Game Theoretic Solutions for Urban Traffic Control

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Abstract: This paper presents game theoretic solutions for urban traffic control. The concept considers the junctions as players in the urban network. In this multi-agent scenario, each junction makes a decision over the green time distribution of its own crossing. A decision made by a junction intends to minimize the number of vehicles within its incoming road-links putting the vehicles to the incoming road-links of neighboring junctions. The coupling effect of decisions made by the junctions (players) generates a conflict situation. The techniques proposed in this paper converts the conflict situation into a game theoretic problem and solve this problem in different hierarchical structures and different cooperation levels (noncooperative Nash strategy and semi-cooperative Stackelberg strategy). Suboptimal solutions for green time distribution due to junctions are also provided through an illustrative example of simple traffic network.

Key–Words: Urban traffic, Optimal control, Game theory, Nash Equilibrium, Noncooperative Stackelberg Equilibrium, Semi-cooperative Stackelberg Equilibrium

1 Introduction

The growing traffic of urban areas requires efficient control system to avoid heavy congestion problems. A most common tool for control applies traffic lights in the junctions (also used the synonym term crossing). The goal of traffic control, based on the measurement of most relevant traffic properties, consists of finding a green time distribution for traffic lights used in the same junction such that the overall behavior of junctions assists to pass as many vehicles through the network as possible.

The literature of developed traffic modeling and proposed control strategies are growing fast, but they still meet a lot of challenges. Some promising approaches apply cell-transformation model which is a discrete approximation to the hydrodynamic model of traffic flow [1], [2]. In general, the computational complexity allows often only heuristic-based or soft computing methods as successful representations of control strategies. In [1], genetic algorithm is applied for optimization, but fuzzy experts [3] and knowledge based methods [4] were also successfully tested in applications. Some recent works reported different techniques, as well. Control strategy on the base of stochastic system modeling is able to release incident-induced traffic congestion in [5]. The methodology in [6] organizes traffic flow into arterial structure which is especially useful to establish green corridors in traffic network [7]. The store-and-forward model described in [8] relies on state-space representation often preferred in control engineering. The control strategy discussed in this paper applies optimal LQ control.

Each junction in the network has a right to make own decision, hence any junction (agent or player, all of them are considered synonyms in this paper) corresponds to an agent in the environment. Interaction among agents are realized by the control strategies they carry out through the chosen green time distribution, as decision. It is possible to define a game in many multi-agent applications. Games assign cost function to every agent for every combination of agents' decisions. The game is cooperative if agents are cooperating for a common goal usually on the base of a command arriving from a higher level supervisor [9]. In many cases, however, the game is noncooperative [10], mainly if agents are opponents in their goal (zero-sum games) or they pursue different goals (nonzero-sum games) or there is a common goal but there is also an individual goal for each agent [11]. In order to find an optimal decision, different types of equilibrium points have been elaborated. Ones of the most widely used equilibrium point in noncooperative games is the Nash equilibrium point. If there exist a hierarchy among agents, Stackelberg strategies also lead to optimal solution.

This paper shows how the urban traffic control problem should be converted to game theoretic problem. As a result, a suboptimal game theoretic solutions are also provided to the problem. One may obtain different solutions due to the cooperation level and the hierarchical structure (Nash strategy, semi-cooperative Stackelberg strategy). An illustrative example of traffic control demonstrates the efficiency of the proposed solution.

The paper is organized, as follows. Section 2 describes a traffic model applied in the simulation test. Section 3 sets up the game theoretic scenario and propose control algorithms to the problem. The simulation results on a regular traffic network (inspired by many North American cities) with size 5×5 are illustrated in Section 4. Finally, Section 5 draws some conclusions.

2 The Traffic Model

For the traffic modeling, the Store-and-Forward model is used. The main notations and the concept are borrowed from [8], however, it contains some minor changes in the notation, assumptions and interpretation to fit the model to game theoretic description.

In the model, the urban network is realized by a graph having edges and nodes. The edges represent road-links, the nodes represent junctions. Consider a junction j. Let I_j be a set containing the incoming road links of j. Similarly, let O_j be a set containing the outgoing road links of j. The model are based on the following assumptions:

(ASF1): The vehicles pass every road link in a constant time. If the inflow is higher than the outflow at the end of the road link, the vehicles are stored (at the end of the road link). For each outcome link, a separated lane is designated from the incoming links of the junctions

(ASF2): Junctions assure at least a minimal green time from their any incoming road link to to their every outgoing road links. The minimal green time of jth junction from wth incoming road link to ith outgoing road link is denoted by $g^j_{w,i,min}$.

(ASF3): The cycle time C_j and the total lost time L_j of junction j are given. In addition, $C_j = C$ for all j.

(ASF4): The relative offsets of cycles between any two junctions are fixed (and consistent to others).

(ASF5): The saturation flows S_z, $z \in I_j$ are known for every junction.

