

BLOCKING PROBABILITY OF ALL-OPTICAL MULTIFIBER NETWORKS

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Abstract

In this paper we present an analytical model that can be applied to the calculation of the blocking probability in all-optical telecommunication networks carrying dynamic traffic. A big advantage of an all-optical network without wavelength conversion in the nodes is the fact that it can provide protocol transparent services to upper transfer layers, but its analysis is much more complicated than that of wavelength converting networks. In addition to handle the problems originating from the non-converted wavelengths, our model supports calculation with arbitrary number of fibers on each link. As different link capacities are allowed, the model may be applied in a wide range of optical networks. To examine the accuracy of the model we compared the numerical results to simulations for different networks and scenarios.

Keywords: all-optical network, performance, multifiber network, analytical model

1. Introduction

A recent research and development area in telecommunication field is the investigation of WDM networks that support dynamic traffic, e.g. ASON of [19]. The motivation is to provide services with higher utilization of optical network resources. On the other hand this solution provides higher performance to the costumers, since their resource needs can be satisfied dynamically and only the real usage of resources has to be payed.

In these optical switching networks the capacity granularity is the optical channel that means a Gb/s connection and can be realized by a beam of one wavelength. Since WDM technology allows the application of more wavelengths per fiber for communication, the capacity of

a fiber can be greater than 1 optical channel, realized by non-identical wavelengths.

Considering only its performance, if wavelength conversion is possible in all nodes of the network, it reduces to a simple circuit switched network with channels of very large capacity. Without the converters only the identical wavelengths of links can be connected in a lightpath and this effects strongly the network performance.

The number of fibers on links is the other multiplicative factor of link capacity and there are more reasons to use multifiber solutions. On one hand, if a link contains only one fiber, the reliability is low since the failure of this one fiber causes the failure of the whole link¹. On the other hand, even assuming the lack of converters there can be more possibilities to continue a lightpath of a given wavelength on the next link, since containing more fibers the link contains more optical channels of the same wavelength.

From the analysis point of view, the multifiber all-optical networks without wavelength conversion can be modelled in a quite difficult way because of the link and wavelength utilization dependencies. These can be significant even in the cases of very simple routing and wavelength assignment algorithms and this paper introduces a model that considers both.

Several previous work were presented in recent years on parts of this problem. Their main concept, solution and applicability are generally very different and mostly even the solutions of the more complex problems are applicable only among very strong constraints.

In [1], [3], [5], [12], [13] and [15] the authors ignore the dependency of link loads and its insertion in their models is not always obvious. A combinatoric approach is the basis of modelling the wavelength sets and their connection possibilities in [3], [4], [5], [12], [13], [14] and [15]. This approach is easily applicable to the examination of *random* wavelength assignment. A very different method, the overflow analysis is used in [6] and [8] that present models for *first-fit* and other assignment of wavelengths. [2] introduces a quite complex derivation of performance bounds that should be able to consider any routing and wavelength assignment algorithm based on the solution of an ILP formalized problem.

Some of the related papers consider not only the *shortest path* routing algorithm, that is the most simple to analyze, but adaptive algorithms too. The papers [6], [7], [12], [13] show results from this area. Mostly they assume the choice among the available path alternatives to be a

¹Of course a cable cut causes the failure of the whole link.

random variable, that can be in some cases a very inaccurate approach of the modelled routing algorithm. In nearly all related works the calls have Poisson arrival process and exponential holding times, but [11] presents an easily applicable model for traffic with different characteristics as well.

The multifiber environment is introduced only in very few of the references. A very clear model is that of [15], but its very important lack is that the network can contain only links with identical number of fibers. The generalized version of this model presented in [16] can handle different trunk sizes on the links, but the link capacities are still assumed to be equal. Note that the equal capacity of all network resources is a considerable constraint even in optical networking. These models can be applied only for Poisson traffic and their computing times are quite large.

Our contribution is an extended model that supports the real multifiber networks, i.e., considers different number of fibers on the links. Without this capability only very simple networks can be analyzed. The basis of our work was the model presented in [14], that gives a good estimation of blocking rate by a theoretical approach. It is able to analyze singlefiber networks carrying Poisson traffic, and captures the link load dependencies using a simple iteration.

This paper is built up as follows. The assumptions and the notation are presented in section 2. Section 3 contains the model description, the formulae of the calculation and some information about its complexity. In section 4 the numerical results and their comparison with simulations are presented and analyzed. Finally section 5 summarizes our work and the possible further directions in which research may advance.

