Analysis and modeling of financial processes with methods of statistical physics

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Summary

In my PhD work I studied two main topics of econo-physics, the price fluctuation of the stock index and the cross-correlations between different stocks.

1. *Determination of the characteristic times for the distribution of the logarithmic return.*

   Observation on real data have shown that the distribution of logarithmic returns can be described for small return values by a symmetric stable Lévy distribution. Hence the distributions of log returns belonging to different time differences, Δt, can be scaled together [1]. However one can observe a cut-off in the distribution at large return values therefore the distribution of the logarithmic returns with large time differences converge to a normal distribution.

   - I have shown that there exists a characteristic time, τ_S, and the distributions of logarithmic return with larger time difference can not be scale together. The value of this characteristic time is in the range of one day, $\tau_S \approx 1$ day.

   I fitted a Lévy distribution on the empirical probability density function and studied whether the exponent of the fitted Lévy distribution changes with time difference, because due to the condition of scaling the value of the exponent should be constant.

   - I have shown that an other characteristic time, τ_G, can be defined as well. This is the time difference for which the distribution of the logarithmic returns can be considered as Gaussian. The value of τ_G is in the range of one month.

   I have fitted a truncated Lévy distribution with exponential cut-off on the empirical distribution and determined so the parameters of the distribution, α, a_α, μ. Substituting these parameters into the analytical form of the kurtosis of the truncated Lévy distribution I studied how the kurtosis as the function of time difference is converging to the value $\kappa = 3$ which characterises the normal distribution. The analytical kurtosis function fitted well the empirically calculated kurtosis values.

- I have studied the conditions of the convergence to Gaussian distribution in case of herding market models. I found that it can have two origins: one of them is if the model is based on a group size distribution which has finite second momentum, the other is if the cut-off in the distribution is due to the finite size of the system. This way I have dissolved a contradiction found in the econo-physics literature.

2. *Modified herding model*

   With the help of the modified version of the herding market model [2] I have studied how precisely the simulated price fluctuations can describe the stylised facts observed on real data.

   I made two modifications on the model. First I generated the trading groups with a preferential growth model.

   - I determined the full time-dependent analytical form of the individual group size distribution, $\mathcal{P}(s,t)$.

   The model can be considered as a simplified version of the Barabási's network model and corresponds to Simon's model which was solved only for the asymptotical limit. The results are particularly important if the time scales of the group evaluation and the trading process can not be separated.

   The other modification on the microscopical model was that the activity is a non-linear function of the ratio of the actual and the fundamental price. Using these modifications the microscopic model gave qualitative agreement with the price fluctuations observed on real data.

   - In my modified single asset model the autocorrelation of the absolute value of logarithmic return decays as a power law. The distribution of the logarithmic return decays as a power law as well and for large return values it has a cut-off which also has power law decay but with an exponent, $\alpha = 3.36$, which is out of the stable regime.

3. *Analysis of the equal time cross-correlation between different stock's returns.*

   For the clustering of stocks I used the Super Paramagnetic model [3]. I modified the method in order to allow not only ferromagnetic but anti-ferromagnetic interactions as well, i.e. the model can describe not only the positive but also the negative cross-correlations. I have performed the clustering procedure for the stocks of the Dow Jones and of the S&P500 index.

   - If I took into account only the positive cross-correlations (ferromagnetic interactions) my results showed a good agreement with those got by Mantegna [4].

- If I took into account the anti-correlations as well (anti-ferromagnetic interactions) I could observe a difference compared to the previous results in the structure of the basic state ($T = 0$). Those stocks which show anti-correlation with almost all the others form a separate group.

4. *Analysis of the time-dependent cross-correlations.*

   - I have shown that there exist some pairs of stocks for which the time-dependent cross-correlation function is asymmetric, i.e. the maximum of the correlation function is at $\tau \neq 0$. The scale of the effect and the magnitude of time shift – which has the order of one minute – is in agreement with the effective market hypothesis.
   - I have defined and created an "influencing directed network" where the nodes denote the different stocks. Those nodes are connected with links for which the time-dependent cross-correlation is asymmetric and the direction of links indicates which is the stock whose price fluctuation follows the other ones. I showed that this network consists of disjunct parts, it has no loops (chain like linking), and on average the larger stocks which are traded more often are attracting the smaller ones, though some exceptions exist (e.g. Nokia).

References

[1] R. N. Mantegna and H. E. Stanley. Scaling behaviour in the dynamics of an economic index. *Nature*, 376, 1995.

[2] R. Cont and J.-P. Bouchaud. Herd behavior and aggregate fluctuations in financial markets. *Journal of Macroeconomic Dynamics*, 4(2):170, 2000.

[3] M. Blatt, S. Wiseman, and E. Domany. Superparamagnetic clustering of data. *Phys. Rev. Lett.*, 76(18):3251, 1996.

[4] R. N. Mantegna. Hierarchical structure in financial markets. *Eur. Phys. J. B.*, 11:193, 1999.