(ASF6): The turning rates $t_{z,w}$, $z \in I_j$, $w \in O_j$ are known or estimated for every junction.

(ASF7): The junctions are arranged in a matrix structure. Every junction has 4 incoming road link and 4 outgoing link road(This often occurs in many North American cities.)

(ASF8): Road links are able to accept new vehicles from their source link without congestion. Based on the assumptions above, one writes that

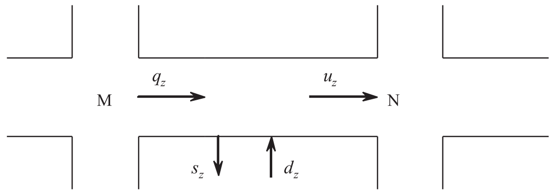

Fig. 1: The schematics of a road link

$$\sum_{w \in I_j} \sum_{i \in O_j} g^j_{w,i} + L_j = C$$
$$g^j_{w,i} \geq g^j_{w,i,min} \quad \forall j \tag{1}$$

where $g^j_{w,i}$ is the effective green time of junction j from incoming road link w to outgoing link i. Note that $i, j \in \{2, \ldots, 4\}$. Of course, the expression (1) generates an inequality constraint when cycle times are set up at traffic lights. Considering a road-link z between junction M and junction N ($z \in I_N, z \in O_M$) as shown in Fig. 1, the discrete dynamics is given by

$$x_z(k+1)x_z(k) + T\,[q_z(k) - s_z(k) + d_z(k)u_z(k)]$$

where x_z is the number of vehicles within link z, q_z is its inflow, u_z is its outflow in the time period $[kT, (k+1)T]$, $k = 1, 2, \ldots$ with control time interval T. The additional terms d_z and s_z denote the demand and the exit flow, respectively. In the most cases, there is a strong relation between the demand and the exit flow described as $s_z(k) = t_{z,0}q_z(k)$. The equation (2) is described now as

$$x_z(k+1)x_z(k) + T\,[(1 - t_{z,0})q_z(k) + d_z(k)u_z(k)]$$

(ASF9): The length of control time interval is at least C.

If x_z is sufficiently high, then (ASF8) and (ASF9) imply that the average value of the outflow is

$$u_z(k) = \frac{S_z G_z(k)}{C} \tag{2}$$

where the effective green time G_z of the road link z is

$$G_z(k) = \sum_{i \in O_j} g^j_{z,i}(k) \quad z \in I_j \tag{3}$$

Exploiting that

$$q_z(k) = \sum_{w \in I_M} t_{w,z} u_w(k) \tag{4}$$

the final form of discrete dynamics related to road link z is

$$x_z(k+1) = x_z(k) \quad (5)$$
$$+ \quad T\left[(1-t_{z,0})\sum_{w\in I_M} t_{w,z}\frac{S_w \sum_{i\in O_M} g^M_{w,i}}{C}\right.$$
$$- \left.\frac{S_w \sum_{i\in O_N} g^N_{w,i}}{C}\right], \quad z\in O_M, z\in I_N$$

Considering the equations (5) together for every road link in the network, one arrives the nonlinear discrete state equation of the urban traffic network. Note that [8] focuses on the system dynamics around the average green time values and applies linear LQ controller. In the next section, we propose a method to solve the problem in game theoretic framework.

3 Control Algorithms

This section establishes a game theoretic framework for urban traffic problem and provides solutions using Nash strategy and semi-cooperative Stackelberg strategy.

3.1 The game theoretic scenario

The idea of the concept is that urban traffic control can be considered as a multi-agent game theoretic problem in which each junction tries to minimize the number of vehicles on its incoming road links (local task with high priority) and taking a solidarity to an extent with its play-mate junctions, it also tries to help them (global task with lower priority). The decisions of the junctions (players) reflect a behavior in the green time distributions from incoming links to outgoing links. For example, if an incoming road link of junction j contains significantly more vehicles than other incoming road links, then junction j endeavors to decrease the load of this link by increasing the length of green times from this road link. Depending on the turning ratios, it increases the load on the incoming road link of some neighboring junctions which generates a conflict situation.

For the more exact discussion, let J denote the set of junctions. In this case the number of players are $\gamma = |J|$. Let $\tau^1_{i_1}$ denote the decision of the first player, let $\tau^2_{i_2}$ denote the decision of the second player etc., where $\tau^j_{i_j}$ usually change somehow the green time distribution

$$g^j(k) = \Big(g^j_{1,1}(k),\ldots,g^j_{1,|O_j|}(k),\ldots,$$
$$g^j_{|I_j|,1}(k),\ldots,g^j_{|I_j|,|O_j|}(k)\Big) \quad (6)$$

of junction j. Let U_j denote the set of all g^j. Note that (ASF7) implies that the number of incoming and outgoing road links of junction j is $|I_j| = |O_j| = 4$. Numerous combinations of functions are allowed to define on $g^j(k)$, as decision set. Let $X^n(\tau^1_{i_1},\ldots,\tau^\gamma_{i_\gamma})$ be the cost of nth player and let $X(\tau^1_{i_1},\ldots,\tau^\gamma_{i_\gamma})$ be the cost vector including the cost of all players. In order to find a game theoretic solution, one has to consider every combination of the decisions of the players (junctions) in the next subsections. It means that if the number of decisions over $g^j(k)$ and/or the size of network i.e. the number of players γ are increasing then it is not possible to find an equilibrium in real time. In order to overcome these problems, our method organizes junctions into groups and operate only with few decisions.

