Budapest University of Technology and Economics
Department of Electromagnetic Theory

PhD Thesis

Hysteresis models from elementary operators and integral equations

Szabó Zsolt

Budapest
2002

I. The scope of the research and its basis

A. Basis studies on hysteresis and magnetic fields

Magnetic phenomena and in particular the hysteresis of magnetic materials have been in focus of quite a few scientists for a long period but due to the complexity of it a general description and adequate modelling has not been realized yet. Nowadays in engineering design, with the current use of the field computation softwares, the need of coupling the numerical techniques with accurate hysteresis models is desired.

In engineering practice to describe accurately arrangements containing ferromagnetic parts, the Maxwell equations must be completed with the relations that describe the magnetic behaviour of ferromagnetic materials. In some situations the consideration of constant permeability or single valued non-linearity does not give the required accuracy or even the adequate approximation of phenomena, for example related to the remanent magnetization or hysteresis losses in electrical machines are not taken into account, so the hysteretic behaviour is necessary to be modelled and in many situations it have to be regarded as vector relationship.

The most popular hysteresis models are the Preisach family types, the Jiles-Atherton model and the Stoner-Wohlfarth model.

The classical Preisach model describes the scalar hysteresis phenomena. The model represents the behaviour of the ferromagnetic material as a collection of elementary shifted rectangular hysteresis operators with different coercive fields and gives a statistical interpretation of the hysteresis with a defined distribution. The vector Preisach model proposed by Mayergoyz is a superposition of uniformly distributed scalar models. The Preisach models represent the most frequently used hysteresis models in electromagnetic field computation because of the possibility of a convenient geometrical representation, easy numerical implementation, clear mathematical formulation and adequate fitting procedures.

The Jiles-Atherton model is based on macromagnetic formulation for multidomain materials taking the domain wall motion as the major reason of hysteresis.

The Stoner-Wohlfarth model describes non-interacting single domain particle with uniaxial anisotropy disregarding the domain walls, so in this case the anisotropy is the reason of hysteresis. The main advantages of the hysteresis models built up as a superposition of Stoner-Wohlfarth particles are the vector nature of the particles and the fact that the reversible and irreversible changes in magnetization are taken into account by a single mechanism. The main disadvantage of the method is the resulting fourth order non-linear equation which has to be solved in order to determine the orientation of an elementary magnetization vector. This can involve large computation times when the bulk hysteresis model is realized.

However exist several approaches it is not easy to select the most suitable hysteresis model for a particular engineering application.

The partial differential equations obtained from Maxwell equations can be reduced to algebraic equations in several ways. The involved field variables, the approximation method and the generated mesh distinct the numerical field solvers. The most frequently applied methods are the finite element, the boundary element, the integral equation, the finite difference, and the global variation methods. Common in these methods is the existence of a mesh, over that the discretization of partial differential equation is made, and the necessity of a matrix inversion or linear equation solver to obtain the result. In the finite element and finite difference methods the whole considered space is necessary to be discretized resulting sparse coefficient matrices.

The integral equation formulation for magnetic field computation may represent some advantages mainly in 3D case and in coupled problems, for example when the magnetic field computation is connected with mechanical analysis. Among the advantages can be mentioned that in simulation of nonlinear magnetic materials the discretization is realized only on ferromagnetic parts of the arrangements, the resulted matrices depend only on the geometry of the arrangement and have to be computed only once, the method does not require the closure of the arrangement corresponding to infinity.

For the problems where the ferromagnetic material is described by a constant permeability the distribution of the magnetization vectors can be obtained in one steep with a matrix inversion. Knowing the distribution of the magnetization any field quantity can be determined in any space point.

Serious difficulties appear when hysteresis is going to be considered. If a convenient hysteresis model is chosen to represent the ferromagnetic material, the coupling of the field computation solver and the hysteresis model has to be realized, and the resulted system of non-linear equation must be solved by iterative schemes. To treat the nonlinearity the fixed point techniques are the most frequently used methods, but the Newton-Raphson method, mostly in those cases when single valued non-linearity or the first magnetization curve is considered, can be applied with success.

B. The scope of the research

The aim of this research work is to develop a general modelling of magnetic hysteresis, finding the main features of the phenomena which can be used to realize bulk vector hysteresis models and to introduce them in field computation programs.

Problems worked out

(1) The Preisach model due to their advantageous properties is one of the principal objects of this work. I have developed a procedure that makes continuous the output of the scalar Preisach model and applied it to build up 2D and 3D vector hysteresis models. To fit the scalar, the 2D and 3D continuous vector Preisach models I have realized an optimisation procedure searching for the parameters of an exponential function.

