and the correlation among different chaotic sample functions is extremely low. However, it also produces a problem associated with estimating long-term statistics of a chaotic process from sample functions of finite duration.

Chaotic digital modulation is concerned with mapping symbols to analog chaotic waveforms. In Chaos Shift Keying (CSK), information is carried in the weights of a combination of basis functions. Differential chaos shift keying (DCSK) is a variant of CSK where information is also carried in the correlation between parts of the basis functions. In the following two sections, we describe in detail the operation of the chaos
shift keying and differential chaos shift keying modulation schemes. We will concentrate on the reception of a single isolated symbol.

*Chaos Shift Keying* [KD93, DKH93] is a digital modulation scheme where chaotic signals generated by different attractors or chaotic signals generated by the same attractor but emerging from different initial conditions are used as basis functions. The number of attractors or initial conditions is equal to the number of basis functions. The attractors may be produced by the same dynamical system for different values of one ore more control parameters or by completely different dynamical systems.

Note that the shape of each basis function is not fixed in chaotic communication. This is why the elements of the signal set which are transmitted through the channel have a different shape during every symbol period $T$, even if the same symbol is transmitted. As a result, the transmitted signal is never periodic.

In the literature we can find many CSK modulation schemes. These can be categorized as:

- **CSK with one basis function**:
  - Antipodal CSK,
  - Unipodal CSK,
  - Chaos On Off Keying (COOK),
- **CSK with two basis function**
  - CSK with two different chaotic signal generator,
  - DCSK (and FM-DCSK).

From these modulation schemes, we will present only those which are related to the thesis, namely: Antipodal CSK and DCSK. For the sake of simplicity, only the binary modulation schemes will be considered.

### 1.4.1 Antipodal Chaos Shift Keying

#### 1.4.1.1 Modulation

Using the notation introduced in Sec. 1.3, the elements of the signal set for the binary antipodal CSK are defined by

$$s_m(t) = s_m g_1(t)$$

where the basis functions $g_1(t)$ is a chaotic waveform.

Bit “1” is represented by $s_1(t) = \sqrt{E_b}g_1(t)$ and bit “0” is given by $s_2(t) = -\sqrt{E_b}g_1(t)$. Equivalently,

$$s_{11} = E_b, \quad s_{21} = -E_b$$

where $E_b$ denotes the average energy per bit.

A block diagram of CSK modulator is shown in Fig. 1.11.

*Figure 1.11* CSK modulator with one basis function.
1.4.1.2 Demodulation

Coherent Correlation Receiver

In a coherent correlation receiver, shown schematically in Fig. 1.12, the reference signal \( \hat{g}_1(t) \) at the receiver is the basis function which has been recovered from the noisy filtered received signal. The observation signal \( z_{m1} \) is given by:

\[
z_{m1} = \int_{T_S}^{T} [\tilde{s}_m(t) + \tilde{n}(t)] \hat{g}_1(t) dt
= \int_{T_S}^{T} [s_{m1}\hat{g}_1(t) + \tilde{n}(t)] \hat{g}_1(t) dt
= s_{m1} \int_{T_S}^{T} \hat{g}_1(t) \hat{g}_1(t) dt + \int_{T_S}^{T} \tilde{n}(t) \hat{g}_1(t) dt
\]

where we assume that the synchronization transient lasts at most \( T_S \) seconds per bit period. In the best case, where synchronization of \( \hat{g}_1(t) \) with \( g_1(t) \) is maintained throughout the transmission, \( T_S = 0 \).

Figure 1.12 Coherent receiver for a CSK signal with one basis function.

The decision circuit decides in favor of bit “1” if \( z_{m1} > 0 \) and bit “0” if \( z_{m1} < 0 \).

A minimum requirement of the circuitry at the receiver which generates a local copy of \( g_1(t) \) is that it must do so whether \( s_{11}g_1(t) \) or \( s_{21}g_1(t) \) is received. Although several strategies for recovering the basis function \( g_1(t) \) have been proposed in the literature under the title “chaotic synchronization,” we are not aware of any chaotic synchronization technique which can recover the basis function in this way, independently of the modulation. Consequently, a coherent CSK receiver has not yet been built.

Note that \( z_{m1} \) is a random variable, whose mean value depends on the energy per bit of the chaotic signal and the “goodness” with which the basis function has been recovered [see the first term in (1.9)]. In contrast to the conventional systems, the energy per bit in chaotic communications vary from sample function to sample function. Hence,

\[
E \left[ \int_{0}^{T} g_1^2(t) dt \right] = 1
\]

where \( E[\cdot] \) denotes the expectation operator. Equation (1.10) identifies another important characteristic of chaotic modulation schemes. The basis functions are not periodic but chaotic signals which can be modeled only as stochastic processes [KKK97a].

This characteristic of chaotic basis function, called autocorrelation estimation problem [KKK97a] increases the variance of observation signal given by (1.9) and corruptions
the noise performance of chaotic communications systems. The optimum noise performance can be achieved with a chaotic modulation scheme only if the estimation problem is eliminated [Kol00c].

This can be achieved by modifying the chaotic basis function(s) such that the energy per bit is kept constant. This technique is used for example in the FM-DCSK [KKK97b] modulation scheme.

The instantaneous power of an FM signal does not depend on the modulation, provided that the latter is slowly-varying compared to the carrier. Therefore, one way to produce chaotic basis functions with constant $E_b$ is to apply a chaotic signal to the input of a frequency-modulator. The output of FM modulator is the basis function or is used to generate the basis function [KKK97b, KKKJ97].

In a noise-free channel with exact recovery of the basis function, a sufficiently wideband channel filter, and permanent synchronization, $\hat{g}_1(t) = \tilde{g}_1(t) = g_1(t)$, and $T_S = 0$. The observation variable in this case is

$$z_m = s_m \int_0^T g_1^2(t)dt \approx s_m.$$  

The performance of a digital communications system is expressed by the bit error rate (BER) [Hay94]. Figure 1.13 shows, by simulation, the theoretical upper bounds on the noise performance of coherent antipodal CSK with constant energy per bit (solid curve). Note that if $E_b$ is kept constant than the noise performance of coherent antipodal CSK is as good as that of coherent BPSK (dotted curve). However, if $E_b$ is not kept constant then the autocorrelation estimation problem [Kol02] corrupts considerably the noise performance of coherent CSK.

![Figure 1.13](image-url)

Figure 1.13 Simulated optimum noise performance of antipodal CSK with a coherent correlation receiver: with constant energy per bit $E_b$ (solid curve) and with non-constant energy per bit (dash-dot curve). Coherent BPSK is shown (dotted) for comparison.

Figure 1.13 also highlights the autocorrelation estimation problem in the case of antipodal CSK when the bit energy has non-zero variance. In this case, the effective noise level at high $E_b/N_0$ is dominated by the variance of the energy per bit $E_b = \int_0^T g_1^2(t)dt$. If $\int_0^T g_1^2(t)dt$ is kept constant, the problem disappears.
1.4.1.3 Theoretical Noise Performance of Coherent Antipodal CSK

Assuming constant $E_b$, the theoretical noise performance for coherent antipodal CSK was reported in [Kol00a]:

$$BER = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right)$$

where $N_0/2$ denotes the power spectral density of the channel noise. As with all conventional coherent modulation schemes, the noise performance does not depend on the bit duration or the RF bandwidth of the channel filter.

The theoretical predictions of (1.11) are compared with the results of computer simulations [KB99] in Fig. 1.14, where the solid curve shows the noise performance predicted by (1.11), while the results of simulations are denoted by ‘+’ marks.

![Figure 1.14](image)

Figure 1.14 Comparison between the theoretical (solid curve) and simulated (‘+’ marks) noise performances of coherent antipodal CSK.

1.4.1.4 Conclusion

Equation (1.11) shows that the noise performance of an antipodal CSK modulator and coherent correlation receiver theoretically matches that of BPSK. However, this performance can be achieved only if the following necessary conditions are satisfied:

- the energy per bit is kept constant, and
- the basis function $g_1(t)$ is recovered exactly at the receiver, independently of the modulation.

The first condition can be satisfied by using FM, for example, as described in [KKK97b]. Although several strategies for recovering the basis function $g_1(t)$ have been proposed in the literature, under the title “chaotic synchronization” [Has95], we are not aware of any chaotic synchronization technique which can regenerate the basis function exactly, independently of the modulation. If the basis function cannot be recovered exactly, the noise performance of antipodal modulation is degraded significantly.

1.4.2 Differential Chaos Shift Keying

Differential chaos shift keying [KVSA96] is a variant of CSK with two basis functions. The important feature of this scheme is that the basis functions consist of repeated
segments of a chaotic waveform, and consequently, in addition to a coherent correlation receiver, a simple differentially coherent technique can be used for demodulation.

1.4.2.1 DCSK Modulation

In binary DCSK, the two elements of the signal set are given by

\[ s_m(t) = s_{m1}g_1(t) + s_{m2}g_2(t) \]  \hspace{1cm} (1.12)

where

\[ (s_{11} \ s_{12}) = \left( \sqrt{E_b} \ 0 \right) \quad \text{and} \quad (s_{21} \ s_{22}) = \left( 0 \ \sqrt{E_b} \right). \]  \hspace{1cm} (1.13)

The DCSK, basis functions have the special form

\[ g_1(t) = \begin{cases} +\frac{1}{\sqrt{E_b}} c(t), & 0 \leq t < T/2 \\ +\frac{1}{\sqrt{E_b}} c(t - T/2), & T/2 \leq t < T \end{cases} \]  \hspace{1cm} (1.14)

\[ g_2(t) = \begin{cases} +\frac{1}{\sqrt{E_b}} c(t), & 0 \leq t < T/2 \\ -\frac{1}{\sqrt{E_b}} c(t - T/2), & T/2 \leq t < T \end{cases} \]

where \( c(t) \) is a chaotic waveform. The first half of the basis function is called the reference chip, while the second one is the information-bearing chip. This is shown schematically in Fig. 1.15.

![Figure 1.15](image-url)  

Figure 1.15 Signal \( g_j(t) \) consists of two segments, the first of which we call the reference chip and the second the information-bearing chip.

To get the best noise performance, the basis functions must be orthonormal [Pro83], i.e., they must be orthogonal and carry constant energy per bit. Observe, that due to their special structure, the DCSK basis functions are always orthogonal, independently of the actual shape of chaotic waveform \( c(t) \). In this sense the FM-DCSK is not a new modulation scheme, it is a variant of DCSK where \( E_b \) is kept constant by an auxiliary FM modulator.

In binary DCSK, bit “1” is sent by transmitting \( s_1(t) = \sqrt{E_b}g_1(t) \), while for bit “0” \( s_2(t) = \sqrt{E_b}g_2(t) \).

Figure 1.16 shows a block diagram of a DCSK modulator. The modulation driver, delay circuit and switch are used to generate the appropriate basis functions according to the modulation input \( b_m \).

1.4.2.2 Differentially Coherent DCSK Demodulator

Although coherent DCSK, can in principle achieve the noise performance of coherent FSK [Kol00c], this level of performance can be reached only if the two basis functions \( g_1(t) \) and \( g_2(t) \) are available at the receiver and if they are orthonormal.

If the recovery of chaotic basis functions is not possible then the special structure of the DCSK basis functions — it consists of a piece of chaotic waveform followed by a
non-inverted or inverted copy of itself — makes it possible to perform the demodulation
by differentially coherent detector, that is, by evaluating the correlation between the
reference and information-bearing chips [KVSA96].

This PhD thesis assumes that the differentially coherent detector configuration is
used, although there are different detector configurations, the coherent DCSK detector
for example, are not considered here.

The block diagram of a differentially coherent DCSK receiver is shown in Fig. 1.17,
where the received signal is delayed by half of the bit duration ($T/2$) and the correlation
between the received signal and the delayed copy of itself is determined.

The observation signal is defined by

$$z_m = \int_{T/2}^{T} [\tilde{s}_m(t) + \tilde{n}(t)][\tilde{s}_m(t - T/2) + \tilde{n}(t - T/2)]dt. \quad (1.15)$$

If a time invariant channel is considered or time-varying channel varies slowly com-
pared to the symbol rate, then the received and filtered DCSK signal is given by

$$\tilde{s}_m(t) = \begin{cases} \tilde{c}(t), & 0 \leq t < T/2, \\ (-1)^{m+1}\tilde{c}(t - T/2), & T/2 \leq t < T, \end{cases} \quad (1.16)$$

where $\tilde{c}(\cdot)$ is the filtered version of the chaotic signal $c(\cdot)$.

Substituting (1.16) into (1.15), the observation signal becomes

$$z_m = (-1)^{m+1} \int_{T/2}^{T} \tilde{c}^2(t - T/2)dt + \int_{T/2}^{T} \tilde{n}(t)\tilde{c}(t - T/2)dt$$

$$+ (-1)^{m+1} \int_{T/2}^{T} \tilde{c}(t - T/2)\tilde{n}(t - T/2)dt + \int_{T/2}^{T} \tilde{n}(t)\tilde{n}(t - T/2)dt \quad (1.17)$$

where $\tilde{n}(t)$ and $\tilde{n}(t - T/2)$ denote the sample functions of filtered noise that corrupt the
reference and information-bearing parts of the received signal, respectively.
At best, $\bar{c}(t) = c(t)$, giving

$$z_m = (-1)^{m+1} \int_{T/2}^{T} c^2(t-T/2)dt + \int_{T/2}^{T} \tilde{n}(t)c(t-T/2)dt$$

$$+ (-1)^{m+1} \int_{T/2}^{T} c(t-T/2)\tilde{n}(t-T/2)dt + \int_{T/2}^{T} \tilde{n}(t)\tilde{n}(t-T/2)dt.$$ (1.18)

Assume that $E_b$ is kept constant. Then $\int_{T/2}^{T} c^2(t-T/2)dt = E_b/2$ and the first term in (1.18) is equal to $\pm E_b/2$. The second, third, and fourth terms, which represent the contributions of the filtered channel noise, are zero-mean random processes. Therefore, the receiver is an unbiased estimator in this case; the threshold level of the decision circuit is zero and is independent of the SNR.

Although the fourth term in (1.18) has zero mean, it has a non-Gaussian distribution. Due to this fourth term, the distribution of the observation signal is not Gaussian and its variance increases with the bit duration $T$ and the RF bandwidth $2B$ of the channel filter. Consequently, the noise performance of DCSK with a differentially coherent receiver decreases with either increasing bit duration or filter bandwidth; this is illustrated in Fig. 1.18.

![Figure 1.18](image)

**Figure 1.18** Simulated noise performance of binary chaotic switching with DCSK basis functions and a differentially coherent DCSK modulation scheme with short ($BT = 1$, solid curve) and long ($BT = 17$, dashed curve) bit durations.

### 1.4.2.3 Theoretical Noise Performance of Differentially Coherent DCSK

The theoretical noise performance of differentially coherent DCSK was reported in [Kol00c]:

$$BER = \frac{1}{2BT} \exp \left( -\frac{E_b}{2N_0} \right) \times \sum_{i=0}^{BT-1} \left( \frac{E_b}{2N_0} \right)^i \sum_{j=i}^{BT-1} \frac{1}{2j} \left( j + BT - 1 \right).$$ (1.19)

To achieve this noise performance, the transmitted energy per bit must be kept constant [Kol00b]. This result shows that the noise performance of differentially coherent DCSK depends on both the bit duration $T$ and the RF bandwidth $2B$ of the channel filter.

Equation (1.19) also shows that, for $BT = 1$, the noise performance of differentially coherent DCSK is as good as that of noncoherent binary FSK. Of course, in this case the DCSK signal becomes a narrow-band signal and the superior multipath performance of DCSK cannot be exploited.
1.4.2.4 Conclusion

Given a noncoherent correlation receiver, the best noise performance can be achieved by orthonormal DCSK basis functions and a differentially coherent receiver.

1.4.3 Comparison of Coherent Antipodal CSK and DCSK

The noise performance of coherent antipodal CSK depends neither on the bit duration nor on the bandwidth of the telecommunications channel. In limit, the noise performance of coherent antipodal CSK is as good as that of conventional coherent BPSK. The main disadvantage of this modulation scheme is that it require chaotic synchronization. Since chaotic synchronization cannot be maintained in noisy channels [CY00] this modulation scheme is inadequate for any realistic telecommunication channel.

We have shown that FM-DCSK need not be considered as a new modulation scheme; it is simply a variant of DCSK where the power of the transmitted signal, and consequently the energy per bit, is kept constant by introducing an auxiliary FM. The main advantage of DCSK is that it do not require chaotic synchronization and it can be implemented with simple circuits.

1.5 OUTLINE OF THE THESIS

This work contains five chapters and three appendices. These are as follows:

Chapter 1 This chapter. It contains a short introduction to the theory and practice of chaotic communications systems.

Chapter 2 In chaotic communications linear filtering cannot remove much of the noise because the spectra of the chaotic signal and the noise overlap. However, by exploiting the short-term predictability of chaotic signals, a part of the additive noise can be removed. This chapter analyzes the noise reduction methods which exploits the determinism in the chaotic signals with an emphasis on their applicability to the DCSK modulation scheme.

Chapter 3 Introduce improved versions of DCSK/FM-DCSK system, give “error correction” methods for these modulation schemes and analyze their noise performance.

Chapter 4 Contains the design of an FM-DCSK transmitter implemented in the framework of the Esprit 31103–INSPECT project, financed by the European Commission.

Chapter 5 Summarizes the results and claims presented in the thesis and defines the field where future work can be focused.

Appendix A Contains the most important terminology used in chaotic communications.

Appendix B Contain a brief description of the Esprit Open Long Term Research Project, financed by the European Commission, “Innovative Signal Processing Exploiting Chaotic Dynamics” (INSPECT), project number 31103. Most of the research presented in this thesis has been carried out in the framework of that project.
Appendix C Provides a detailed description of the multipath models used in system level simulations.

Appendix D Provides a list of published or accepted publications authored or coauthored by the candidate until finishing the PhD thesis, and the list of citations by independent authors.
2 Noise Reduction in Chaotic Communications Schemes

2.1 Recovery of the Transmitted Signal from the Noisy Received Signal

In chaotic communications systems, digital information is carried by inherently wideband chaotic signals. To obtain optimum noise performance, the Signal to Noise Ratio (SNR) at the input of the demodulator has to be maximized. Linear filtering cannot remove much of the noise because the spectra of the chaotic signal and the noise overlap.

Over the past few years several methods have been proposed to decontaminate noisy chaotic signals by exploiting the short-term predictability of chaotic signals [KY90, FS91, KS93, KS97, Aba96]. A possible application area of these methods is the removal of the unwanted channel noise in chaotic communications, since an improvement in SNR results in an improvement in the noise performance.

This section is devoted to the analysis and comparison of noise reduction algorithms known from the literature in the context of chaotic communications. We will show first that the methods published by Lee and Williams in 1997 are closely related to the well-known standard gradient method. Then appropriate performance measures will be proposed and finally the attainable noise performance improvement in DCSK will be evaluated.

Since the main emphasis is on the analysis of noise reduction algorithms, the modulation carried by the chaotic signal is not considered in the remainder of the section. The problem of noise reduction in the case of modulated chaotic signals is analyzed in Sec. 2.3.

Problem Exposition

Let us assume that a chaotic signal generator at the transmitter generates an $M$-dimensional chaotic signal which is given by the following difference equation:

$$ m_{k+1} = f(m_k) \quad k = 1, 2, \ldots, N. \quad (2.1) $$

Let $m$ denote a system orbit made of $N$ consecutive $M$-dimensional points of the system (2.1), i.e.,

$$ m = [m_1, m_2, \ldots, m_N]^T. \quad (2.2) $$
At the receiver we may observe only a contaminated version of $m$, i.e., $m_k$, $k = 1, 2, \ldots, N$, is corrupted with an additive $M$-dimensional Gaussian noise $n_k$. The corrupted version of the orbit is denoted by $r$. Thus,

$$r_k = m_k + n_k \quad k = 1, 2, \ldots, N. \quad (2.3)$$

All of these three signals, $m$, $n$ and $r$, are defined on $\mathbb{R}^M \times \mathbb{R}^N$.

The goal of the noise reduction methods is to find an estimate $\hat{m}$ of the noise-free orbit $m$ from a given $r$, which gives the lowest bit error rate (BER), i.e., the best overall noise performance in the chaotic communications system.

**Problem Solution**

In order to achieve this goal, the estimation problem is considered as an optimization problem in which a cost function has to be minimized with respect to $\hat{m}$. The key idea is to exploit the a priori known dynamics of the chaotic signal generator to improve the SNR. To achieve this goal a cost function made of the sum of two terms is chosen where

- The first term $C_1(\hat{m}, r)$ ensures that the global shape of $\hat{m}$ is close to $r$.
- The second term $C_2(\hat{m})$ shows how well the estimated orbit fits in with the dynamics of the system.

The cost function to be minimized is a linear combination of the first and second terms such as

$$C(\hat{m}, r) = (1 - \Gamma)C_1(\hat{m}, r) + \Gamma C_2(\hat{m}) \quad (2.4)$$

where $\Gamma \in [0, 1]$ is a scalar weight. The goal is to find an iterative method in $\hat{m}^{(i)}$ which converges towards a minimum of (2.4).

### 2.1.1 Methods Proposed by Lee and Williams

The problem of noise reduction in contaminated chaotic signals has been addressed recently in [LW97]. The authors have generalized the ideas proposed by Kostelich and Yorke [KY90] and derived two noise reduction methods.

Heuristic arguments have been used by Lee and Williams to develop their methods. The goal of this section is to discuss Methods I and II of Lee and Williams shortly and then in Sec. 2.2 these methods will be compared to the standard gradient (Cauchy’s steepest descent) optimization technique.

#### 2.1.1.1 Design of Cost Function

In both methods, Lee and Williams expressed the cost function as:

$$C(\hat{m}, r) = C_1(\hat{m}, r) + \Gamma C_2(\hat{m}) \quad (2.5)$$

and presented two practical examples in which the closeness of $\hat{m}$ and $r$ in $C_1(\hat{m}, r)$ is measured by the Euclidean and correlation distances. In both examples they assumed that $\Gamma$ in (2.5) is 1.

First let the closeness of $\hat{m}$ and $r$ be expressed by the Euclidean distance. In this
case the cost function is obtained as

$$C_{\text{euc}}(\bar{m}, r) = C_1(\bar{m}, r) + C_2(\bar{m})$$

$$= \sum_{k=1}^{N} ||\bar{m}_k - r_k||^2$$

$$+ \sum_{k=L_2}^{N-L_1-1} \left\{ \sum_{l=1}^{L_1} ||f^l(\bar{m}_k) - \bar{m}_{k+l}||^2 + \sum_{l=1}^{L_2} ||f^{-l}(\bar{m}_k) - \bar{m}_{k-l}||^2 \right\}$$

(2.6)

where $L_1$ and $L_2$ specify the regions over which the forward and backward dynamics, respectively, of system (2.1) are considered in the cost function.

The closeness of $\bar{m}$ and $r$ can also be expressed by their correlation. Since the correlation of the two vectors also depends on their norms, to cancel this effect Lee and Williams used the following normalized cost function

$$C_{\text{corr}}(\bar{m}, r) = \left[ 1 - \frac{\sum_{k=1}^{N} \bar{m}_k^T r_k}{\sqrt{\sum_{k=1}^{N} ||\bar{m}_k||^2 \sum_{k=1}^{N} ||r_k||^2}} \right] + \sum_{k=1}^{N-1} ||f(\bar{m}_k) - \bar{m}_{k+1}||^2.$$

(2.7)

A close inspection of (2.6) and (2.7) shows that the cost functions proposed by Lee and Williams are not optimum, because

- The terms which contains the Euclidean distance of orbits are unnormalized, thus their values depends on $N$.

- In equation (2.7) the first term is expressed by the correlation distance, while the second term is an unnormalized Euclidean distance. Using these different measures of closeness in the same cost function corrupts the effect of noise reduction.

- Section 2.3 will show that the weighting factor $\Gamma$ has a strong influence on the noise reduction process. The selection of $\Gamma = 1$ does not guarantee the best efficiency in noise reduction.

A proper cost function that eliminates the shortcomings discussed above will be introduced in Sec. 2.3.

