Budapest University of Technology and Economics
Department of Telecommunications

Approximating non-Markovian Behavior by Markovian Models

András Horváth

Scientific supervisor

Prof. Dr. Miklós Telek

Budapest, 2003

Abstract

The results presented in the dissertation can be divided into two categories.

The first set of results, which is presented in the 1. set of theses, presents properties of discrete-time, acyclic Phase-type distributions (ADPH). These results emphasize similarities and differences as compared to continuous-time, acyclic Phase-type distributions (ACPH). Moreover, an algorithm is given for the estimation of the parameters of ADPH distributions based on the maximum likelihood principle.

The second set of results is presented in the 2. and 3. set of theses. We give methods to model typical phenomena of communication networks (heavy-tailed distribution, long-range dependence) with Markovian models.

1 Background

Communication networks are characterized by dynamic change. New technologies and architectures introduce new services. When introducing new techniques it is important, both from the point of view of thrift and from the point of view of quality of service, to have a clear view on the performance issues.

Performance analysis can be performed through the analysis of the stochastic model describing the system under study. Markovian models gained widespread use in practice because their numerical analysis is easy (at least compared to other model classes that do not enjoy the Markov property) and several algorithms and tools are at hand for the modeler.

A Markov chain can be discrete-time, in which case the distribution of the duration between state changes is geometric, or continuous-time when the duration between state changes is exponentially distributed. This property poses strong limitation on what kind of phenomena can be modeled with sufficient accuracy with Markov chains. We can overcome this problem to some extent by representing one state of the system by several states of the Markov chain. In this case the duration between system state changes corresponds to the time that the Markov chain spends in a set of its states. The intensities inside that set of states which represents the same system state have to be set such a way that the time spent there corresponds to the duration that the system spends in this system state. Important task is to develop fitting algorithms and to analyze how precise the approximations are.

Apart from fitting distributions, it is a relevant problem to fit processes too. This can be necessary, for example, when we need to model the arrival of clients to a queue. If we wish to use a Markovian model to describe the system, we need a Markovian model to describe the arrival of the costumers. In this case too, the accuracy of the approximation has to checked.

2 Research aims

The objective of the research can be divided into to categories.

2.1 Discrete-time, acyclic Phase-type distributions

Phase-type distributions are given by the time to absorption of a Markov chain [9]. If the Markov chain is discrete-time, the resulting distribution is discrete, if it is continuous-time, the time to absorption follows a continuous distribution. Since the '70's numerous articles have been published that deal with properties and application of continuous-time Phase-type distributions (CPH) [10]. On the contrary, less research considered discrete-time Phase-type distributions (DPH).

In case of acyclic Markov chains, the resulting distributions are called discrete- or continuous-time, acyclic Phase-type distributions (ADPH or ACPH). Experiments suggest that restricting ourselves to acyclic PH distributions does not pose limitation from practical point of view.

For what concerns ADPH distributions the research aims were the following:

- To describe the properties of ADPH distributions and to compare them to the properties of ACPH distributions.
- To develop an algorithm that constructs an ADPH approximation for any general distribution. To test the algorithm and to compare the accuracy of ACPH and ADPH approximations.

The results connected to ADPH distributions are summarized in the 1. set of theses.

2.2 Traffic modeling by Markovian models

Recent studies on traffic of communication systems have shown evidence of several phenomena whose modeling with Markovian models applying the traditional estimating procedures results in low accuracy. Such phenomena are the heavy-tailed distributions or the long-range dependent processes. The research aims connecting to this area were:

- To analyze the effect of the distance measure applied in Phase-type fitting on the accuracy of the resulting approximation when fitting heavy-tailed distributions.

- To develop an algorithm for efficient and accurate fitting of heavy-tailed distributions with Phase-type distributions. To test the algorithm and to study the accuracy of the approximations.
- To develop methods for the approximation of long-range dependent processes with Markovian models. To test the methods and their accuracy.

The related results are summarized in the 2. and 3. sets of theses.

3 Methodology

The methods utilized during the research were determined by the above mentioned aims. The necessary theoretical background is composed of the following factors.

- Theory of stochastic processes with particular interest to Markovian stochastic processes.
- As I aimed at constructing algorithms that are capable of generating traffic similar to that of real networks, it was necessary to study the statistical tests that provide the description of real traffic data. Moreover, it was important to gain familiarity with the parameter estimation techniques.

For what concerns the methodology of the research the following components are worth to mention.

- Measurement studies regarding traffic of telecommunication networks are available on the world wide web [12].
- For the analysis of the available data the statistical tests had to be implemented.

In order to study the applicability of the developed procedures, the following components were necessary.

- Implementing the procedure.
- Studying the goodness of the approximations from the point of view of the relevant statistical tests.
- Studying the goodness of the approximations by plugging them into complex stochastic models. This was done, for example, by plugging the approximation of the service time distribution into a queuing model and checking the goodness of the fitting from the points of view of overall performance indices of the system.