2. Assumptions, Notation

Since we used [14] as reference and starting point, most of the notation and the basic computation concept itself is taken from that paper. Some of the derivations are not detailed in this work since the calculation introduced in the description of that model have to be applied without any changes.

We assume the all-optical communication network to be a set of N switching nodes and J links those connect them. In the following we refer an individual node by n and j is for an individual link. A continuous series of links called path or route is noted by R . To identify the links of a given route R the term j means its j^{th} link as well in the according text environment.

A different number of fibers is allowed on each link but identical wavelength set is assumed on each fiber. The number of fibers on link j is M_j , an integer value greater than 0 and its maximum value for all links is M . Let C be the number of different wavelengths, i.e., optical channels available on a fiber and so C_j , the full capacity of link j can be calculated as $M_j C$. The optical channels on the links are assumed to be bidirectional, realized by pairs of fibers for example, and from graph theory point of view the network graph is non-directed. This assumption accords to the services of switched optical networks providing bidirectional connection, i.e., if a call is assigned to a channel its pair will be occupied for reverse direction traffic. Do not confuse this very natural assumption (think on telephone calls as analogy) with a symmetrical traffic matrix!

If an individual wavelength has to be identified, i is used for the i^{th} wavelength. This is its global serial number, i.e., refers the same wavelength on each fiber of each link.² In the corresponding terms i is also used to denote a set of wavelengths, where the cardinality is even i .

A free wavelength on the link means that there is at least one fiber of the link on that the given wavelength is not occupied. The same term is used for routes and a free wavelength on a route is a wavelength that is free on each link of the route. Our model supports fix routing, i.e., a call can be routed on a fixed path, that can be the shortest or any else. In our simulation scenarios the shortest path is used in the routing. If there is at least one free wavelength on the route assigned to the source-destination pair, the call will be accepted, else it will be refused. The maximum hop number of route is noted by H .

If there are more free optical channels on the route, the choice is randomly, that in the multifiber environment means a weighted random choice among the free wavelengths. The weights correspond always to the number of the free optical channels of the given wavelength on the whole route. In this way a uniform distribution of calls can be realized among the wavelengths.

Calls arrive in dynamic fashion according to a Poisson process as the simplest approach, but the model can be extended to flows with different squared coefficient of variation. The duration of calls is exponentially distributed and its mean value is set to 1 in order to obtain a solution, that supports normalized link loads. Since the flows can be referred by

²There is no meaning to use different wavelength sets in fibers because they have to be converted in the switching nodes and it does not correspond to our mean concept about the switching without wavelength conversion. For example a fiber with 30 different wavelength is not obvious to connect with one of only 20 wavelengths. This problem could be handled applying a decomposition of the network to layers containing only fully contactable wavelength sets. This papers does not covers the possible solution of decomposition.

their fix assigned route, i.e., R can be used for the identification of a flow as well.

3. Model description

Let us describe how the traffic and the network components are modelled. As mentioned before, some definition are not again presented, since they are yet introduced in the paper defining the original model.

3.1 Traffic model

The traffic model is based on the state dependent arrival model used in many publications and described in [14] too, but there are some small changes in the notation because of the multifiber environment.

Let X_j an integer value random variable representing the number of free channels on link j . According to this, let $q_j(m) = Pr \{X_j = m\}$ be the probability that on link j there are exactly m free channels that can be of any wavelengths. This probability can be easily computed in the Poisson case knowing the $\alpha_j(m)$ state dependent arrival intensity values:

$$q_j(m) = \frac{C_j(C_j - 1) \cdots (C_j - m + 1)}{\alpha_j(1)\alpha_j(2) \cdots \alpha_j(m)} q_j(0) \quad (1)$$

where $q_j(0)$ has to be calculated according to the fact, that $q_j(m)$ is a distribution on m .

The traffic type affects our model only at this point so if a non-Poisson extension is needed, the transition rates of this birth-death process have to be changed. This paper does not cover the solution of this problem. The extension could be done using the approach presented in [11].

3.2 Modelling the network

For the occupancy of the wavelength i on link j let us introduce the random variable $Y_{i,j}$ denoting the number of the fibers where this wavelength is not available. Note that a wavelength is told to be free, if $Y_{i,j} < M_j$.