2.1.1.2 Gradient of Cost Function

At the minimum of cost function, its gradient is equal to zero. The estimate of the noise free orbit is that one where the gradient becomes zero.

Unfortunately, this approach does not guarantee the achievement of the global minimum, the estimate, generally determined by iterations, usually ends up at the closest local minimum.
For $C_{\text{euc}}(\hat{m}, r)$ the $k$th element of the gradient is

$$
\frac{\partial C_{\text{euc}}(\hat{m}, r)}{\partial m_k} = 2 \left\{ m_k^{\text{r}} - r_k^{\text{r}} + \sum_{l=1}^{L_1} D^T f^l(\hat{m}_k)[f^l(\hat{m}_k) - \hat{m}_{k+1}]^{\text{r}} - \sum_{l=1}^{L_1} [f^l(\hat{m}_{k-1}) - \hat{m}_k]^{\text{r}} + \sum_{l=1}^{L_2} D^T f^{-l}(\hat{m}_k)[f^{-l}(\hat{m}_k) - \hat{m}_{k-1}]^{\text{r}} - \sum_{l=1}^{L_2} [f^{-l}(\hat{m}_{k+1}) - \hat{m}_k]^{\text{r}} \right\}
$$

(2.8)

where

$$
D^T f^l(\hat{m}_k) = \prod_{i=0}^{l-1} D^T f(\hat{m}_{k+i})
$$

and $D^T f(\hat{m}_k)$ is the transpose of Jacobian matrix

$$
D^T f(\hat{m}_k) = \begin{pmatrix}
\frac{\partial f(\hat{m}_k)}{\partial m_{k,1}} & \frac{\partial f(\hat{m}_k)}{\partial m_{k,1}} & \ldots & \frac{\partial f(\hat{m}_k)}{\partial m_{k,1}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f(\hat{m}_k)}{\partial m_{k,M}} & \frac{\partial f(\hat{m}_k)}{\partial m_{k,M}} & \ldots & \frac{\partial f(\hat{m}_k)}{\partial m_{k,M}}
\end{pmatrix}.
$$

Similarly the gradient of cost function for the case when the closeness of $\hat{m}$ and $r$ is expressed by their correlation can be expressed in closed form. To get compact equations, we introduce the following notations

$$
R_{m} = \sum_{k=1}^{N} ||\hat{m}_k||^2 \quad R_{r} = \sum_{k=1}^{N} ||r_k||^2 \quad R_{mr} = \sum_{k=1}^{N} \hat{m}_k^T r_k.
$$

Then for the $k$th element of the gradient of $C_{\text{corr}}(\hat{m}, r)$ we obtain

$$
\frac{\partial C_{\text{corr}}(\hat{m}, r)}{\partial m_k} = r_k \sqrt{R_m R_r} - \hat{m}_k R_{mr} \sqrt{R_r / R_m} + 2D^T f(\hat{m}_k)[f(\hat{m}_k) - \hat{m}_{k+1}] - 2[f(\hat{m}_{k-1}) - \hat{m}_k]
$$

(2.9)

where $D^T f(\hat{m}_k)$ is the transpose of Jacobian.

### 2.1.1.3 Removal of Noise

To find an estimate $\hat{m}$ of the noise-free trajectory, a minimum of the cost function has to be found. This can be done by calculating the gradient of $C(\hat{m}, r)$ and setting it to zero

$$
\frac{\partial C(\hat{m}, r)}{\partial \hat{m}} = \frac{\partial C_1(\hat{m}, r)}{\partial \hat{m}} + \frac{\partial C_2(\hat{m})}{\partial \hat{m}} = 0.
$$

(2.10)

The elements of the gradient vector are expressed in closed form by (2.8) and (2.9). The solution of (2.10) gives the estimate of the noise free orbit.

Unfortunately, both (2.8) and (2.9) contain nonlinear terms which prevent the solution of (2.10). Lee and Williams have proposed two iterative techniques to solve (2.10) which they call Methods I and II [LW97]. These differ only in the iterative schemes used to solve (2.10).
2.1.1.4 Method I of Lee and Williams

Inspecting (2.8) and (2.9), Lee and Williams have recognized that the first term of each gradient is an estimate \( \hat{n} = h(\hat{m}, r) = r - \hat{m} \) of the noise or “deviation” as this term is called later in [LW97]. To get this term, the gradients given by (2.8) and (2.9) are set to 0 and rearranged in the form

\[
\hat{m} - r + h(\hat{m}, r) = 0. \tag{2.11}
\]

Comparing (2.8) and (2.9) to (2.11), \( h(\hat{m}^{(i-1)}, r) \) is obtained.

The application of Method I is illustrated by two examples in [LW97].

In Example 1, the Euclidean distance is considered in \( C_1(\hat{m}, r) \). The kth element of the deviation corresponding to (2.11) is derived from (2.10) and (2.8) rearranging them into the form of (2.11)

\[
h_k(\hat{m}^{(i-1)}, r) = \hat{n}_k
\]

\[
= \sum_{l=1}^{L_1} D^T f^l(\hat{m}_k)[f^l(\hat{m}_k) - \hat{m}_{k+1}] - \sum_{l=1}^{L_1} [f^l(\hat{m}_{k-1}) - \hat{m}_k]
+ \sum_{l=1}^{L_2} D^T f^{-l}(\hat{m}_k)[f^{-l}(\hat{m}_k) - \hat{m}_{k-1}] - \sum_{l=1}^{L_2} [f^{-l}(\hat{m}_{k+1}) - \hat{m}_k]. \tag{2.12}
\]

In Example 2, the correlation distance is considered in \( C_1(\hat{m}, r) \). The kth element of the deviation corresponding to (2.11) is derived from (2.9) and is given by (16) of [LW97] as

\[
h_k(\hat{m}^{(i-1)}, r) = \hat{n}_k
\]

\[
= 2\sqrt{R_m R_r} D^T f(\hat{m}_k)[f(\hat{m}_k) - \hat{m}_{k+1}]
- 2\sqrt{R_m R_r}[f(\hat{m}_{k-1}) - \hat{m}_k] + \left( \frac{R_{mr}}{R_m} - 1 \right) \hat{m}_k. \tag{2.13}
\]

To find the solution of (2.10), Lee and Williams have proposed the following iterative scheme, called Method I:

\[
\hat{m}_k^{(0)} = r
\]

\[
\hat{m}_k^{(i)} = \hat{m}_k^{(i-1)} - K_{m_1}^{(i-1)} h(\hat{m}^{(i-1)}, r) \tag{2.14}
\]

where \( K_{m_1} \) is a \( N \times N \) diagonal matrix whose diagonal elements \( K_{m_1}(n, n) \) take two values according to some heuristic consideration on the norm of \( h_k(\hat{m}^{(i-1)}, r) \) [LW97]. \( K_{m_1}(n, n) = K_2 \) if \( ||h_k(\hat{m}^{(i-1)})|| < \delta \) and \( K_{m_1}(n, n) = K_1 K_2 \) if \( ||h_k(\hat{m}^{(i-1)}, r)|| > \delta \).

The constant \( K_1 \) is chosen close to 0, \( \delta \) is some positive threshold, and \( K_2 \) determines the stability and convergence rate of iteration.

2.1.1.5 Method II of Lee and Williams

In Method I, appropriate values have to be found for two weighting constants \( K_1 \) and \( K_2 \). To reduce the number of required constants from two to one, Lee and Williams have proposed Method II [LW97], in which another approximation is used to get an approximate solution to (2.10).

Lee and Williams recognized by inspection that \( \hat{m}_k \) appears both in (2.8) and in (2.9). Let (2.10) be rearranged in the form

\[
\hat{m} - g(\hat{m}, r) = 0 \tag{2.15}
\]
and assume that an estimate $\hat{m}^{(i-1)}$ of the solution to (2.10) is known. Then an improved temporarily estimate of the noise free orbit can be expressed from (2.15)

$$\hat{m}_k^{(\text{temp})} = g_k(\hat{m}^{(i-1)}, r).$$

In Example 1, the Euclidean distance is considered in $C_1(\hat{m}, r)$. The $k$th element of $\hat{m}_k^{(\text{temp})}$ is derived from (2.10) and (2.8) rearranging them into the form of (2.15). The result is given after (15), page 504, in [LW97] as

$$\hat{m}_k^{(i,\text{temp})} = g_k(\hat{m}^{(i-1)}, r) = \frac{1}{1 + L_1 + L_2} \left\{ r_k - \sum_{l=1}^{L_1} D_l^T f^l(\hat{m}_k)[f^k(\hat{m}_k) - \hat{m}_{k+l}] - \sum_{l=1}^{L_2} D_l^T f^{-l}(\hat{m}_k)[f^{-l}(\hat{m}_k) - \hat{m}_{k-l}] + \sum_{l=1}^{L_1} f^l(\hat{m}_{k-l}) + \sum_{l=1}^{L_2} f^{-k}(\hat{m}_{k+l}) \right\}.$$  

(2.16)

In Example 2, the correlation distance is used to describe the closeness of $\hat{m}$ and $r$. Then the temporarily estimate takes the form (see (17) of [LW97])

$$\hat{m}_k^{(i,\text{temp})} = g_k(\hat{m}^{(i-1)}, r) = \frac{R_m}{R_{mn} + 2R_m\sqrt{R_mR_r}} \left\{ r_k - 2\sqrt{R_mR_r} D^T f(\hat{m}_k)[f(\hat{m}_k) - \hat{m}_{k+1}] + 2\sqrt{R_mR_r} f(\hat{m}_{k-1}) \right\}. \quad (2.17)$$

Using these approximate solutions, an update scheme called Method II has been proposed by Lee and Williams

$$\hat{m}^{(0)} = r$$

$$\hat{m}^{(i)} = \hat{m}^{(i-1)} + K_{m2} \left( g(\hat{m}^{(i-1)}, r) - \hat{m}^{(i-1)} \right)$$  

(2.18)

where $K_{m2}$ is a positive scalar.

### 2.2 Analysis of Methods I and II Proposed by Lee and Williams

A few heuristic arguments have been used by Lee and Williams to develop their methods [LW97]. The goal of this section is to relate Methods I and II of Lee and Williams to the standard class of optimization techniques. It clarifies the origination of the two methods by showing that they are closely related to noise reduction method that will be developed here from the standard gradient optimization technique. In addition, the optimality of the two methods is evaluated, proving that one of the methods is identical with the standard gradient method with varying step size while the other is a suboptimal one. The noise reduction performance of the Lee and Williams methods are also compared by computer simulation to that of the standard gradient method.

Our goal is to compare algorithms (2.14) and (2.18) with the gradient algorithm applied in the context of least-squares minimization. Hence we have to rewrite the functions $h(\hat{m}^{(i-1)}, r)$ and $g(\hat{m}^{(i-1)}, r)$ as functions of $\frac{\partial C(\hat{m}, r)}{\partial \hat{m}} \hat{m} - \hat{m}^{(i-1)}$. 
Theoretically the gradient of the cost function is set to 0. But recall that due to the nonlinear terms in (2.8) and (2.9), (2.10) could not be solved directly but an iteration had to be used to get an approximate solution. The approximate solution is enhanced by iteration performed by either Method I or II. Of course, the approximate solution differs from the exact one, i.e., the gradient differs from 0.

### 2.2.1 Least-Squares Optimization Using the Gradient Method

Our purpose is to show that Methods I and II are closely related to a least-squares optimization technique which uses a gradient descent approach. For the sake of comparison, let us recall here the equations of the gradient descent approach. The gradient method updates a current estimate of the system orbit in the opposite direction to the gradient of the cost function

\[
\begin{aligned}
\hat{m}^{(i)} &= \hat{m}^{(i-1)} - \mu \left[ \frac{\partial C(\hat{m}, r)}{\partial \hat{m}} \right]_{\hat{m}=\hat{m}^{(i-1)}}. \\
\end{aligned}
\]  

(2.19)

Provided that \( \mu > 0 \) is chosen small enough, the gradient method will converge to a local minimum of the cost function given by (2.4). Thus, a gradient algorithm can be obtained as:

\[
\begin{aligned}
\hat{m}^{(0)} &= r, \\
\hat{m}^{(i)} &= \hat{m}^{(i-1)} - \mu \left[ \frac{\partial C(\hat{m}, r)}{\partial \hat{m}} \right]_{\hat{m}=\hat{m}^{(i-1)}}. \\
\end{aligned}
\]  

(2.20)

### 2.2.2 Comparison of Method I with the Gradient Method

First let the closeness of \( \hat{m} \) and \( r \) be expressed by the Euclidean distance. Equation (2.8) may be rearranged as

\[
\begin{aligned}
\frac{1}{2} \frac{\partial C_{\text{euc}}(\hat{m}, r)}{\partial \hat{m}_k} &= -(r_k - \hat{m}_k) + \sum_{l=1}^{L_1} D_T f_l(\hat{m}_k)[f_l(\hat{m}_k) - \hat{m}_{k+l}] \\
&\quad - \sum_{l=1}^{L_1} [f^l(\hat{m}_{k-l}) - \hat{m}_k] + \sum_{l=1}^{L_2} D_T f^{-l}(\hat{m}_k)[f^{-l}(\hat{m}_k) - \hat{m}_{k-l}] \\
&\quad - \sum_{l=1}^{L_2} [f^{-l}(\hat{m}_{k+l}) - \hat{m}_k]. \\
\end{aligned}
\]  

(2.21)

The estimate \( h(\hat{m}^{(i-1)}, r) \) of the deviation can be obtained by comparing (2.21) with (2.12).

Substituting \( h(\hat{m}^{(i-1)}, r) \) into (2.14) the iterative scheme becomes

\[
\begin{aligned}
\hat{m}^{(0)} &= r, \\
\hat{m}^{(i)} &= \hat{m}^{(i-1)} - K_{m_1}^{(i-1)} \left[ \frac{1}{2} \frac{\partial C_{\text{euc}}(\hat{m}, r)}{\partial \hat{m}} \right]_{\hat{m}=\hat{m}^{(i-1)}} + r - \hat{m}^{(i-1)}. \\
\end{aligned}
\]  

(2.22)

This equation can be rearranged as:

\[
\begin{aligned}
\hat{m}^{(0)} &= r, \\
\hat{m}^{(i)} &= \hat{m}^{(i-1)} - \frac{K_{m_1}^{(i-1)}}{2} \left[ \frac{\partial C_{\text{euc}}(\hat{m}, r)}{\partial \hat{m}} \right]_{\hat{m}=\hat{m}^{(i-1)}} - K_{m_1}^{(i-1)} \hat{n}^{(i-1)}.
\end{aligned}
\]  

(2.23)
where \( \hat{n}^{(i-1)} = r - \hat{m}^{(i-1)} \) is the estimation of the noise at iteration \((i - 1)\).

Comparing (2.20) with (2.23) we conclude that Method I is very closely related to the gradient technique with a varying step size of

\[
\mu^{(i)} = \frac{K_{m_1}^{(i-1)}}{2}. \tag{2.24}
\]

However, the extra term \( K_{m_1}^{(i-1)} \hat{n}^{(i-1)} \) in (2.24) makes Method I to be different from the standard gradient method and prevents it to achieve a minimum of cost function.

Let the closeness of \( \hat{m} \) and \( r \) be expressed by the correlation distance. Equation (2.9) may be rearranged as

\[
\sqrt{R_m R_r} \frac{\partial C_{corr}(\hat{m}, r)}{\partial \hat{m}_k} = -(r_k - \hat{m}_k) + 2\sqrt{R_m R_r} D^T f(\hat{m}_k)[f(\hat{m}_k) - \hat{m}_{k+1}]
- 2\sqrt{R_m R_r} [f(\hat{m}_{k-1}) - \hat{m}] + \left( \frac{R_{mr}}{R_m} - 1 \right) \hat{m}_k. \tag{2.25}
\]

The estimate \( h(\hat{m}^{(i-1)}, r) \) of the deviation can be obtained by comparing (2.25) with (2.13). Substituting \( h(\hat{m}^{(i-1)}, r) \) into (2.14) the iterative scheme becomes

\[
\begin{align*}
\hat{m}^{(0)} &= r \\
\hat{m}^{(i)} &= \hat{m}^{(i-1)} - K_{m_1}^{(i-1)} \left( \sqrt{R_m R_r} \left[ \frac{\partial C_{corr}(\hat{m}, r)}{\partial \hat{m}} \right] \right) \hat{m} = \hat{m}^{(i-1)} + r - \hat{m}^{(i-1)}.
\end{align*} \tag{2.26}
\]

Thus,

\[
\begin{align*}
\hat{m}^{(0)} &= r \\
\hat{m}^{(i)} &= \hat{m}^{(i-1)} - K_{m_2}^{(i-1)} \sqrt{R_m R_r} \left[ \frac{\partial C_{corr}(\hat{m}, r)}{\partial \hat{m}} \right] \hat{m} = \hat{m}^{(i-1)} = K_{m_1}^{(i-1)} \hat{n}^{(i-1)}
\end{align*} \tag{2.27}
\]

where \( \hat{n}^{(i-1)} = r - \hat{m}^{(i-1)} \) is the estimation of the noise at iteration \((i - 1)\).

Comparing (2.20) with (2.27) we conclude again that Method I is very closely related to the gradient technique with a varying step size of

\[
\mu^{(i)} = K_{m_1}^{(i-1)} \sqrt{R_m R_r}. \tag{2.28}
\]

Observe again that an extra term \( K_{m_1}^{(i-1)} \hat{n}^{(i-1)} \) appears in (2.24).

These extra terms make Method I a suboptimum optimization technique by preventing it to achieve the minimum of the cost function. However, if the additive noise level is small enough, these extra terms become negligible and Method I will converge close to a minimum of the cost function.
2.2.3 Comparison of Method II with the Gradient Method

First let the closeness of $\hat{m}$ and $r$ be expressed by the Euclidean distance. The gradient (2.8) may be rearranged as

$$
\frac{1}{2(1 + L_1 + L_2)} \left[ \frac{\partial C_{euc}(\hat{m}, r)}{\partial \hat{m}_k} \right] \hat{m}_k = \hat{m}^{(i-1)} = \hat{m}_k - \frac{1}{1 + L_1 + L_2} \left\{ r_k - \sum_{l=1}^{L_1} D^T f^l(\hat{m}_k)[f^l(\hat{m}_k) - \hat{m}_k] 
- \sum_{l=1}^{L_2} D^T f^{-l}(\hat{m}_k)[f^{-l}(\hat{m}_k) - \hat{m}^{-1}] + \sum_{l=1}^{L_1} f^l(\hat{m}_{k-1}) + \sum_{l=1}^{L_2} f^{-l}(\hat{m}_{k+1}) \right\}.
$$

Comparing (2.29) with (2.16), $g_k(\hat{m}^{(i-1)}, r)$ is obtained as

$$
g_k(\hat{m}^{(i-1)}, r) = \hat{m}^{(i-1)} - \frac{1}{2(1 + L_1 + L_2)} \left[ \frac{\partial C_{euc}(\hat{m}, r)}{\partial \hat{m}_k} \right] \hat{m}_k = \hat{m}^{(i-1)}.
$$

Substituting (2.30) into the iterative update scheme defined by (2.18) we get

$$
\hat{m}^{(0)} = r
$$

$$
\hat{m}^{(i)} = \hat{m}^{(i-1)} - \frac{K_{m_2}}{2(1 + L_1 + L_2)} \left[ \frac{\partial C_{euc}(\hat{m}, r)}{\partial \hat{m}_k} \right] \hat{m}_k = \hat{m}^{(i-1)}.
$$

Comparing (2.31) with (2.20) we conclude that Method II is the gradient descent algorithm with a constant step size of

$$
\mu = \frac{K_{m_2}}{2(1 + L_1 + L_2)}.
$$

If the correlation distance is used in the cost function then (2.9) may be rearranged as

$$
\frac{R_m\sqrt{R_mR_r}}{R_{mr} + 2R_m\sqrt{R_mR_r}} \left[ \frac{\partial C_{corr}(\hat{m}, r)}{\partial \hat{m}_k} \right] \hat{m}_k = \hat{m}^{(i-1)} = \hat{m}_k - \frac{R_m}{R_{mr} + 2R_m\sqrt{R_mR_r}} \left\{ r_k 
- 2\sqrt{R_{mr}R_r} D^T f(\hat{m}_k)[f(\hat{m}_k) - \hat{m}_k] + 2\sqrt{R_{mr}R_r} f(\hat{m}_{k+1}) \right\}.
$$

Comparing (2.32) with (2.17), $g(\hat{m}^{(i-1)}, r)$ is expressed as a function of the gradient. Substituting it into (2.18) we obtain

$$
\hat{m}^{(0)} = r
$$

$$
\hat{m}^{(i)} = \hat{m}^{(i-1)} - \frac{R_m\sqrt{R_mR_r}}{R_{mr} + 2R_m\sqrt{R_mR_r}} \left[ \frac{\partial C_{corr}(\hat{m}, r)}{\partial \hat{m}_k} \right] \hat{m} = \hat{m}^{(i-1)}
$$

which is again the gradient descent algorithm, where the step size

$$
\mu^{(i)} = \frac{R_m\sqrt{R_mR_r}}{R_{mr} + 2R_m\sqrt{R_mR_r}}
$$

varies during the iteration.
2.2.4 Performance Evaluation and Comparison

In this section the performance of the two methods proposed by Lee and Williams and the conventional gradient method is evaluated and compared. For the performance evaluation of Methods I and II the weighting constants $K_1$, $K_2$ and $K_m^2$ proposed by Lee and Williams in [LW97] were used, while for the conventional gradient method we searched for the step size $\mu$ which offered the best convergence properties.

To perform the computer simulations, two two-dimensional chaotic systems have been used, namely the Henon and the Ikeda maps\(^1\).

For the Henon map the governing mapping are:

\[
m_{1,n+1} = 1 - 1.4m_{1,n}^2 + m_{2,n} \\
m_{2,n+1} = 0.3m_{1,n},
\]

![Graphs of noise free and noisy attractors for the 2-D Henon map.](image)

\(\text{(a)}\) Noise free attractor  \hspace{1cm} \(\text{(b)}\) Noisy attractor

\(\text{(c)}\) Attractor cleaned by method I  \hspace{1cm} \(\text{(d)}\) Attractor cleaned by method II  \hspace{1cm} \(\text{(e)}\) Attractor cleaned with the gradient method

\(\text{Figure 2.1}\) The (a) noise free, the (b) noisy (SNR=7 dB) and the (c)-(e) cleaned attractors for the 2-D Henon map. The attractor was enhanced by (c) Method I, (d) Method II and the (e) gradient descent algorithm.

Each noise reduction method was iterated for 200 times with the following parameters

- Method I: $K_1 = 0.06667$ and $K_2 = 0.003$;
- Method II: $K_m^2 = 0.08$;

\(^1\)For a chaotic communications system the use of a one-dimensional system instead of these maps is more realistic. However, to be able to compare our simulation results with those given in [LW97], the same mappings and the same number of points are considered here as in [LW97].
ANALYSIS OF METHODS I AND II PROPOSED BY LEE AND WILLIAMS

• gradient method: \( \mu = 0.05 \).

The effect of the different cleaning methods on a contaminated chaotic signal generated by the Henon map is shown in Fig. 2.1. The figures depict the noise-free, noisy (SNR = 7dB) and enhanced attractors for each method studied here.

For the Ikeda map the governing mapping is

\[
m_{n+1} = 1 + 0.9 m_n \exp \left\{ j \left( 0.4 - \frac{6.0}{1 + |m_n|^2} \right) \right\}
\]

where the first coordinate \( x_{1,n} \) and the second coordinate \( x_{2,n} \) are defined as the real and imaginary parts of the state vector \( m_n \), respectively.

\[\begin{array}{c}
\text{Noise free attractor} \\
\text{Noisy attractor}
\end{array}\]

(a) (b)

\[\begin{array}{c}
\text{Attractor cleaned by method I} \\
\text{Attractor cleaned by method II} \\
\text{Attractor cleaned with the gradient method}
\end{array}\]

(c) (d) (e)

Figure 2.2 The (a) noise free, the (b) noisy (SNR=7dB) and the (c)-(e) cleaned attractors for the 2-D Ikeda map. The attractor was enhanced by (c) Method I, (d) Method II and the (e) gradient descent algorithm.