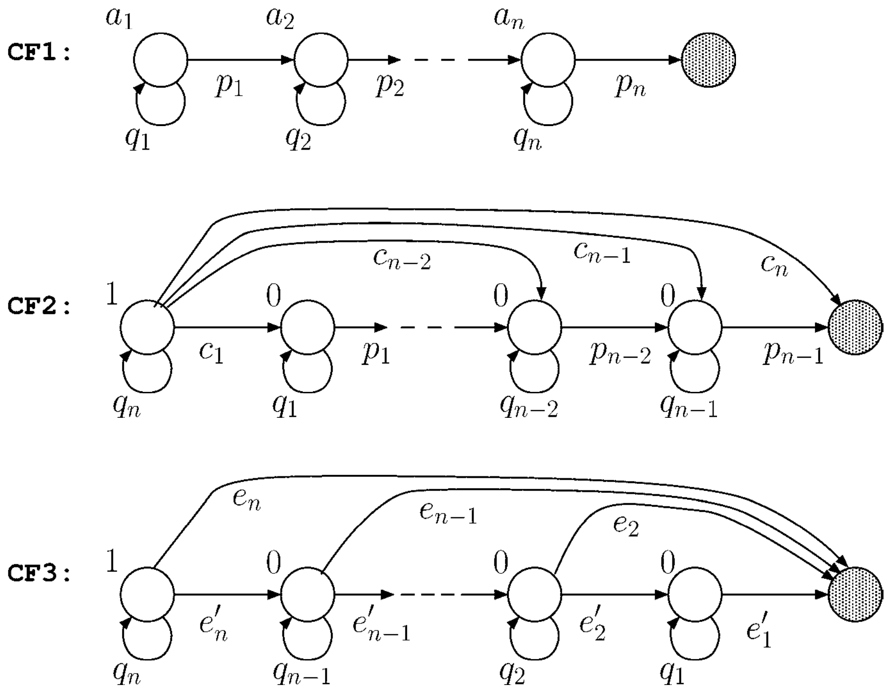

Figure 1: Canonical forms for ADPH distributions

4 New results

1. set of theses: discrete-time, acyclic Phase-type distributions

In this set of theses I summarize the results regarding discrete-time, acyclic Phase-type distributions with particular interest to emphasize similarities and differences to continuous-time, acyclic Phase-type distributions. The next three theses delineate the most important issues.

Thesis 1.1 *I have shown that any discrete-time, acyclic Phase-type distribution can be transformed into any of the canonical forms depicted in Figure 1. Any of these canonical forms gives the minimal representation of the given ADPH distribution in the sense that it describes the distribution uniquely and with the minimal number of parameters. The next equations provide the connection between the canonical forms.*

$$c_k = a_k p_n, \quad s_i = \sum_{j=1}^{i} a_j, \quad e'_i = \frac{a_i}{s_i} p_i, \quad e_i = \frac{s_{i-1}}{s_i} p_i. \tag{1}$$

The previous thesis gave a property of ADPH distributions. A very similar property is known for ACPH distributions [5].

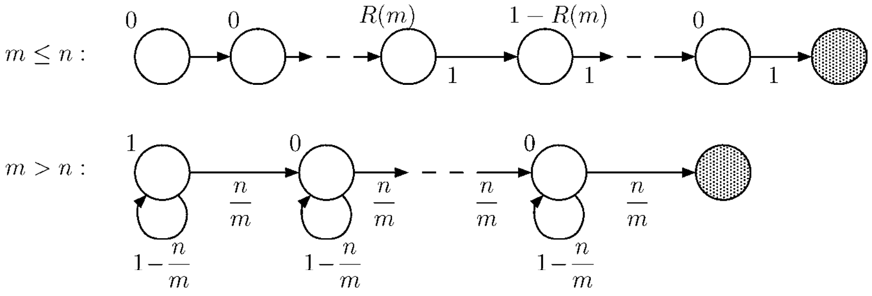

Figure 2: Structure exhibiting the minimal coefficient of variation

An obvious difference between the discrete-time and the continuous-time cases is that with ADPH distributions one can realize a distribution with 0 coefficient of variation (i.e., a deterministic duration) while it is not possible with ACPH distributions. The next thesis indicates more precisely the difference between the discrete-time and the continuous-time cases from the point of view of the minimal coefficient of variation.

Thesis 1.2 *The minimal coefficient of variation that can be realized by an n-phase ADPH distribution depends on the mean and is given by*

$$cv^2_{min}(m) = \begin{cases} \dfrac{R(m)(1-R(m))}{m^2} & \text{if } m \le n \ , \\ \\ \dfrac{1}{n} - \dfrac{1}{m} & \text{if } m > n \ , \end{cases} \tag{2}$$

where m denotes the mean of the distribution and $R(m)$ the fractional part of m. The structure that exhibits the minimal coefficient of variation depends on the relation of n and m; it is depicted in Figure 2.