Let us analyze the states of a set of wavelengths on a link. The probability that the wavelength set i is free on link j , can be calculated using a combinatoric approach according to the random wavelength assignment as:

$$\beta_{i,j}^{mul} = \sum_{m=i}^{C_j} q_j(m) \frac{\sum_{k=0}^{\min\left(i, \left\lfloor \frac{C_j-m}{M_j} \right\rfloor\right)} (-1)^k \binom{i}{k} \binom{C_j-M_j k}{m}}{\binom{C_j}{m}} \quad (2)$$

To achieve the probability that set i is free on link j , given m free channels on link j , we have to consider all possible cases with m free channels ($k = 0$), minus the cases where at least one member of the set is fully occupied, i.e., occupied on all M_j fibers of the link. The right number of this cases can be obtained using the including-excluding method for the cases that at least k member are not free. The upper index mul just refers that this value is used in a multifiber environment. In the case where M_j is equal to 1, it can be proved that we get $\beta_{i,j}$ of [14].

The calculation of the blocking probability on a route can be given easily. Let the random variable X_R represent the number of free wavelengths on route R . Using the inclusion-exclusion rule we get:

$$\begin{aligned} B_R = Pr\{X_R = 0\} &= 1 - \left(\sum_{i=1}^C (-1)^{i-1} \binom{C}{i} g_i^R \right) \\ &= \sum_{i=0}^C (-1)^i \binom{C}{i} g_i^R, \end{aligned} \quad (3)$$

where g_i^R is the probability, that a given i set of wavelengths is free on route R . The i value as index in the sum is equal the number of wavelengths in this set. Note that in a multifiber environment the meaning of g_i^R is:

$$\begin{aligned} g_i^R &= Pr\{(Y_{1,1} < M_1, Y_{1,2} < M_2, \dots, Y_{1,j_R} < M_{j_R}), \\ &\dots (Y_{i,1} < M_1, Y_{i,2} < M_2, \dots, Y_{i,j_R} < M_{j_R})\}. \end{aligned} \quad (4)$$

In the above expression index j in $Y_{j,i}$ is for the j^{th} link of the j_R hops long path R and index i means the i^{th} member of the wavelength set.

3.3 Wavelength independent case

Now there are two possible approach that can be used in the calculation of the free wavelength set probability g_i^R . For the simpler solution we have to assume the independency of the occupied wavelengths on the link of the route. This model is referred as **WIMM**, that is for wavelength independent multifiber model.

Assuming this independency the load blocking effect of links is not omitted, i.e., the reduced-load algorithm will still be used in the blocking calculation. This iterative method is for capturing the reduction of link load caused by the blocking events of the flows on the other links of the route corresponding to the flow.

In the wavelength independent case the g_i^R can be easily given as:

$$g_i^R = \prod_{j:j \in R} \beta_{i,j}^{mul}. \quad (5)$$

3.4 Arrival intensities

In the following part we show how the $\alpha_j(m)$ arrival intensity on the j^{th} link in case of m free bandwidth can be calculated, assuming the load reductions mentioned before.

Let us introduce the conditional probability $g_i^R(X_j = m)$ with the same meaning as it was used in [14], i.e., the probability that a fixed i wavelength set is free on route R given that exactly m free channels exist on link j . In the wavelength independent case it can be calculated as:

$$g_i^R(X_j = m) = \prod_{l:l \in R, l \neq j} \beta_{i,l}^{mul} \frac{\sum_{k=0}^{\min(i, \frac{C_j-m}{M_j})} (-1)^k \binom{i}{k} \binom{C_j - M_j k}{C_j - M_j k - m}}{\binom{C_j}{C_j - m}}. \quad (6)$$

Now the same way as it was applied in (3) we can compute the conditional blocking on route R given $m > 0$ free channels on link j :

$$Pr \{X_R > 0 | X_j = m\} = \sum_{i=1}^{\min(m, C)} (-1)^i \binom{C}{i} g_i^R(X_j = m) \quad (7)$$

and 0 for $m = 0$.

The reduced intensities of link we can get summing the reduced intensities of all flows using this link, i.e.,

$$\alpha_j(m) = \sum_{R:j \in R} \lambda_R Pr \{X_R > 0 | X_j = m\} \binom{C}{i} g_i^R(X_j = m). \quad (8)$$

The iterative algorithm is implemented as follows:

1. Set the mean blocking probability B_p to 0. For each link j set $\alpha_j(m)$ to $\sum_{R:j \in R} \lambda_R$, for $m = 0..C_j$.
2. Calculate $q_j(m)$ and $\beta_{i,j}^{mul}$ for each link j and $m = 0..C_j$ using (1) and (2).
3. Determine $\alpha_j(m)$ for each link j and $m = 0..C_j$ using (8).
4. Calculate the mean blocking probability B using (5) and (3).
5. If $|B - B_p| < \epsilon$ stop the iteration, else $B_p = B$ and jump to 2.