In order to compare the performance of the different cleaning methods each of them was iterated for 200 times with the parameters

- Method I: \( K_1 = 0.06667 \) and \( K_2 = 0.002 \);
- Method II: \( K_{m_2} = 0.05 \);
- gradient method: \( \mu = 0.05 \).

The noise-free, noisy (SNR=7 dB) and enhanced attractors for the Ikeda map are shown in Fig. 2.2.
To evaluate and compare the performance of the different noise reduction methods, a measure of noise reduction has to be introduced in accordance with the terminology used in electrical engineering to characterize the noisy and distorted signals.

Signals to be processed are corrupted by additive noise in many engineering applications. To get the best system performance, the signal-to-noise ratio (SNR) has to be maximized. The effectiveness of different noise reduction methods can be compared if the SNR after enhancement of the chaotic signal is plotted as a function of the input SNR. If Methods I and II are really closely related to the gradient descent algorithm then the noise reduction capability of the three methods has to be almost the same.

In our computer simulations the noise-free chaotic orbit $m$ is known. Let the output noise be defined as the difference between the enhanced and original noise-free chaotic orbits

$$n_{out} = \hat{m} - m.$$ 

From $n_{out}$ and $m$ the output SNR can be calculated.

The output SNR achieved by the different noise reduction techniques are shown in Fig. 2.3. In order to get a smooth function, the results of 50 trials have been averaged for every input SNR. The noise-free chaotic orbits $m$ have been generated by the Henon and Ikeda maps.

![Figure 2.3](image)

**Figure 2.3** Effectiveness in noise reduction achieved by Method I (dash-dot line), Method II (dashed line) and the gradient method (solid line) for the (a) Henon and (b) Ikeda maps.

Our theoretical investigations have shown that Methods I and II are closely related to the gradient descent algorithm. The results of simulations shown in Fig. 2.3 confirm this conclusion, the performance of the three noise reduction methods are very close to each other. In our simulations the parameter $\mu$ of the gradient method is set (by trial and error) to an optimum constant value. It has been verified numerically that if the step size is chosen according to (2.31) and (2.33), then Method II is equivalent to the gradient method, while if $\mu$ is chosen according to (2.24) and (2.28), then Method I is very close to the gradient method.

### 2.2.5 Conclusions

Two noise reduction techniques called Methods I and II have been proposed recently by Lee and Williams [LW97]. In this part of the thesis we have related these new methods...
to the standard class of optimization techniques.

We have shown that Method II is equal to the standard gradient descent algorithm. Method I is a suboptimal approach closely related to the standard gradient algorithm. Method I should not be used in practice because it has an extra term which prevents from finding the minimum of the cost function. Numerical experiments support these theoretical conclusions.

2.3 NOISE REDUCTION IN DCSK

This section evaluates the effectiveness of noise reduction in a DCSK data communications system. One of the most robust chaotic modulation schemes proposed to date is the differentially coherent DCSK [KVSA96]. A further advantage of DCSK is that due to the special structure of transmitted signal, the noise reduction technique can be applied without any major modification of the DCSK receiver. The operation of a DCSK telecommunications system has been presented in Sec. 1.4.2. For the sake of simplicity a discrete time DCSK system is considered in this section. However, the presented methods and results can be extended to the continuous time DCSK.

2.3.1 Operation of Discrete Time DCSK System

The block diagram of a DCSK telecommunications system is shown in Fig. 2.4. At the transmitter, a discrete-time low-pass chaotic signal $m$ is the input to a DCSK modulator. This modulator maps each transmitted bit $b_m$ to two chaotic sample functions. The first sample function serves as reference while the second one carries the information. For bit “1” the same sample function is transmitted twice in succession while for bit “0” the information-bearing part is an inverted copy of the reference.

As it has been shown in Section 2.1, the noise reduction algorithms work on vectors. Each vector contain a certain number of the iterations of the chaotic signal generator. In the discrete-time DCSK the reference and information bearing sample functions contain chaotic sequences of a finite duration. To get the simplest receiver configuration, the noise reduction is applied to a complete sample function, i.e.,

$$m = [m_1, m_2, \ldots, m_{N_S/2}].$$

The output $s$ of the DCSK modulator is corrupted by Additive White Gaussian Noise (AWGN) $n$ in the channel, i.e., the received signal is

$$r = s + n.$$
In DCSK, the demodulation is performed by determining the correlation between the reference and information-bearing sample functions. The output signal $z$ of the correlator is sampled once per $N_S$ samples; this signal is called the observation signal and is denoted by $z_m$. The recovered bit $\hat{b}_m$ is determined based on the values of observation signal; a positive value of $z_m$ indicates that bit “1” was received while in case of a negative value of $z_m$ the decision is done in favor of “0”.

### 2.3.2 Chaos Generator

To check the effectiveness of noise reduction on DCSK, the following maps have been used to generate the discrete-time chaotic signal

- odd piecewise linear map,
- translated logistic map, and
- symmetric tent map.

The return maps of these generators are shown in Fig. 2.5.

![Figure 2.5](image)

**Figure 2.5** Discrete-time chaos generators, which have been used during the simulations: (a) an odd piecewise linear map, (b) the translated logistic map, and (c) the symmetric tent map.

To limit the maximum value of the transmitted signal, the parameters of each chaos generator have been chosen such that the chaotic output signal lies in the interval $[-0.5, 0.5]$.

The results of computer simulations have shown that the improvement in noise performance is almost identical for these mappings. To avoid the repetition of almost identical figures, only the results achieved by the piecewise linear map are presented here.

When the odd piecewise linear map has been used to generate the input $m$ of the DCSK modulator then the chaotic signal is generated by the following mapping

$$m_{k+1} = \begin{cases} -\frac{1}{0.2}(m_k + 0.3) - 0.5 & \text{if } -0.5 < m_k < -0.3 \\ \frac{1}{0.5}m_k & \text{if } -0.3 \leq m_k \leq 0.3 \\ -\frac{1}{0.2}(m_k - 0.3) + 0.5 & \text{if } 0.3 < m_k < 0.5. \end{cases} \quad (2.34)$$

Since the discrete-time chaotic system which generated $m$ at the transmitter is a priori known at the receiver, a part of the additive channel noise can be eliminated by using the noise reduction methods discussed in the previous section.
2.3.3 Design of Cost Function

The cost function used to enhance a noisy DCSK signal is a modified version of (2.4). The first term of the cost function $C_1(\hat{m}, r)$ ensures that the global shape of $\hat{m}$ remains in the neighborhood of the received signal $r$:

$$C_1(\hat{m}, r) = \frac{1}{N_S} \sum_{k=1}^{N_S} (\hat{m}_k - r_k)^2. \quad (2.35)$$

The second error term quantifies how well $\hat{m}$ fits the dynamics of the chaotic signal generator, i.e., it exploits the deterministic relationship $m_{k+1} = f(m_k)$ which exists between the consecutive points of the transmitted signal. Thus,

$$C_2(\hat{m}) = \frac{1}{N_S - 1} \sum_{k=1}^{N_S-1} (\hat{m}_{k+1} - f(\hat{m}_k))^2. \quad (2.36)$$

Since the maps presented in Fig 2.5, as almost all of the chaotic mappings, do not have an inverse, the backward iteration ($f^{-1}$) is not considered in (2.36).

The cost function is a linear combination of $C_1(\hat{m}, r)$ and $C_2(\hat{m})$

$$C(\hat{m}, r) = (1 - \Gamma)C_1(\hat{m}, r) + \Gamma C_2(\hat{m}) \quad (2.37)$$

where $\Gamma \in [0, 1]$ is a scalar weight. The effect of this weighting factor on the performance of the noise reduction method is discussed in Sec. 2.3.7.

Note the difference between the cost functions defined by (2.4) and (2.37). In (2.35) and (2.36), $C_1(\hat{m}, r)$ and $C_2(\hat{m})$ are scaled by the length of the received vector, and both terms are weighted using by $\Gamma$ to overcome the problems listed in Paragraph 2.1.1.1.

2.3.4 Selection of Optimization Technique

In the literature, many optimization methods [PTVF97] have been proposed. Their performance have been compared in [Ják99b, Ják98c, Ják98a, Ják98b]. The comparison of the different optimization methods in a telecommunications environment is shown in Table 2.1, where the computation time is the required running time to clean up a noisy chaotic signal of 1000 samples generated by the piecewise linear map on a Sun Ultra3 270MHz computer. The SNR improvement is the difference between the final and initial SNR where the initial SNR was 10 dB. These results have been published in [Ják99b] and are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Optimization method</th>
<th>Computation time</th>
<th>SNR Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>downhill simplex</td>
<td>16 minutes</td>
<td>6.1 dB</td>
</tr>
<tr>
<td>Powell’s</td>
<td>140 seconds</td>
<td>5.8 dB</td>
</tr>
<tr>
<td>conjugate gradient</td>
<td>11 seconds</td>
<td>6 dB</td>
</tr>
<tr>
<td>variable metric</td>
<td>0.22 seconds</td>
<td>6 dB</td>
</tr>
<tr>
<td>simulated annealing</td>
<td>4 hours</td>
<td>6.5 dB</td>
</tr>
</tbody>
</table>

Table 2.1  Comparison of the efficiency of the different optimization methods in the noise reduction process.
Although the simulated annealing algorithm gives the best results in terms of noise reduction capability [JDK98], the variable metric (quasi-Newton) method is more efficient in terms of computational efficiency and robustness against parameter mismatch when it is applied for noise reduction in a DCSK system [Ják99b]. Therefore the latter algorithm is used in the remaining part of this section for finding the minimum of cost function.

### 2.3.5 Receiver Configurations

To include the noise reduction algorithm described above in a DCSK receiver, the structure of the DCSK demodulator has to be changed.

The correlator used to demodulate a DCSK signal and the block containing the noise reduction algorithm can be combined in at least two different ways. The reference part of the received signal can be enhanced easily, because it does not depend on the transmitted bit. Rather, it always contains the output signal of the chaos generator.

Figure 2.6 shows the DCSK demodulator with noise reduction, where only the reference part of the received signal is cleaned. Note that this signal is equal to the delayed received signal vector. In this structure, the received noisy information-bearing signal vector is correlated with the enhanced reference signal. This DCSK demodulator is referred to as the first modified receiver in the remaining part of the chapter.

![Figure 2.6 DCSK receiver with noise reduction. Only the reference part of the received signal is cleaned.](image)

Enhancement of the information-bearing sample function is not straightforward because, depending on the modulation, this part is equal to either a non-inverted or an inverted copy of the reference signal. The altering sign prevents the calculation of term $C_2(\bar{m})$ in the cost function. However, if the mapping $f(\cdot)$ is an odd function, i.e.,

$$-m_{k+1} = f(-m_k)$$

then $C_2(\bar{m})$ becomes independent of modulation and the same noise reduction algorithm can be used to enhance both the original chaotic sample function and its inverted copy. In our case, the odd piecewise-linear chaos generator, given by (2.34), meets this requirement. Thus, both the reference and information-bearing parts of the transmitted signal can be enhanced using the same cost function. If the logistic map or the tent map is used as chaos generator, then only the reference part of the transmitted signal can be enhanced.

Figure 2.7 shows how the receiver has to be modified if the noise reduction algorithm is used to enhance both the reference and information-bearing parts of a DCSK signal. This DCSK demodulator is referred to as second modified receiver in the rest of the chapter.

The effect of noise reduction on the noise performance of the first and second modified receivers will be examined in the next section.
2.3.6 Performance Measures

Determination of the Bit Error Rate (BER) by computer simulation requires a very long simulation time. This is why we would like to predict the effectiveness of noise reduction schemes without determining the BER.

The first term of the cost function, see (2.35), depends on the additive channel noise. Because of this term, the noise enhancement becomes a stochastic process which has to be characterized by its probability distribution and moments. If a Gaussian approximation is used then it is enough to consider the mean and the variance of the stochastic process.

The assumption of Gaussian distribution is only an approximation but it can be used if our goal is the evaluation of effectiveness of different noise reduction techniques only.

The quality of a digital communication system is characterized by the BER. The BER depends on the probability distribution of the observation signal denoted by \( z_m \) in Fig. 2.4. Using the Gaussian approximation again, we may say that the BER is determined by the variance: the lower the variance, the better the BER [Hay94]-[Pro83]. The variance of noise enhancement increases the variance of the observation signal.

The variance of noise reduction process can be reduced by increasing \( N_S \), i.e., the length of the noisy received signal to be enhanced. However, the greater length results in a longer computational time. Moreover, increasing \( N_S \) has additional unwanted effects, namely:

- if the chip duration is kept constant while \( N_S \) is increased, the date rate is reduced;
- if the bit duration is kept constant while \( N_S \) is increased, the bandwidth of the transmitted signal increases.

In communications systems the bandwidth of the radiated signal and the data rate are fixed by the system specifications. Hence, it is not possible to choose an arbitrary \( N_S \). The results of our simulations have shown that the noisy signal to be enhanced has to contain at least 10 samples to overcome this problem.

Our goal is to exploit the noise reduction techniques presented above to improve the overall system performance of the DCSK modulation scheme, i.e., to reduce the BER for a given \( E_b/N_0 \).

2.3.6.1 Dynamical Error

Of the performance measures proposed to date, the so-called “dynamic error” approach defined as the inconsistency in the dynamics of the enhanced data, has been introduced
in [KY88] and used in [LW97]

$$\text{err} = \frac{1}{N_S - 1} \sum_{k=1}^{N_S-1} |\hat{m}_{k+1} - f(\hat{m}_k)|.$$

However, the relationship between this performance measure and the noise performance of a chaotic communication system is not known. In particular, a reduction in the dynamical error does not necessarily yield an improvement in the BER since even if the enhanced signal matches with the dynamics of chaotic signal generator its waveform may differ completely from the one which was radiated by the transmitter. A second disadvantage is that the dynamical error cannot be measured directly in a physical system.

2.3.6.2 SNR Improvement

To predict the noise performance of a telecommunications system, the signal-to-noise ratio (SNR) must be known at the input of the demodulator. This is why the SNR of the enhanced chaotic signal versus the SNR of the noisy received signal has been introduced in [JDK98] and used in [Ják99b, DSH99] to characterize the efficiency of a noise reduction method. The SNR of the noisy received chaotic signal is referred to as the initial SNR in the following remainder.

In our simulations, the noise-free chaotic signal \( \mathbf{m} \) is known. Let the noise in the enhanced signal be defined as the difference between the enhanced and the original chaotic signals

$$\mathbf{n}_{\text{out}} = \hat{\mathbf{m}} - \mathbf{m}. $$

Then the initial SNR is defined as

$$10 \log_{10} \left( \frac{\mathbf{m}^T \mathbf{m}}{\mathbf{n}^T \mathbf{n}} \right)$$

and the SNR of the enhanced signal is given by

$$10 \log_{10} \left( \frac{\mathbf{m}_{\text{out}}^T \mathbf{m}}{\mathbf{n}_{\text{out}}^T \mathbf{n}_{\text{out}}} \right)$$

where \( \mathbf{m}^T \) denotes the transpose of \( \mathbf{m} \).

As an example, a chaotic signal having a length of 100000 samples was generated by means of the piecewise linear map. This chaotic signal was corrupted by white Gaussian noise and then the Broyden-Fletcher-Goldfarb-Shanno variant of the variable metric method [PTVF97] was used to minimize the cost function, i.e., to obtain the enhanced chaotic signal. Finally the SNR of the enhanced chaotic signal was calculated as a function of the initial SNR.

The results of these investigations are shown in Fig. 2.8. Note that the noise reduction algorithm does indeed improve the SNR; the factor of SNR enhancement varies from 2.5 dB to 12 dB, depending on the initial SNR. Because the BER depends on the SNR measured at the demodulator input, we expect that these noise reduction techniques can be used to improve the noise performance of chaotic communication systems.

In Fig. 2.8 the SNR improvement of the noise reduction algorithm versus the initial SNR is plotted. To evaluate the system level performance, the effect of noise reduction
on the BER has to be determined as a function of $E_b/N_0$. The relation between the
$E_b/N_0$ and SNR is given by

$$[\text{SNR}]_{dB} = \left[ \frac{E_b}{N_0} \right]_{dB} - 10 \log_{10} N_S. \quad (2.38)$$

![Figure 2.8](image)

*Figure 2.8* The effect of noise reduction using the variable metric method.

The length of the noisy signal to be enhanced has to be greater than 10 to ensure
that the additional variance of the observation signal remains sufficiently small (see
Sec. 2.3.6.3). The SNR at the input of the receiver depends on the particular application;
for an IEEE 802.11 compliant *wireless local area network* (WLAN) the SNR may become
as low as 0 dB [And97].

Figure 2.8 shows that a 2.5 dB noise reduction can be achieved if $SNR = 0$ dB. A
2.5 dB improvement in $E_b/N_0$ should result in a significant improvement in the noise
performance of a DCSK system.

However, our system level simulations have shown that in spite of the improved SNR,
the noise performance of the entire DCSK system has not been improved by using either
the first or second modified receiver configuration.

The reason for the failure of the noise reduction can be understood by analyzing the
noise-free, noisy, and enhanced signals in the time domain.

The noise-free chaotic signal varies in the interval $[-0.5, 0.5]$, i.e., the function $f(\cdot)$
maps the interval $[-0.5, 0.5]$ onto itself.

However, due to the additive channel noise, the value of the received signal is not
bounded. For low SNR, the noise completely overwhelms the chaotic signal, causing
the received signal to vary over an extremely wide range. The noise reduction method
will pull back this signal into the interval $[-0.5, 0.5]$ to fulfill the system dynamics.
Certainly, this SNR improvement does not mean that the correlation of the enhanced
signal with the original one would be improved. If the magnitude of the enhanced signal
is bounded then almost *any* signal gives a better SNR than the noisy received one.

This effect can be observed in Fig. 2.9, where the signals $\mathbf{m}$, $\mathbf{r}$ and $\hat{\mathbf{m}}$ are shown for a
short time interval. Comparing the upper and lower figures we see that an improvement
in SNR does not improve the correlation of the noise-free and enhanced signals. Since
the demodulation is performed by evaluating the correlation, the improvement in SNR
cannot be used as a performance measure for a DCSK system.
2.3.6.3 Variance of the Observation Signal

Since the BER depends on the probability distribution of the observation signal, a more suitable application-oriented performance measure of the effectiveness of noise reduction is the probability distribution of the observation signal. To get a simple scalar number, the Gaussian approximation is applied and the variance of observation signal is used as a performance measure.

In DCSK, due to the varying energy per bit of chaotic signals, the observation signal has a non-zero variance, even in the noise-free case, and the estimation problem [KKK97a] appears. If the chaotic signal is corrupted by noise in the telecommunication channel, then the variance of \( z_j \) becomes even higher. Due to channel noise, the noise reduction algorithm itself is also a random process and it also contributes to the variance of the observation signal. As a result, the variance of the observation signal depends on:

- the chaotic signal itself,
- the additive channel noise, and
- the noise reduction algorithm.

Since these are independent, their variance have to be added.

The observation signal has a non-Gaussian distribution, i.e., the BER cannot be calculated directly from the variance of \( z_m \). However, the variance has a strong effect on the noise performance; the larger the variance, the larger the BER. The computer simulations performed have shown that the easily computable variance of \( z_m \) can be used successfully as a performance measure.

The correlation between the reference and information-bearing parts is:

\[
R_{x_{\text{ref}}, x_{\text{inf}}} = \frac{1}{N_S} x_{\text{ref}}^T x_{\text{inf}}
\] (2.39)

where \( x_{\text{ref}} \) is the cleaned reference part in both the first and second modified DCSK receivers. Signal \( x_{\text{inf}} \) denotes the noisy and enhanced information-bearing parts for the first and second modified DCSK receivers, respectively.

Figure 2.9  Signal waveforms in the time domain. The top figure represents the transmitted (noise-free) signal, the middle figure shows the received (noisy) signal while in the bottom figure, the enhanced signal is plotted. Note that the noisy signal is plotted on a different scale.
Unfortunately, $R_{x_{ref},x_{inf}}$ also depends on the energy of the received bit. To cancel this effect, the correlation in (2.39) has to be scaled by the signal power. This quantity is called normalized observation signal:

$$R_{x_{ref},x_{inf}} = \frac{x_{ref}^T x_{inf}}{\sqrt{x_{ref}^T x_{ref} x_{inf}^T x_{inf}}}.$$ (2.40)

The histogram of the normalized observation signal in a DCSK communications system for the noise-free case is shown in Fig. 2.10(a). If an additive channel noise is present then the variance of the histograms increases, as shown in Fig. 2.10(b). The two histograms overlap, this overlap results wrong decisions. A larger variance in the observation signal yields a worse BER.

This effect can be observed in Figs. 2.11, 2.12 and 2.13, where histograms of the observation signal are plotted for a DCSK system implemented without and with noise reduction, see Figs.(a) and Figs.(b), respectively. Figures 2.11, 2.12 and 2.13 show the histograms for initial SNR values of 10 dB, 17 dB and 35 dB, respectively.

Comparing Figs. 2.11(a) and (b) we can see that although the noise reduction method could improve the SNR of the underlying chaotic signal, it could not reduce the variance of the observation signal. Even worst, the “nose reduction” in this case deteriorates the system performance.

If the initial SNR is about 17 dB then the variance of the observation signal is the same for the original DCSK and enhanced DCSK systems, as shown in Figs. 2.12(a) and (b), respectively. At this SNR, the noise reduction does not have any effect on the noise performance.

If the initial SNR is about 35 dB, then the noise reduction algorithms reduce the variance of the observation signal, as shown in Figs. 2.13(a) and (b). In this case, the noise performance of the modified DCSK receivers given in Figs. 2.6 and 2.7 becomes better than that of the original one.

From these observations we can define a better application-directed performance measure for the noise reduction schemes. Namely, to obtain a lower BER, the noise reduction technique must reduce the variance of the observation signal.
Figure 2.11 Histograms of the normalized observation signal in the DCSK system (a) without and (b) with noise reduction. The initial SNR is 10 dB.

Figure 2.12 Histograms of the normalized observation signal in the DCSK system (a) without and (b) with noise reduction. The initial SNR is 17 dB.

Figure 2.13 Histograms of the normalized observation signal in the DCSK system (a) without and (b) with noise reduction. The initial SNR is 35 dB.
2.3.7 Effect of Weighting Factor $\Gamma$

The cost function in (2.37) is a trade-off between two terms which can be minimized simultaneously only in the noise-free case.

The first term in (2.37) forces the enhanced orbit to be as close as possible to the received noisy one. In a strict sense this term does not contribute to the noise reduction. Worse still, in the case of a noisy input signal, the initial noisy trajectory can be very far from the transmitted one. Furthermore, because chaotic orbits are dense in the state space, there is a chance that another, completely different, orbit is recovered. The first term simply forces the algorithm to select an orbit which is close to the received noisy signal; it does not have to be the “right” orbit. Consequently, the performance of noise reduction method deteriorates when the SNR is low.

The second term forces the enhanced orbit to be a system trajectory, i.e., to fit to the system dynamics. But this orbit can be any of the system orbits, even one which is very “far” from the original chaotic signal.

The factor $\Gamma$ plays a key role in (2.37), it weights these two “contradictory” terms: closeness to the received signal, and consistency with the system dynamics.

The variance of the normalized observation signal for the original, first modified, and second modified versions of the DCSK receiver are plotted as a function of $\Gamma$ in Figs. 2.14, 2.15 and 2.16 for $E_b/N_0$ of 10, 17, and 35 dB, respectively.

![Figure 2.14](image_url)  
Figure 2.14 Variance of the normalized observation signal measured for the original (solid curve), first modified (dash-dot curve) and second modified (dashed curve) DCSK receiver versus $\Gamma$. $E_b/N_0$ is 10 dB.

These figures show that if $E_b/N_0$ is below a certain threshold, then the noise reduction method does not reduce the variance of the correlation. Even worse, the variance is increased. Therefore, if $E_b/N_0$ is less than 17 dB, then the variance of correlation cannot be improved by these noise reduction techniques.