For the continuous-time case the minimal coefficient of variation is given by Aldous and Shepp[1]: it depends only on the number of phases and it is $1/n$. The difference between the discrete-time and the continuous-time cases from the point of view of the minimal coefficient of variation is illustrated in Figure 3 for fixed number of phases and in Figure 4 for fixed mean. It can be deduced from the figures that, if the difficulty of the fitting is caused by the low coefficient of variation and it is important to model precisely the coefficient of variation, application of ADPH distributions compared to ACPH distributions can be beneficial.

In order to apply ADPH distributions in complex stochastic models parameter estimation procedures are required.

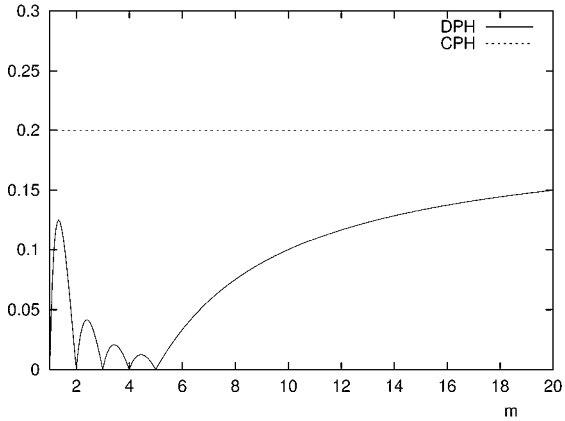

Figure 3: Minimal coefficient of variation for $n = 5$

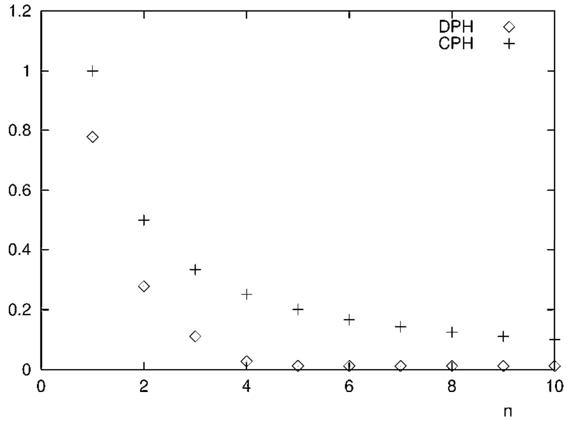

Figure 4: Minimal coefficient of variation for $m = 4.5$

Thesis 1.3 *I have developed an estimation algorithm that provides the parameters of the approximating ADPH distribution for a given set of samples. The procedure is based on the maximum likelihood principle.*

Novelty of the procedure lies in the time-domain derivative of the likelihood function that leads to a more stable estimation procedure compared to the algorithm based on the transform-domain derivative.

The validation of the parameter estimation procedure was carried out by fitting distributions whose ACPH approximation is known [3]. The experiments confirm the theoretical result given in Thesis 1.2: for distributions with low coefficient of variation ADPH distributions gives better fitting than ACPH distributions. Moreover, it turned out that for distributions with low coefficient of variation Thesis 1.2 provides a good suggestion for the number of phases.

The results presented in the 1. set of theses, together with other achievements regarding ADPH distributions, have been published in [J2]. The estimation algorithm is implemented in the tool PhFit [C8]. A comparison of ADPH and ACPH distributions in complex modeling context is presented in [J1].

2. set of theses: approximating heavy-tailed distribution by PH distributions

From the 1980's several algorithms have been developed for the estimation of parameters of Phase-type distributions. All of these algorithms perform the fitting according to a predefined distance measure [7].

Thesis 2.1 *I have developed and implemented an estimation procedure that is more general than the previous ones in the sense that it optimizes the parameters of the Phase-type distribution according to an arbitrary distance measure.*

The non-linear optimization problem is solved by consecutive linearization of the goal function starting from a random point of the parameter space. In every step of the algorithm, we need the derivatives of the goal function according to the parameters. These derivatives are calculated numerically.

Recent studies on traffic of telecommunication systems have given evidence that some of the important random quantities of typical networks follow heavy-tailed distribution [4]. With the general parameter estimator I have carried out experiments in order to find a good measure for fitting heavy-tailed distributions. The experiments evidenced that none of the commonly used distance measures performs well for heavy-tailed distributions.

Feldman and Whitt presented a heuristic fitting method for approximating heavy-tailed behavior with hyperexponential distribution [6]. Shortcoming of their method is that it can be applied only to distributions with monotone decreasing probability density function.

Thesis 2.2 *I have developed a fitting procedure for the efficient approximation of heavy-tailed behavior by ACPH distributions. The procedure combines the general fitting method with the heuristic method of Feldman and Whitt. The combined algorithm relaxes the restriction posed by the heuristic method.*

The combined algorithm starts with determining the parameters of the hyperexponential distribution that considers only the tail of the distribution to be fitted. Then the general method is performed to take into account the body of the distribution. The general method had to be appropriately modified in order to be combined with the heuristic method of Feldman and Whitt.

The goodness of the new algorithm was tested both by comparing the original heavy-tailed distribution and its approximation and by applying the approximation in complex stochastic models. The experiments showed that the suggested combined method is applicable in practice.