A group includes at most 4 members as depicted in Fig. 2. As the figure shows, it is possible the groups to contain different number of junctions. Every group defines a subgame solved parallel to other subgames. The concept relies on the idea that the effect of a junction's decision to the cost of another junction is decreasing if the distance between the junctions increases.

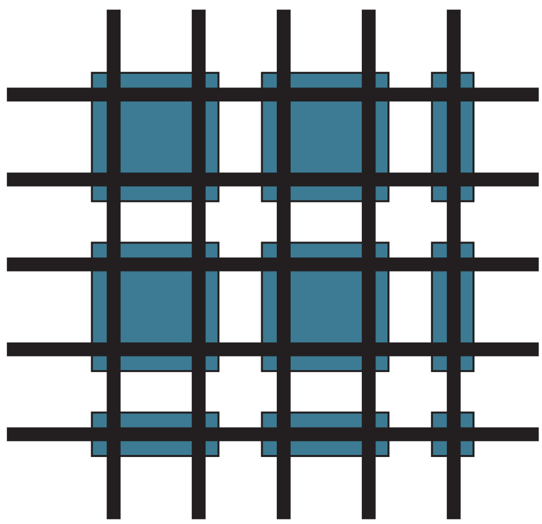

Fig. 2: The grouping of junctions into subgames

Decisions are restricted to 4 different choices in the proposed technique, each choice prefers exactly one incoming road link of the junction at the other incoming road links' expense. The total green time of the junctions (and the cycle time) is maintained at constant value with this strategy during the whole traffic control. Of course, it is possible to choose other functions over $g^j(k)$.

The algorithm of urban traffic control on game theoretic basis requires some additional notations. Let

Z be the set of all road links. Let $H(j,f)$ be the (an arbitrary ordered) set of junctions having a minimal distance f from junction j. Distance f is measured by road links, therefor it is an integer. Denote $H_p(j,f)$ the pth element of $H(j,f)$ and let $|H(j,f)|$ be the number of elements in $H(j,f)$.

3.2 Game theoretic control with Nash strategy

Nash strategy is a common way to reach a reasonable equilibrium in noncooperative games.

Definition 1 A decision vector $(\tau_{i_1}^{1*},\ldots,\tau_{i_\gamma}^{\gamma*})$ is said to be a Nash equilibrium strategy in the kth control time period, if the inequalities

$$\begin{aligned} X^1(k,\tau_{i_1}^{1*},\tau_{i_1}^{2*}\ldots,\tau_{i_\gamma}^{\gamma*}) &\leq X^1(k,\tau_{i_1}^{1},\tau_{i_1}^{2*}\ldots,\tau_{i_\gamma}^{\gamma*}) \\ &\vdots \\ X^\gamma(k,\tau_{i_1}^{1*},\tau_{i_1}^{2*}\ldots,\tau_{i_\gamma}^{\gamma*}) &\leq X^\gamma(k,\tau_{i_1}^{1*},\ldots,\tau_{i_{\gamma-1}}^{\gamma-1*}\tau_{i_\gamma}^{\gamma}) \end{aligned} \quad (7)$$

are satisfied. In the sequel, the notation $X_{NN} = (X_{NN}^1,\ldots,X_{NN}^\gamma)$, $X_{NN}^j := X^j(k,\tau_{i_1}^{1*},\tau_{i_1}^{2*}\ldots,\tau_{i_\gamma}^{\gamma*}), j = 1,\ldots,\gamma$ is used for the cost of Nash equilibrium.

Nash strategy realizes a rational and useful choice for every agents. If any agents deviates from its Nash strategy (while the others insist on their Nash strategy) then the cost function of renegade agent increases. The proposed algorithm for urban traffic control is the following.

Algorithm 1 (Urban Traffic control with Nash strategy)
Input: $g_{w,z,min}^j$, L_j, C, S_z, $x_z(0)$, T, $t_{z,0}$, $t_{w,z}$, $d_z(0)$, $g_{w,z}^j$, $\forall j \in J, \forall z \in O_j, \forall w \in I_j$.
Output: $g_{w,z}^j(k)$, $\forall j \in J, \forall z \in O_j, \forall w \in I_j$.
Steps:
Step 1) Initialization.
Δg the quantum of the change in green time. $R > 0$ The radius (measured in edges) in which a junction considers the cost of other junctions, as well.