In the magnetic research the other frequently applied hysteresis model is the Stoner-Wohlfarth model. As the generalization of the Stoner-Wohlfarth particle with uniaxial anisotropy and biasteroid particle with biaxial anisotropy I have introduced a new elementary vector hysteresis operators to investigate the possibility to construct different interacting and non-interacting bulk hysteresis models. In this way I have realized a more phenomenological description of the vector hysteresis which can be regarded as a natural vector generalization of the Preisach and Stoner-Wohlfarth models.

(2) To compute different arrangements with ferromagnetic parts I have applied the integral equation method. I have proved that this method can be applied with success to describe magnetic arrangements where the hysteresis of the ferromagnetic materials is considered. According to the implementation of the hysteresis model to the integral equation method a nonlinear iteration is needed to find the correct solution. To realize the connection between the integral equations and hysteresis models of ferromagnetic materials I have developed an iterative procedure on the basis of Picard-Banach iterative scheme. To prove the validity and the convergence of the developed nonlinear iteration, implemented to the integral equation method for handling the magnetic nonlinearity, I have simulated the magnetization process of different simple 2D and 3D ferromagnetic arrangements.

(3) I have proved that with the realized method a simulation can be developed for positioning relatively small pieces with magnetic fields. I have investigated the behaviour of a non-magnetic cube with ferromagnetic coating on two opposite sides immersed in homogeneous external magnetic field. As a result of the investigation I have determined the magnetic field strength, necessary for positioning the object when friction is taken into account.

Finally, at low frequency, under DC excitation I have determined the field reducing effects inside of a ferromagnetic box if holes are present on the sides of the cube and I have compared it with results obtained from a ferromagnetic box with closed geometry. I have also investigated the magnetic field strength created by the remanent magnetizations.

II. The applied methods

A. The scalar and vector Preisach model

The model represents the behaviour of the ferromagnetic materials as a collection of elementary shifted rectangular hysteresis operators with different coercive fields. The switching fields α and β where the magnetization jumps up from $-M_s$ to $+M_s$ and jumps down from $+M_s$ to $-M_s$ respectively, characterize each operator, see Fig. 1. The operators can be defined also by their coercive field $h_c = (\alpha - \beta)/2$ and interaction field $h_m = (\alpha + \beta)/2$. The bulk magnetization can be expressed by the integral

$$M(t) = \iint_{\alpha \geq \beta} \mu(\alpha,\beta)\gamma\left(\alpha,\beta,H(t)\right) d\alpha d\beta \,, \qquad (1)$$

where $\mu(\alpha,\beta)$ is the Preisach distribution function, γ is the elementary hysteresis operator with the switching fields α, β, and $H(t)$ is the applied magnetic field, the input of the model, while $M(t)$ is the magnetization, the output of the hysteresis model. An easy and suggestive representation of the model can be obtained by the following geometrical interpretation. For each point of the half plane $\alpha \geq \beta$ corresponds only one elementary hysteresis operator γ, whose switching up and down field values are respectively equal to α and β. This way the Preisach triangle can be introduced. It will be assumed that $\mu(\alpha,\beta)$ is a finite distribution function on the Preisach triangle and is equal to zero outside. A staircase line $L(t)$ can be considered, as it is plotted in Fig. 2.2 that divides the Preisach triangle in two parts, one part S^+ where all operators are switched up and one S^- where all operators are switched down.

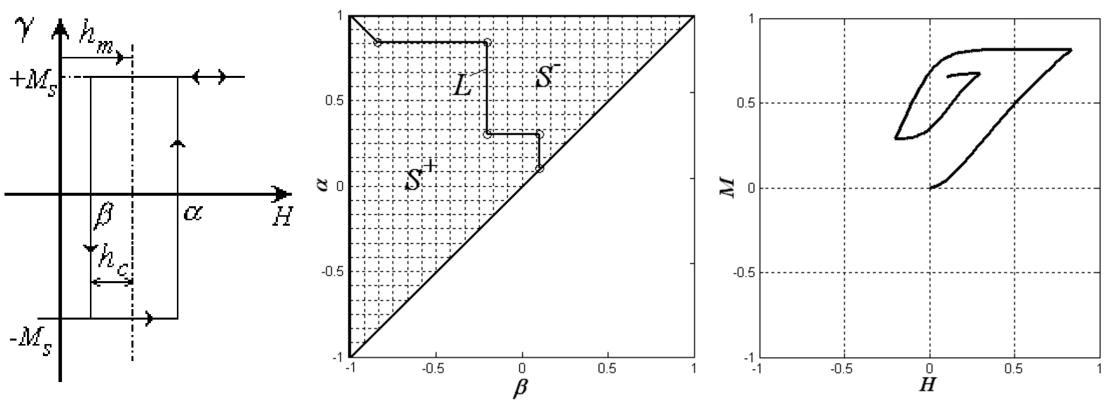


Fig.1 The elementary hysteresis operator, the Preisach triangle with the staircase line and the corresponding hysteresis characteristic

The staircase line changes its shape in accordance to the applied field, memorizing the history of the magnetization process (past local extrema of the input). The turning points of the staircase line have coordinates on the α, β plane corresponding to the variation of local minimum and maximum values of the field strength. The last segment is horizontal and moves up if the input is increased; it is vertical and moves to the left if the input is decreased.