Characteristics of the combined estimation algorithm is illustrated in Figures 5 and 6 through the fitting of a distribution with Pareto tail. The expression X+Y in the legend indicates that X phases were used to fit the body while Y phases were used to approximate the tail. Additional part of the legend gives the upper bound of fitting the tail. With the combined algorithm, because it separates the fitting of the body and the tail, one can set the precision of fitting the body and the precision of fitting the tail independently of each other. The figures show that the PH distribution follows the original tail behavior only up to a finite time point. However this time point can be freely chosen by setting appropriately the parameters of the heuristic fitting algorithm.

Figures 7 and 8 illustrate the goodness of the fitting in the context of queuing. We consider a queue in which clients arrive according to a Poisson process and gain service according to a distribution with Pareto tail. The figures give the body and the tail of the queue length distribution of the original queuing model and the queue length distribution of the queuing model in which the distribution with Pareto tail is approximated by a PH distribution. The characteristics of the tail fitting are reflected by the tail of the queue length distribution.

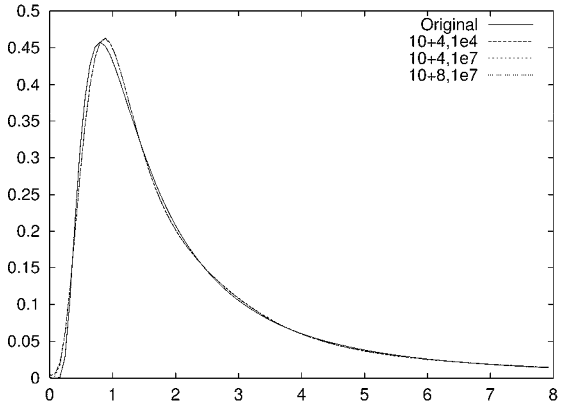

Figure 5: Fitting a heavy-tailed distribution (body)

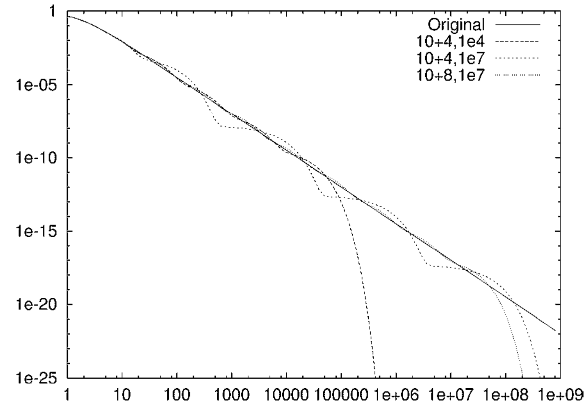

Figure 6: Fitting a heavy-tailed distribution (tail)

The results of the 2. set of theses are presented in [C16]. The combined algorithm is included in the tool PhFit [C8].

3. set of theses: fitting self-similar and multi-fractal processes by Markovian models

Besides the presence of heavy-tailed distributions, long-range dependence of traffic processes of networks was also reported in recent studies [8]. When fitting a long-range dependent process, there is emphasis on capturing its self-similar or multi-fractal nature. In the opening phase of research connected to this subject, it was agreed on that Markovian models are not

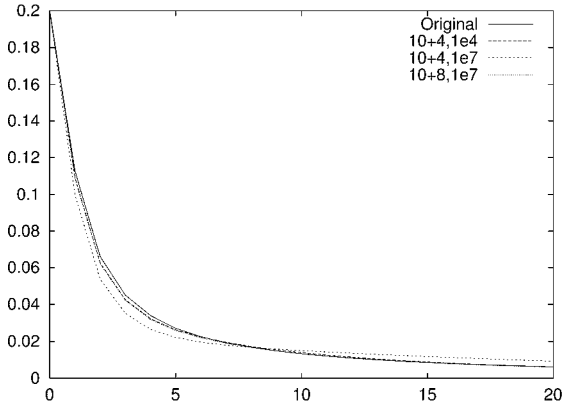

Figure 7: Queue length distribution (body)

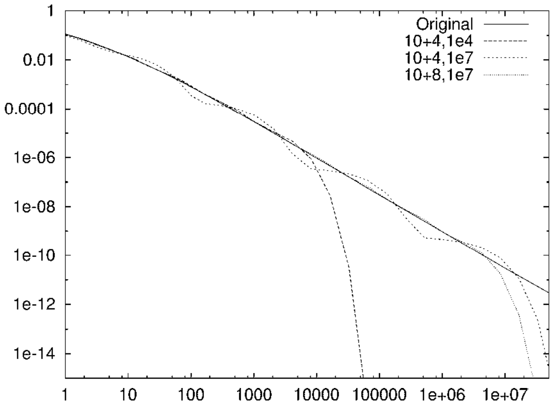

Figure 8: Queue length distribution (tail)

capable of capturing this kind of behaviors to a sufficient extent. My research in the field took the aim of proposing Markovian traffic models that exhibit self-similar or multi-fractal nature throughout all those timescale that are important from the point of view performance analysis.

