**Gábor Csernák**

# LIFETIME ESTIMATION OF CHAOTIC TRANSIENTS IN APPLIED MECHANICAL PROBLEMS

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1. **We revealed some deficiencies of the methods which can be used for the estimation of the mean lifetime of chaotic transients.**

   **The mean lifetime of chaotic transients is usually considered as the reciprocal of the so-called escape rate $\kappa$. We showed, that this relation is not exact in case of maps. The proposed new formula for the kickout number of maps is $K_m = 1/(1 - e^{-\kappa})$. At small $\kappa$ values, $K_m$ is approximately $1/\kappa$ (see Sections 3.2, 4.2.2, and [27,28]).**

2. **Two fixed points of the Lorenz system lose stability by means of a Hopf bifurcation. We derived the parametric equation of the unstable limit cycles that appear in the neighbourhood of these fixed points (Section 4.1.2, and [19]). We estimated the mean lifetime of chaotic transients in the Lorenz model by several methods of the literature (Section 4.2). We approximated the 1D Lorenz map by unimodal piecewise linear maps, and developed two methods, which provide the accurate value of the mean kickout number $K_m$ in case of such maps (Section 4.3, and [20,21,22]). These methods have two fundamental advantages over the procedures of the literature. First, they are not iterative procedures. Second, the accuracy of them does not depend on the actual value of the mean lifetime.**

   The first presented method is based on the use of the so-called transition matrix and evolution operator. The matrices required for the application of the method can be constructed using simple algorithms. Besides the numerical results, this method provides explicit formula for the mean kickout number, if the matrix of the evolution operator can be diagonalized. We proved, that this can be done in the typical cases, and showed a symmetric special case – which should be avoided –, when the matrix cannot be diagonalized (Appendix D).

   The other method is based on the construction of appropriate tree-structures. This recursive method provides explicit formulae for the mean kickout number. The quantitative results of the two methods are equal.

3. **We derived the equation of motion of a mechanical model which arises in computer-controlled systems in the presence of considerable internal and external friction [28]. We showed, that the solutions can be described by a generalized version of the so-called micro-chaos map (Section 5.2, and [28]). We determined the mean lifetime of chaotic transients in case of two versions of the micro-chaos map (Sections 5.3.1, 5.3.2, and 5.3.3, and [25,27,28]).**

   According to our experience, only infinite or very large transition matrix can be constructed for the generalized micro-chaos map. It makes impossible to use some of the methods described in the literature. However, our recursive method could be applied for this system, and we were able to determine the life expectancy of transient chaotic oscillations for certain parameter domains.

4. **We showed for the two versions of the micro-chaos map, that the graph of the mean lifetime is piecewise linear as a function of the system parameter $|I_0|$. The domain of definition of this graph exhibits a fractal structure and the mean lifetime function is similar to the so-called devil's staircase (Sections 5.3.1, 5.3.2, and 5.3.3, and [27]). We showed in both cases, that the diagram, presenting the number of steps needed to escape from different initial points, exhibits a multi-scale fractal structure induced by singularities (Sections 5.3.1, 5.3.2, and 5.3.3). As a by-product of our recursive method, we were able to determine the fractal dimension of this diagram in some parameter domains. Using this result, we developed a new, fast approximation method for the estimation of the mean lifetime (Section 5.3.4).**

5. **If the mathematical model, map, describing the analysed dynamical system or machine is not known, the measured data can be analysed by nonlinear time series analysis. Using some quantities that can be estimated by nonlinear time series analysis, we propose two methods for the approximate determination of the escape rate (Section 6.1). We checked our procedures in case of the two versions of the micro-chaos map for numerically generated time series (Section 6.3, and [26]).**

   There are no time series analysis methods in the literature for the direct estimation of the escape rate $\kappa$, but the Lyapunov exponent, the correlation dimension, and the correlation entropy are eminently measureable from time series. By exploiting Pesin's relations, i.e., that the escape rate can be expressed approximately by the Lyapunov exponent and the correlation dimension, or by the Lyapunov exponent and the correlation entropy, we found two new methods for the determination of the escape rate, by which measured data can be processed.

   We applied our methods in case of the micro-chaos map. The obtained results support the applicability of nonlinear time series analysis methods for the estimation of the lifetime of chaotic transients.

6. **We searched for indications that refer to transient chaotic behaviour in case of a harmonically excited dry-friction oscillator. We discovered a bifurcation phenomenon, during which the number of stops per excitation period changes as the excitation frequency is changed (Section 7.3, and [23,24]). Using analytical methods, we found a periodic solution showing the transition from a non-sticking solution to a solution with one stop per cycle. A linear stability analysis was performed for this solution (Section 7.4, and [23,24]).**

   During the examination of the oscillator, we numerically determined the number of stops $N$ per excitation period and the amplitude of the oscillations versus the excitation frequency at some values of the friction parameter. A non-trivial connection was found between these diagrams, namely, $N$ decreases at the local minima of the amplitude-frequency diagram. Using numerical results, a qualitative explanation was given for this phenomenon.

The explored bifurcation phenomenon does not refer to the existence of chaotic or transient chaotic oscillations, which were proved to exist in the literature in case of more advanced friction models.