To represent any hysteresis loop the Preisach distribution function or its integral, the so called Everett function, must be known

To realize a Preisach model with continuous output I have developed a procedure which memorizes the turning points of a staircase line corresponding to any magnetic field strength value. The staircase line evolves continuously on the Preisach triangle. The Everett function is defined on a discretized pattern and its corresponding values determined by the staircase line can be computed by linear or cubic interpolation. Once the values of the Everett function are known for each turning point, the staircase line is looked over and the magnetization computed.

The vector Preisach model is built up as a superposition of continuously distributed scalar Preisach models. In numerical realization of the model a finite number of directions can be taken into account. The projections of the magnetic field strength are determined in each considered direction. The magnetizations are computed in the specified directions by continuous scalar Preisach models. The vector sum of the magnetizations represents the output of the vector model.

In 2D case a quite good approximation can be obtained with six or eight directions. For 3D model to realise the uniform distribution of scalar Preisach models with minimum number of directions I have considered regular polyhedrons. With cube 7, with icosahedron 16 uniformly distributed directions can be defined.

The experimental construction of the Preisach model is difficult because sophisticated apparatus are necessary and the measured Everett function is sensitively influenced by the noises that unavoidably appear in the measurement process. Therefore analytical functions or distributions can be applied to construct the model

$$\mu(\alpha,\beta)=\begin{cases} e^{-\frac{(\alpha-\beta-c)^2}{10^a}-\frac{(\alpha+\beta-d)^2}{10^b}}, & \alpha+\beta\le 0 \\ e^{-\frac{(\alpha-\beta-c)^2}{10^a}-\frac{(\alpha+\beta+d)^2}{10^b}}, & \alpha+\beta>0 \end{cases}, \qquad \alpha\in[-1,1], \quad \beta\in[-1,1]. \tag{2}$$

With the parameters a,b,c,d the scalar Preisach model can be fitted to the measured data. I have developed a procedure to determine the parameters of the distribution for a given magnetization loop applying an optimization algorithm minimizing the following mean square error

$$\frac{1}{n}\sum_{i=1}^{n}(M_m^i-M_c^i)^2=\varepsilon\to\min, \tag{3}$$

where M_m^i are measured values on the major hysteresis loop, and M_c^i are values on the major loop computed from Preisach model corresponding for the same H^i.

B. Reversible and irreversible magnetization process, uniaxial and biaxial anisotropy

In the Stoner-Wohlfarth model the ferromagnetic material is regarded as a collection of small non-interacting particles, with constant magnetization, and uniaxial crystal anisotropy. The energy of a single Stoner-Wohlfarth particle can be expressed as

$$w=K\sin^2\theta-\mu_0 HM_s\cos(\theta_0-\theta), \tag{4}$$

where K is the anisotropy constant, θ is the angle between the easy axis and the particle magnetization, θ_0 is the angle between the applied magnetic field H and easy axis. The first term represents the anisotropic energy, the second the interaction energy between the external magnetic field and the particle magnetization. The minimization of (4) indicates when the orientation of an individual magnetic moment is stable within a crystal structure, thus the equilibrium position of the magnetization can be obtained from the following equations

$$\frac{\partial w}{\partial \theta} = 0, \quad \frac{\partial^2 w}{\partial \theta^2} \geq 0. \tag{5}$$

Depending on the applied field intensity the first equation can have two or four real solutions for $\cos\theta$. In the first case one energy minimum exists thus the magnetization can have one equilibrium position. In the second case four real solutions exist, two energy minima can be found and the magnetization can have two equilibrium orientations. Consequently in the H_x, H_y plane two regions can be separated in function of the solutions of (5). On the boundary of these regions

$$H_x^{2/3} + H_y^{2/3} = \alpha^{2/3} \tag{6}$$

which represents an asteroid shape. With the asteroid (6), graphical interpretation can be given for the magnetization process of the Stoner-Wohlfarth particle. The solutions of (5) are the tangent lines to the asteroid. Inside of the asteroid four tangent lines exist, while outside two tangent lines can be drawn. The direction of the magnetization is parallel with the tangent line, which represents the actual energy minimum.