3.5 Wavelength dependent case

Assuming the dependency among links not only in the meaning of their load, but considering that caused by the lack of wavelength conversion, the model becomes much more complicated. This effect exists in the all optical networks, because an accepted call will occupy the same wavelength on all of the links of its route. The model is referred as **WDMM**, that is for wavelength dependent multifiber model.

As it is mentioned in more of the references too, the influence of the wavelength dependency is not negligible if there are several routes that contain some series of links in common. In general the sparse networks have this characteristic, e.g. rings, because of the small number of links.

In the calculation of g_i^R , i.e., the probability that a given set i of wavelengths is free on route R , the freedom probabilities of the links contained by the route are not considered as independent values. It is caused by the dependencies among the random variables $Y_{i,j}$.

In our model we take some assumptions about this dependency. The first two assumptions are taken from [14] but instead of the number of occupied fibers of the links for a wavelength, we assume independency only between some indicator random variables representing the freedom of wavelengths on the links. Let us define the random variable $Y_{i,j}^{free}$ as

$$Y_{i,j}^{free} = \begin{cases} 1 & \text{if } Y_{i,j} < M_j, \\ 0 & \text{else.} \end{cases}$$

The corresponding assumptions are now as it follows:

Assumption 1. is on the conditional independency of different wavelengths on the following links, referred as **A1** and can be formalized as:

$$Y_{i,j}^{free} \prod Y_{k,j-1}^{free} \quad k \neq i \quad \text{given} \quad Y_{k,j}^{free} \quad \text{or} \quad Y_{i,j-1}^{free}$$

Assumption 2. is on the conditional independency of the state of the same wavelength on not following links, referred as **A2** and can be formalized as:

$$Y_{i,j}^{free} \prod Y_{i,l}^{free} \quad j \neq l \quad \text{given} \quad Y_{i,j-1}^{free}$$

Our aim is now to determine the value of g_i^R that can be then applied in the expression (3) to calculate the blocking probabilities.

Some new notations will be introduced for the simpler description of the expressions. Actually most of them accord to the original notation in [14], but with an extended meaning because of the multifiber environment, i.e.,

$$\gamma_{j,j-1}^{(0)} = Pr \{Y_{i,j} < M_j | Y_{i,j-1} < M_{j-1}\}$$

and

$$\gamma_{j,j-1}^{(1)} = Pr \{Y_{i,j} < M_j | Y_{i,j-1} = M_{j-1}\}.$$

These values are identical for each wavelength and can be computed based on probabilities that characterize the continuing and not continuing calls from a given link to the following one. We introduce a third assumption regarding conditional independencies:

Assumption 3. is on the conditional independency of the state of a wavelength from the non-continuing calls on the previous links and referred as **A3**. $Y_{i,j}^{cont}$ stands for the number of those calls assigned to this wavelength that continue to the following link $j+1$ and formalization is:

$$Y_{i,j} \prod (Y_{i,j-1} - Y_{i,j-1}^{cont}) \quad \text{given} \quad Y_{i,j-1}^{cont}$$

Now using the assumption **A3** with a simple summarization of the possible cases we get, that

$$\begin{aligned} \gamma_{j,j-1}^{(0)} &= 1 - Pr \{Y_{i,j} = M_j | Y_{i,j-1} < M_{j-1}\} \\ &= 1 - \frac{\sum_{k=0}^{M_{j-1}-1} \sum_{l=0}^{\min(M_j, k)} P_n^{(j)}(M_j - l | l) P_c^{(j-1)}(l, k - l)}{1 - P_l^{(j-1)}(M_{j-1})} \end{aligned} \quad (9)$$

and

$$\begin{aligned} \gamma_{j,j-1}^{(1)} &= 1 - Pr \{Y_{i,j} = M_j | Y_{i,j-1} = M_{j-1}\} \\ &= 1 - \frac{\sum_{k=0}^{\min(M_j, M_{j-1})} P_n^{(j)}(M_j - k | k) P_c^{(j-1)}(k, M_{j-1} - k)}{P_l^{(j-1)}(M_{j-1})} \end{aligned} \quad (10)$$