If $E_b/N_0$ is above this threshold then the noise reduction method reduces the variance of the observation signal for certain values of $\Gamma$. This effect can be observed in Fig. 2.16, from which the optimum value of $\Gamma$ can be determined. Fig. 2.16 shows that if $E_b/N_0 = 35$ dB then the optimum value of $\Gamma$ is 0.18. The low value of $\Gamma$ shows that the effect of the second term in the cost function which forces the enhanced signal to fit the dynamics of chaotic signal generator has to be relatively low to save the correlation between the transmitted and enhanced chaotic signals.

The optimum value of $\Gamma$ which ensures the minimum variance of the correlation given by (2.40) is plotted in Fig. 2.17 as a function of $E_b/N_0$. Note that the optimum value of $\Gamma$ depends on $E_b/N_0$ and that the noise reduction technique can improve the noise
Figure 2.15 Variance of the normalized observation signal measured for the original (solid curve), first modified (dash-dot curve) and second modified (dashed curve) DCSK receiver versus $\Gamma$. $E_b/N_0$ is 17 dB.

Figure 2.16 Variance of the normalized observation signal measured for the original (solid curve), first modified (dash-dot curve) and second modified (dashed curve) DCSK receiver versus $\Gamma$. $E_b/N_0$ is 35 dB.

Figure 2.17 Optimum value for $\Gamma$ for the first modified (dash-dot curve) and second modified (dashed curve) DCSK receivers versus $E_b/N_0$. 
performance of the DCSK modulation scheme only if $E_b/N_0 > 17$ dB.

Figure 2.17 also shows that noise reduction has little effect on the variance if $E_b/N_0$ is less than 18 dB. The curve has a high slope about this point but it remains almost constant if we increase $E_b/N_0$ further. This is why, for the computer simulations discussed in the remaining part of this section, the weighting factor $\Gamma$ was set to 0.12.

### 2.3.8 Attainable Improvement in Noise Performance

The variance of the correlation for the original, first, and second modified DCSK receivers is plotted in Fig. 2.18. Note that the curves cross each other about 17 dB, as expected. Below this value, the noise reduction algorithm causes a slight degradation in the noise performance, while above this threshold a slight improvement can be achieved.

![Figure 2.18 Variance of the normalized observation signal measured in the original (solid curve), first modified (dash-dot curve) and second modified (dashed curve) DCSK receivers.](image)

To evaluate the improvement in BER and to verify the applicability of the performance measure proposed here, the BER has also been determined by computer simulation for the original and improved DCSK systems. The BER curves are plotted in Fig. 2.19 for the original, first, and second modified DCSK receivers. Note that, in accordance with the results of variance evaluation method, the BER can be improved slightly if $E_b/N_0 > 16.5$ dB.

The good agreement between the thresholds shown in Figs. 2.18 and 2.19 proves that the variance of the observation signal, proposed here and which requires significantly less computation than a complete BER calculation, can be used as a performance measure. The slight difference between the thresholds shown in Figs. 2.18 and 2.19 is due to the fact that the noise reduction changes not only the variance of the normalized observation signal but also the shape of its probability distribution.

### 2.3.9 Robustness to Parameter Mismatch

Until now, all works in this field has assumed that the system dynamics $f(\cdot)$ used to enhance the noisy signal at the receiver are identical to those that are used to generate the chaotic signal at the transmitter. Except in the case of an all-digital implementation\(^2\), there is always a parameter mismatch between the chaotic mappings used at the

\(^2\)Since a digital system is a finite-state machine, it cannot produce a real chaotic signal.
transmitter and receiver due to different temperatures of environments, imperfections in technology, aging, etc. In real applications, it is not enough if a noise reduction algorithm is fast, requires relatively small memory, and makes a relatively significant improvement in BER. It must also be robust against the parameter mismatch.

Robustness against parameter mismatch is shown in Fig. 2.20, where the variance of the normalized observation signals of the DCSK system for a parameter mismatch of 3% is plotted.

Figure 2.20 shows that even this relatively large parameter mismatch causes only a small degradation in the noise reduction performance. Observe that the parameter mismatch reduces the effectiveness of noise reduction by shifting the crossing point of the curves to higher $Eb/N0$. For example, when the parameter mismatch is 3%, the threshold at which the noise reduction methods start to become effective is about 20 dB.

2.3.10 Conclusions

In chaotic communications, the digital information is carried by inherently wideband chaotic signals. Since chaotic signals are generated by deterministic circuits, their dynamics are known at the receiver. This knowledge can be exploited for noise reduction at the receiver by means of the deterministic optimization technique. As a result, the noise performance of chaotic modulation schemes can be improved.
This part of the thesis has proposed two receiver configurations which improve the noise performance of a discrete-time DCSK modulation scheme. A two-component cost function is minimized, where the first component ensures that the enhanced orbit remains near the noisy received signal, while the second one guarantees that the enhanced signal is close to a system orbit.

Evaluation of the noise performance of a telecommunication system by computer simulation requires a very long simulation time. The performance measures proposed up to date (dynamical error, SNR improvement) cannot be used in chaotic communications systems because an improvement in these measures does not necessarily imply any improvement in BER. By recognizing that the demodulation at a DCSK receiver is performed by evaluating the correlation between the reference and information-bearing parts of the received signal, a more adequate performance measure has been introduced. In the proposed method, the variance of the normalized observation signal is evaluated. A larger reduction in this variance results in a better noise performance.

The noise reduction methods studied here optimize a cost function which is a weighted sum of two terms. The optimum value for the weighting factor $\Gamma$ has been determined. We have concluded that if $E_b/N_0$ is below a certain threshold then the noise performance cannot be improved. Beyond this threshold, a slight performance improvement can be achieved by the proposed new DCSK receiver configurations.

Except in the case of an all-digital implementations, a parameter mismatch is always present in an implemented system. We have shown by computer simulation that the noise reduction methods can tolerate the parameter mismatch even if it is as high as 3%. However, parameter mismatch reduces the effectiveness of noise reduction.

2.4 RESULTS PRESENTED IN CHAPTER 2

2.4.1 Summary of Claims

Claim 1.1: I have related Methods I and II proposed by Lee and Williams to the standard gradient descent algorithm. I have shown that both methods are closely related to the standard gradient method. Numerical experiments support these theoretical conclusions.

- I have shown analytically that Method I is close to a gradient method with a varying step size. However, Method I differs from the gradient method because the iteration scheme contains an extra term. This additional term degrades the performance of the algorithm by preventing it to reach the desired minimum of the cost function.
- I have shown analytically that Method II is the standard gradient descent algorithm with a varying step size.

Claim 1.2: I have elaborated new detector configuration for DCSK modulation scheme that exploit noise reduction.

- I have applied the noise reduction schemes to the DCSK telecommunications system by modifying the receiver configuration. I have elaborated the first modified and second modified receiver structures which enhance the even
and odd chaotic signals, respectively. I have developed a proper cost function which has to be minimized at the receiver.

- I have shown that although the simulated annealing algorithm gives the best results in terms of noise reduction capability, the variable metric (quasi-Newton) method is more efficient in terms of computational efficiency.
- I have introduced an adequate computationally efficient performance measure for the noise reduction algorithms which can be used in chaotic communications schemes.

**Claim 1.3:** I have analyzed the performance of noise reduction methods in the DCSK communications scheme.

- I have shown that a threshold effect exists, namely if $E_b/N_0$ is below a certain threshold then the noise reduction technique cannot improve the noise performance. Beyond this threshold, a slight performance improvement can be achieved.
- I have shown that the noise reduction methods tolerate a parameter mismatch even if it is as high as 3%. However, the parameter mismatch reduces the effectiveness of noise reduction.

### 2.4.2 Publications of the Author, Related to Chapter 2

#### 2.4.2.1 Journal Papers


#### 2.4.2.2 Refereed Conference Contributions


2.4.2.3 Other Publications


3
Enhanced Versions of DCSK Telecommunications Systems

3.1 BASIC IDEA OF ENHANCED DCSK SYSTEMS

Although FM-DCSK offers the best noise performance among the chaotic modulation schemes published to date, even its noise performance lags a few dBs behind that of the conventional ones. Consequently, any improvement in the noise performance of FM-DCSK has a great importance.

The techniques shown in this chapter can be applied to both FM-DCSK and DCSK systems to improve their noise performance. However, because the best noise and multipath performances is offered by the FM-DCSK modulation scheme, the simulation results given in this chapter have been determined only for the FM-DCSK system.

The time slots of the original DCSK signal are shown in the upper part of Fig. 3.1. In this figure, \( R_i \) and \( I_i \) denote the reference and the information-bearing parts of the \( i \)th bit, respectively.

If the differentially coherent decision is used then the drawback of DCSK and FM-DCSK modulation schemes compared to conventional ones is that every information bit is transmitted by \( \text{two} \) sample functions. Consequently, the bit rate is halved and the transmitted energy per bit is doubled compared to the conventional binary modulation schemes where every sample function represents one bit [Hay94]. In DCSK, the sample function is a piece of a chaotic signal, while in FM-DCSK, it is a piece of an FM...
modulated chaotic signal. These, signal pieces will be referred later to as chips.

A possible enhancement of the original DCSK technique is as follows: instead of transmitting only one information-bearing chip after one reference chip, \( N \) bits are transmitted using the same reference.

The waveform of the improved DCSK technique is shown in the lower part of Fig. 3.1, where \( T_S \) denotes the duration of one chip and \( E_S \) is the energy carried by one chip. Observe that in a block containing \((N+1)\) chips every chip, except the first one, carries information.

The advantages and disadvantages of the enhanced modulation scheme proposed here are as follows:

- **advantages**
  - the bit duration \( T \) is decreased from \( 2T_S \) to \( \frac{N+1}{N}T_S \), i.e., the data rate is increased;
  - the transmitted energy per bit \( E_b \) is reduced from \( 2E_S \) to \( \frac{N+1}{N}E_S \);

- **disadvantages**
  - the periodic component at frequency of \( 1/T_S \) and its harmonics may increase in the output spectrum;
  - a more complex system configuration is required;
  - worst system performance in a time-varying channels.

These advantages and disadvantages of enhanced DCSK modulation scheme will be studied in detail in Sec. 3.1.2 and Sec. 3.1.3.

The transmission and reception of the information are done in blocks carrying \( N \) bits. For the sake of simplicity, we will consider only the transmission of one block, i.e., the transmission of \( N \) bits in the remaining part of this chapter.

To get compact equations a few notations have to be introduced. Let \( s_m(t) \) denote the transmitted signal in the \( m \)th time slot \( t \in [mT_S, (m+1)T_S) \). Hence, \( s_0(t) = y(t) \) is the reference chip and \( s_m(t) = \pm s_0(t - mT_S) \) is the information bearing chip for bit \( b_m \). Let \( \tilde{r}_m(t) \), \( m = 0, 1, \ldots, N \), denote the signal received in the \( m \)th time slot. Let the correlation between the \( k \)th and \( l \)th chips be denoted by

\[
 z_{k,l} = \tilde{r}_k(t) \ast \tilde{r}_l(t) \int_0^{\frac{T}{N+1}} \tilde{r}_k \left( t - \frac{k}{N+1}T \right) \tilde{r}_l \left( t - \frac{l}{N+1}T \right) dt. \tag{3.1}
\]

The observation signal of the \( m \)th bit at the receiver is defined as

\[
 z_{0,m} = \tilde{r}_0(t) \ast \tilde{r}_m(t).
\]

### 3.1.1 Transmitter and Receiver Configurations

It has been shown in [Kol00c] that the FM-DCSK is not a new modulation scheme, but a variant of DCSK where the energy per bit is kept constant. In the remaining part of the thesis we will not distinguish between the DCSK and FM-DCSK modulation schemes, the later abbreviation will be used only if we want to refer to the FM-DCSK system configuration. However, it will be always assumed that \( E_b \) is kept constant and consequently, the estimation problem [KKK97a] is eliminated.
The enhancement techniques to be discussed in this chapter has been elaborated for the improvement of noise performance of differentially coherent detector [KVSA96]. Other detectors such as coherent [Kol00c] or energy [KK03] are not considered here.

3.1.1.1 General Transceiver Architecture

A possible implementation of the enhanced DCSK modulator is shown in Fig. 3.2, where a delay line with \( N \) taps is used.

\[
\begin{align*}
&y(t) \\
&T_S \\
&b_1 \\
&T_S \\
&b_2 \\
&\vdots \\
&T_S \\
&b_N \\
&s(t)
\end{align*}
\]

Figure 3.2 Enhanced DCSK modulator configuration implemented using a delay line having \( N \) taps.

The transmission of each \( N \) bits is preceded by a reference chip \( s_0(t) \), after which the information-bearing chips \( s_m(t) \) are transmitted. This is done by changing the switch position in Fig. 3.2 at each \( T_S \) time instants. To get compact figure, we have omitted the blocks in Fig. 3.2 which converts the incoming bits “0” and “1” into a sequence of “-1” and “+1”, respectively. Consequently, the value of \( b_i \) is “±1”.

The block diagram of the demodulator contains \( N \) delay lines and correlators as shown in Fig. 3.3.

\[
\begin{align*}
&z_1 \\
&f_{T_S} \cdot dt \\
&z_2 \\
&f_{T_S} \cdot dt \\
&\vdots \\
&f_{T_S} \cdot dt \\
&z_N \\
&\text{Decision} \\
&\text{circuit} \\
&\hat{b}
\end{align*}
\]

Figure 3.3 Enhanced DCSK receiver configuration.

The correlator outputs sampled at \( kT_S, k = 1, 2, \ldots, N + 1 \) constitute the elements of observation vector. They are denoted by \( z_1, z_2, \ldots, z_N \) in Fig 3.3 and their values are given in Table 3.1. The estimated binary information is denoted by a vector \( \hat{b} = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_N) \).

The transmitted information is carried by the sign of the correlation between the reference and information bearing chips. This information is available for the \( m \)th bit at the output of the \( m \)th correlator at the \((m + 1)\)th sampling time instant as shown in Table 3.1.
In the enhanced version of DCSK, $r_m(t)$, $m = 1, 2, \ldots, N$ are noninverted or inverted copies of $r_0(t)$, i.e., each chip is transmitted $(N + 1)$ times. This redundancy will be exploited later to improve further the noise performance of DCSK.

Another source of redundancy which will be also exploited to get better noise performance is that in the receiver shown in the Fig. 3.3, the correlation among each par of chips are determined as shown in Table 3.1.

The advantage of this receiver architecture is that it makes possible the exploitation of each kind of redundancy being in the enhanced DCSK signal. The price to be paid is the higher complexity. A solution that requires the simplest receiver configuration is presented in the following subsection.

### 3.1.1.2 Simplified Transceiver Architecture

One of the disadvantages of the enhanced DCSK scheme is its higher sensitivity to the time varying channels. Because the information to be transmitted is carried by the correlations between the reference and information chips, in the original DCSK system the channel parameters has to be more or less constant only for a time interval of $T_S$. In the enhanced DCSK system this time interval is $N$ times longer.

To get ride of this problem we have to modify the DCSK modulator. Let the $N$ bits $(b_1, b_2, \ldots, b_N)$ to be transmitted mapped to the chips as follows:

- the first chip $s_0(t) = y(t)$ is the reference for $b_1$;
- chips 1, 2, \ldots, $(N - 1)$ serves not only as information bearing chips, but also as reference for the next bits;
- the $N$th chip is the information bearing chip for $b_N$.

Consequently, the observation signal for the $m$th bit is obtained as

$$z_{m,m-1} = \tilde{r}_m(t) \ast \tilde{r}_{m-1}(t).$$

To generate the waveform described above we have to either modify the transmitter shown in Fig. 3.2 or have to change the coding of the bits to be sent from NRZ (non-return-to-zero) to RZ (return-to-zero) [OS89, SHL95]. If we convert the bit stream $(b_1, b_2, \ldots, b_N)$ to $(c_1, c_2, \ldots, c_N)$ in such a way that

$$c_1 = b_1, \quad c_2 = b_2 \otimes c_1, \ldots, \quad c_N = b_n \otimes c_{N-1}$$

and use the transmitter architecture shown in Fig. 3.2, then the correlation between the $m$th and $(m - 1)$ chips will carry bit $b_m$. 

### Table 3.1 Elements of observation vectors at $T_S$, $2T_S, \ldots, (N + 1)T_S$ time instants.

<table>
<thead>
<tr>
<th>Time instant</th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>\cdots</th>
<th>$z_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_S$</td>
<td>-</td>
<td>-</td>
<td>\cdots</td>
<td>-</td>
</tr>
<tr>
<td>$2T_S$</td>
<td>$\tilde{r}_0(t) \ast \tilde{r}_1(t)$</td>
<td>-</td>
<td>\cdots</td>
<td>-</td>
</tr>
<tr>
<td>$3T_S$</td>
<td>$\tilde{r}_1(t) \ast \tilde{r}_2(t)$</td>
<td>$\tilde{r}_0(t) \ast \tilde{r}_2(t)$</td>
<td>\cdots</td>
<td>-</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
<td>\ddots</td>
<td>\vdots</td>
</tr>
<tr>
<td>$(N + 1)T_S$</td>
<td>$\tilde{r}_{N-1}(t) \ast \tilde{r}_N(t)$</td>
<td>$\tilde{r}_{N-2}(t) \ast \tilde{r}_N(t)$</td>
<td>\cdots</td>
<td>$\tilde{r}_0(t) \ast \tilde{r}_N(t)$</td>
</tr>
</tbody>
</table>
Demodulation of the received signal can be performed by the conventional differentially coherent DCSK demodulators with the only difference that the line in the correlator has to be equal to the chip duration instead of $T/2$. This solution offers the simplest receiver configuration as shown in Fig. 3.4.

Comparing this receiver with the general receiver configuration presented in the previous section we can observe two advantages:

- simplicity: it has only one delay element and one correlator instead of $N$,
- less sensitivity to the time varying channels. The system realized with this receiver architecture is as robust as the original FM-DCSK system against time varying channels.

The disadvantage of the simplest enhanced DCSK modulation scheme shown in Fig. 3.4 is that redundancy carried by the correlation of each pair of chips is not exploited for further noise performance improvement. The noise performance improvement exploiting this extra information will be discussed in Sec. 3.3. If that extra information is not exploited then the receiver configurations shown in Figs. 3.3 and 3.4 has the same noise performance in both AWGN and indoor multipath radio channels. These noise performances will be discussed in Subsection 3.1.3.

### 3.1.2 Spectrum of Transmitted FM-DCSK Signal

In the enhanced version of FM-DCSK system the reference chip is followed by $N$ information-bearing chips. In order to compare the spectral properties of the original FM-DCSK and that of its enhanced version, the power spectral density (PSD) of the transmitted signal is determined by computer simulation.

The PSDs calculated for a random information bit sequence are plotted in Figs. 3.5 (a) and (b) for the original and enhanced FM-DCSK ($N = 9$), respectively. The comparison of the two figures do not show any significant difference between the PSDs of the two signals.

Our further investigations have showed that the power spectral densities differ from each other if a pure sequence of bits “0” or “1” is transmitted. For the case of a pure sequence of bit “1,” the spectrum for the original and enhanced FM-DCSK systems are plotted in Figs. 3.6 (a) and (b), respectively. Recall that for a pure sequence of bit “1,” each information-bearing chip is equal to the reference one. Figure 3.6 (a) shows that the PSD of original FM-DCSK signal becomes zero at certain frequencies. The distance between the zeros is determined by the length of chip duration, i.e., it is equal to $1/T_S = 1/(T/2) = 1$ MHz for 500 kbit/s ($T = 2 \mu s$) data rate.

For the case of enhanced FM-DCSK, the PSD of transmitted signal is plotted in Fig. 3.6 (b), where the chip duration is $T_S = TN/(N + 1) = 1.8 \mu s$ to get the same data
ENHANCED VERSIONS OF DCSK TELECOMMUNICATIONS SYSTEMS

Figure 3.5 Power spectral density of the transmitted signal in the (a) original and (b) enhanced FM-DCSK ($N = 9$) systems when a random bit sequence is transmitted. The number of information-bearing sample functions belonging to one reference is equal to 9 in the latter case.

Figure 3.6 Power spectral density of transmitted signal in the (a) original and (b) enhanced FM-DCSK ($N = 9$) systems when a pure sequence of bit “1” is transmitted.

We have shown that the enhancement of FM-DCSK system does not degrade the spectral properties of the transmitted signal when a random bit sequence is transmitted. As it will be shown below, the enhanced FM-DCSK system offers better noise performance both in AWGN and in multipath channels that the conventional one.
3.1.3 Performance Evaluation

3.1.3.1 Conditions of Performance Comparison

The system performances of the enhanced FM-DCSK modulation schemes will be evaluated and compared by computer simulation in this section. To make a realistic comparison, first the conditions of the comparison have to be fixed.

From the application point of view the most important parameters of a digital communication system are the data rate, the energy which is required to transmit one bit information and the occupied bandwidth. Consequently, the first condition is that the compared enhanced and original FM-DCSK

- have the same data rate;
- requires the same energy per bit and
- occupies the same RF bandwidth.

The chaotic communication systems are intended to be used in indoor and mobile applications, where the system performance is determined by the multipath propagation. Consequently, the comparison will be done using multipath channel models elaborated by the PCS Joint Technical Committee [PL95] for the indoor

- office,
- residential and
- commercial

areas. For a detailed description of these channel models refer to Appendix C.

3.1.3.2 Performance Evaluation in AWGN channel

An exact expression for the noise performance of original FM-DCSK was given in [Kol00c]. In the simplest enhanced version of FM-DCSK, denoted by FM-DCSK/S, the energy per bit and the bit duration is reduced by sending $N$ information bearing chips with one reference chip. Otherwise, there is no difference between FM-DCSK and FM-DCSK/S systems. Consequently, the noise performance of FM-DCSK/S can be expressed by modifying equation (26) of [Kol00c]. Substituting the new values of bit duration $T_s(N+1)/N$ and energy per bit $E_s(N + 1)/N$ we obtain:

$$\text{BER} = \frac{1}{2^{2BT} \frac{N}{N+1}} \exp \left( - \frac{E_b N}{N_0 (N + 1)} \right) \sum_{i=0}^{\frac{2BT}{N+1}-1} \left( \frac{E_b N}{N_0 (N+1)} \right)^i \sum_{j=i}^{\frac{2BT}{N+1}-1} \frac{1}{2j} \left( j + 2BT \frac{N}{N+1} - 1 \right).$$  \hspace{1cm} (3.2)

If $N$ tends towards infinity (3.2) becomes

$$\text{BER} = \frac{1}{2^{2BT}} \exp \left( - \frac{E_b}{N_0} \right) \sum_{i=0}^{2BT-1} \left( \frac{E_b}{N_0} \right)^i \sum_{j=i}^{2BT-1} \frac{1}{2j} \left( j + 2BT - 1 \right).$$  \hspace{1cm} (3.3)
This is exactly the noise performance of suboptimum non-coherent DPSK system [Okuan]. Note, that assuming $N = \infty$ is impractical, but (3.2) give us a performance bound for the FM-DCSK/S system.

The noise performance of FM-DCSK/S system for different values of $N$ is plotted in Fig. 3.7. By increasing the number of information-bearing chips to $N = 9$ the noise performance is improved by 1.6 dB at a BER$= 10^{-3}$.

![Figure 3.7](image)

**Figure 3.7** Noise performance of the FM-DCSK/S system for different values of $N$: curves belonging to $N = 9, 4, 3$ and 2 are denoted by “+”, “x”, “◦” and “□” marks, respectively. The noise performance of the original FM-DCSK and the theoretical performance limit of FM-DCSK/S systems, marked by “∗” and “♦”, respectively, are shown for comparison.

Table 3.2 shows the required $E_b/N_0$ for different given BER values as a function of $N$. If $N$ is small then the required $E_b/N_0$ can be reduced significantly by increasing $N$, i.e., a significant improvement in the noise performance can be achieved. However, above a certain limit increasing $N$ has little effect on the system noise performance, i.e., a threshold effect can be observed.