Real crystal structures are more complicated and present higher order anisotropy. Considering cubic crystal structures, for example iron, which place the individual magnetic moments in a body-centred cubic lattice, for magnetization confined to (001) plane the total energy can be expressed as

$$w = K \sin^2\theta \cos^2\theta - \mu_0 H M_s \cos(\theta_0 - \theta). \tag{7}$$

The stable orientation of the magnetization vector in the crystal structure can be obtained in the same way as in case of the Stoner-Wohlfarth particle. In this case 8, 6, 4 or 2 real solutions can be obtained depending on the magnitude and orientation of the magnetic field intensity. The curve, which separates the regions with different energy extremes is a biasteroid (wind-rose)

$$H_x = \alpha \cos^3\theta (5 - 6\cos^2\theta),$$
$$H_y = \alpha \sin^3\theta (5 - 6\sin^2\theta). \tag{8}$$

Outside of the biasteroid only one energy minimum exists, thus the magnetization can have only one stable orientation and can be selected easily. Inside the biasteroid 4, 3 or 2 energy minima exist.

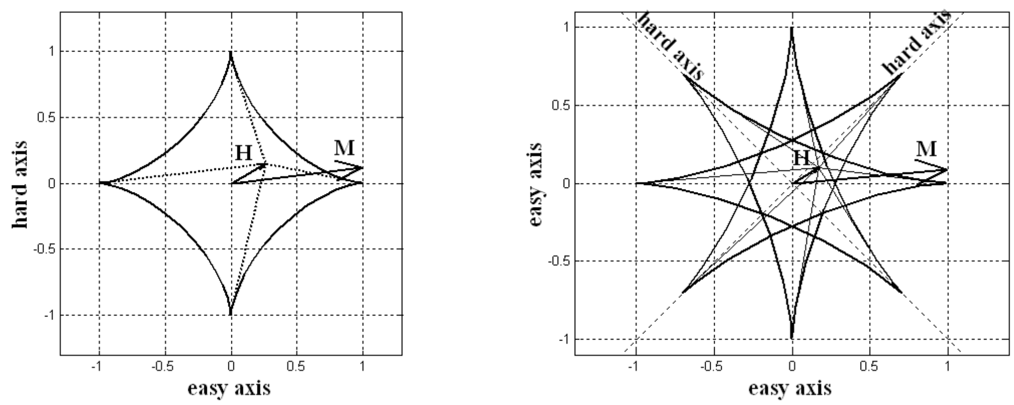


Fig. 2 Magnetization process with uniaxial and biaxial anisotropy

To realize a bulk hysteresis model, a collection of particles with uniaxial or biaxial anisotropy can be considered with different easy axis orientations and weights associated to each particle.

C. **Hypocycloid based generalized elementary vector hysteresis operators**

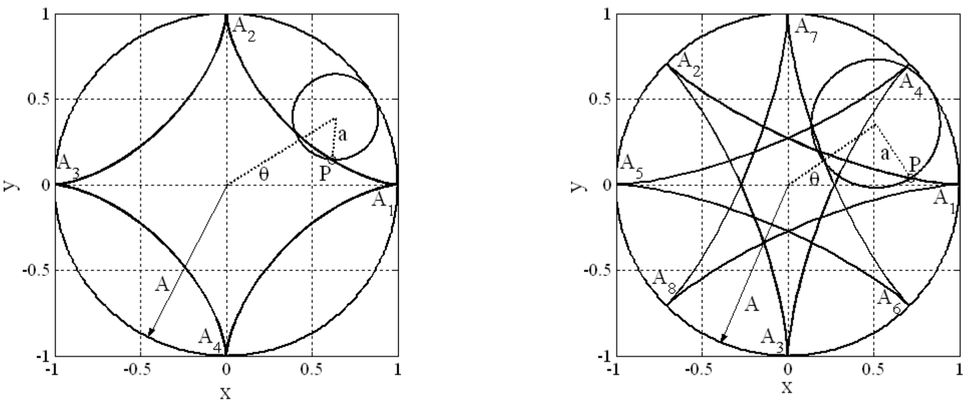
The Stoner-Wohlfarth and the biasteroid particle can be regarded as elementary hysteresis operators. However from geometrical point of view both represent hypocycloid. The parametric equation of a hypocycloid is

$$x = (A-a)\cos\theta + a\cos\left(\frac{A-a}{a}\theta\right)$$

$$y = (A-a)\sin\theta + a\sin\left(\frac{A-a}{a}\theta\right). \quad (9)$$

The shape of the hypocycloid depends by the proportion $m = A/a$. The asteroid shape is obtained for $m = 4$, the biasteroid for $m = 8/3$ (see Fig 4). The tangent lines to a hypocycloid can be obtained solving the following non-linear equation

$$H_x \sin\left(\frac{2a-A}{2a}\theta\right) - H_y \cos\left(\frac{2a-A}{2a}\theta\right) + (A-2a)\sin\frac{A}{2a}\theta = 0. \quad (10)$$