The notations in the above expressions represent the following probabilities for wavelength i . Their calculation is based on a combinatoric approach in the case of the occupancy of one wavelength on one link:

$$\begin{aligned} P_l^j(k) &= Pr \{Y_{i,j} = k\} \\ &= \sum_{m=M_j-k}^{C_j-k} Pr \{Y_{i,j} = k | X_j = m\} Pr \{X_j = m\} \\ &= \sum_{m=M_j-k}^{C_j-k} q_j(m) \frac{\binom{M_j}{k} \binom{C_j-M_j}{m-(M_j-k)}}{\binom{C_j}{m}} \end{aligned} \quad (11)$$

Estimating the distribution of calls on a wavelength of a link among the possible source-destination pairs by the their relative intensities, a binomial approach can be applied to the description of following links and their continuing calls. λ_j is the entire arrival intensity on link j , while $\lambda_{j,j+1}$ represents the intensity of the arrival of those calls that should use both link j and $j+1$ (continuous calls).

$$\begin{aligned} P_c^j(l, k) &= Pr \{Y_{i,j}^{cont} = l, (Y_{i,j} - Y_{i,j}^{cont}) = k\} \\ &= P_l^j(k+l) \left(\frac{\lambda_{j,j+1}}{\lambda_j} \right)^l \left(1 - \frac{\lambda_{j,j+1}}{\lambda_j} \right)^k \binom{k+l}{l} \end{aligned} \quad (12)$$

and applying the same binomial model of the distribution of continuing and non-continuing calls, we estimate the conditional probability:

$$\begin{aligned} P_n^j(k|l) &= Pr \{(Y_{i,j} - Y_{i,j-1}^{cont}) = k | Y_{i,j-1}^{cont} = l\} \\ &= \begin{cases} 0 & \text{if } k+l > M_j, \\ 1 & \text{if } l = M_j \text{ and } k = 0, \\ P_l^j(k+l) \left(\frac{\lambda_{j-1,j}}{\lambda_j} \right)^l \left(1 - \frac{\lambda_{j-1,j}}{\lambda_j} \right)^k \binom{k+l}{l} & \text{otherwise.} \end{cases} \end{aligned} \quad (13)$$

The values λ_j and $\lambda_{j,k}$ can be calculated easily knowing the offered traffic on the routes. With the technic used in (8) too, the arrival intensities can be calculated given m free channels on the link. To obtain λ_j and $\lambda_{j,k}$ these values shall be summed up weighted by the $q_j(m)$ values.

Since in our solution the fixpoint of the network blocking is searched by iteration, the above expressions have to be recalculated in each iteration, because of the changing values of the λ_j and $\lambda_{j,j+1}$ intensities.

Applying Bayes' rule and assumption **A1**, the g_i^R value for route R consisting of two links A and B can be computed as:

$$g_i^{A,B} = \beta_{i,B}^{mul} \prod_{k=1}^i \frac{\gamma_{B,A}^{(0)} \eta_{k,A}}{\gamma_{B,A}^{(0)} \eta_{k,A} + \gamma_{B,A}^{(1)} (1 - \eta_{k,A})} \quad (14)$$

where $\eta_{i,j}$ means the same as in [14]:

$$\eta_{i,j} = \begin{cases} \beta_{i,j}^{mul} & \text{if } i = 1 \\ \frac{\beta_{i,j}^{mul}}{\beta_{i-1,j}^{mul}} & \text{else.} \end{cases}$$

In the computation for a longer route of H hops our starting points are these two link routes (note, that for one link routes $g_i^j = \beta_{i,j}^{mul}$), and using our assumption **A2** we get easily:

$$g_i^R = \prod_{j=1}^H \beta_{i,j}' \quad (15)$$

and considering **A3** too, the conditional freedom of the i wavelength set on route R , given exactly m free optical channels on the route along:

$$g_i^R(X_j = m) = \prod_{l \in R, l \neq j} \beta_{i,l}' \frac{\sum_{k=0}^{\min(i, \lfloor \frac{C_j - m}{M_j} \rfloor)} (-1)^k \binom{i}{k} \binom{C_j - M_j k}{C_j - M_j k - m}}{\binom{C_j}{C_j - m}} \quad (16)$$

where

$$\beta_{i,j}^t = \begin{cases} \beta_{i,H}^{mul} & \text{if } j \text{ is the last link of the route} \\ \frac{g_i^{j,j+1}}{\beta_{i,j+1}^{mul}} & \text{else.} \end{cases}$$

In the case of the **WDMM** approach the calculating algorithm will be changed on some points according to the differences to the **WIMM**.