<table>
<thead>
<tr>
<th>System</th>
<th>$N$</th>
<th>Required $E_b/N_0$ for the desired BER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BER$=10^{-1}$</td>
</tr>
<tr>
<td>original FM-DCSK</td>
<td>1</td>
<td>9.5 dB</td>
</tr>
<tr>
<td>enhanced FM-DCSK</td>
<td>2</td>
<td>8.9 dB</td>
</tr>
<tr>
<td>enhanced FM-DCSK</td>
<td>3</td>
<td>8.6 dB</td>
</tr>
<tr>
<td>enhanced FM-DCSK</td>
<td>4</td>
<td>8.3 dB</td>
</tr>
<tr>
<td>enhanced FM-DCSK</td>
<td>9</td>
<td>8.2 dB</td>
</tr>
<tr>
<td>enhanced FM-DCSK</td>
<td>∞</td>
<td>7.8 dB</td>
</tr>
</tbody>
</table>

**Table 3.2** Noise performance of the FM-DCSK/S system. The required $E_b/N_0$ for a given BER is shown as a function of $N$. The noise performance of the original FM-DCSK and the theoretical performance bounds are also shown for comparison.
3.1.3.3 Performance Degradation in PCS JTC Channels

The performance of the FM-DCSK system operating in multipath channel has been evaluated in [Kis03]. This is why this section is not aimed to give a detailed analysis of the performance degradation of FM-DCSK in multipath channels.

As shown in Sec. 3.1.2, the FM-DCSK/S introduce some kind of periodicity in the radiated signal by repeating the reference signal or its inverted copy \( N \) times. This periodicity rises a special question: whether the multipath performance of FM-DCSK/S is better or worse as that of conventional FM-DCSK.

To answer this question the performance degradations of the FM-DCSK/S in different multipath channels are given in this section. For this purpose, we have chosen the channel models proposed by the Personal Communication Services (PCS) Joint Technical Committee (JTC). For a detailed description of the PCS JTC channel models see Appendix C. In a nutshell: PCS JTC has developed channel models for office, residential and commercial environments. To describe the various propagation conditions, three different channel profiles, denoted by channels A, B and C, are given for each area. Depending on the attenuation of radio channel, channel A, or channels A and B, or all of the three channels have to be used with a specified probability in the simulation. In the worst-case situation the channel attenuation exceeds a certain limit and all of the three channel profiles have to be used in the simulation.

Since, we are interested only in the multipath performance of FM-DCSK/S, instead of analyzing all of the possible channel conditions we will focus only on the worst-case scenario\(^1\), i.e., the comparison is done here for the worst propagation conditions.

The performance degradations of the original FM-DCSK and FM-DCSK/S systems in indoor office, residential and commercial environments are shown in Figs. 3.8, 3.9 and 3.10, respectively.

![Figure 3.8](image)

**Figure 3.8** Worst-case performance degradation of the original and enhanced \((N = 9)\) FM-DCSK systems in indoor office application. Multipath performances in indoor office area are plotted by dotted curves for the original FM-DCSK (right curve) and the enhanced FM-DCSK (left curve). For comparison, the noise performances of original FM-DCSK (right curve) and enhanced FM-DCSK (left curve) systems without multipath are also shown by solid curves.

Figures 3.8, 3.9 and 3.10 shows that the multipath performance of enhanced FM-

\(^1\)We have compared the multipath performance of the original and enhanced FM-DCSK systems for all of the possible propagation conditions given by PCS JTC. The conclusions drawn from the results of simulation are similar to the conclusions have got for the worst case scenario. Hence, for brevity those results are omitted here.
Figure 3.9  Worst-case performance degradation of the original and enhanced \((N = 9)\) FM-DCSK systems in the indoor residential application. Multipath performances in indoor residential area are plotted by dashed curves for the original FM-DCSK (right curve) and the enhanced FM-DCSK (left curve). For comparison, the noise performances of original FM-DCSK (right curve) and enhanced FM-DCSK (left curve) systems without multipath are also shown by solid curves.

Figure 3.10  Worst-case performance degradation of the original and enhanced \((N = 9)\) FM-DCSK systems in indoor commercial application. Multipath performances in indoor commercial area are plotted by dash-dot curves for the original FM-DCSK (right curve) and the enhanced FM-DCSK (left curve). For comparison, the noise performances of original FM-DCSK (right curve) and enhanced FM-DCSK (left curve) systems without multipath are also shown by solid curves.
DCSK is as good as that of original FM-DCSK. Moreover, the same improvement in noise performance can be achieved by FM-DCSK/S in multipath environment as in the case of AWGN channel. This improvement is about 1.6 dB at BER = 10^{-3} for N = 9.

In the next sections further improved versions of FM-DCSK modulation scheme will be presented. The results of computer simulations have shown that the enhanced FM-DCSK schemes discussed later also have excellent multipath performance, the improvement in noise performance achieved in AWGN channel is preserved in multipath channels. Since, the aim of this chapter is to elaborate enhanced FM-DCSK modulation schemes, and not to provide a detailed analysis of them over multipath channels, for brevity those results will be omitted in the remaining part of this chapter.

3.1.3.4 Conclusions

Taking into account the complexity of the transmitter, the PSD of the radiated signal and the results shown in Fig. 3.7, and Table 3.2 we conclude that to get the best system parameters in the enhanced FM-DCSK systems the following conditions have to be satisfied:

- \( N \leq 9 \);
- simplest receiver configuration shown in Fig. 3.4 has to be used;
- an RZ to NRZ converter and a scrambler has to be used in the transmitter.

The noise performance of FM-DCSK/S modulation scheme is about 1.6 dB better in a single-ray AWGN channel than that of the original FM-DCSK. The results shown in the previous section shows that this improvement is preserved in the various multipath channels.

3.2 EXTRA INFORMATION AVAILABLE IN OBSERVATION SIGNAL

The decision in the DCSK receiver is done by determining the correlation between the \( k \)th and \( l \)th chips, i.e., by evaluating \( z_{k,l} = \tilde{r}_k(t) \star \tilde{r}_l(t) \). In the enhanced DCSK, each chip is radiated \((N+1)\) times and at the receiver shown in Fig. 3.3 the correlation between each pair of chips is evaluated. This means that the decision for each information bit can be done using many different decision strategies, where correlations between different pairs of chips are considered. The selection of optimum decision strategy will be shown in Sec. 3.3.

The noise corrupting the received signal varies from chip to chip. To get the best noise performance, those correlations have to be used to make a decision which assures the highest probability of correct decision.

Until this time only the sign of correlation of chips has been considered to make a decision. We will show in this paragraph that the absolute value of this correlation is a measure of probability of making a correct decision: the higher its value, the higher the probability of making a correct decision. Consequently, the absolute value of correlation may be used to select the way of decision which assure the highest probability of making a correct decision, i.e., to improve the noise performance of the DCSK system further.

Let \( P_+(z_{k,l}) \) denote the probability that \( z_{k,l} \) is the value of the observation signal if \( s_k(t) = s_l(t) \) is transmitted. Similarly, let \( P_-(z_{k,l}) \) denote the probability that \( z_{k,l} \) is the value of the observation signal if \( s_k(t) = -s_l(t) \) is transmitted. Assuming equiprobable
bits the probability distribution functions (PDFs) of $P_+(z_{k,l})$ and $P_-(z_{k,l})$ are shown in Fig. 3.11.

If the observation signal has a Gaussian distribution then it can be fully characterized by the mean $\mu$ and the standard deviation $\sigma$ of $P_+(z_{k,l})$ and $P_-(z_{k,l})$. Although the observation signal has a non-Gaussian distribution [Kol00c], for moderate noise level it can be approximated by a Gaussian one [AGS98b, AGS98a, STV00]. Hence, the probability distribution of $z_{k,l}$ is obtained as

$$P_+(z_{k,l}) = \frac{1}{\sigma\sqrt{2\pi}} e^{(z_{k,l}-\mu)^2/(2\sigma^2)} \quad (3.4)$$

and

$$P_-(z_{k,l}) = \frac{1}{\sigma\sqrt{2\pi}} e^{(z_{k,l}+\mu)^2/(2\sigma^2)} \quad (3.5)$$

where $\mu = E_S$ while $\sigma$ depends on the noise level, bit duration and the bandwidth of the signal.

At the demodulator we assume that either $s_k(t) = s_l(t)$ or $s_k(t) = -s_l(t)$ has been transmitted. For a given $z_{k,l}$ assuming $s_k(t) = s_l(t)$ the probability that our assumption is correct is obtained by

$$P_{C+}(z_{k,l}) = \frac{P_+(z_{k,l})}{P_+(z_{k,l}) + P_-(z_{k,l})}. \quad (3.6)$$

Substituting (3.4) and (3.5) into (3.6) we get

$$P_{C+}(z_{k,l}) = \frac{1}{1 + e^{-2z_{k,l}\mu/\sigma^2}}. \quad (3.7)$$

Similarly, assuming $s_k(t) = -s_l(t)$, for a given $z_{k,l}$ the probability that our assumption is correct becomes

$$P_{C-}(z_{k,l}) = \frac{1}{1 + e^{2z_{k,l}\mu/\sigma^2}}. \quad (3.8)$$

Observe that $P_{C+}(z_{k,l}) + P_{C-}(z_{k,l}) = 1$ and that for a given $z_{k,l}$ the probabilities for making a wrong decision are $P_{W+}(z_{k,l}) = P_{C-}(z_{k,l})$ and $P_{W-}(z_{k,l}) = P_{C+}(z_{k,l})$. These probability density functions are shown in Fig. 3.12.

From Fig. 3.12 we conclude that to minimize the probability of making a wrong decision at the receiver for $z_{k,l} \geq 0$ we have to assume that $s_k(t) = s_l(t)$ has been transmitted, while $s_k(t) = -s_l(t)$ should be assumed for $z_{k,l} < 0$. Hence, the probability of correct decisions is expressed by

$$P_{C}(z_{k,l}) = \begin{cases} 
\frac{1}{1+e^{-2z_{k,l}\mu/\sigma^2}} & \text{if } z_{k,l} \geq 0 \\
\frac{1}{1+e^{2z_{k,l}\mu/\sigma^2}} & \text{if } z_{k,l} < 0,
\end{cases} \quad (3.9)$$
Observe that depending on the absolute value of the correlation two regions can be distinguished.

In the region about the origin, the probability of making a wrong or correct decision is almost the same, i.e., the low absolute value of correlation implies a high probability of wrong decision.

If the absolute value of the correlation is about $E_S$ or it is greater than $E_S$ then the probability of making a wrong decision is low.

As a result we conclude that the absolute value of correlation is a measure of the probability of making a correct decision. A low absolute value of the correlation indicates that that correlation is not suitable to make a correct decision.

The problem with (3.10) is that it assumes that we know $\mu$ and $\sigma$ a priori. However, at the receiver due to the unknown channel attenuation and noise we do not know $\mu$ and $\sigma$. Fortunately, we have a large number of observation signals, so we can compute a good estimate of these parameters.

If only a block of $N$ bits are transmitted, we have $N(N + 1)$ correlation as shown in Table 3.1. In this case $\mu$ and $\sigma$ can be estimated by

$$\hat{\mu} = \frac{1}{N(N + 1)} \sum_{k=0}^{N-1} \sum_{l=k+1}^{N} |z_{k,l}| \quad (3.11)$$

and

$$\hat{\sigma} = \sqrt{\frac{1}{N(N + 1) - 1} \sum_{k=0}^{N-1} \sum_{l=k+1}^{N} (|z_{k,l}| - \hat{\mu})^2}. \quad (3.12)$$

In the simulation results presented in the remaining part of this section we have assumed a more realistic communication environment where a large number of bits ($10^7 - 10^8$) is transmitted. Consequently, a very good estimation for $\mu$ and $\sigma$ have been obtained.

Note, that (3.11) and (3.12) give an estimate for the parameters $\mu$ and $\sigma$ from the actual values of $|z_{k,l}|$. This approach can be used for the estimation from the received modulated signal since $P_+(z_{k,l}) = P_+(-z_{k,l})$.

Observe that for $s_k(t) = s_l(t)$ the assumption

$$P_+(z_{k,l}) \approx P_+(|z_{k,l}|) \quad (3.13)$$

causes a distortion if $z_{k,l}$ is about zero. However, if $P_+(z_{k,l})$ is small enough for the observation signal values being close to zero then $\mu$ and $\sigma$ can be estimated from (3.11) and (3.12). Fortunately, in a telecommunications system we are interested only in such
a noise level for which at least a BER of $10^{-2}$ can be achieved. For this moderate SNR both assumption (3.13) and the Gaussian approximations are valid, consequently, (3.11) and (3.12) can be used to get estimates for $\mu$ and $\sigma$.

Having calculated $\hat{\mu}$ and $\hat{\sigma}$ from (3.11) and (3.12) respectively, an estimation for probability of correct decision as a function of the absolute value of correlation can be obtained from (3.10)

$$
\hat{P}_C(|z_{k,l}|) = \frac{1}{1 + e^{-2|z_{k,l}|/\hat{\sigma}}}
$$

(3.14)

The analysis of observation signal has resulted in two important conclusions that can be used to improve further the noise performance of enhanced DCSK:

- as shown in Fig. 3.12, the absolute value of correlation is a measure of probability of correct decision. The larger the value of $|z_{k,l}|$, the higher the probability of correct decision.

- an approximate formula given by (3.14) has been developed which gives the probability of correct decision as a function of the absolute value of the observation signal.

### 3.3 NON-REDUNDANT ERROR CORRECTION IN THE EXTENDED DCSK SYSTEM

Section 3.1 improved the noise performance of FM-DCSK in such a way that $N$ information bearing chips were transmitted after one reference chip. Section 3.2 showed that the absolute value of correlation is a measure of probability of correct decision. In this section we present three methods which improve further the noise performance of the enhanced FM-DCSK system. These methods are referred later as “error correction” methods. However, please note that there is a significant difference between the error correction methods used in conventional digital communications systems and these methods.

At the receiver in our DCSK system, similarly to the conventional digital communications systems equipped with error correction, we have more information (observation signals) that is required to make the decisions for the transmitted bits. Thus we are able to find an optimum decision strategy.

The main difference between our and the conventional digital communications systems equipped with error correction can be found at the transmitter part.

In conventional digital communications extra bits are added to each block of bits, these extra bits introduce an algorithmic redundancy in the transmitted information that makes possible the application of error correction algorithms at the receiver. As an example see the Viterbi algorithm [LM93]. In these systems we can design these additional bits in such a way that their Hamming [LM93] distance becomes maximum.

In the enhanced FM-DCSK we do not add any redundant bit to the transmitted information, the extra information for the error correction is provided by the special structure of the enhanced FM-DCSK signal. This is why our methods are called “Non-redundant error corrections”.

Our error correction do not exclude the application of standard error correction algorithms. In a built system, if required, the standard error correction blocks can be put on the top of our error corrections block and a further improvement in noise performance can be achieved.
3.3.1 Noise Reduction by Averaging in the Improved FM-DCSK System

A further improvement in noise performance of the FM-DCSK/S system can be achieved if we exploit the fact that every chaotic sample function is transmitted \((N + 1)\) times. Every received chip is corrupted by noise. If the modulation is removed from the noisy received chips then the noise content of the reference signal can be reduced considerably by averaging the \((N + 1)\) received noisy chips.

Let \(\tilde{r}_l(t)\) denotes the received noisy chip in the \(l\)th time slot. To remove the modulation from \(\tilde{r}_l(t)\), it has to be multiplied by \(\text{sign}(\tilde{r}_0(t) \ast \tilde{r}_l(t))\), i.e., by \(\text{sign}(z_{0,l})\). Then an enhanced reference chip can be generated by averaging the \((N + 1)\) chips

\[
\hat{r}_0^*(t) = \frac{1}{N + 1} \left\{ \tilde{r}_0(t) + \sum_{l=1}^{N} \text{sign}(z_{0,l})\tilde{r}_l(t) \right\}
\]

(3.15)

The noise reduction given by (3.15) works efficiently only if the modulation is removed correctly from the noisy incoming signal. If the estimation of the modulation determined by \(\text{sign}(z_{0,l})\) is wrong then that term corrupts the efficiency of noise reduction instead of improving it.

To alleviate this problem, we introduce a scalar weighting factor \(C_{0,l}\) which depends on the probability of wrong decisions made by the \(\text{sign}(\cdot)\) function. Thus, the enhanced reference chip is obtained as

\[
\hat{r}_0(t) = \frac{1}{N + 1} \left\{ \tilde{r}_0(t) + \sum_{l=1}^{N} C_{0,l}\text{sign}(z_{0,l})\tilde{r}_l(t) \right\}
\]

(3.16)

where \(C_{0,l}\) has to be small if the probability of wrong decision is high.

An approximate analytical expression for the probability that a decision is correct is given by (3.14). Let us use this measure to obtain \(C_{0,l}\).

First we estimate the mean \(\mu\) and the standard deviation \(\sigma\) of \(z_{k,l}\) using (3.11) and (3.12), respectively. If \(N\) is small then the correlations from the previous block of bits also has to be considered. Having \(\mu\) and \(\sigma\), the probability of the correct sign of \(z_{0,l}\) is computed by (3.14). Because the probability \(\hat{P}_C(|z_{0,l}|)\) covers the interval \([0.5, 1.0]\) it has to be rescaled. Then \(C_{0,l}\) is obtained as

\[
C_{0,l} = 2 \left[ \hat{P}_C(|z_{0,l}|) - 0.5 \right].
\]

(3.17)

Substituting (3.17) into (3.16), the noise-free reference chip can be estimated by:

\[
\hat{r}_0(t) = \frac{1}{N + 1} \left\{ \tilde{r}_0(t) + \sum_{l=1}^{N} 2[\hat{P}_C(z_{0,l}) - 0.5]\text{sign}(z_{0,l})\tilde{r}_l(t) \right\}.
\]

(3.18)

Finally, this enhanced reference chip is used to perform the demodulation

\[
\hat{b}_m = 0.5\text{sign}(\hat{r}_0(t) \ast \hat{r}_m(t)) + 0.5.
\]

This modulation scheme, where the noise performance is improved further by averaging will be referred later as FM-DCSK/AV. In order to determine the performance of
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the FM-DCSK/AV system, several simulations have been performed for different values of $N$. The results of these simulations are summarized in Fig. 3.13. Observe, that with this method a 3.9 dB improvement in the noise performance can be achieved for $N = 9$.

The source of the noise performance improvement is twofold:

- $N$ information bearing chips are transmitted using one reference chip and
- the noise corrupting the reference chip is reduced by averaging.

These effects can be observed in Table 3.2, where the required $E_b/N_0$ is given for three values of BER as a function of $N$. The table shows the improvement in noise performance for both the FM-DCSK/S and FM-DCSK/AV modulation schemes.

<table>
<thead>
<tr>
<th>$N$</th>
<th>BER=$10^{-1}$</th>
<th>BER=$10^{-2}$</th>
<th>BER=$10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.5 dB</td>
<td>12.7 dB</td>
<td>14.3 dB</td>
</tr>
<tr>
<td>2</td>
<td>8.9 dB</td>
<td>8.5 dB</td>
<td>11.9 dB</td>
</tr>
<tr>
<td>3</td>
<td>8.6 dB</td>
<td>8.0 dB</td>
<td>11.6 dB</td>
</tr>
<tr>
<td>4</td>
<td>8.3 dB</td>
<td>7.4 dB</td>
<td>11.4 dB</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>8.2 dB</td>
<td>6.6 dB</td>
<td>11.2 dB</td>
</tr>
</tbody>
</table>

Table 3.3 The required $E_b/N_0$ for different desired BER as a function of $N$. The table shows the performance of the enhanced FM-DCSK systems without and with averaging. For reference, the noise performance of the original FM-DCSK is shown in the first row ($N = 1$).

Comparing the advantages and disadvantages of the coherent antipodal CSK (for a description see Sec. 1.4.1), FM-DCSK and FM-DCSK/AV, we conclude:

- coherent antipodal CSK
  - advantages:
only one chip is required to transmit one bit information,

if the synchronization is achieved and maintained at the receiver, we have an almost noise free reference signal,

best theoretical noise performance;

– disadvantage: the problem of chaotic synchronization in noisy environment has not yet been solved and most probably the problem of identical synchronization required in communication cannot be solved because of theoretical reasons;

• differentially coherent FM-DCSK

  – advantages:
    * do not require synchronization,
    * excellent multipath performance;

  – disadvantages:
    * the reference signal at the receiver is corrupted by noise,
    * half of the bit energy and half of the bit duration is wasted because the reference chip does not carry information only provides reference for the demodulation;

• FM-DCSK/AV

  – advantages:
    * do not require synchronization,
    * excellent multipath performance,
    * the wasted bit energy and bit duration are only $E/(N+1)$ and $T/(N+1)$, respectively,
    * the larger $N$, the smaller the noise corrupting the recovered reference;

  – disadvantage: complex receiver architecture is required.

For a hypothetical case when $N$ tends towards infinity the noise performance of FM-DCSK/AV tends toward that of coherent antipodal CSK modulation scheme, i.e., the additional energy and time to transmit the reference becomes negligible and the reference signal becomes noise-free. This gives the performance bound of this system.

The choice of $N$ is a trade-off among a few factors:

• the larger $N$ provides better noise performance;

• the noise performance improvement as a function of $N$ has a saturation, i.e., above a certain $N$ any further increase in $N$ results in a very small improvement in noise performance;

• the larger the $N$, the more complex the receiver required for the demodulation.

To overcome the disadvantages of the FM-DCSK/AV, two solutions are proposed in the next section for noise performance improvement.
3.3.2 Non-Redundant Error Correction Using the Graph Theory

To demonstrate the role of error correction algorithms let us consider an enhanced FM-DCSK system with \( N = 2 \). Assume that the enhanced FM-DCSK receiver is used and that the elements of observation vectors are:

\[
\begin{align*}
    z_{0,1} &= +0.95E_b \\
    z_{0,2} &= -0.1E_b \\
    z_{1,2} &= +1.03E_b
\end{align*}
\]

The segmentation of this enhanced FM-DCSK signal and the observation signals are shown in Fig. 3.14.

![Figure 3.14](image)

**Figure 3.14** Enhanced FM-DCSK signal segmentation and the observation signals for \( N = 2 \), i.e., one reference are followed by two information-bearing chips.

If the decision is done from the sign of \( z_{0,i}, i = 1, 2 \) then the following bits appears at the demodulator output

\[
\begin{align*}
    z_{0,1} &= +0.95E_b & \Rightarrow & & b_1 = 1 \\
    z_{0,2} &= -0.1E_b & \Rightarrow & & b_2 = 0
\end{align*}
\]

However the sign of \( z_{1,2} \) is positive. This correlation suggests that the two transmitted bits are identical. From the above considerations we can conclude that one of the signs of correlations is wrong.

Since \( z_{0,2} \) has the smallest absolute value, taking into account the results shown in Sec. 3.2 we conclude that most likely \( z_{0,2} \) is the correlation with the wrong sign. So, instead of using \( z_{0,1} \) and \( z_{0,2} \) to make the decision we should use \( z_{0,1} \) and \( z_{1,2} \). The error correction by averaging presented in Sec. 3.3.1 probably eliminates this error, but since \( N \) is small it is not guaranteed.

Extending the above analysis for the general case we may expect that powerful error correction can be achieved if we select from all of the possible observation signals only those which have most likely correct sign and to perform the decision using these observation signals.

To select the observation signals providing the highest probability of correct decision we represent the received chips and their cross correlations by a graph and use combinatorial optimization methods to find the optimum decision strategy. During this optimization process instead of correlating a chip only with the reference, we correlate it with all of the received chips as shown in Fig 3.3. In contrast with the FM-DCSK/AV where each chip has to be stored, in this case only the elements of the observation vector which are scalar numbers have to be stored to perform the error correction. Furthermore, the analytical expression about the correctness of a decision presented in Sec. 3.2 is used during the optimum search process.

For a block of one reference and \( N \) information bearing chips, let us define a graph in the following way
• the \((N+1)\) vertices of the graph represents the received chips \(\tilde{r}_l(t), l = 0, 1, \ldots, N\);

• \((N+1)N/2\) edges which connects these vertices represents the elements of the observation vector \(z_{k,l}\);

• a weight giving the probability of correctness of correlation is associated to each edge.

Hence, in this graph vertex “0” represent the reference chip, while vertex “1” stands for the \(l\)th information chip. Edge \((k,l)\) which connects vertex \(k\) with vertex \(l\) represents the correlation between \(\tilde{r}_k(t)\) and \(\tilde{r}_l(t)\), i.e., the element \(z_{k,l}\) of the observation vector. This graph is shown in Fig. 3.15.