The Markovian arrival process presented in the next thesis exhibits the so-called “pseudo self-similar” property, that is, it exhibits self-similarity through several timescales.

Thesis 3.1 *As known from [11], if the probability density function of the renewal period of a renewal process is given as*

$$f(x) = \frac{c \cdot a^c}{(x+a)^{c+1}}, \ x \geq 0 \tag{3}$$

then the process exhibit second order asymptotic self-similarity with Hurst-parameter

$$H = \frac{3-c}{2}. \tag{4}$$

I have shown that if the renewal period of a renewal process follows a PH distribution that is an approximation of a random variable with probability density function given in (3) and the parameters of the approximation is chosen appropriately, then the renewal process exhibit “pseudo self-similarity” through several timescales. The degree of “pseudo self-similarity” is given by (4).

One can check for self-similarity of a process by the simple variance-time plot which looks at the behavior of the variance of the aggregated process [2]. If the variance-time plot is close to linear with slope β, then the process exhibits self-similarity with Hurst-parameter $H = 1 + (\beta/2)$. Figure 9 depicts examples for variance-time plots of various PH renewal processes together with the linear lines corresponding to various Hurst-parameters. The Markovian renewal processes exhibit "pseudo self-similarity" through several timescales.

Applying Thesis 3.1 a Markovian arrival process can be built whose long-term behavior is defined by the Hurst-parameter. For modeling real traffic, it is necessary to construct such arrival process whose some other characteristics can also be set.

Thesis 3.2 *I have developed a procedure for the construction of a Markovian arrival process whose following parameters can be set as required:*

- *average arrival intensity,*
- *variance of number of arrivals for two time intervals,*
- *Hurst-parameter.*

The procedure starts with realizing the required long-term behavior applying Thesis 3.1. Then, by superposing the resulting PH renewal process with a two-state Markov-modulated Poisson process, the algorithm realizes the required short-term descriptors.

The traffic model constructed by the above algorithm generates a point process which is very similar to real data traffic from many points of view. This similarity is illustrated in Figure 10 where the variance-time plot of the original trace and its approximations are

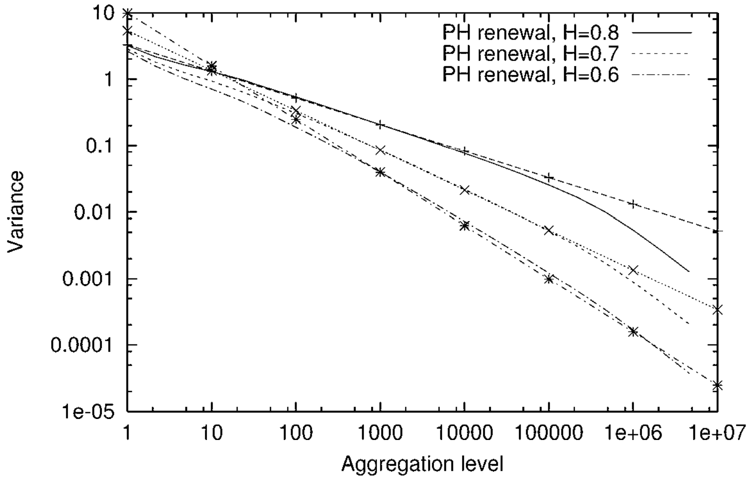

Figure 9: Variance-time plots of PH renewal processes

depicted. The approximation denoted by (x, y) sets the variance of the number of arrivals for the time intervals of length ax and ay where a denotes the average interarrival time. Figure 11 shows the queuing behavior of the original and the approximating traces in the case when the service time is deterministic and the utilization is 0.8. Even if, from the statistical point of view, the matching is satisfactory, there is relevant mismatch in the queuing behavior. In some cases, the reason can be that the long-term behavior of the traffic process is described by a single parameter (the Hurst-parameter) while it is, in reality, more complex. The statistical methods applied in the theory of multi-fractals serve to describe this complex long-term behavior.