3.6 Computation complexity

Let us present now the complexity of the algorithm. The binomial coefficients can be obtained using the Pascal-triangle in $O(C^2M^2)$ steps and the calculation of $\beta_{i,j}^{mul}$ is in order of $O(JC^3M)$.

The call blocking computation in the wavelength independent case is $O(HC)$. The feedback by the conditional arrival intensities can be performed by another $O(HC)$ order calculation.

The wavelength-dependent case requires other calculation too. The helping variables P_l , P_c , P_n , $\gamma^{(0)}$, $\gamma^{(1)}$, β^t are computable in order of $O(JC^2M^2) + O(HC) + O(HC) + O(J^2M^2) + O(J^2M) + O(JC^2)$. Summarizing these complexities the whole calculation is in order of $O(JC^3M) + O(JC^2M^2) + O(J^2M^2)$ in each step of the iteration.

This order is considerable and the computation is obviously more complex than that of the singlefiber case, which is in order of $O(JC^2) + O(2HC) + O(C)$. On the other hand the complexity of the model for multifiber networks presented in [15] is at least $O(HC^5M^3)$ which is considerably more complex than our method.

4. Numerical results and comparison with simulation

In the following the numerical results will be compared with simulation. Our aim is to examine the accuracy of **WIMM** and **WDMM** in cases of scenarios with different topologies, fiber capacities, traffic patterns and network loads.

Accordingly to section 2, Poisson arrival and exponential holding time is assumed for the calls. In all studies the load was set to values to get blocking probabilities in the range of 10^{-4} to 10^{-1} . On the figures the network load is indicated in (optical) Erlang, i.e., calls/timeunit. The applied routing choice was that of the shortest path with random wavelength assignment.

Our first study was performed in a simple 13 node ring with $C_j = 24$ optical channels on each link (uniform ring), $C = 24$ too and all links have the same value of $M_j = 1$ that agrees with our assumption in section

2 about the wavelength number in fibers. Uniform traffic pattern was used and the blocking probability values corresponding to the different network loads can be seen on Figure 1a.

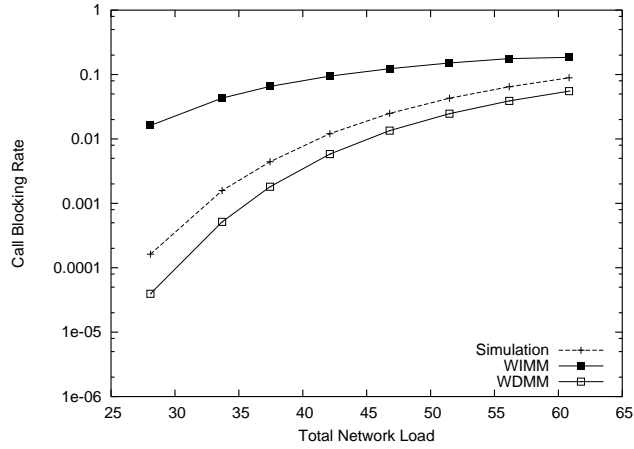


Figure 1a. Uniform 13-node ring, uniform traffic, 1 fiber - 24 wavelengths

One can see that the wavelength-independent model overestimates widely the simulation, in particular assuming small loads. On the other hand a smaller underestimation can be observed using the wavelength-dependent model.

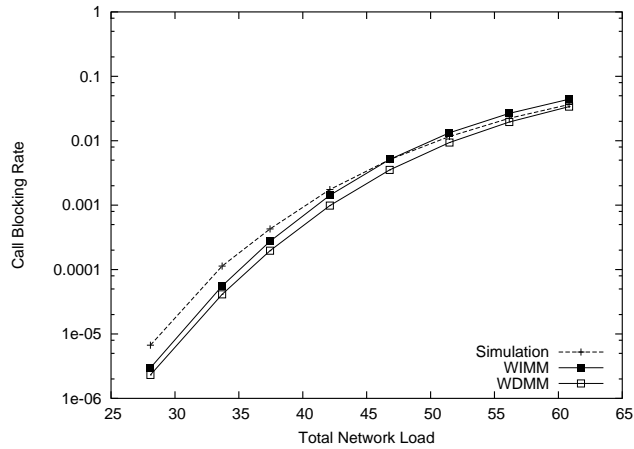


Figure 1b. Uniform 13-node ring, non-uniform traffic, 4 fibers - 6 wavelengths

Using still the same ring topology, but setting C to 6 and, accordingly, M_j to 4 for each link, and applying a random generated traffic pattern³ we obtained the results shown on Figure 1b.