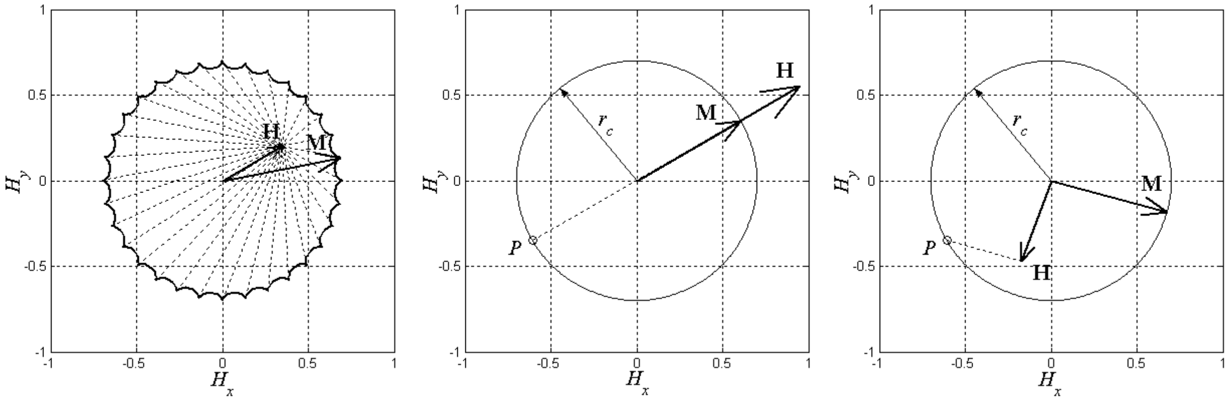
$m = 4$ $m = 8/3$

Fig. 3 Special hypocycloids the asteroid and the biasteroid

A generalized elementary vector hysteresis operator can be introduced, which consists from a hypocycloid shape and the associated selection rules that choose the tangent line which give the direction of the magnetization.

To solve (10) is time consuming, because fourth or higher order equation results. To overcome this disadvantage a simplified elementary operator can be introduced. If the parameter m of the elementary hysteresis operator is integer, the hypocycloid has m vertices and the branches do not intersect each other. Increasing the value of m the hypocycloid shape converge to a circle. In Fig. 5 the working principle of a simplified elementary hysteresis operator can be seen, for comparison a hypocycloid operator with $m = 32$ is presented.

When bulk hysteresis models are constructed, the elementary hysteresis operators can be rotated and, or translated with respect to the origin. The translation of the hysteresis operator means the modification of the switching criteria to obtain elementary hysteresis operators which produce non-symmetric hysteresis loops.



(a) (b) (c)

Fig. 4 The simplified elementary operator

D. Assembly of the elementary vector hysteresis operators

For easy representation of the bulk behaviour I have introduced a model matrix, so the hysteresis operators are considered elements of this matrix. The model matrix generally can be expressed as

$$\mathcal{M} = \begin{bmatrix} \mathcal{H}_{11} & \dots & \mathcal{H}_{1m} \\ \vdots & \mathcal{H}_{ij} & \vdots \\ \mathcal{H}_{n1} & \dots & \mathcal{H}_{nm} \end{bmatrix} = [\mathbf{M}_{ij}], \ i = 1 \dots n, \ j = 1 \dots m, \tag{11}$$

where

$$\mathbf{M}_{ij} = \mathcal{H}\left(m_{ij}, M_{ij}^{s}, \xi_{ij}, \alpha_{ij}, \beta_{ij}\right), \tag{12}$$

represents the elementary vector hysteresis operators. The parameter m defines the type of the hypocycloid, the parameter M_s is the magnitude of the magnetization, the parameter ξ is the angle, which expresses whether the operator is rotated in comparison with x axis, and the parameters α, β represent the switching up and down fields in direction of easy magnetization axes. The output of the bulk model is the sum of the magnetisations generated by the elementary vector hysteresis operators.

Applying the generalized elementary operator and the introduced model matrix I have realized different isotropic and anisotropic bulk hysteresis models.