Step 2) Measure the characteristic of the actual traffic at kth control time interval: S_z, $t_{w,z}$, $d_z(k)$.

Step 3) Compute the potential decisions of each junction:

$$\tau_i^j = \Big(\delta_{1,1}^j(k),\ldots,\delta_{1,|O_j|}^j(k),\ldots, \delta_{|I_j|,1}^j(k),\ldots,\delta_{|I_j|,|O_j|}^j(k)\Big) \quad (8)$$

Decision i, $i = 1,\ldots,|I_j|$ of junction j, $j \in J$ satisfies that $\delta_{p,w}^j(k) = (|I_j|-1)\Delta g$, if $p = i$, else $\delta_{p,w}^j(k) = -\Delta g$. If $g_{p,w}^j(k)+\delta_{p,w}^j(k) < g_{p,w,min}^j$ then τ_i^j is set to a zero vector. Note that (ASF7) make $|I_j| = 4$ fix $\forall j$. It means that each decision prefers only one incoming road link increasing its green time by $3\Delta g$ while the green times from other incoming road link of junction j are decreasing by $-\Delta g$. During this operation the total green time in junction j does not change and does not change the cycle time either. The potential green time from the incoming road p to the outgoing road w at junction j after decision τ_i^j would be

$$\hat{g}_{p,w}^j(k) = g_{p,w}^j(k)+\delta_{p,w}^j(k). \quad (9)$$

Step 4) Compute to every decision of every junction $t \in J$:

$$X^t(k,\tau_{i_1}^1,\ldots,\tau_{i_\gamma}^\gamma) = \sum_{h=0}^{R}\frac{1}{h+1}\sum_{p=1}^{|H(t,h)|}\sum_{z\in I_{H_p(t,h)}} x_z(k,\tau_{i_1}^1,\ldots,\tau_{i_\gamma}^\gamma) \quad (10)$$

where $x_z(k)$ is computed by considering (5)

$$\begin{aligned} x_z(k+1) &= x_z(k) \\ &+ T\Bigg[(1-t_{z,0})\sum_{w\in I_M} t_{w,z}\frac{S_w\sum_{i\in O_M}\hat{g}_{w,i}^M}{C} \\ &- \frac{S_w\sum_{i\in O_N}\hat{g}_{w,i}^N}{C}\Bigg], \quad z\in O_M, z\in I_N \end{aligned} \quad (11)$$

Step 5) Build the normal form of the game. In order to achieve normal for one should evaluate the vector vector function

$$X(\tau_{i_1}^1,\ldots,\tau_{i_\gamma}^\gamma) = \Big(X^1(\tau_{i_1}^1,\ldots,\tau_{i_\gamma}^\gamma),\ldots,X^\gamma(\tau_{i_1}^1,\ldots,\tau_{i_\gamma}^\gamma)\Big) \quad (12)$$

for every combination of decisions. The vectors should be arranged in a matrix with dimension $|I_1| \times |I_2|\ldots|I_\gamma|$.

Step 6) Find a Nash equilibrium point of the game by solving (7). If more than one Nash equilibrium exists, agents select one by a known strategy. If there is no Nash equilibrium point in pure strategies, a possible alternative to find a mixed equilibrium point as described in Proposition 3.5 in [10].

Step 7) Modify the green times according to Nash equilibrium point. It means that $g_{p,w}^j(k) = \hat{g}_{p,w}^j(k)$ where $\hat{g}_{p,w}^j(k)$ is selected from $\tau_{i_j}^{j*}$. Step 8) Repeat the procedure from Step 2) for the next control time interval.

Nash equilibrium applied in the Algorithm tries to achieve a balanced vehicle load on the road links of a subnetwork. Since groups play subgames parallel, the solution is suboptimal. It is easy to extend the algorithm to dynamic and hierarchical games, however, it threaten the chance of real time realization.

3.3 Game theoretic control with semi-cooperative Stackelberg strategy

Semi-cooperative Stackelberg strategy (SCSS) introduced in [12] allows to define a hierarchy of junctions in a group. In the hierarchy, there is a leader equipped by highest priority and there are followers. In the group of followers a further priorities may be defined. SCSS exploits also the fact that junctions pursue not only individual goals, but also they have partly a common goal of minimizing vehicles in the traffic network. SCSS is derived from noncooperative Stackelberg strategy. Without loss of generality, let the leader be the player with index 1.