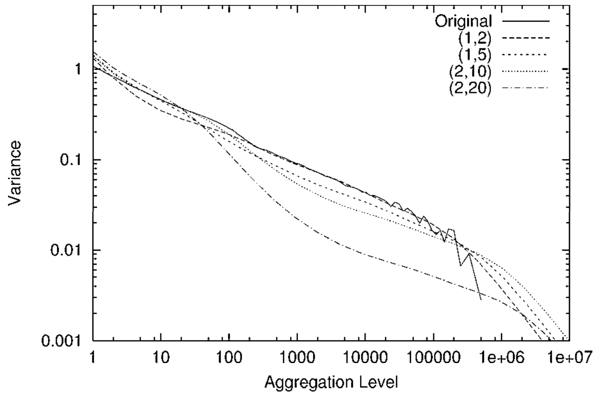

Figure 10: Variance-time plot of the original trace and its approximations

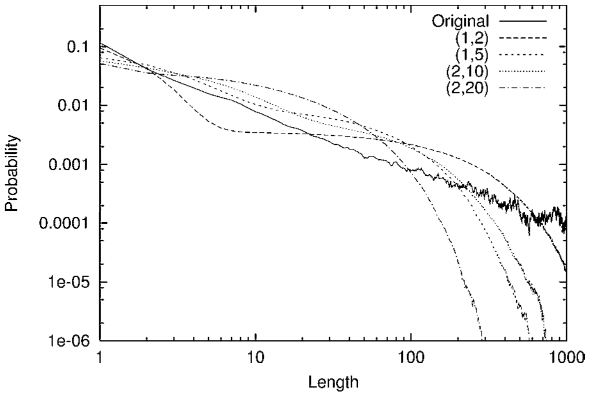

Figure 11: Queue length distribution given by the original trace and its approximations

Thesis 3.3 *For capturing more precisely the long-term behavior of traffic processes, I have suggested a Markovian arrival process whose structure recalls the Haar wavelet transformation. Moreover, I have proposed a fitting algorithm that determines the parameters of the Markovian arrival process based on the second moments of the Haar wavelet coefficients.*

As evidenced by statistical tests, the above mentioned Markovian traffic model exhibits multi-fractal behavior through several timescales. Furthermore, the proposed fitting algorithm results in Markovian traffic models that matches the original traffic trace both from the point of view of statistical tests and the point of view of queuing behavior. The fitting is illustrated in Figure 12 where the second moments of the Haar wavelet coefficients are depicted for the original trace and its approximation as the function of the aggregation level. Figure 13 shows the queue length distribution for deterministic service times and 80% of utilization.

The results summarized in the 3. set of theses is presented in [C17] and [C9].

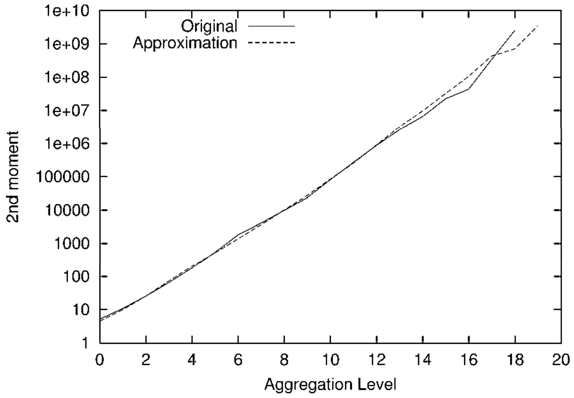

Figure 12: Second moment of the Haar wavelet coefficient of the original trace and its approximation

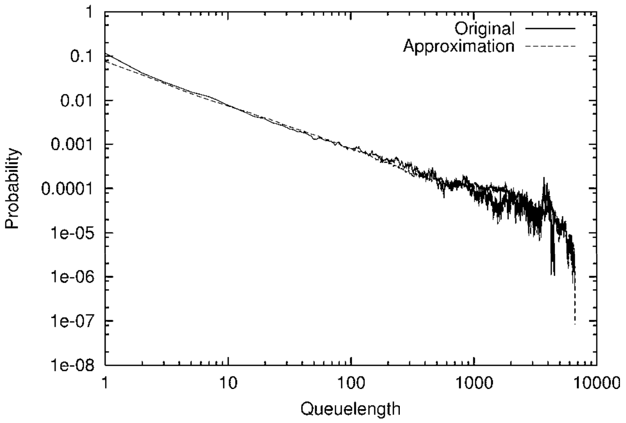

Figure 13: Queue length distribution given by the original trace and its approximation

5 Application of the results

The 1. set of theses regard ADPH distributions. These results describe properties of these distributions which indicates that application of ADPH distribution can be advantageous compared to the application of ACPH distributions in certain modeling situations. Such case is the modeling of durations with low coefficient of variation or with finite support.

The 2. and 3. set of theses present algorithms for Markovian approximation of typical non-Markovian behavior of telecommunication networks. The proposed techniques provide the possibility of analyzing systems by Markovian methods. This is beneficial since Markovian models of queuing systems are numerically more tractable (for example, with matrix-geometric techniques) than other models proposed in the literature.

References

[1] D. Aldous and L. Shepp. The least variable Phase-type distribution is Erlang. *Stochastic Models*, 3:467–473, 1987.

[2] J. Beran. *Statistics for long-memory processes.* Chapman and Hall, New York, 1994.

[3] A. Bobbio and M. Telek. A benchmark for PH estimation algorithms: results for Acyclic-PH. *Stochastic Models*, 10:661–677, 1994.

[4] M. E. Crovella, M. S. Taqqu, and A. Bestavros. *A Practical Guide To Heavy Tails*, chapter Heavy-Tailed Probability Distributions in the World Wide Web, pages 3–26. Chapman & Hall, New York, 1998.

[5] A. Cumani. On the canonical representation of homogeneous Markov processes modelling failure-time distributions. *Microelectronics and Reliability*, 22:583–602, 1982.