E. Magnetic field computation with integral equation method and nonlinearity treatment

For the ferromagnetic parts of an investigated arrangement the Maxwell equations of the magnetostatic case completed with the material equation can be expressed as

$$rot\ \mathbf{H} = \mathbf{J}, \ div\ \mathbf{B} = 0, \ \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}). \tag{13}$$

On the base of Helmholtz theorem the magnetic field intensity can be divided into two components

$$\mathbf{H} = \mathbf{H}_s + \mathbf{H}_m, \tag{14}$$

where $\mathbf{H}_s$ is divergence free and represents the source field created by the current sources, $\mathbf{H}_m$ is curl free and represents the magnetic field strength created by the appearing magnetizations
The magnetic field intensity in 3D can be expressed as

$$\mathbf{H}^f = \mathbf{H}_s^f - \frac{1}{4\pi} grad_f \int_{V_e} \mathbf{M}_s \cdot grad_s \left(\frac{1}{r}\right) dV . \tag{15}$$

where r indicates the distance between the source point s and the field point f. Dividing the ferromagnetic material in elementary volumes and considering the magnetization constant over each element after simple mathematical manipulations, introducing the following notations

$$G_{xx}^f = \frac{1}{4\pi} \int_{V_s} \frac{3(x_f - x_s)^2 - r^2}{r^5} dV , \qquad G_{xy}^f = \frac{1}{4\pi} \int_{V_s} \frac{3(x_f - x_s)(y_f - y_s)}{r^5} dV ,$$

$$G_{yy}^f = \frac{1}{4\pi} \int_{V_s} \frac{3(y_f - y_s)^2 - r^2}{r^5} dV , \qquad G_{yz}^f = \frac{1}{4\pi} \int_{V_s} \frac{3(y_f - y_s)(z_f - z_s)}{r^5} dV ,$$

$$G_{zz}^f = \frac{1}{4\pi} \int_{V_s} \frac{3(z_f - z_s)^2 - r^2}{r^5} dV , \qquad G_{xz}^f = \frac{1}{4\pi} \int_{V_s} \frac{3(x_f - x_s)(z_f - z_s)}{r^5} dV , \tag{16}$$

the following system of equations is obtained

$$\mathbf{H} = \mathbf{H}_s + \boldsymbol{G}\mathbf{M} . \tag{17}$$

where $\boldsymbol{G}$ is a matrix with elements (16), which only depends on the geometry of the arrangement. In (17) the magnetic field intensity and the magnetizations are both unknowns. If constant susceptibility is considered and (17) is required to be satisfied at the mass centre of each element, the magnetization can be computed in one step as

$$\mathbf{M} = \left(\chi^{-1} - \boldsymbol{G}\right)^{-1} \mathbf{H}_s , \tag{18}$$

where χ is the susceptibility tensor. Knowing the magnetization distribution, any field quantity can be determined in any required point.

The elements of the geometry matrix are evaluated for different shapes of elementary cells. As long as the field point f is outside of the elementary volume s the integrals can be evaluated without any difficulties. However the field point f is inside of the elementary volume s, the integrals become singulars, because at $f = s$ the distance $r = 0$. For simple elementary cells (rectangular element in 2D, parallelepiped element in 3D) the integrals can be evaluated analytically. In this case to obtain the expressions of the integrals on the self element a limit calculation can be applied and in this way the singularity disappears. When the shape of the elementary volume is complicated and is convenient to apply numerical methods to compute the integrals, on the self element special treatment is necessary to obtain the correct values of the integrals. The singularity is excluded, surrounding the source point by a small hole (square in 2D and cube in 3D) and in this way the integration can be performed transforming the integrals to the surface of the elementary cells. The obtained results are independent of the shape of the hole, square and cube were selected for the simplicity of the computations.

The non-linear iteration can be constructed on the basis of (17) completed with the nonlinear characteristic.

$$\mathbf{H} = \mathbf{H}_s + \boldsymbol{G}\mathbf{M}\,,$$
$$\mathbf{M} = \mathcal{H}\left\{\mathbf{H}\right\}, \tag{19}$$

must be solved iteratively, where the magnetic field intensity $\mathbf{H}$ and the magnetization $\mathbf{M}$ are both unknown and the operator $\mathcal{H}$ denotes the nonlinear characteristic. Applying the Piccard-Banach procedure, results the following iterative scheme

$$\mathbf{H}^{k+1} = \mathbf{H}^{k}(1-\tau) + \tau(\mathbf{H}_s + \boldsymbol{G}\mathbf{M}^{k})\,. \tag{20}$$

The iteration always starts from demagnetized state

$$\mathbf{H} = 0\,,\ \mathbf{M} = 0\,, \tag{21}$$

so the magnetization is known and, from (20), the magnetic field intensity is computed solving the linear equation system by Gauss-Seidel method. Owing to the obtained values of the field strength and the nonlinear characteristic, a new approximation for the magnetization is determined

$$\mathbf{M}^{k+1} = \mathcal{H}\left\{\mathbf{H}^{k+1}\right\}, \tag{22}$$

and a new iteration step is started by substituting the resulted magnetization in (20) and computing the magnetic field strength.