Definition 2 Strategy $\tau_{k_1}^{1,SS}$ is a *semi-cooperative Stackelberg equilibrium strategy for the leader* if

$$\begin{aligned} X_1^{SS} &:= \min_{\tau_{k_2}^2,\dots,\tau_{k_\gamma}^\gamma \in R_F\left(\tau_{k_1}^{1,SS}\right)} X^1\left(\tau_{k_1}^{1,SS},\tau_{k_2}^2,\dots,\tau_{k_\gamma}^\gamma\right) \\ &= \min_{\tau_{k_1}^1\in U_1} \min_{\tau_{k_2}^2,\dots,\tau_{k_\gamma}^\gamma \in R_F\left(\tau_{k_1}^1\right)} X^1\left(\tau_{k_1}^1,\tau_{k_2}^2,\dots,\tau_{k_\gamma}^\gamma\right) \end{aligned} \tag{13}$$

where $R_F\left(\tau_{k_1}^{1,SS}\right)$ is *the optimal response set of followers' group (called player Union)* and is defined for any $\tau_{k_1}^1 \in U_1$ by

$$\begin{aligned} &R_F\left(\tau_{k_1}^1\right) = \Big\{ \left(\xi^2,\dots,\xi^\gamma\right) \in U_2\times\cdots\times U_\gamma : \\ &X^2\left(\tau_{k_1}^1,\xi^2,\xi^3,\dots,\xi^\gamma\right) \le X^2\left(\tau_{k_1}^1,\tau_{k_2}^2,\xi^3,\dots,\xi^\gamma\right) \\ &X^3\left(\tau_{k_1}^1,\xi^2,\xi^3,\dots,\xi^\gamma\right) \le X^3\left(\tau_{k_1}^1,\xi^2,\tau_{k_3}^3,\xi^4,\dots,\xi^\gamma\right) \\ &\qquad\vdots \\ &X^\gamma\left(\tau_{k_1}^1,\xi^2,\xi^3,\dots,\xi^\gamma\right) \le X^3\left(\tau_{k_1}^1,\xi^2,\xi^3,\dots,\xi^{\gamma-1},\tau_{k_\gamma}^\gamma\right) \\ &\forall\left(\tau_{k_2}^2,\dots,\tau_{k_\gamma}^\gamma\right) \Big\} \end{aligned} \tag{14}$$

Any $\left(\tau_{k_2}^{2,SS},\dots,\tau_{k_\gamma}^{\gamma,SS}\right) \in R_F\left(\tau_{k_1}^{1,SS}\right)$ is an optimal decision in the followers, however only the decision

$$\begin{aligned} &\left(\tau_{k_2}^{2,SS},\dots,\tau_{k_\gamma}^{\gamma,SS}\right) \\ &= \arg \min_{\tau_{k_2}^2,\dots,\tau_{k_\gamma}^\gamma \in R_F\left(\tau_{k_1}^{1,SS}\right)} X^1\left(\tau_{k_1}^{1,SS},\tau_{k_2}^2,\dots,\tau_{k_\gamma}^\gamma\right) \end{aligned} \tag{15}$$

is optimal in the coordination game and followers, respectively. If more than one decision satisfy (15), team-mates select one decision with social agreement (e.g. lexicographical order). Decision $\left(\tau_{k_2}^{2,SS},\dots,\tau_{k_\gamma}^{\gamma,SS}\right)$ is called *the semi-cooperative optimal response of followers' group (i.e. player Union)*. The cost function of the team-mates is

$$\begin{aligned} X_{SS}^1 &= X^1\left(\tau_{k_1}^{1,SS},\tau_{k_2}^{2,SS},\dots,\tau_{k_\gamma}^{\gamma,SS}\right) \\ X_{SS}^2 &= X^2\left(\tau_{k_1}^{1,SS},\tau_{k_2}^{2,SS},\dots,\tau_{k_\gamma}^{\gamma,SS}\right) \\ &\vdots \\ X_{SS}^\gamma &= X^\gamma\left(\tau_{k_1}^{1,SS},\tau_{k_2}^{\gamma,SS},\dots,\tau_{k_\gamma}^{\gamma,SS}\right) \end{aligned}$$

Definition 2 says that followers choose their optimal decision to the leader's announced decision $\tau_{k_1}^1$. Based on this, leader can select his optimal decision $\tau_{k_1}^{1,SS}$ that results the minimum cost for him even if the optimal followers' decision selected is the worst possible for the leader. The optimal response of followers at the lowest priority level is obtained by a subgame which is played only by the followers with the lowest priority. This game is called followers'game. The semi-cooperative optimal response of followers' group is important because leader can assumes a cooperation from the followers to a certain extent if they have no unique optimal response. It is equivalent with a situation in which the leader has the right to resolve the trade-off between the (Nash) equilibrium points of the followers game by selecting the one for his own favor. Indeed, a team operation like urban traffic control can really expect this kind of minimal cooperation from the junctions. The proposed algorithm for urban traffic control is the following.

Algorithm 2 (Urban Traffic control with SCSS)
Input: equivalent to the Input of Algorithm 1
Output: equivalent to the Output of Algorithm 1
Steps:
Step 1)-Step 2) are equivalent to Step 1)-Step 2) from Algorithm 1.