[6] A. Feldman and W. Whitt. Fitting mixtures of exponentials to long-tail distributions to analyze network performance models. *Performance Evaluation*, 31:245–279, 1998.

[7] A. Lang and J. L. Arthur. Parameter approximation for Phase-type distributions. In S. R. Chakravarty and A. S. Alfa, editors, *Matrix-analytic methods in stochastic models*, Lecture notes in pure and applied mathematics, pages 151–206. Marcel Dekker, Inc., 1996.

[8] W. E. Leland, M. Taqqu, W. Willinger, and D. V. Wilson. On the self-similar nature of ethernet traffic (extended version). *IEEE/ACM Transactions in Networking*, 2:1–15, 1994.

[9] M. Neuts. Probability distributions of phase type. In *Liber Amicorum Prof. Emeritus H. Florin*, pages 173–206. University of Louvain, 1975.

[10] M.F. Neuts. *Matrix Geometric Solutions in Stochastic Models.* Johns Hopkins University Press, Baltimore, 1981.

[11] B. Ryu and S. B. Lowen. Point process models for self-similar network traffic, with applications. *Stochastic models*, 14, 1998.

[12] The internet traffic archive. http://ita.ee.lbl.gov/index.html.

List of publications

Articles published (or accepted for publication) in international journals

[J1] A. Bobbio, A. Horváth, and M. Telek. **The scale factor: A new degree of freedom in Phase Type approximation**. *Performance Evaluation*, 2003. Accepted for publication. To appear.

[J2] A. Bobbio, A. Horváth, M. Scarpa, and M. Telek. **Acyclic discrete Phase Type distributions: Properties and a parameter estimation algorithm**. *Performance Evaluation*, 54(1):1–32, 2003.

[J3] M. Gribaudo, A. Horváth, A. Bobbio, E. Tronci, E. Ciancamerla, and M. Minichino. **Fluid petri nets and hybrid model-checking: A comparative case study**. *Reliability Engineering & System Safety*, 81(3):239–257, 2003.

[J4] A. Horváth and M. Telek. **Time domain analysis of non-markovian stochastic Petri nets with pri transitions**. *IEEE Transactions On Software Engineering*, 28(20):933–943, 2002.

[J5] M. Gribaudo and A. Horváth. **Fluid stochastic Petri nets augmented with flush-out arcs: A transient analysis technique**. *IEEE Transactions On Software Engineering*, 28(10):944–955, 2002.

[J6] M. Telek and A. Horváth. **Transient analysis of age-MRSPNs by the method supplementary variables**. *Performance Evaluation*, 45(4):205–221, 2001.

[J7] S. Bernardi, S. Donatelli, and A. Horváth. **Implementing compositionality for stochastic Petri nets**. *International Journal on Software Tools for Technology Transfer*, 3(4):417–430, 2001.

[J8] M. Gribaudo, M. Sereno, A. Horváth, and A. Bobbio. **Fluid stochastic Petri nets augmented with flush-out arcs: Modelling and analysis**. *Discrete Event Dynamic Systems*, 11:97–111, 2001.

Articles published in proceeding of international conferences

[C1] M. Telek, A. Horváth, and G. Horváth. **Analysis of inhomogeneous Markov reward models**. In *NSMC '03 (International Conference on the Numerical Solution of Markov Chains)*, pages 305–322, Urbana, Illinois, USA, Sep. 2003.

[C2] A. Bobbio, M. Gribaudo, and A. Horváth. **Modeling a car safety controller using fluid stochastic Petri nets**. In *Proc. of the 6th International Workshop on Performability Modeling of Computer and Communication System*, Monticello, Illinois, USA, Sep. 2001. Extended abstract.

[C3] M. Gribaudo and A. Horváth. **Modeling hybrid positive systems with hybrid Petri nets**. In *Proc. of 1st Multidisciplinary Int. Symp. on Positive Systems: Theory *and Applications (POSTA'03)*, volume 294 of *Lecture Notes in Control and Information Sciences*, pages 71–78, Rome, Italy, Aug. 2003.

[C4] M. Gribaudo and A. Horváth. **Translating hybrid Petri nets into hybrid automata**. In *Proceedings of Workshop on Stochastic Petri nets and related formalisms*, pages 92–102, Eindhoven, The Netherlands, June 2003.

[C5] A. Horváth and M. Telek. **Markovian modeling of real data traffic: Heuristic Phase Type and map fitting of heavy tailed and fractal like samples**. In *Tutorial of the IFIP WG 7.3 Int. Symposium on Computer Performance Modeling, Measurement and Evaluation (PERFORMANCE 2002)*, volume 2459 of *Lecture Notes in Computer Science*, Rome, Italy, Sept 2002.

[C6] M. Gribaudo, A. Horváth, A. Bobbio, E. Tronci, E. Ciancamerla, and M. Minichino. **Model-checking based on fluid Petri nets for the temperature control system of the icaro co-generative plant**. In *Proc. The 21st Int. Conference on Computer Safety, Reliabiltiy and Security (SAFECOMP 2002)*, volume 2434 of *Lecture Notes in Computer Science*, Catania, Italy, Sept 2002.