To stop the iterations the error estimate is made for the magnetic field strength, computing the differences

$$\mathbf{H}^{k+1} - \mathbf{H}^{k} = \boldsymbol{\varepsilon} \tag{23}$$

where $\boldsymbol{\varepsilon}$ is an array of error values. The iteration is convergent, if

$$\left\|\boldsymbol{\varepsilon}\right\| < \delta \tag{24}$$

with δ small positive number. The behaviour of the iteration and the speed of the convergence depend on the choice of the parameter τ . To assure contraction τ must be selected as

$$0 < \tau \le \frac{2}{\chi_{\max} + 1}\,, \tag{25}$$

where $\chi_{\max}$ is the maximum susceptibility of the hysteresis characteristic.

III. Summary of the new scientific results

1.Thesis

I have developed isotropic and anisotropic bulk vector hysteresis models as superposition of elementary hysteresis operators.

1.1. I have realized the fitting of the scalar, 2D and 3D vector Preisach models to measured data by an optimisation procedure searching for the parameters of an exponential function.

1.2. I have developed a generalized elementary vector hysteresis operator. The operator consists from a hypocycloid shape, the tangent lines to it give the possible directions of the magnetisation vector; and the associated selection rules, which determine the actual direction of the magnetisation vector. I have introduced a fast and simple elementary operator as a specific case of the generalized operator. The scalar Preisach hysteresis operator, the Stoner-Wohlfarth particle and the biasteroid particle can be derived as special cases of the new hysteresis operator.

1.3. I have introduced the model matrix with elementary vector hysteresis operator elements to build up different interacting and non-interacting macroscopic bulk hysteresis models, to follow the phenomenological description of the magnetisation process. To take into account the interaction field I have translated the elementary operators while to represent different material textures I have rotated them.

2. Thesis

I have implemented the vector hysteresis characteristic to the integral equation method and I have realized a convergent iterative procedure to solve the resulted nonlinear equations to determine the magnetic field quantities in all referred points of the geometry region.

2.1. For the implementation of the vector hysteresis models of ferromagnetic materials to the integral equation method I have worked out a convergent iterative procedure on the basis of the Picard-Banach theory.

2.2. On the basis of the presented examples I have proved that the developed numerical method is applicable to describe arrangements with various geometries and results the correct solution for linear and nonlinear magnetic materials. I have showed what effect the number of elements has on the correctness of the solution.

3. Thesis

3.1. I have developed a procedure that realise the connection between the magnetic field computation by integral equation method taking into account the hysteresis and the equation of motion. Applying the algorithm I have showed that small pieces can be positioned with success with magnetic fields. If the value of the external magnetic field strength is adequate the piece after a number of oscillations will have the required position.

3.2. I have developed 2D and 3D models of ferromagnetic boxes with and without holes taking into account the hysteresis. On the basis of the investigated arrangements I have showed that the field strengths created by the remanent magnetizations are not too high however in some situations may represent a disturbing factor when passive shielding of external magnetic fields is realized. The achieved computations with hysteresis showed again the well known fact that the geometry is not necessary to be closed when shielding is realized, but depending on the arrangement may contain holes and perforations without decreasing the shielding efficiency.

IV. Utilizations and future developments

On the basis of the presented principles and procedures a field computation code which takes into account the hysteresis had been realized. This program package can be very useful in engineering design, to simulate different arrangements and realize the optimization of the product.

In the first part of the presented work a general description of the hysteresis phenomena had been realized on the basis of the introduced elementary hysteresis operator. In the future research I want to extend the realized description introducing 3D elementary hysteresis operator and apply it to build up bulk hysteresis models. The presented exponential distribution cannot realize always the required fitting precision. Consequently a future problem is to develop how to select the suitable elementary operator to a given material and how to determine the introduced distributions from measured values. On other interesting problem is to examine the possibility of a similar simplification with a global treatment of the operators as it is realized with the Preisach triangle and staircase line in case of the scalar Preisach model. The establishing Magnetic Measurements Laboratory at our department can be a splendid place to solve the above mentioned insufficiencies.

In the second part of the dissertation a quasistatic solution of the magnetic field problems based on the integral equation method with iterative treatment of the hysteresis was realized. As it was presented the iterative procedure based on the Piccard-Banach method need a relatively long computation times. As a future investigation the variation and the optimal selection of different iterative techniques can represent also an interesting and important topic of research. To take into consideration the eddy currents with nonlinearity is also an important future task. Other approximation based for example on magnetic vector potential and high order shape functions can be examined as well. To determine a principle how to select a minim number of elements necessary for a prescribed accuracy should be very useful.