![Figure 3.15](image.png)

Figure 3.15  The selection of the optimum decision strategy is done using this graph. The vertices of the graph denote the chips, while its edges represent the correlations between the connecting vertices. The graph has \(N + 1\) vertices and \((N + 1)N/2\) edges.

We characterize each edge \((k,l)\) with a weight called “cost”. This cost is a function of the probability of making a wrong decision if we use \(z_{k,l}\). Hence,

\[
w_{k,l} = f(\hat{P}_W(|z_{k,l}|)) = f[1 - \hat{P}_C(|z_{k,l}|)]
\]

where \(w_{k,l}\) is the weight associated to edge \((k,l)\), while \(\hat{P}_C(z_{k,l})\) is given by (3.14). The scaling function \(f(\cdot)\) has to be introduced to convert the \(P_W(|z_{k,l}|)\) to a cost which is adequate for standard optimum search methods. This function depends on the optimum search algorithm and will be described later. A large cost means a large probability of making a wrong decision, i.e., using that edge in the detection results in a poor BER.

Using these considerations we can develop two powerful error correction methods. The first method is based on searching the optimum decision path for each individual bit. This method and the definition of the decision path is discussed in Sec. 3.3.2.1. The second method is based on using the smallest cost decision tree to demodulate in one step a whole block of \(N\) bits. This method and the definition of the decision tree is discussed in Sec. 3.3.2.2.

Both methods assume that the receiver configuration shown in Fig. 3.3 is used. Hence, the decision circuit produces \(N(N+1)/2\) observation signals denoted by \(z_{k,l}\). Using these scalar quantities the graph shown in Fig. 3.15 is constructed. The challenge is: find the optimum decision strategy starting from this graph.
3.3.2.1 Error Correction Using the Shortest Path Algorithm,—a Maximum-Likelihood Detection

It follows from the special structure of the enhanced FM-DCSK signal that the decision for each bit can be done along many paths in the graph which starts from vertex “0” and terminates at the desired vertex. These paths are referred later to as decision paths.

For example, if \( N = 3 \) and the decision has to be done for \( m = 3 \), any of the following five decision paths can be used:

\[
\text{sign}[\tilde{r}_0(t) \ast \tilde{r}_3(t)] \quad \text{or}
\]

\[
\text{sign}[\tilde{r}_0(t) \ast \tilde{r}_1(t)] \text{sign}[\tilde{r}_1(t) \ast \tilde{r}_3(t)] \quad \text{or}
\]

\[
\text{sign}[\tilde{r}_0(t) \ast \tilde{r}_2(t)] \text{sign}[\tilde{r}_2(t) \ast \tilde{r}_3(t)] \quad \text{or}
\]

\[
\text{sign}[\tilde{r}_0(t) \ast \tilde{r}_1(t)] \text{sign}[\tilde{r}_1(t) \ast \tilde{r}_2(t)] \text{sign}[\tilde{r}_2(t) \ast \tilde{r}_3(t)] \quad \text{or}
\]

\[
\text{sign}[\tilde{r}_0(t) \ast \tilde{r}_2(t)] \text{sign}[\tilde{r}_2(t) \ast \tilde{r}_1(t)] \text{sign}[\tilde{r}_1(t) \ast \tilde{r}_3(t)]
\]

where the decision is done in favor of bit “1” if these expressions are greater than zero.

In the noise free case, all of the above expressions give the same result, but if we receive noisy chips these expressions may differ from each other.

Using the results presented in Sec. 3.2 and assuming that the observation signals are independent random process we get that the probability of the correct decision for bit \( b_3 \) is:

\[
\begin{align*}
\bullet & \quad P_C(|z_{0,3}|) \text{ using the first decision path;} \\
\bullet & \quad P_C(|z_{0,1}|)P_C(|z_{1,3}|) \text{ using the second decision path;} \\
\bullet & \quad \ldots \\
\bullet & \quad P_C(|z_{0,2}|)P_C(|z_{2,1}|)P_C(|z_{1,3}|) \text{ using the last decision path.}
\end{align*}
\]

The application of ML decision rule to the \( m \)th bit requires the selection of that path from all of the possible decision paths for which the probability of correct decision is the maximum.

In the following we will extend the above example \((N = 3)\) for the general case.

If \( N \) bits are transmitted using the same reference chip then the number of possible decision paths is

\[
\sum_{i=1}^{N} \frac{(N-1)!}{(N-i)!}.
\]

For a given decision path let us denote the indexes of the vertices along the path with \( k_1, k_2, \ldots k_L \), i.e., the decision path starts from \( k_1 = 0 \), goes through vertices \( k_l \) and ends in \( k_L = m \). Let us denote this decision path by \( G \), i.e.,

\[
G = (G_1, G_2, \ldots G_{L-1}) = \{(k_1, k_2), (k_2, k_3), \ldots (k_{L-1}, k_L)\}
\]

where \( G_l \) denotes edge \((k_l, k_{l+1})\). As in the previous sections, let us denote the correlation between the \( k_l \)th and \( k_{l+1} \)th chips with \( z_l \).

The decision for bit \( b_m \) is made using the signs of observations signals along the given decision path \( G \) starting at \( k_1 = 0 \) and ending in \( k_L = m \), i.e.,

\[
\hat{b}_m = 0.5 + 0.5 \prod_{l \in G} \text{sign}(z_l).
\]
The probability that (3.19) gives the correct estimation for \( b_m \) is determined by the probabilities of correctness of the signs of \( z_l \). Assume that \( z_l \) are independent random variables then the probability that \( \hat{b}_m = b_m \) is given by

\[
P_{\text{correct}} = \prod_{l \in G} \hat{P}_C(|z_l|) \tag{3.20}
\]

where \( \hat{P}_C(|z_l|) \) is the probability that the sign of observation signal \( z_l \) is correct, it is given by (3.14) in Section 3.2.

For the whole \( N(N+1)/2 \) observation signals, to make a ML decision for bit \( b_m \), we have to use those observation signals which forms a decision path starting from vertex “0” and ending on vertex “m” and ensures the smallest probability of wrong decision, i.e., (3.20) along that path is maximum.

For the selection of the decision path that ensures the highest probability of correct decision the shortest path\(^2\) algorithm [Kre99] is used. Since this algorithm searches the path which has the minimum cost, a cost has to be associated with each edge to select the path having the minimum cost. We define the cost associated to edge \( l = (k_l, k_{l+1}) \) as

\[
w_l = -\log \left[ \hat{P}_C(z_l) \right]. \tag{3.21}
\]

In equation (3.21) \( \hat{P}_C(z_l) \) is transformed using the “\(- \log(\cdot)\)” function. The reason of this transformation is that the shortest path algorithm detects the path in a graph which starts from a specified vertex, ends on another one, and the \textit{sum} of the weights along that path is minimum. Using the cost (3.21) we detect that path along which the \textit{product} of \( P_C(z_l) \) is maximum.

Substituting (3.14) into (3.21) we get

\[
w_l = -\log \left[ \frac{1}{1 + e^{-2|z_l|/\hat{\sigma}^2}} \right] \tag{3.22}
\]

This algorithm ensures that

\[
\sum_{l \in G} w_l = -\sum_{l \in G} \log \left[ \hat{P}_C(z_l) \right] \tag{3.23}
\]

is minimum for the selected path \( G \). Hence, if (3.23) is minimum then the expression

\[
\sum_{l \in G} \log \left[ \hat{P}_C(z_l) \right] \tag{3.24}
\]

is maximum, i.e., the probability of making a correct decision is maximum.

The steps of this enhanced FM-DCSK demodulation procedure are as follows:

- each reference chip is followed by \( N \) information bearing chips, i.e., \( N \) information bits is transmitted by one block;
- the correlation between each pair of received chips are calculated at the receiver;

\(^2\)In our simulations we have used Dijkstra’s algorithm [Kre99] to find the shortest path.
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- $\mu$ and $\sigma$ are determined using (3.12) and (3.11), respectively;
- the weights associated with each edge are calculated by means of (3.22);
- using Dijkstra’s shortest path algorithm, the path giving the ML decision is selected for each bit $b_m$;
- the decision for $b_m$ is done using the optimum decision path found in the previous step.

This enhanced FM-DCSK system will be referred later to as FM-DCSK/SP.

In order to determine the performance of FM-DCSK/SP system, several simulations have been performed. The results of these simulations are shown in Fig. 3.16.

![Figure 3.16](image)

Figure 3.16 Noise performance of the FM-DCSK/SP system for different values of $N$: $N = 9$ ("+") marks), $N = 4$ ("x" marks), $N = 3$ ("o" marks) and $N = 2$ ("□" marks). The noise performance of the original FM-DCSK system ("∗" marks) is also shown for comparison.

Figure 3.16 shows that an improvement of 4.4 dB at $BER = 10^{-3}$ can be achieved in noise performance by the FM-DCSK/SP system. Note that this performance improvement is twofold:

- $N$ information bearing chips are transmitted using one reference chip and
- the optimum decision path is selected among the $\sum_{i=1}^{N} (N - 1)!/(N - i)!$ possible decision paths for each of the transmitted bits by the shortest path algorithm.

Table 3.4 shows the required $E_b/N_0$ for different desired BER as a function of $N$. The table shows the performance of the improved FM-DCSK system without and with the shortest path technique proposed in this section.

We conclude that FM-DCSK/SP is a ML solution to select the optimum decision path for each of the transmitted bits. It improves the noise performance of the original FM-DCSK system by 4.4 dB at a BER=$10^{-3}$. This performance improvement means an additional 2.8 dB improvement compared to the performance of the FM-DCSK/S system.

3.3.2.2 Error correction Using the Minimum Cost Spanning Tree Algorithm

The error correction presented in the previous section is the best in the term of noise performance. However, the determination of the weights and the search for the optimum decision path in the case of each transmitted bit need a lot of computations. The goal
Table 3.4 The required $E_b/N_0$ for different desired BER as a function of $N$. The table shows the performance of the improved FM-DCSK system without and with the shortest path technique presented in this section. For comparison, the noise performance of the original FM-DCSK is shown in the first row ($N = 1$).

of this section is to find a suboptimum method that provides almost the same noise performance at much smaller computation cost.

To achieve this goal we search for the smallest number of edges on the graph which:

- make us possible to detect all of the $N$ bits, and
- give us the highest probability for making a correct decisions for a block of $N$ bits simultaneously.

To find these edges the following conditions should hold:

- we have to have at least $N$ edges,
- we have to have at least one decision path from vertex “0” to all of the vertices, and
- $P_C$s of these edges have to be as large as possible.

A spanning tree of a graph with $(N + 1)$ vertices is by definition a subgraph build from $N$ connected edges which do not have cycle. Hence, the answer for our problem is to find the minimum cost spanning tree.

In FM-DCSK/SP we had to do an optimum search for each individual bits. Here, the minimum cost spanning tree algorithm [Kre99] is used to select the spanning tree offering the minimum probability of wrong decisions for the block of $N$ bits, and the decision is done using this tree. This is why this error correction requires less computation time.

To minimize the BER, the weights associated with each edge must have a low value if the probability of correct decision is high. Recall that the absolute value of correlation characterize the probability of correct decision, consequently the weight for the edge connecting the $k$th and $l$th vertices is defined as

$$w_{k,l} = \frac{1}{|\tilde{r}_k(t) \star \tilde{r}_l(t)| + \epsilon} \quad (3.25)$$

$3$Connected edges mean that there is a path from an arbitrary chosen vertex to any other vertex.

$4$A cycle is a path of at least three edges that are closed.

$5$In our simulations we have used Prim’s algorithm to find the minimum cost spanning tree.
where $\epsilon$ is a small positive number. Its duty is to prevent the division by zero. Note that a large weight is associated with an edge if the probability of correct decision is low. More accurate weights to achieve a slightly better BER can be developed from (3.14), but this weight (3.25)

- does not require the estimation of $\mu$ and $\sigma$,
- does not contain exponential and logarithmic terms, and
- computer simulations confirmed that the improvement in noise performance achieved by the application of (3.14) is negligible.

The steps of this enhanced FM-DCSK technique are as follows:

- each reference chip is followed by $N$ information bearing one;
- the correlations between each pair of received chips are calculated;
- the weights associated with each edge are calculated from (3.25);
- using Prim’s minimum cost spanning tree algorithm, the spanning tree having the lowest probability of wrong decision is selected for the block of $N$ bits;
- the decision is done for each bit using edges of the selected spanning tree.

This enhanced FM-DCSK technique is referred later to as FM-DCSK/ST.

In order to determine the noise performance of the FM-DCSK/ST technique several simulations have been performed. The results of these simulations are shown in Fig. 3.17.

![Figure 3.17](image)

*Figure 3.17* Noise performance of the FM-DCSK/ST system for different values of $N$: $N = 9$ (“+” marks), $N = 4$ (“x” marks), $N = 3$ (“o” marks) and $N = 2$ (“□” marks). The noise performance of the original FM-DCSK system (“*” marks) is also shown for comparison.

From Fig. 3.17 we may determine the performance improvement of FM-DCSK/ST related to the original FM-DCSK. However, as in the previous cases, this performance improvement arises not only due to the error correction. Noise improvement can be achieved if $N$ information-bearing chips follow the reference chip and noise reduction is not done at the receiver. To have a better sight to the relative performance improvement of FM-DCSK/ST the results presented in Table. 3.2 and Fig. 3.17 are summarized in Table 3.5.
Table 3.5 shows the required $E_b/N_0$ for different desired values of BER as a function of $N$. The table shows the performance of the enhanced FM-DCSK system without and with the application of the spanning tree technique proposed in this section.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$E_b/N_0$ for BER=10$^{-1}$</th>
<th>$E_b/N_0$ for BER=10$^{-2}$</th>
<th>$E_b/N_0$ for BER=10$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.9 dB</td>
<td>12.7 dB</td>
<td>14.3 dB</td>
</tr>
<tr>
<td>2</td>
<td>8.6 dB</td>
<td>11.9 dB</td>
<td>13.5 dB</td>
</tr>
<tr>
<td>3</td>
<td>8.3 dB</td>
<td>11.4 dB</td>
<td>13.0 dB</td>
</tr>
<tr>
<td>4</td>
<td>8.2 dB</td>
<td>11.2 dB</td>
<td>12.7 dB</td>
</tr>
</tbody>
</table>

Table 3.5 The required $E_b/N_0$ for different desired BER as a function of $N$. The table shows the performance of the enhanced FM-DCSK system without and with the spanning tree algorithm. For comparison the noise performance of the original FM-DCSK is also shown in the first row ($N = 1$).

We conclude that the FM-DCSK/ST improves the noise performance of the original FM-DCSK system by 4 dB at a BER=10$^{-3}$. This performance improvement means an additional 2.4 dB improvement related to the noise performance of the FM-DCSK/S system.

### 3.4 CONCLUSIONS

Four enhanced FM-DCSK systems have been proposed in this chapter. To compare the effectiveness of these techniques to each other, the noise performances of these systems were evaluated by computer simulation for the following system parameters: $2B=17\,\text{MHz}$ and $T = 2\,\mu\text{s}$. The results of simulations are shown in Fig. 3.18.

From Fig. 3.18 we conclude that the best improvement in noise performance is achieved by application of the shortest path algorithm. An improvement of 4.4 dB can be achieved at BER=10$^{-3}$ as shown in Fig. 3.18 (d).

FM-DCSK/S improve the noise performance of the FM-DCSK system by reducing the bit duration and bit energy by a factor of $2N/(N + 1)$. The redundancy in FM-DCSK/S signal is exploited in the further improved FM-DCSK systems (FM-DCSK/AV, FM-DCSK/SP and FM-DCSK/ST).

FM-DCSK/AV gives a “smooth transition” from FM-DCSK/S to an ideal coherent antipodal CSK system by reducing the noise corrupting the reference chip. Although, FM-DCSK/AV improves further the noise performance of the FM-DCSK/S system, it requires a relatively large $N$ to work efficiently. If $N$ is sufficiently large then its noise performance becomes almost the same as that of the FM-DCSK/SP system.

FM-DCSK/SP uses the shortest path algorithm to improve further the noise performance of the FM-DCSK/S system. It is a maximum likelihood method that offers the best noise performance. The computational time required by FM-DCSK/SP is larger than that of the FM-DCSK/ST technique.

FM-DCSK/ST uses the minimum cost spanning tree algorithm to perform the error
correction. It is the most efficient in term of computation time. However, its noise performance is a bit behind that of FM-DCSK/SP if $N$ is large. For small $N$ the noise performances of FM-DCSK/ST and FM-DCSK/SP are almost the same.

Considering both the noise performance improvement and the implementation issues we conclude that the best enhanced FM-DCSK system is FM-DCSK/ST.

3.5 RESULTS PRESENTED IN CHAPTER 3

3.5.1 Summary of Claims

Claim 2.1: I have proposed and analyzed enhanced versions of DCSK/FM-DCSK systems. In these enhanced modulation schemes one reference chip is used to transmit $N$ information-bearing chips contrary to the original differentially coherent DCSK/FM-DCSK where each reference chip is followed only by one information-bearing chip.

- I have introduced an enhanced version of DCSK/FM-DCSK systems in which one reference chip is used to transmit $N$ information-bearing chips. In this...
enhanced DCSK/FM-DCSK system the bit energy and the bit duration of the original FM-DCSK system are reduced by a factor of \(2N/(N+1)\). I have defined two receiver architectures for the enhanced FM-DCSK system and I have analyzed the power spectral density of the radiated signal.

- I have given an expression in analytical form for the noise performance and the noise performance bound of the enhanced FM-DCSK system in the case of an AWGN channel. The noise performance of the FM-DCSK/S system proposed by me is 1.6 dB better for \(N = 9\) than that of the original FM-DCSK system at \(BER=10^{-3}\). I have shown that this noise performance improvement is preserved for JTC PCS multipath indoor radio channels.

Claim 2.2: I have studied the statistical property of the observation signals in the FM-DCSK systems and using these results I have elaborated noise reduction strategies.

- I have shown that in the original and enhanced FM-DCSK systems the absolute value of the observation signal can be used as a measure of the probability of correct decision. I have given an analytical expression for the probability of correct decision.
- I have elaborated an averaging technique for the enhanced FM-DCSK receiver in which the reference chip is estimated from the noisy received chips. The noise performance of this method tends toward a coherent antipodal Chaos Shift Keying modulation scheme, where perfect synchronization is assumed. The noise performance of this method is 3.9 dB better than that of the original FM-DCSK.

Claim 2.3: I have introduced two non-redundant error correction techniques exploiting combinatorial optimization methods known from the graph theory. During the optimization process I have exploited that property of the enhanced FM-DCSK system that every chip is transmitted \((N+1)\) times. Furthermore, I used the analytical expression on the correctness of a decision to find the optimum decision strategy.

- Based on the shortest path algorithm I have elaborated a receiver configuration for the enhanced FM-DCSK system. I have shown that this receiver has the best noise performance, but its computational time is longer than that of the minimum cost spanning tree method. The noise performance of this method is 4.4 dB better than that of the original FM-DCSK.
- Based on the minimum cost spanning tree algorithm I have introduced a receiver configuration for the enhanced FM-DCSK system. This system is the best in term of computation time and offers a noise performance that is almost the same as to that of the shortest path algorithm. The noise performance of this method is 4 dB better than that of the original FM-DCSK/S.

3.5.2 Publications of the Author, Related to Chapter 3

3.5.2.1 Book Chapter

- M. P. Kennedy, G. Kolumbán, and Z. Jákó. Chaotic modulation schemes. In M. P. Kennedy, R. Rovatti, and G. Setti, editors, Chaotic Electronics in Telecommuni-
Journal Papers


Refereed Conference Contributions


Other Publications

4

Design of FM-DCKS
Transmitter

4.1 INTRODUCTION

One of the goals of the INSPECT project (for a summary of the project see Appendix B) was the development of a prototype FM-DCKS radio system. This chapter describes the system level development and specifications of the FM-DCKS transmitter done by the author in the framework of this project. The receiver part of the FM-DCKS modem was developed by Gábor Kis and is described in [Kis03].

The targeted main system parameters for the FM-DCKS transmitter, specified in the INSPECT proposal were:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation scheme</td>
<td>FM-DCKS</td>
</tr>
<tr>
<td>Data rate</td>
<td>up to 500 kbit/s</td>
</tr>
<tr>
<td>Targeted BER at $E_b/N_0$</td>
<td>$14 \text{ dB } 10^{-3}$</td>
</tr>
</tbody>
</table>

Other important constrains, such as the center frequency and the bandwidth of the transmitted RF signals, comes from frequency regulations determined by communications commissions and industrial standards. These constrains are described in the following.

Many national communications commission has made available three frequency bands, 902–928 MHz, 2400–2483.5 MHz, and 5728–5750 MHz to satisfy the demand for using new technologies in commercial applications. In these frequency bands, called Industrial Scientific and Medical (ISM) bands, license and service provider fees are not required and new radio communications techniques for which standards have not yet been approved can be tested. Hence, the radiated FM–DCSK signal should be in one of these frequency bands.

One of the applications operating at these frequencies is the WLAN. Since, FM–DCSK was designed as a cheap alternative to conventional WLAN modems, most of the FM-DCKS transmitter parameters, such as RF bandwidth and spectral mask, has been chosen to be similar to that of the conventional WLAN transmitters. The parameters for a WLAN modem are given in the IEEE ISO 802.11b standard [Ins97].
4.2 TRANSFORMATION OF THE FM-DCSK TRANSMITTER

The general block diagram of the FM-DCSK transmitter is shown in Fig. 4.1. This block diagram shows the signal processing tasks to be performed. Unfortunately, most of these signal processing tasks component are not present in conventional communication transmitters, consequently these blocks are not available on the market.

Since the goal of the INSPECT project was to develop a prototype FM-DCSK radio system in order to demonstrate the feasibility of chaotic communications, we have decided to build the FM–DCSK modem using of-the-shelf, and already made circuits instead of trying to build the whole system from the scratch.

After a comprehensive search on the ICs available on the market we concluded that:

- Intersil (formerly Harris) has the most performant “antenna-to-bit” WLAN chipset solution, called PRISM. Many parts of it can be used to implement the FM–DCSK transmitter.

- CNM (one member of the INSPECT project) has an implemented discrete time “Bernoulli shift” chaos generator realized in AMS 0.35 µm 2p-3m CMOS technology.

Hence, the transmitter design priority shifted from the low cost transmitter structure shown in Fig. 4.1 to a transformed FM–DCSK transmitter which uses the chaos generator built by the CNM, their expertise in digital IC design and the PRISM chipset.

For this purpose the transmitter has been partitioned into two parts:

- a core low frequency FM-DCSK transmitter which is implemented by custom made and of-the-shelf circuits,

- an IF/RF part, which converts the low frequency output of the previous block into the ISM band. These circuits are built using the PRISM chipset.

The block diagram of the transformed FM-DCSK transmitter is shown in Fig. 4.2. Note that it is split into two parts. The first part is an FM–DCSK modulator, its output signal is centered about 36 MHz. This signal is up-converted, amplified and filtered to get the 2.4 GHz ISM signal by the second part, i.e., by the circuits of PRISM chipset.

The low frequency part of the INSPECT FM-DCSK radio system is described in the next section, while the IF/RF part is presented in Sec. 4.4.
4.3 LOW-FREQUENCY PART OF THE FM–DCSK TRANSMITTER

The low-frequency FM–DCSK modulator is shown in Fig. 4.3. The chaotic signal is generated by a discrete time chaos generator. This signal is converted into phase increments and a delayed copy of the phase increments by the DCSK phase modulator. The phase increments are used by the direct digital synthesizers (DDS) to produce the signals modulated in frequency by the chaotic signal and the delay is necessary to produce the information bearing chips. The frequency modulation of the chaotic signal is done using two DDS circuits. The outputs of the DDS circuits are processed by the DCSK amplitude modulator sub-block, after which the signal is filtered by an anti aliasing filter. Everything, except the DDS subsystem, has been implemented as a custom made IC by CNM. The DDS subsystem has been realized by two of-the-shelf DDS ICs, manufactured by Analog Devices.

The low-frequency FM–DCSK transmitter provides the FM-DCSK signal centered about 36 MHz and having a bandwidth of 17 MHz.

The detailed description of each of the components shown in Fig. 4.3 is given in the following subsections.