Step 3) Set up hierarchy for junctions. The highest priority is assigned to the junctions with highest cost in each group. In each group, there must be only one junction at a priority level except the lowest level. In the first control time interval, every junction is on the same level.

Step 4)-Step 6) are equivalent to Step 3)-Step 5) from Algorithm 1.

Step 7) Find a Semi-cooperative Stackelberg equilibrium point of the game by solving (13) and (14).

Step 8)-Step 9) are equivalent to Step 7)-Step 8) from Algorithm 1.

The following statement holds for Nash and SCSS strategies.

Theorem 3 *For a given γ person game of team-mates in target tracking problem, let X^1_{SS} denote the semi-cooperative Stackelberg cost of the leader and let X^1_{NN} denote any Nash equilibrium cost of the leader. Then, $X^1_{SS} \leq X^1_{NN}$.*

Proof: The proof can be found in [12].
The benefit of SCSS comes from the procedure of leader selection. If the junction with the highest cost at the previous discrete time in a group is chosen to leader then costs deviations from the average cost in a group can be decreased in relative to Nash equilibrium. It results a balanced vehicle loads for junctions.

4 Simulation Results

The simulation results on 5×5 sized traffic network, with $T = 60$ sec, $C = 300$ sec, $t_{z,0} = 0.01$, $d_z = 0.01$, $S_z = 1$, $x_z(0) = 30$, $\Delta g = 3$ sec, $g^j_{w,z,min} = 5$ sec are illustrated in Fig. 3-Fig. 9.

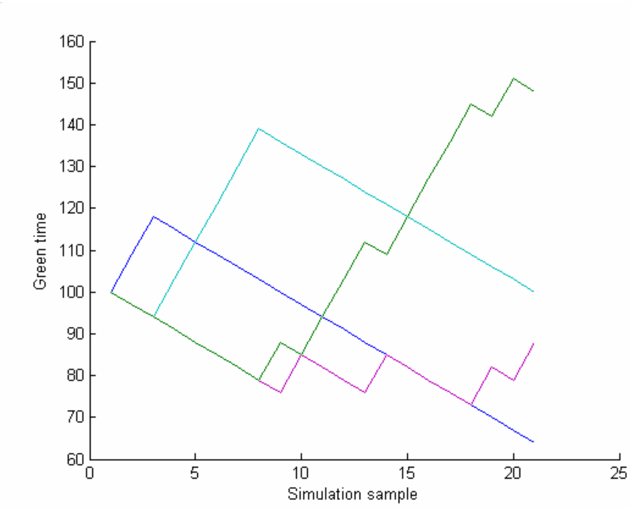

Fig. 3: Green times in the junction (2,4)

Typical green time distributions are illustrated on two junctions with coordinates (2,4) and (3,4). By definition, green times are never set up to a constant values. It is seen from the figures that there is no incoming road link of the illustrated junctions that are absolutely dominating the game. In fact, a dominating incoming road link may occur if it has heavy load permanently in relative to others.

Fig. 5 - Fig. 7 provide a possible way to compare the traffic control strategies to a constant green time set up. The strategies start from the same green time values, however the proposed Algorithms allow to adapt to network load by changing green times of the traffic lights. The computation of cost function

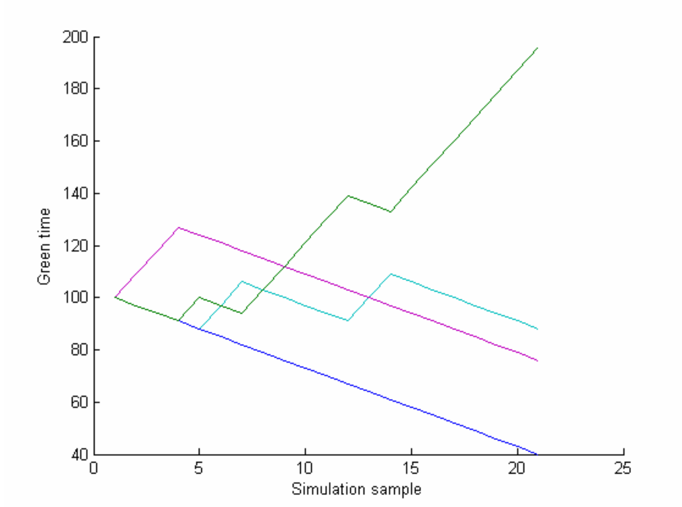

Fig. 4: Green times in the junction (3,4)

(10) is carried out for each junction. The cost values of individual junctions with constant green times are shown in Fig. 5, with Nash equilibrium point are shown in Fig. 6 and with SCSS are shown in Fig. 7. It is observed that the cost values are increasing in every case which comes from the fact that too many vehicles entry into the whole network and the traffic lights cannot clear the road links from the cars. Still, this phenomena provides better simulation environment to see the differences between the two concepts. Using constant green times for traffic light, relatively many junctions achieve cost values around 3000 while the highest cost values occur for Nash strategy and SCSS are lower. The performance of Nash strategy and SCSS are similar due to the fact that they lead to the same equilibrium point in more than 90 % of the discrete time intervals. Additionally, it occurs only in extreme cases that more than one Nash equilibrium exist in the followers' game therefore leader has rarely the freedom to select an equilibrium point for his own favor. It is also seen that Nash strategy results also acceptable balanced costs in the groups of junctions. Of course, it is possible to define other cost functions, too. They may provide other green time distributions in the network. The definition of the cost function depends on the user. It is possible, for example, to define a game in which a priority for vehicles with distinguished signal appears in the decisions. The new scenario can be easily integrated in game theoretic framework.