The future developments of the realized field computation code must include friendly input and output facilities, automatic mesh generator and a post processing unit able to determine for example force and stress distributions.

The transfer of the realized code to parallel version should be inevitable for solving engineering problems with complicated geometries.

V. List of my publications referred in the dissertation

1. Zs. Szabó, A. Iványi, Computer Aided Simulation of Stoner-Wohlfarth Model, *Journal of Magnetism and Magnetic Materials*, vol. 215-216, 2000, pp. 33-36. L R
2. Zs. Szabó, A. Iványi, Demagnetizing Field in Ferromagnetic Sheet, *Physica B*, vol. 306, 2001, pp. 172-177. L
3. Zs. Szabó, A. Iványi, The Stoner-Wohlfarth Particle in Rotational Magnetic Field, *Proceedings of the Engineering of Modern Electric Systems, EMES'99*, Baile Felix, Romania, 27-29 May 1999, pp.115-121. L SR
4. Zs. Szabó, A. Iványi, Anisotropic Sheet with Assembly of Stoner-Wohlfarth Particles, *12th Conference on the Computation of Electromagnetic Fields, COMPUMAG'99*, Sapporo, Japan, 25-28 October 1999, pp. 388-389. L

5. Zs. Szabó, A. Iványi, Vectorial Models of Magnetic Materials as Assembly of Elementary Operators, *Proceedings of the 7th International Conference on Optimization of Electric and Electronic Equipment*, Brasov, Romania, 11-12 May 2000, pp.89-95. L
6. Zs. Szabó, A. Iványi, Vectorial Hysteresis and Temperature Dependence, *Proceedings of 9th International IGTE Sympossium on Numerical Field Calculation in Electrical Engineering and European TEAM Workshop*, Graz, Austria, 11-13 September 2000, pp. 389-395. L SR
7. Zs. Szabó, Iványi A, Monte Carlo Based Meshless Method in Electromagnetic Field Computation, *13th Conference on the Computation of Electromagnetic Fields, COMPUMAG'01*, Evian, France, 2-5 July 2001, pp 80-81. L
8. Zs. Szabó, A. Iványi, Sorting Process with Static Fields, *XI. International Symposium on Theoretical Electrical Engineering, ISTET'01,* Linz, Austria, 19-22 August 2001, CD Proceeding.
9. Zs. Szabó, J. Füzi, A. Iványi, Ferromagnetic Cylinder with Hysteresis in Homogeneous Magnetic Field, 8*th International Conference on Optimization of Electric and Electronic Equipment*, Brasov, Romania, 16-17 May 2002, pp. 45-50.
10. Zs. Szabó, J. Füzi, A. Iványi, Hysteresis as a superposition of simplified elementary vector operators, *The Tenth Biennial IEEE Conference on Electromagnetic Field Computation, CEFC*, Perugia, Italy, June 16-19 2002, accepted for presentation.
11. Zs. Szabó, A. Iványi, Anizotrop Anyagok Hiszterézis Karakterisztikájának Vektoriális Szimulációja, *Híradástechnika*, 1999/9, vol. 1, pp. 15-38.
12. Zs. Szabó, Iványi A, A Stoner-Wohlfarth Modell Számítógépes Szimulációja, Műszaki Tudományos Füzetek, *Fiatal Műszakiak Tudományos Ülésszaka*, Kolozsvár, Romania, 19-20 Március 1999, pp.73-76.
13. Zs. Szabó, A. Iványi, J. Füzi, Gy. Székely, Toroid Alakú Vasmagot Tartalmazó Tekercs Mágneses Veszteségei, Műszaki Tudományos Füzetek, *Fiatal Műszakiak Tudományos Ülésszaka*, Kolozsvár, Romania, 19-20 Március 1999, pp.77-80.
14. Zs. Szabó, A Stoner-Wohlfarth Hiszterézis Modell, Tavaszi Szél '99 Posztergaléria, *Fiatal Magyar Tudományos Kutatók és Doktoranduszok III. Világtalálkozója*, Budapest 1999, pp. 54-55.
15. Zs. Szabó, Iványi A, Anizotrop Anyagok Vektor Hiszterézis Modellje, Műszaki Tudományos Füzetek, *Fiatal Műszakiak Tudományos Ülésszaka*, Kolozsvár, Romania, 24-25 Március 2000, pp.69-72.
16. Zs. Szabó, Iványi A, Mágneses térrel való pozicionálás, Műszaki Tudományos Füzetek, *Fiatal Műszakiak Tudományos Ülésszaka*, Kolozsvár, Romania, 23-24 Március 2001, pp.43-46.