4.3.1 Discrete Time Chaos Generator

The chaos generator is a discrete time “Bernoulli shift” chaos generator, already presented in Sec. 1.2.3.2. A short description of the chaos generator is repeated here for convenience.
The governing difference equation of the Bernoulli shift is

\[ x_{k+1} = \begin{cases} 2x_k + 1 & x_k < 0 \\ 2x_k - 1 & x_k \geq 0 \end{cases} \]  

(4.1)

Its return map, i.e., \( x_{k+1} \) in function of \( x_k \) is shown in Fig. 4.4.

![Bernoulli shift return map](image)

Figure 4.4 “Bernoulli shift” return map.

Figure 4.5 shows the block diagram of chaos generator implemented by CNM. The main features of this chaos generator are [DRRV99]:

- clock frequency: from 10 MHz up to 20 MHz,
- resolution: 10 bits,
- realized in AMS 0.35 µm 2p-3m CMOS technology,
- power supply: 2.7 V.

Observe, that this chaos generator has one important parameter that has a strong influence on the overall system performance. That parameter is the clock frequency of the sample and hold (S/H) circuit. The effect of clock frequency on the overall system performance is determined in Sec. 4.5.

### 4.3.2 FM–DCSK Modulator Implemented by DDS

#### 4.3.2.1 Direct Digital Synthesis (DDS)

In its simplest form, a direct digital synthesizer is a finite state machine (phase accumulator) with a sinusoidal readout function:

\[
\begin{align*}
\theta_{k+1} &= (\theta_k + \Delta \theta_k) \mod 2\pi \\
y_k &= \sin(\theta_k)
\end{align*}
\]
where $\theta_k$ denotes the phase at time instant $t = k/f_{DDS}$, $k$ is an integer and $f_{DDS}$ is the clock frequency of DDS circuits.

To get a frequency modulated signal, the phase increment has to be varied. The mean value of the phase increment per clock cycle $\Delta \theta_k = \text{mean}(\Delta \theta_k)$ determines the center frequency $f_c$ of the output

$$f_c = \frac{\Delta \theta_c}{2\pi} f_{DDS}.$$

For example, if $\Delta \theta_c = \pi/2$, then $f_c = f_{DDS}/4$.

The block diagram of the DDS system is shown in Fig. 4.6.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure4_6.png}
\caption{Block diagram of Direct Digital Synthesizer (DDS).}
\end{figure}

4.3.2.2 Frequency Modulation with DDS

In FM-DCSK, the carrier is a frequency modulated signal. The instantaneous frequency of DDS output is controlled by the phase increment of the phase accumulator. By increasing the phase increment, the output frequency is increased; by reducing the phase increment, the output frequency decreases. Thus, frequency modulation may be accomplished by adding an additional input $u$ to the phase accumulator:

$$\theta_{k+1} = (\theta_k + \Delta \theta_c (1 + u_k)) \mod 2\pi$$

$$y_k = \sin(\theta_k).$$

If $u_k$ is constant then the output frequency is given by

$$f(u_k) = (1 + u_k)\frac{\Delta \theta_c}{2\pi} f_{DDS} = (1 + u_k)f_c.$$

Let $\Delta \theta_c = \pi/2$ as before and consider two cases: $u_k^- = -1/4$ and $u_k^+ = 1/4$. The corresponding output frequencies are:

$$f(u_k^-) = \frac{3}{4}f_c \quad \text{and} \quad f(u_k^+) = \frac{5}{4}f_c.$$

4.3.2.3 FM-DCSK with DDS

Since the DDS can operate as an FM modulator, an FM-DCSK signal can be obtained by preceding two DDS with a DCSK phase modulator and following them by a DCSK amplitude modulator as shown in Fig. 4.3. The upper and lower branches of DDS subsystems provide the FM signals for the reference and information bearing parts, respectively.

The operation of the DCSK phase and amplitude modulators are described in the next sections.
4.3.3 DCSK Phase Modulator

The block diagram of the DCSK phase modulator is shown in Fig. 4.7. The first part of the DCSK phase modulator is called amplitude to phase converter. The amplitude to phase converter converts the input signal from \([-1, 1]\) into phase increments in \([2\pi(f_c - BW/2)/f_{DDS}, 2\pi(f_c - BW/2)/f_{DDS}]\). The term \(\pi BW/f_{DDS}\) determines the RF bandwidth, while the offset \(2\pi f_c/f_{DDS}\) determines the signal center frequency. This transformation yields an FM modulated chaotic signals at the output of the DDS circuits.

![DCSK phase modulator block diagram.](image)

The output of the amplitude to phase converter is fed into the first DDS to get the reference part of the FM-DCSK signal, while a delayed copy of the same signal goes into the second DDS to produce the information bearing part of the transmitted signal. In this manner the output of the second DDS will be a delayed copy of the first DDS output.

4.3.4 The DCSK Amplitude Modulator and the Anti Aliasing Filter

These are the last two blocks of the low-frequency FM-DCSK transmitter shown in Fig. 4.3. The DCSK amplitude modulator selects the output of the first DDS (reference part of the FM-DCSK signal) for the first half of the bit duration. For the second half of the bit duration depending on the transmitted bit either a delayed copy or a delayed and inverted copy of the reference signal is selected by the DCSK amplitude modulator.

The clock frequency of DDS is 180 MHz and the resolution of D/A converter is 10 bit. To remove the unwanted spectral components, the output of D/A converter has to be filtered by an anti aliasing filter.

This filter is a third-order band-pass Butterworth filter, with the center frequency of 36 MHz and a bandwidth of 17 MHz.

\(^1\)Please note that instead of this formula the frequency deviation is used in Sec. 4.5.
4.4 IF AND RF PARTS OF THE FM-DCSK TRANSMITTER

The RF part of the transmitter is based on the PRISM chip-set. This chip-set has been developed by Intersil to implement an IEEE 802.11 compliant wireless local area network. It contains all the components needed to generate an RF signal in the 2.4 GHz ISM band.

The block diagram of the IF/RF part of the FM-DCSK transmitter implemented in the framework of the INSPECT project is shown in Fig. 4.8.

Please, note that expect the first up converter mixer (MC13143), the whole architecture is identical with that proposed by the Intersil company for the WLAN transmitter. This is why, a detailed analysis of the IF/RF part of the transmitter is not carried out in the thesis.

The signal path from the output of low-frequency FM-DCSK modulator to antenna and a brief description of the circuits used in the IF/RF part is given in the remaining part of this section.

At the first stage the FM-DCSK modulated signal, which is centered around 36 MHz is up converted to the 280 MHz intermediate frequency (IF) using the MC13143 (Motorola) mixer. This is a low power wide-band mixer. It has a 1 dB compression point of -1 dBm and a third-order interception point of 16 dBm. These features, together with an extremely wide input and output bandwidth, make it suitable for use as a first mixer in this application.

The output spectrum is formed by a channel filter (TQS432–Toyocom). This filter also reduces the the excess out-of-band noise floor that could be amplified by the next stages causing potential adjacent channel interference and also suppresses the side lobes resulting from the digital modulation process.

This filter has a bandwidth of 17 MHz and a center frequency of 280 MHz. It is
a surface acoustic wave (SAW) device manufactured by Toyocom. It has a maximum insertion loss of 10 dB. It has very high bandstop attenuation, namely 50 dB at $f_0 \pm 38$ MHz.

The SAW filter is followed by a HFA3664 (Intersil) circuit. This is an up-converter followed by a silicon bipolar AGC preamplifier. It uses a double balanced mixer with a conversion gain of 4.5 dB and requires a low local oscillator (LO) power of -8 dBm. Its LO leakage is as low as -25 dBm. It has a noise figure of 18 dB, an IP3 of -1 dBm and a 1 dB compression point of -6.5 dBm.

A bandpass filter is used after the mixer in order to:

- provide the first post-selection of transmitting channels,
- reduce the out-of-band noise and mixing spurs, and
- reduce the LO leakage.

This is a SAW filter manufactured by Fujitsu (F6CE). It has a bandwidth of 97 MHz centered at 2448.5 MHz, an insertion loss of 4 dB and an out-of-band attenuation of 20 dB.

The next circuit is an AGC RF preamplifier. This AGC preamplifier has a maximum gain of 24 dB with an output compression point of 8 dBm and an AGC range of 30 dB. It has a noise figure of 8 dB, an IP3 of 14 dBm and a 1 dB compression point of 10 dBm.

The third filter is placed after this preamplifier to attenuate further the LO leakage and also to remove any spurious output from the preamplifier. It is a two-pole dielectric filter manufactured by Toko (TDF2A) with a bandwidth of 100 MHz. It has 2 dB in-band insertion loss and 20 dB attenuation at $f_0 \pm 280$ MHz.

The final stage is the HFA3925 (Intersil) power amplifier. This is a GaAs device with a maximum linear output of 21 dBm, a 24 dBm compression point, a gain of 24 dB, an IP3 of 24 dBm and a noise figure of 3 dB. This drives the antenna directly with 100 mW output power. The antenna is a simple half wave dipole.

The LO signals for the two mixers are provided by the dual frequency synthesizer control circuit HFA3524 (Intersil) and two VCOs. The first mixer uses a local oscillator frequency of 244 MHz. This is generated using a discrete VCO. The second mixer uses a local oscillator frequency of 2150 MHz generated by an integrated VCO, manufactured by Motorola (KXN1332a). The VCO power is not sufficiently high to drive the up-converter directly, so a HFA3424 low noise amplifier is used to amplify the oscillator output.

The level diagram and the part list for the IF/RF part of the FM-DCSK transmitter are shown in Tables 4.4 and 4.4, respectively.

### 4.5 DETERMINATION OF MAIN SYSTEM PARAMETERS

Hitherto most of the parameters of INSPECT FM-DCSK transmitter have been fixed. The summary of fixed system parameters are given in Table 4.3.

Table 4.3 shows that three system parameters have not yet been exactly specified. These parameters are:

- the bit duration,
- the clock frequency of the chaotic signal generator $f_{\text{chip}}$, and
Table 4.1 Level diagram for the IF/RF part of the transmitter.

<table>
<thead>
<tr>
<th>Component</th>
<th>part no.</th>
<th>manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>First up-converter</td>
<td>MC13143</td>
<td>Motorola</td>
</tr>
<tr>
<td>Channel select filter</td>
<td>TQS432E-7R</td>
<td>Toyocom</td>
</tr>
<tr>
<td>Image reject filter</td>
<td>F6CE-2G4500-L2WA</td>
<td>Fujitsu</td>
</tr>
<tr>
<td>Second mixer and pre-amp</td>
<td>HFA3664</td>
<td>Harris</td>
</tr>
<tr>
<td>Transmit filter</td>
<td>TDF2A-2450T-10</td>
<td>Toko</td>
</tr>
<tr>
<td>Power amplifier</td>
<td>HFA3925</td>
<td>Harris</td>
</tr>
<tr>
<td>LNA</td>
<td>HFA3424</td>
<td>Harris</td>
</tr>
<tr>
<td>Frequency synthesizer</td>
<td>HFA3524</td>
<td>Harris</td>
</tr>
<tr>
<td>VCO</td>
<td>KXN1332A</td>
<td>Motorola</td>
</tr>
</tbody>
</table>

Table 4.2 Parts list for the IF/RF part of the transmitter.

Modulation scheme                | FM-DCSK         |
Architecture                      | super heterodyne|
Center frequency                  | 2.4 GHz         |
RF bandwidth                      | \( \leq 17 \) MHz |
Data rate                         | \( \leq 500 \) kbit/s |
Center frequency of the low-frequency FM-DCSK modulator | 36 MHz |
Chip rate                         | from 10 MHz up to 20 MHz |
Frequency deviation of the FM modulator | depends on the RF bandwidth |
Resolution of the chaotic signal generator | 10 bits |
Targeted BER at \( Eb/N_0 = 14 \) dB (AWGN channel) | \( 10^{-3} \) |

Table 4.3 Main system parameters of INSPECT FM-DCSK transmitter.
• the bandwidth of the transmitted signal.

When developing a new telecommunications system, a trade-off between the main system parameters is inevitable. For example, a larger transmitted bandwidth offers better multipath performance, but it yields worse noise performance in AWGN channel.

Because the INSPECT FM-DCSK transmitter has been developed for WLAN applications, the shape of the spectrum of the transmitted signal was optimized according to this application. The radiated signal has to be a wide-band signal, with a spike free spectrum and its power spectral density must satisfy the spectral mask given in the IEEE 802.11 standard [Ins97]. The system parameters have to be chosen in such a way that the spectrum of INSPECT FM-DCSK signal satisfy these requirements.

Since an analytical expression for the noise performance of FM-DCSK operating in WLAN environment is not yet available, the effects of the main system parameters on the noise performance will be determined mostly by computer simulation in the following subsections.

4.5.1 Spectrum of the Transmitted FM-DCSK Signal

Since the clock frequency of the chaos generator, also called chip rate, has the strongest influence on the shape of radiated signal, this subsection is aimed to determine the optimum value for the chip rate.

The objectives of the wide-band and spike free signal are twofold: (1) to overcome the multipath propagation problem and (2) to reduce the transmitted power spectral density to avoid interfering with other radio communications sharing the same frequency band. Depending on the parameters of the FM-DCSK modulator we can implement either a fast or a slow FM-DCSK system.

In this subsection we will assume that the bandwidth of the transmitted signal is set to 17 MHz, according to the IEEE 802.11 standard. The performance of the FM-DCSK system for different bandwidths will be analyzed in Sec. 4.5.3 and Sec. 4.5.4.2 for AWGN and multipath channels, respectively.

4.5.1.1 Fast and Slow Spreading Techniques

The generation of FM-DCSK signal is shown in Fig. 4.3: the chaotic signal is fed into an FM modulator to generate a wide-band band-pass signal, then the DCSK modulation is applied to this wide-band signal.

As described in Sec. 4.3.1, the frequency of chaotic signal generator may vary from 10 MHz to 20 MHz. We will refer the FM-DCSK system operating at these limits to as slow and fast FM-DCSK systems.

Hence, in our simulations the chip rates take two values: 20 MHz and 10 MHz, respectively, in the case of fast and slow FM-DCSK systems. To get the best multipath and interference performance, the spectra of the fast and slow FM-DCSK signals must be free from any spikes. To avoid interference with other radio channels, the power generated outside the 17 MHz RF bandwidth has to be as low as possible.

Simulated spectra of the chaotic FM modulator output and the FM-DCSK signal are plotted in Fig. 4.9 for the fast FM-DCSK system. To show the relationship between the bit duration and the skirt of the FM-DCSK signal, Fig. 4.9 (b) shows the spectrum for $T = 2 \mu$s and $T = 16 \mu$s. Note that DCSK modulation does not cause periodic spikes in the spectrum, but it increases the skirt considerably. By increasing the bit duration the
skirt can be lowered, but increasing $T$ reduces the attainable data rate. The other way to maintain the data rate is to suppress the unwanted skirt by means of a band-pass filter at the transmitter.

Figure 4.9  Output spectra of the (a) FM and (b) DCSK modulators in a fast FM-DCSK system. $T$ is set to 2 $\mu$s and 16 $\mu$s to illustrate the effect of bit duration on the height of the skirts. The chip rate is 20 MHz.

Simulated spectra of the slow FM-DCSK system are shown in Fig. 4.10. Note that the in-band shapes of the spectra of the fast and slow FM-DCSK signals differ considerably from each other. Because the bit duration is set to 2 $\mu$s in our simulations, even the slow FM-DCSK system visits ten different frequencies during the transmission of one information bit. Figure 4.10 shows the output spectra of both the FM modulator (a) and the DCSK modulator (b). Note again that the effect of DCSK modulation in both cases is to raise the skirt.

Figure 4.10  Spectra of a slow FM-DCSK system (a) at the output of the FM modulator and (b) at the output of the DCSK modulator. The chip rate is 10 MHz.

Differences Between Fast and Slow FM-DCSK Systems

Both the fast and slow spreading techniques reduce the average power spectral density of a FM-DCSK signal, but the way in which they do this is very different. This difference has serious implications for radio communications systems sharing the same frequency band.

In the fast system, the instantaneous frequency of the transmitted signal is constant
only for a very short time interval (the chip time). Thus, the transmitted energy is never concentrated about a certain frequency; rather, it is spread continuously over the entire RF channel bandwidth. If this signal interferes with another radio channel in the same band then it simply increases the background “noise level”, and so reduces the quality of that channel, but it does not interrupt it. If the power spectral density of the fast FM-DCSK system remains under the noise level of the other users then its presence may not even be noticed.

In the slow FM-DCSK system, the chip rate is relatively low and the instantaneous frequency of the transmitted signal remains constant for a relatively long time interval. Consequently, the slow FM-DCSK signal appears like a relatively narrow-band signal that changes its center frequency. A narrow-band radio system in the same frequency band will experience a pop or burst of noise when the slow FM-DCSK signal hits its channel frequency. In this case, the spreading shares the pain of channel interruption over all narrow-band radio systems using the same frequency band.

Note that in this sense the slow FM-DCSK is qualitatively very similar to slow Frequency Hopping Spread Spectrum (FH-SS) systems [Dix94]. However, there is a fundamental difference: in slow FH-SS, a couple of bits are transmitted at every frequency, while the slow FM-DCSK visits many different frequencies within a bit duration. Thus in the case of slow FH-SS a couple of bits can be lost due to multipath, but this problem does not appear in the slow FM-DCSK.

Figs. 4.9 (b) and 4.10 (b) show that the in-band shapes of the radiated spectra are different for the fast and slow FM-DCSK systems. This is why, as will be shown later, the fast FM-DCSK system performs better than the slow one in the WLAN application.

**Performance of Fast and Slow FM-DCSK Systems in AWGN Channel**

The exact expression for the noise performance of FM-DCSK in AWGN channel is given in [Kol00c]:

\[
BER = \frac{1}{2^{BT}} \exp \left( -\frac{E_b}{2N_0} \right) \times \sum_{i=0}^{BT-1} \left( \frac{E_b}{2N_0} \right)^i \sum_{j=i}^{BT-1} \frac{1}{2^j} \left( j + BT - 1 \right).
\]

(4.2)

Where \( B \) is half of the RF bandwidth, while \( T \) denotes the bit duration. From (4.2) we conclude that as long as both the fast and the slow FM-DCSK systems have identical bit duration and bandwidth, their noise performances do not differ. However, please note that this conclusion holds only for AWGN channels.

The performance of the fast and slow FM-DCSK systems in multipath channels will be analyzed in Sec. 4.5.4.

**4.5.1.2 Choosing the Maximum Frequency Deviation of the FM Modulator**

In the FM-DCSK transmitter the carrier is modulated in frequency by the chaotic signal. We may expect that the gain of FM modulator \( k_f \) determines the bandwidth of the transmitted signal and has no other significant influence on the spectral properties of the transmitted signal. However, by simulating the system with different \( k_f \) it has turned out that the ratio of \( f_{\text{chip}}/k_f \) has a strong effect on spectral shape of the transmitted signal. I have observed by computer simulations that if the ratio of \( f_{\text{chip}}/k_f \) is close to an integer then spikes appear in the radiated signal spectrum. This effect is shown in Figs. 4.11 and 4.12.
For a chip rate of 20 MHz the spectrum at the FM modulator output is given in Figs. 4.11 and 4.12 for $k_f = 10$ MHz/V and $k_f = 7.8$ MHz/V, respectively.

As shown in Fig. 4.11, if the clock rate of the chaos generator is equal to $2k_f$, strong periodic components appear at the FM modulator output. For good multipath and interference performance, the spectrum of the FM-DCSK signal must be free from any spikes. To overcome this problem $k_f$ was set to 7.8 MHz/V; the resulted spectrum is shown in Fig. 4.12.

Figures 4.9 (b) and 4.10 (b) shows that the digital modulation reises the skirt of the spectrum, but the spectrum of this signal remains free of spikes. However, this does not mean that the spectrum of the signal is IEEE 802.11 compliant.

The IEEE/ISO standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications [Ins97] requires the following: “The transmitted spectral products shall be less than -30 dBr (dB relative to the peak at $f_c$) for $f_c - 22$ MHz $< f < f_c - 11$ MHz, $f_c + 11$ MHz $< f < f_c + 22$ MHz, -50 dB for $f < f_c - 22$ MHz, and $f > f_c + 22$ MHz, where $f_c$ is the channel center frequency.”

At the input to the PRISM chipset neither a conventional SS transmitter nor the slow and fast FM-DCSK transmitters satisfy this condition.

To suppress the signal level outside the ISM channel bandwidth, a filter is used after the DCSK modulator. Hence, the spectral components covering the adjacent channels are suppressed by the SAW filter in the PRISM chipset. The spectrum of the filtered
FM-DCSK signal is shown in Fig. 4.13

Figure 4.13  Spectrum of the filtered fast FM–DCSK signal after the SAW filter (TQS432-Toyocom).

The spectrum of the filtered signal satisfies the WLAN requirement but an amplitude modulation appears due to filtering. The only disadvantage of the AM is that it requires a linear power amplifier at the transmitter, otherwise the spectrum component in the sideband regrow. Fortunately, both the preamplifier (AGC) and the power amplifier in the PRISM chipset are linear.

4.5.2 Effect of Bit Duration

At the demodulator of a coherent conventional telecommunications system, the received signal is compared, in some sense, to a noise-free reference. The noise-free reference is recovered by synchronization in correlation receivers, while it is stored as the impulse response of a matched filter in matched filter receivers. If the energy per bit is kept constant, then the noise performance of these modulation schemes is independent of the bit duration and the RF bandwidth of the transmitted signal [Hay94]. Equation (4.2) shows that in FM-DCSK the bit duration has a strong influence on its noise performance.

The effect of bit duration on the noise performance of FM-DCSK is shown in Fig. 4.14, where the RF bandwidth of the FM-DCSK signal is 17 MHz.

Figure 4.14  Noise performance of FM-DCSK modulation for bit duration of 1 µs (dotted curve), 2 µs (solid curve), 4 µs (dashed curve), and 8 µs (dashed curve).

Figure 4.14 shows that the bit duration has a very strong influence on the noise performance. From Fig. 4.14 we conclude that reducing the bit duration improves the noise performance of FM-DCSK and results in a higher data rate. However, it also increases the skirt of the generated FM-DCSK signal (see Fig. 4.9) and makes the system
more sensitive to timing recovery errors. As a trade-off among the requirements given above the bit duration has been chosen to 2 \( \mu s \).

### 4.5.3 Effect of RF Bandwidth

Equation (4.2) shows that the noise performance of FM-DCSK also depends on the bandwidth of transmitted signal. To demonstrate this effect we show the results of computer simulations for three different RF bandwidths: 8 MHz, 12 MHz and 17 MHz. The bit duration is 2 \( \mu s \) in every case.

Figure 4.15 shows that the RF bandwidth of the FM-DCSK signal has a little effect on the noise performance of the system. For example, if \( \text{BER}=10^{-3} \) is specified, then the required values of \( E_b/N_0 \) are 13.7, 14.2 and 14.6 dB for RF bandwidths of 8, 12 and 17 MHz, respectively. Although reducing the RF bandwidth improves the noise performance, it degrades the multipath performance. Thus, the choice of RF bandwidth is a trade-off between noise performance and multipath performance.

![Figure 4.15](image-url)

**Figure 4.15** The effect of RF bandwidth on the noise performance of the FM-DCSK modulation scheme. Curves are shown for three RF bandwidths: 8 MHz (dashed curve), 12 MHz (dotted curve) and 17 MHz (solid curve).

### 4.5.4 Performance of FM-DCSK Systems in Multipath Channels

The performance of the FM–DCSK system operating in multipath channel is addressed in [Kis03]. This is why this section is not aimed to give a detailed analysis of the performance degradation of FM-DCSK in multipath channels\(^2\). In contrast, we are interested only in determining the optimum values of system parameters of FM-DCSK transmitter.

For this purpose, we have chosen the simplest WLAN multipath channel model, i.e., a two-ray channel. Typical values of the delay \( \Delta \tau \) are 91 ns for large warehouses and 75 ns for office buildings [JP98]. For this channel we can get the worst-case performance degradation if the delay is 75 ns and the attenuation along the two paths are identical [Kis03]. Hence, in the remaining part of this chapter we will consider this worst-case scenario, and will determine the optimum values for those parameters which have influence on the multipath performance, namely: the chip rate and the RF bandwidth.