The efficiency of the whole network can be measured by the total cost of the junctions. The total cost in case of constant green times is depicted in Fig. 8, in case of Nash equilibrium strategy is depicted in in Fig. 9 and in case of SCSS is depicted in Fig. 10. As observed from the figures, the strategy of constant green

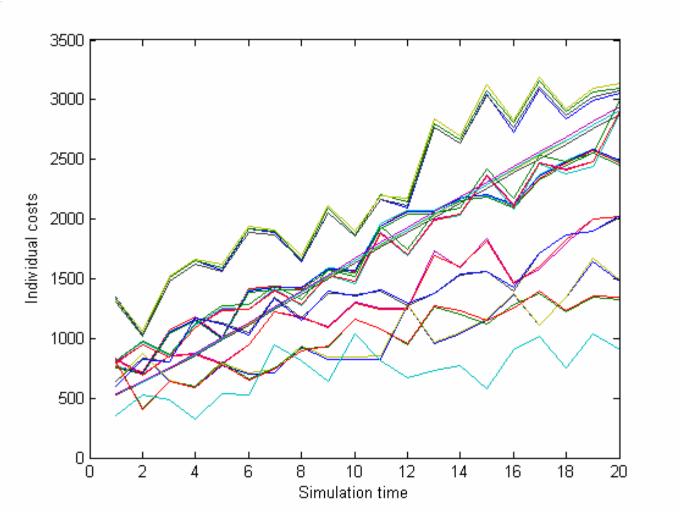

Fig. 5: Induvidual costs with constant green times

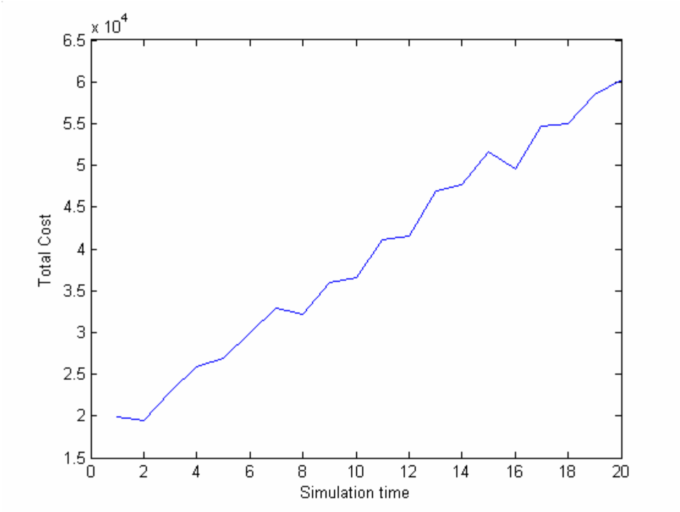

Fig. 8: The total cost with constant green times

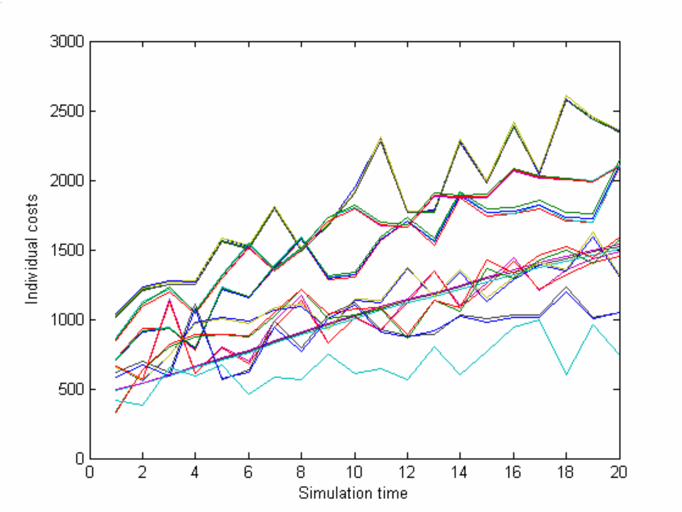

Fig. 6: Induvidual costs with Nash strategy

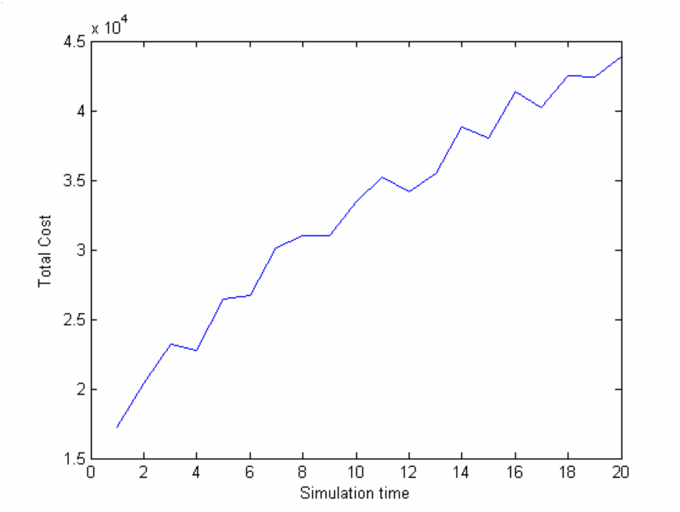

Fig. 9: The total cost with Nash strategy

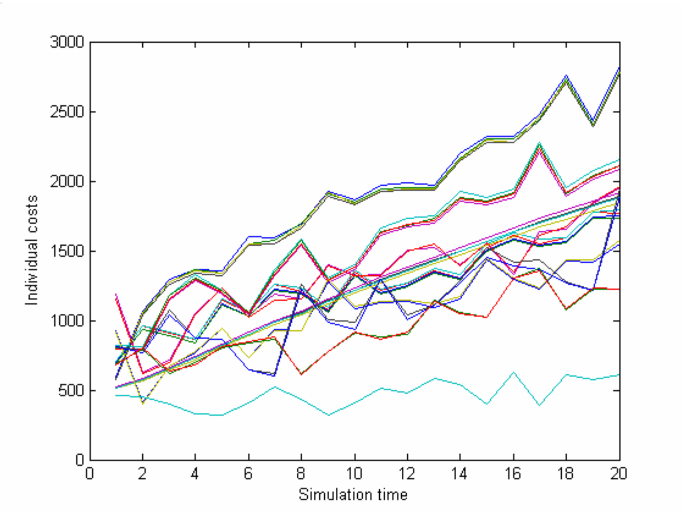

Fig. 7: Induvidual costs with SCSS

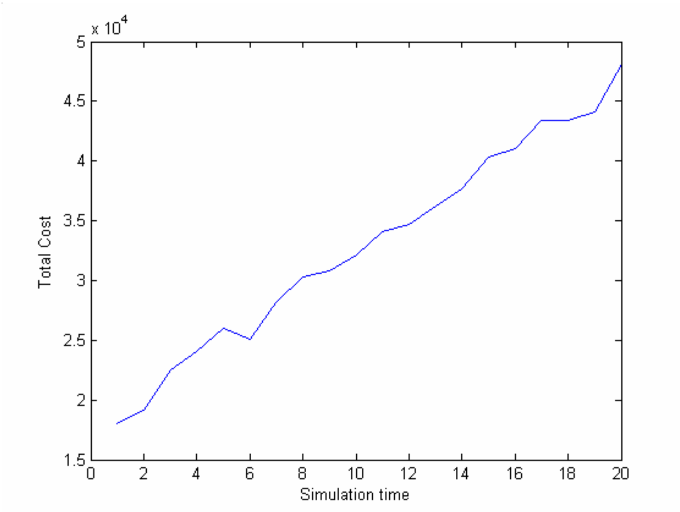

Fig. 10: The total cost with SCSS

times leads to around 30% worse performance than Nash strategy or SCSS. In addition, SCSS achieves a smoother total cost in time, which is the indirect consequence of a balanced cost distribution in the junction groups.

The control strategy implemented in Matlab2006a and executed on PC with 3Ghz Pentium processor has been fulfilled the real time requirements. The performance of game theoretic traffic control can be further improved by considering bigger groups and by more sophisticated decisions. Sophisticated methods require increasing number of alternatives in decision, i.e. improvements spoil the chance of real time realization. Similar case occurs if junction plays dynamic games exploiting the effect of decisions in time.

5 Conclusion

A game theoretic framework using Nash and semi-cooperative Stackelberg strategies have been proposed in the paper. Simulation results were underlying the intuition that a game theoretic strategy is able to outperform constant green times strategies. Nash and semi-cooperative Stackelberg strategies provided similar results due to the fact that they led to the same equilibrium point at most of the times. According to leader selection strategy, however, balanced cost distribution can be achieved by SCSS which results smoother total cost in time and more predictable behavior of the traffic network. The solution is computationally expensive and can be realized only if some simplifications are carried out. The most important simplifications appear in bundling junctions into groups and decreasing the number of decisions. The proposed methods allow to integrate optimal path planning methods of vehicles into the game theoretic framework of traffic control.

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