The elevated number of fibers causes that the difference between the two models becomes significantly smaller. In the low-load area both models underestimate the blocking probability got by simulation.

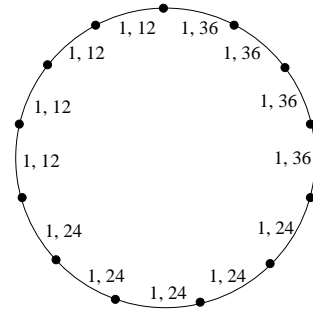


Figure 2. 13-node non-uniform ring

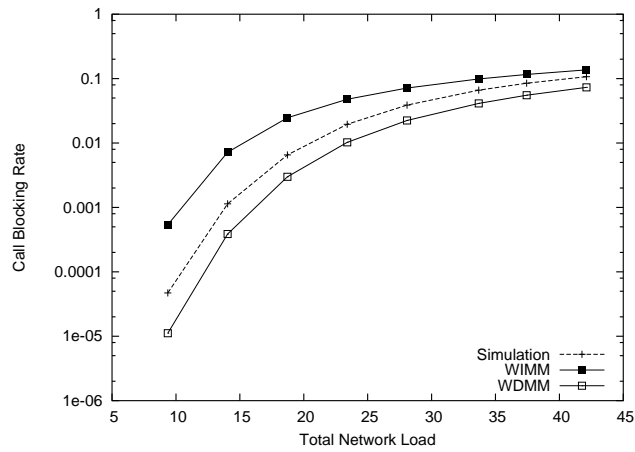


Figure 3a. Non-uniform 13-node ring, uniform traffic, 12 wavelengths

Similar behavior can be observed in our next examination cases using a ring of 13 nodes but with different C_j link capacities (non-uniform ring) which is presented on Figure 2 (the values written on the links represent their length and capacity in optical channels).

³Of course, the same pattern was applied for every case of network load.

The sum of the capacities in this network is the same as in the case of the previously studied uniform ring, i.e., $24 * 13 = 312$ optical channels. First the C value is set to 12, and the number of fibers on link j will be $M_j = C_j/C$ accordingly. The results obtained with uniform traffic pattern are presented on Figure 3a.

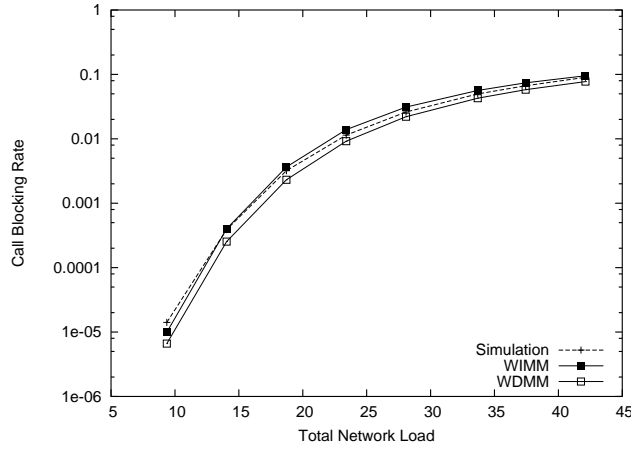


Figure 3b. Non-uniform 13-node ring, non-uniform traffic, 4 wavelengths

For the case of setting the C to 4 and applying a random generated traffic pattern the results are shown on Figure 3b.

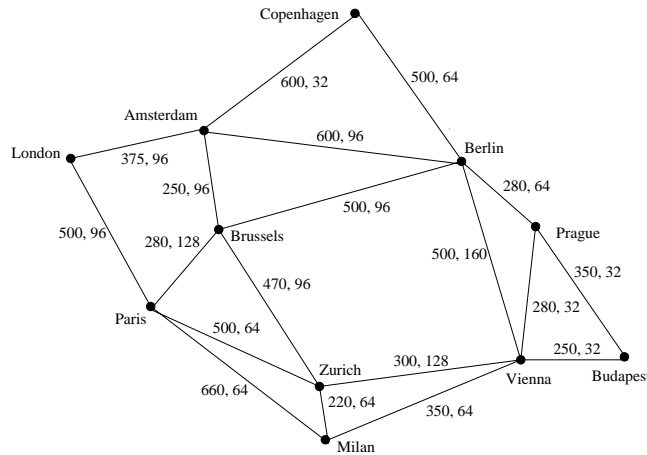


Figure 4. Central-West European Network

The third network that we used in our comparison was a hypothetical Central-West European network. The topology of this theoretical but realistic network consists of 11 nodes and 19 optical links as presented on Figure 4. Its design was based on previous publications [18] and [17] regarding the study of possible pan-European optical networks. The link capacities were determined considering the population-distance-based traffic relation matrix in [18]. The traffic pattern was composed according to this matrix, but in the place of Prague Budapest was used.