\(^2\)For detailed description of the multipath channel models to be used in WLAN environment see Appendix C.
4.5.4.1 Determination of Chip Rate

Figures 4.9 and 4.10 show that the average power spectral densities of the fast and slow FM-DCSK signals are different. In this context “average” means that very long sample functions (of length 1000\(T\)) were used to calculate these spectra.

The consequence of this difference is highlighted in Fig. 4.16, where the multipath performance of fast and slow FM-DCSK is plotted for different center frequencies of the FM-DCSK signal. The curves plotted belong to the best and worst results for both cases. Note that the average performance degradation for the fast and slow FM-DCSK systems is the same, but the variation in performance loss as a function of the relative positions of the center frequency and the multipath-related nulls (for a description see Appendix C) is higher in the slow FM-DCSK system. This effect is caused by the different shapes of the spectra of fast and slow FM-DCSK.

In Fig. 4.16, the bit duration is 2 \(\mu s\), \(2B = 17\) MHz, \(\Delta \tau = 75\) ns and the transmitted signal propagates via two paths, the gain of each path being equal to 1/2.

Because the multipath performance of the fast FM-DCSK is better than that of the slow one, I have concluded that the 20 MHz chip rate has to be used instead of the 10 MHz chip rate in the implemented INSPECT FM-DCSK transmitter.

4.5.4.2 Determination of RF Bandwidth

The bandwidth of the transmitted FM-DCSK signal has the strongest influence on the multipath performance of the system. This effect is illustrated in Fig. 4.17, which shows the worst-case performance degradation in the fast and slow FM-DCSK systems when the RF bandwidth is reduced from 17 MHz to 8 MHz. The attenuation along the two propagation paths is the same and the multipath-related nulls coincide with the center frequency of the FM-DCSK signal. The performance degradation is unacceptably large, showing that 8 MHz bandwidth is not sufficient if data communication has to be established in a multipath environment where \(\Delta \tau\) may be as short as 75 ns.

4.5.5 Concluding Remarks

Those system level parameters that were not exactly known in Table 4.3 have been determined in this section. The optimum values of these parameters have been determined...
Figure 4.17 Worst-case performance degradation caused by multipath propagation in the fast (dashed curve with ‘+’ marks) and slow (dotted curve with ‘+’ marks) FM-DCSK systems when the RF channel bandwidth is reduced from 17 MHz to 8 MHz. For comparison, the noise performance of FM-DCSK with 8 MHz RF channel bandwidth without multipath is also shown (solid curve with ‘×’ marks).

Table 4.4 Parameters of the INSPECT FM-DCSK transmitter.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation scheme</td>
<td>FM-DCSK</td>
</tr>
<tr>
<td>Architecture</td>
<td>super heterodyne</td>
</tr>
<tr>
<td>Center frequency</td>
<td>2.4 GHz ISM band</td>
</tr>
<tr>
<td>RF bandwidth</td>
<td>17 MHz</td>
</tr>
<tr>
<td>Data rate</td>
<td>500 kbit/s</td>
</tr>
<tr>
<td>Center frequency of the low-frequency FM-DCSK modulator</td>
<td>36 MHz</td>
</tr>
<tr>
<td>Chip rate</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Gain of the FM modulator</td>
<td>7.8 MHz/V</td>
</tr>
<tr>
<td>Resolution of the chaotic signal generator</td>
<td>10 bits</td>
</tr>
</tbody>
</table>

by computer simulations using AWGN and multipath channel models. Analyzing the results of the simulations I have concluded:

- the channel bandwidth has to be 17 MHz for the best multipath performance;
- the fast FM-DCSK has to be used instead of the slow one to get better multipath performance;
- the gain of FM modulator has to be set to $k_f = 7.8$ MHz/V in order to avoid spikes in the spectrum;
- 500 kbit/s data rate, i.e., 2 $\mu$s bit duration, has to be used.

The parameters of the INSPECT FM-DCSK transmitter are given Table 4.4.

4.6 RESULTS PRESENTED IN CHAPTER 4

4.6.1 Summary of Claims

Claim 3: I have elaborated design rules for the transmitter part of the FM–DCSK modem.
I have transformed the FM-DCSK transmitter to a low-frequency FM-DCSK transmitter and an IF/RF subsystem. The advantage of this new transmitter architecture is that most of its components can be implemented using off-the-shelf IC such as DDS (from Analog Devices) and super-heterodyne WLAN chipset (from Intersil). Hence, only a part of the low-frequency FM-DCSK transmitter had to be implemented by CNM using mostly low-frequency digital circuits.

I have analyzed the effect of the main system parameters, such as the chip rate, the bit duration and the bandwidth of the transmitted signal on the overall system performance. Assuming a WLAN environment I have determined the optimum system parameters for the INSPECT FM-DCSK transmitter.

4.6.2 Publications of the Author, Related to Chapter 4

4.6.2.1 Book Chapter


4.6.2.2 Journal Papers


4.6.2.3 Patent

- M. P. Kennedy, G. Kolumbán, G. Kis, and Z. Jákó, “Binary digital communication system using a chaotic frequency-modulated carrier,” Irish patent number: S80913, granted on June 1 1999.

4.6.2.4 Refereed Conference Contributions

- M. P. Kennedy, G. Kolumbán, G. Kis, and Z. Jákó, “Recent advances in communicating with chaos,” in Proc. IEEE–ISCAS’98, vol. IV, (Monterey, USA),


4.6.2.5 Conference Article in Hungarian


4.6.2.6 Other publication

Conclusions

There are many radio data communications applications where the conventional narrowband systems fail to operate. For example, if the telecommunications have to be established in a multipath environment. The frequency hopping and direct sequence spread spectrum techniques offer a solution to these problems. However, those techniques require a complex circuitry, the synchronization of spreading codes has to be achieved and maintained at the receiver and due to the complex system configuration these systems have a high power consumption.

The chaotic communications, a brand new approach, offers an alternative solution to these problems. In chaotic communications, the digital information to be transmitted is mapped directly to chaotic, i.e., to an inherently wideband signal. Then this wideband chaotic signal is transmitted via the telecommunications channel. Since the chaotic signal is a wideband signal, the chaotic telecommunications systems are not sensitive to the multipath propagation and due to the low spectral density of transmitted signal they do not cause interference in the narrowband systems operating in the same frequency band. Due to its simple circuitry, the chaotic telecommunications technique offers a cheap alternative to the wideband communications.

Over the past ten years many different modulation schemes have been proposed for chaotic communications. Of these, the Differential Chaos Shift Keying (DCSK) and FM-DCSK techniques, offer the best system performance. However, even the noise performance of these schemes is a few dB behind that of the conventional ones in additive white Gaussian noise channels. Hence, any improvement in the noise performance of DCSK/FM-DCSK has a great importance.

The goal of this thesis was to propose suitable noise performance improvement methods for these systems and to give design rules for the transmitter part of the FM-DCSK system. After a short introductory Chapter (Chapter 1) the above fields have been addressed in the thesis.

In chaotic communications linear filtering cannot remove much of the noise because the spectra of the chaotic signal and the noise overlap. However, by exploiting the short-term predictability of chaotic signals, a part of the additive noise can be removed. Chapter 2 analyzes the noise reduction methods which exploits the determinism in the chaotic signals with an emphasis on their applicability to the DCSK modulation scheme.
The new scientific results given in Chapter 2 are summarized in the following claims:

**Claim 1.1:** I have related Methods I and II proposed by Lee and Williams to the standard gradient descent algorithm. I have shown that both methods are closely related to the standard gradient method. Numerical experiments support these theoretical conclusions.

**Claim 1.2:** I have elaborated new detector configuration for DCSK modulation scheme that exploit noise reduction.

**Claim 1.3:** I have analyzed the performance of noise reduction methods in the DCSK communications scheme.

Chapter 3 introduces improved versions of DCSK/FM-DCSK system, gives “error correction” methods for these modulation schemes and analyzes their noise performance. The new scientific results given in Chapter 3 are summarized in the following claims:

**Claim 2.1:** I have proposed and analyzed enhanced versions of DCSK/FM-DCSK systems. In these enhanced modulation schemes one reference chip is used to transmit $N$ information-bearing chips contrary to the original differentially coherent DCSK/FM-DCSK where each reference chip is followed only by one information-bearing chip.

**Claim 2.2:** I have studied the statistical property of the observation signals in the FM-DCSK systems and using these results I have elaborated noise reduction strategies.

**Claim 2.3:** I have introduced two non-redundant error correction techniques exploiting combinatorial optimization methods known from the graph theory. During the optimization process I have exploited that property of the enhanced FM-DCSK system that every chip is transmitted $(N + 1)$ times. Furthermore, I used the analytical expression on the correctness of a decision to find the optimum decision strategy.

Chapter 4 contains the system level design of an FM-DCSK transmitter implemented in the framework of the Esprit 31103–INSPECT project. The claim of this chapter is:

**Claim 3:** I have elaborated design rules for the transmitter part of the FM–DCSK modem.

For a more detailed description of these claims see Sections 2.4, 3.5 and 4.6.
References


REFERENCES


Ins97. The Institute of Electrical and Electronics Engineers (IEEE). *Std 802.11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications*, 1997.


JKD00. Z. Jákó, G. Kolumbán, and H. Dedieu. On some recent developments of noise cleaning algorithms for chaotic signals. *IEEE Trans. Circuits and


REFERENCES


REFERENCES


Appendix A: Terminology Used in this Work

In the thesis a few acronyms have been used. The meaning of them has been defined at the first use. Here is the list of these acronyms:

AM: Amplitude Modulation
AMS: Austrian Micro Systems
APLL: Analog Phase Locked-Loop
ASK: Amplitude Shift Keying
AUT: University of Thessaloniki
AWGN: Additive White Gaussian Noise
BER: Bit Error Rate
BME: Budapest University of Technology and Economics
BME-MIT: BME Department of Measurement and Information Systems
BPSK: Binary Phase Shift Keying
CDMA: Code Division Multiple Access
CMOS: Complementary Silicon-Metal-Oxygen
CNM: Centro Nacional de Microelectrónica
COOK: Chaos On Off Keying
CPM: Continuous Phase Modulation
CSK: Chaos Shift Keying
DCSK: Differential Chaos Shift Keying
DCSK/AV: Enhanced DCSK with Averaging
DCSK/SP: Enhanced DCSK with Shortest Path
DCSK/ST: Enhanced DCSK with Spanning Tree
DDS: Direct Digital Synthesis
DPSK: Differential Phase Shift Keying
DS-SS: Direct Sequence Spread Spectrum
DS-CDMA: Direct Sequence Code Division Multiple Access
EPFL: Swiss Federal Institute of Technology Lausanne
$E_b/N_0$: Bit energy over the noise power spectral density
FDMA: Frequency Division Multiple Access
FH: Frequency Hopping
FH-SS: Frequency Hopping Spread Spectrum
FM: Frequency Modulation
FM–DCSK: Frequency Modulated Differential Chaos Shift Keying
FSK: Frequency Shift Keying
HUT: Helsinki University of Technology
IF: Intermediate Frequency
INSPECT: Innovative Signal Processing Exploiting Chaotic Dynamics
ISI: Inter-Symbol Interference
ISM: International Scientific and Medical frequency band
JTC: Joint Technical Committee
LPF: Low-Pass Filter
LSB: Least Significant Bit
LTR: Long Term Research Programme (EU)
MAC: Medium Access Control
ML: Maximum Likelihood
NRZ: Non Return to Zero
ODE: Ordinary Differential Equation
OTKA: Hungarian Scientific Research Fund
PCS: Personal Communication Services
PD: Phase Detector
PDF: Probability Distribution Function
PHY: Physical Layer
PM: Pulse Modulation
PSD: Power Spectral Density
QAM: Quadrature AM
QPSK: Quadrature Phase Shift Keying
RF: Radio Frequency
RZ: Return to Zero
SNR: Signal to Noise Ratio
TDMA: Time Division Multiple Access
UB-DEIS: University of Bologna
UCC: University College Cork
VCO: Voltage Controlled Oscillator
WLAN: Wireless Local Area Network
Appendix B: Esprit Project 31103 - INSPECT, Open LTR

In 1997 an Esprit Open Long Term Research Project, financed by the European Commission, entitled “Innovative Signal Processing Exploiting Chaotic Dynamics” (INSPECT), project number 31103 was started. The project has been carried out as a collaboration between seven European universities and it was successfully finished in 2001.

Because most of the research presented in this thesis has been carried out in the framework of this project, this appendix is devoted to give a brief description of it.

B.1 PROJECT COORDINATOR

University College Cork (UCC), represented by Dr. Michael Peter Kennedy, Department of Microelectronic Engineering.

B.2 CONTRACTORS

- University College Cork (UCC), Ireland
- Swiss Federal Institute of Technology Lausanne (EPFL), Switzerland
- Helsinki University of Technology (HUT), Finland
- Budapest University of Technology and Economics (BME), Hungary
- University of Bologna (UB-DEIS), Italy
- University of Thessaloniki (AUT), Greece
- Centro Nacional de Microelectrónica (CNM), Spain
B.3 PROJECT SUMMARY

The aim of the INSPECT project was to develop chaotic nonlinear dynamical solutions for applications in digital communications and signal processing which are competitive with current technology in terms of simplicity, cost-effectiveness, and design flexibility.

The objectives of the project were:

- to develop chaotic spreading codes for code-division multiple access (CDMA) communication systems,
- to determine performance limits for chaotic modulation schemes,
- to demonstrate the viability of chaotic communications.

B.4 APPROACH

The performance of chaotic modulation and spreading was evaluated quantitatively, comparing the state of the art in nonlinear dynamical systems research with conventional techniques in each application domain. The relative advantages and disadvantages of the proposed solutions were identified and documented. Prototype hardware and software systems were developed to demonstrate proof-of-concept and to facilitate technology transfer to industry.

B.5 ACCOMPLISHED RESULTS

The results accomplished in the framework of this project are:

- The performance of digital communication techniques using chaos has been evaluated using standard communication system performance measures.
- The merits and limitations of the technology were identified.
- The advantages of chaotic codes in CDMA systems was quantified.
- The optimum performance of Differential Chaos Shift Keying (DCSK) and FM-DCSK modulation schemes has been determined.
- Prototype integrated circuits were designed and realized for the implemented FM-DCSK modulator and demodulator subsystems.
- A prototype FM-DCSK radio communications system has been constructed and characterized.

B.6 DUTY OF BME-MIT IN THE INSPECT PROJECT

DCSK/FM-DCSK modulation-Theoretical investigations: the objective was to develop a DCSK/FM-DCSK radio system simulator and using this simulator to determine the theoretical optimum performance of DCSK/FM-DCSK modulation schemes under poor propagation conditions.
Several theoretical problems have been addressed in order to optimize the performance of FM-DCSK:

- development of new modulation schemes using chaotic signals as carrier,
- based on the analytic signal approach simulation models for chaotic signals and wideband RF channels were developed,
- design rules and methods for the elaboration of a chaotic communication system using FM-DCSK modulation scheme were developed.
B.7 IMPLEMENTED FM-DCSK TRANSMITTER

In the framework of this project, based on the design rules developed in Chapter 4 CNM realized a custom made integrated circuit for the low-frequency FM-DCSK transmitter. The micro-photograph of this IC is shown in Fig. B.1.

In parallel with this work HUT also implemented an FM-DCSK using FPGA and the PRISM chipset. The photograph of this transmitter is shown in Fig. B.2.

Please note that the results presented in Chapter 4 gives the design rules only for the transmitter shown in Fig. B.1. The design rules for the transmitter shown in Fig. B.2 differs slightly from those presented in Chapter 4.

Figure B.1 Micro photograph of the low-frequency FM-DCSK modem realized by CNM (IC part no: IMSE_125FMDCSK)

Figure B.2 FM-DCSK realized by HUT using the Prism chipset.
Appendix C: 
Multipath Channel Models

C.1 OPERATION IN A MULTIPATH ENVIRONMENT

In many applications such as WLAN, mobile communications, and indoor radio, the received signal contains components which have traveled from the transmitter via multiple propagation paths with differing delays; this is called multipath propagation [Hay94]. The components arriving via different propagation paths may add destructively, resulting in deep frequency-selective fading. Conventional narrow-band systems fail catastrophically if a multipath-related null, defined below, resulting from deep frequency-selective fading coincides with the carrier frequency.

In the applications mentioned above, the distance between the transmitter and receiver is relatively short, i.e., the attenuation of the telecommunications channel is moderate. The effect which limits the performance of communications in such an environment is not the noise $N_0$, but deep frequency-selective fading caused by multipath propagation. In these applications, the most important system parameter is the sensitivity to multipath.

C.2 MODEL OF THE MULTIPATH CHANNEL

The tapped delay line model of a time-invariant multipath radio channel having $N$ propagation paths is shown in Fig. C.1. The radiated power is split and travels along the $K$ paths, each of which is characterized by a delay $T_l$ and gain $k_l$, where $l = 1, 2, \ldots, K$.

C.3 WLAN SPECIFIC TWO-RAY MULTIPATH CHANNEL MODELS

A time-invariant multipath radio channel having two propagation paths can be modeled as shown in Fig. C.1. In the worst-case situation for a two-ray channel the attenuation for the two paths are identical. In this situation the two received signals cancel each
other completely at the carrier frequency $\omega_c$, i.e.,

$$\Delta \tau \omega_c = (2n + 1)\pi, \quad n = 1, 2, 3, \ldots \quad (C.1)$$

where $\Delta \tau$ denotes the additional delay of the second path.

Let the multipath channel be characterized by the frequency response shown in Fig. C.2. Note that the multipath-related nulls, where the attenuation becomes infinitely large, appear at

$$f_{null} = \frac{2n + 1}{2\Delta \tau}, \quad n = 1, 2, 3, \ldots \quad (C.2)$$

Let the bandwidth of fading be defined as the frequency range over which the attenuation of the multipath channel is greater than 10 dB. Then the bandwidth of fading can be expressed as

$$\Delta f_{null} \approx \frac{0.2}{\Delta \tau} \quad (C.3)$$

Equations (C.2) and (C.3) show that the center frequencies of the multipath-related nulls, the distances between them, and their bandwidths, are determined by $\Delta \tau$; a shorter delay accentuates the problem.

Typical values of $\Delta \tau$ are 91 ns for large warehouses and 75 ns for office buildings [JP98]. If $\Delta \tau = 75$ ns then the distance between two adjacent multipath-related nulls is 13.33 MHz. In the case of the three IEEE 802.11-compliant telecommunications channels in the 2.4 GHz ISM band [JP98], if off-the-shelf channel selection filters are used, then the RF bandwidth of the FM-DCSK signal should be 17 MHz. This means that at most two multipath-related nulls may appear in any of the three channels.

Equation (C.2) shows that the frequencies of the multipath-related nulls are determined by the additional delay of the second path $\Delta \tau$. The number of multipath-related
nulls appearing in a WLAN channel and their positions relative to the FM-DCSK center frequency depend on the exact value of $\Delta \tau$. In a real application, $\Delta \tau$ may vary, thus changing the frequencies of the multipath-related nulls. To quantify this effect during the simulations, the additional delay of the second path is kept constant but the center frequency of the FM-DCSK signal is varied.

**C.4 PERFORMANCE EVALUATION IN PCS JTC CHANNELS**

The Personal Communication Services (PCS) Joint Technical Committee (JTC) has recommended a comprehensive multipath channel model to check and compare the performance of personal communications and mobile telecommunications systems in both indoor and outdoor applications [PL95]. In indoor application, which is a potential application field of the enhanced FM-DCSK systems, channel models have been developed for office, residential and commercial environments.

To describe the various propagation conditions, three different channel profiles, denoted by channels A, B and C, are given for each area. Each channel profile is described by the tapped delay line model shown in Fig. C.1. The JTC model assumes that the channel profile and the channel attenuation are correlated and it provides a statistical procedure for selecting the channel profiles as a function of channel attenuation.

Let the probabilities of selecting channel profiles A, B and C be denoted by $P(A)$, $P(B)$ and $P(C)$, respectively. In the JTC recommendation, these probabilities are given as a function of channel attenuation.

The piecewise-linear curves that relate the probability of selecting a particular channel profile to the channel attenuation are shown in Fig. C.3. The curves are characterized by two parameters $PL_1$ and $PL_2$ that correspond to the channel attenuation at the two break points of the piecewise-linear curves. Note that the probabilities $P(A)$, $P(B)$ and $P(C)$ for a given attenuation are given by the distance between the curves, as shown in Fig. C.3.

![Figure C.3](image)

**Figure C.3** Probabilities of selecting channel profiles A, B and C as a function of channel attenuation in the JTC multipath channel model.

Figure C.3 shows, in a qualitative manner, the probability of selecting a channel profile. The exact values of the five parameters $P(A)$, $P(B)$, $P(C)$, $PL_1$ and $PL_2$ are given in Table C.1 for the indoor office, residential and commercial areas. More details on the JTC multipath channel model are given in [PL95].

If the attenuation of the radio channel is less than $PL_1$ (see Table C.1) then only
channel profile \( A \) has to be considered. The propagation conditions in channels \( B \) and \( C \) are much worse. In the worst case situation, the attenuation of the radio channel is greater than \( PL_2 \) and all of the tree channels should be considered with the probability of \( P(A) \), \( P(B) \) and \( P(C) \).

Since the performance degradation of FM-DCSK are addressed in detail in [Kis03], in this thesis only the worst case of simulations results are given, i.e., in all JTC multipath channel models it is assumed that the attenuation of the radio channel is greater than \( PL_2 \).

Excess delay and attenuation of the taps in the channel profiles recommended for the indoor office, residential and commercial areas by the PCS Joint Technical Committee are given in Tables C.2, C.3 and C.4, respectively.
<table>
<thead>
<tr>
<th>Tap</th>
<th>Channel A</th>
<th>Channel B</th>
<th>Channel C</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Excess Delay</td>
<td>Excess Delay</td>
<td>Excess Delay</td>
</tr>
<tr>
<td></td>
<td>(nsec)</td>
<td>(nsec)</td>
<td>(nsec)</td>
</tr>
<tr>
<td></td>
<td>Relative Attenuation</td>
<td>Relative Attenuation</td>
<td>Relative Attenuation</td>
</tr>
<tr>
<td></td>
<td>(dB)</td>
<td>(dB)</td>
<td>(dB)</td>
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<tr>
<td>8</td>
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<td>750</td>
</tr>
</tbody>
</table>

Table C.3 Excess delay and attenuation of the taps in the three channel profiles recommended for the indoor residential area by the PCS Joint Technical Committee.

<table>
<thead>
<tr>
<th>Tap</th>
<th>Channel A</th>
<th>Channel B</th>
<th>Channel C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Excess Delay</td>
<td>Excess Delay</td>
<td>Excess Delay</td>
</tr>
<tr>
<td></td>
<td>(nsec)</td>
<td>(nsec)</td>
<td>(nsec)</td>
</tr>
<tr>
<td></td>
<td>Relative Attenuation</td>
<td>Relative Attenuation</td>
<td>Relative Attenuation</td>
</tr>
<tr>
<td></td>
<td>(dB)</td>
<td>(dB)</td>
<td>(dB)</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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</tr>
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<td>350</td>
<td>750</td>
</tr>
</tbody>
</table>

Table C.4 Excess delay and attenuation of the taps in the three channel profiles recommended for the indoor commercial area by the PCS Joint Technical Committee.
Appendix D: Publications

D.1 PUBLICATIONS DIRECTLY RELATED TO THE THESIS

D.1.1 Book Chapter


D.1.2 Journal Papers


**D.1.3 Conference Articles**


D.1.4 Patent


D.1.5 Conference Articles in Hungarian


D.1.6 Other publications


D.2 PUBLICATIONS NOT DIRECTLY RELATED TO THE THESIS


D.3 CITATIONS BY INDEPENDENT AUTHORS

A. Citations to the Book Chapter


B. Citations to Journal Papers


C. Citations to Conference Articles


First Asia-Pacific Workshop on Chaos Control and Synchronization, Shanghai Jiao Tong University, Shanghai, China, June 28–29, 2001, pp. 28-29.