[C7] A. Bobbio, A. Horváth, and M. Telek. **The scale factor: A new degree of freedom in Phase Type approximation**. In *Proc. of 3rd Int. Performance & Dependability Symposium (IPDS '02)*, Washington, DC, USA, June 2002.

[C8] A. Horváth and M. Telek. **PhFit: A general Phase Type fitting tool**. In *Proc. of 12th Performance TOOLS*, volume 2324 of *Lecture Notes in Computer Science*, Imperial College, London, April 2002.

[C9] A. Horváth and M. Telek. **A Markovian point process exhibiting multifractal behaviour and its application to traffic modeling**. In *Proc. of 4th Int. Conference on Matrix-Analytic Methods in Stochastic models*, Adelaide, Australia, July 2002.

[C10] A. Horváth and M. Gribaudo. **Matrix geometric solution of fluid stochastic Petri nets**. In *Proc. of 4th Int. Conference on Matrix-Analytic Methods in Stochastic models*, Adelaide, Australia, July 2002.

[C11] A. Bobbio and A. Horváth. **Model checking time Petri nets using NuSMV**. In *Proc. of the 5th Int. Workshop on Performability Modeling of Computer and Communication System*, Erlangen, Germany, Sep. 2001. Kibővített absztrakt.

[C12] A. Bobbio and A. Horváth. **Petri nets with discrete Phase Type timing: A bridge between stochastic and functional analysis**. In *Proc. of 2nd Workshop on Models for Time-Critical Systems*, volume 52 No. 3 of *Electronic Notes in Theoretical Computer Science*, Aalborg, Denmark, Aug. 2001.

[C13] A. Horváth and M. Telek. **Time domain analysis of NMSPNs with PRI transitions**. In *Proc. of 9th Intern. Workshop on Petri Nets and Performance Models*, Aachen, Germany, Sep. 2001. IEEE-CS Press.

[C14] M. Gribaudo and A. Horváth. **Fluid stochastic Petri nets augmented with flush-out arcs: A transient analysis technique**. In *Proc. of 9th Intern. Workshop on Petri Nets and Performance Models*, Aachen, Germany, Sep. 2001. IEEE-CS Press.

[C15] S. Bernardi, C. Bertoncello, S. Donatelli, G. Franceschinis, R. Gaeta, M. Gribaudo, and A. Horváth. **GreatSPN in the new millenium**. In *Tool Session of 9th Intern. Workshop on Petri Nets and Performance Models*, Aachen, Germany, Sep. 2001.

[C16] A. Horváth and M. Telek. **Approximating heavy tailed behavior with Phase Type distributions**. In *Proc. of 3rd Int. Conference on Matrix-Analytic Methods in Stochastic models*, Leuven, Belgium, June 2000.

[C17] A. Horváth, G. I. Rózsa, and M. Telek. **A MAP fitting method to approximate real traffic behavior**. In *Proc. of 8th IFIP Workshop on Performance Modeling and Evaluation of ATM and IP*, Ilkley, England, June 2000.

[C18] S. Bernardi, S. Donatelli, and A. Horváth. **Compositionality in the GreatSPN tool and its application to the modelling of industrial applications**. In *Proc. of Workshop on the practical use of High Level Petri Nets*, Aarhus, Denmark, June 2000.

[C19] A. Horváth, A. Puliafito, M. Scarpa, and M. Telek. **Analysis and evaluation of non-Markovian stochastic Petri nets**. In *Proc. of 11th Performance TOOLS*, volume 1786 of *Lecture Notes in Computer Science*, pages 171–187, Schaumburg, IL, USA, March 2000. Springer.

[C20] A. Bobbio, S. Garg, M. Gribaudo, A. Horváth, M. Sereno, and M. Telek. **Modeling software systems with rejuvenation, restoration and checkpointing through fluid stochastic Petri nets**. In *Proc. of 8th Intern. Workshop on Petri Nets and Performance Models*, Zaragoza, Spain, Sep. 1999. IEEE-CS Press.

[C21] M. Telek and A. Horváth. **Supplementary variable approach applied to the transient analysis of age-MRSPNs**. In *Proc. of 3rd Int. Computer Performance & Dependability Symposium (IPDS '98)*, pages 44–51, Durham, North Carolina, USA, Sep. 1998. IEEE Comp. Soc. Press.

[C22] A. Horváth, A. Puliafito, M. Scarpa, M. Telek, and O. Tomarchio. **Design and implementation of a WEB-based non-Markovian stochastic Petri net tool.** In *Advances in Computer and Information Sciences, Proc. of ISCIS'98*, Concurrent System Engineering Series, pages 101–109. IOS Press, Ohmsha, Antalya, Turkey, 1998.

Articles in hungarian

[M1] Arató D., Bajor P., Horváth A. **Távközlőhálózatok megbízhatósági elemzése.** *Magyar Távközlés*, VIII.,1.,9-14., 1997. Január