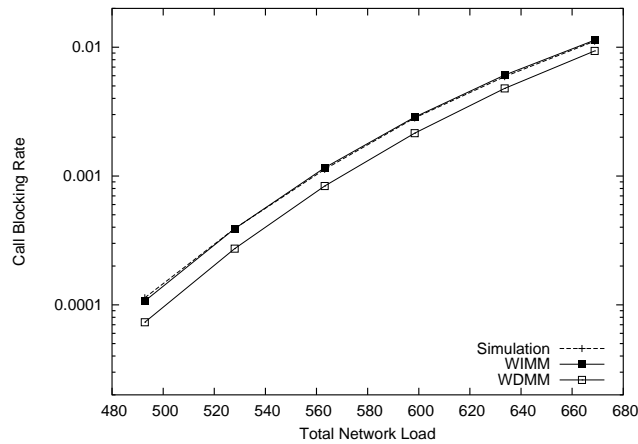


Figure 5a. Central-West European network, 32 wavelengths

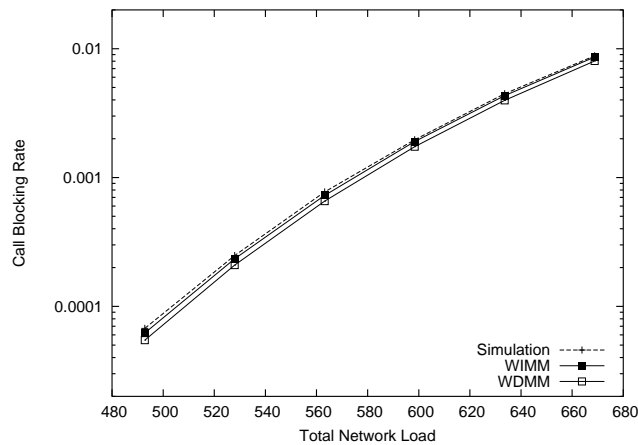


Figure 5b. Central-West European network, 16 wavelengths

In the first case C , the number of different wavelengths was set to 32 and then to 16 wavelengths. The results are presented on Figures 5a - 5b, respectively.

Our experience is that even **WIMM** estimates very well the performance of this mesh network and the underestimation of **WDMM** can be observed in these cases as well.

The simulations ran till the blocking probability as significant statistical variable reached the confidence value 0.99 with 0.01 accuracy. The average number of all simulated calls during a simulation was in range of $8 * 10^6$. In the computations using the analytical models the iteration was stopped as the difference to the previous step decreased under 10^{-6} . The average number of iteration steps was around 30 for the **WIMM** and 10 for the **WDMM**.

5. Summary and conclusion

In this paper we introduced an analytical model for dynamic all-optical multifiber networks with different link capacities. One of the most important characteristic of network performance, the blocking probability can be estimated with the presented methods. Two versions of the model were elaborated, the first one does not take into consideration the wavelength occupation dependencies on the following links of routes (**WIMM**), the other does it (**WDMM**). Both versions consider the load dependencies of the links using iteration to find the fixpoint. To consider wavelength dependency is not obvious at all and the model contains some assumptions and estimations of probability terms.

Summarizing our experiences in the above studies we can have that the model considering the wavelength dependency in the network provides a light underestimation of call blocking ratio in all scenarios. The model considering only link load dependency estimates it with different error rates. This inaccuracy depends on load, traffic pattern and topology of the network. The difference is less considerable if:

- Only the minor part of link traffic is continuing, i.e., the following links of a path are not loaded by the traffic of the same source and destination, e.g. in ring networks.
- The number of fibers on the links is elevated, e.g. more than 1 on each link.
- The load, and accordingly the blocking probability, is considerable, e.g. the call blocking ratio is more than 0.01.

Our further aim is to compare the accuracy of the models to that of models presented in the related works for the supported network cases.

The observable inaccuracy is originating from the taken assumptions and the calculations of composed probabilities. To achieve better estimations we need to apply at this point more complicated approaches. Further studies shall be performed on the iteration method as well.

Further works will be concentrated on these objectives and on the extension possibilities mentioned in the paper, will develop models supporting other traffic types, other routing and wavelength assignment algorithms.

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