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PhD Thesis

# Two-dimensional quantum field theory in and out of equilibrium

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# Introduction

Despite its long history of over 120 years, quantum theory continues to pose an abundance of conceptual and computational challenges to physicists to this day. A particularly flourishing contemporary branch of research aims to acquire an understanding of the complex phenomena related to quantum many-body dynamics, where low-dimensional systems provide insight due to the enhanced role of quantum fluctuations. Apart from the possibility to observe strong quantum correlations, low-dimensional dynamics is also appealing from a merely theoretical point of view due to the notion of integrability, a property which allows for the exact treatment of quantum many-body physics. Integrable models often appear near quantum phase transitions, where universal dynamics in the vicinity of the critical point is captured by quantum field theory models.

Recent developments in experimental techniques rekindled the interest in low-dimensional quantum field theories. On the one hand, the finely-tuned conditions of quantum integrability are now attainable in quasi-one-dimensional crystals, bringing the corresponding theoretical curiosities within touching distance. On the other hand, low-dimensional quantum models can be routinely engineered in cold atomic gases, with the possibility of manipulating the interaction parameters in time to observe the out-of-equilibrium dynamics. The experiments triggered fundamental questions regarding the equilibration of closed quantum systems, and inspired a considerable theoretical effort to accurately describe various aspects of non-equilibrium quantum physics. In this regard, immense attention focuses on the quantum quench—a paradigmatic protocol relevant both to theory and experiments—which is equivalent to the sudden change of coupling parameters in the system. The complete solution to the quench problem is only available in free theories, while the role of interactions is far less understood. To identify universal characteristics in the non-equilibrium time evolution, I utilise the tools of quantum field theory in conjunction with integrability in this thesis, contributing to the theoretical understanding of low-dimensional quantum systems.

# Aims

My central aim in this thesis was to develop a description of various non-equilibrium settings in an interacting two-dimensional quantum field theory. To this end, I mostly build on the well-known ground state properties of the Ising field theory (IFT). The IFT is a quantum field theory that captures the universal dynamics of the low energy degrees of freedom near a quantum critical point that belongs to the Ising universality class. The model has a two-dimensional parameter space, and two integrable directions therein. Despite its simple formulation, it has a rich structure: it contains a free field theory, a strongly interacting integrable model (the $E_8$ model), and a non-integrable region with a well-understood particle spectrum.

In particular, my first aim was to characterise the time evolution following a quantum quench in the interacting $E_8$ theory by a comparison of numerical and analytical approaches. In this regard, I investigated the differences between two types of quenches: first, when the post-quench dynamics is integrable, and second, when the quench breaks the integrability of the system. As a related second goal, I aimed to develop a perturbative expansion to express the initial state following generic quantum field theory quenches. The evaluation of the formulae in the $E_8$ model motivated the exact calculation of matrix elements, i.e. the solution of the form factor bootstrap.

A third aim was to explore the effects of a quantum critical point on the non-equilibrium dynamics, in other words, to investigate the Kibble–Zurek mechanism in an interacting quantum field theory. This mechanism predicts universal dynamical scaling based on very generic arguments, which were seldom put to test in the presence of interactions.

# Methods

To pursue the above aims, I used numerical and analytical tools. On the numerical side, I employed the truncated conformal space approach (TCSA), a Hamiltonian truncation method based on the conformal symmetry of the critical point. While the original formulation of the method can be readily used to the quench protocol, an algorithmic development was required to study the Kibble–Zurek mechanism. In collaboration, I constructed an improved algorithmic structure explicitly utilising the chiral factorisation of the conformal Hilbert space, leading to a more economic memory usage. These results are about to be published in the form of a ready-to-use code package. On the analytical side, I adapted the tools of form factor perturbation theory to the non-equilibrium settings under consideration. This framework uses that the matrix elements of the scaling fields can be calculated exactly from integrability.

# Original results

1. I worked out the solution of the form factor bootstrap in the $E_8$ integrable quantum field theory. I performed a rederivation of the recurrence relations generating the multi-particle form factors, and by solving them extended the set of matrix elements available in the earlier literature. The solution for the $\sigma$ field was published in Ref. [1], while the analogous results for the $\epsilon$ operator were published in Ref. [2] with a synopsis of the form factor bootstrap. Finally, my results provided theoretical support for the experimental

observation of the $E_8$ particle spectrum, published in Ref. [3].

2. I analysed the time evolution following a global quantum quench in the $E_8$ field theory. With the tools of numerical modelling I compared two analytical approaches to the post-quench dynamics with each other, and argued that the expansion on the post-quench basis is more suited to describe the time evolution, which carries the fingerprints of the equilibrium post-quench model. Most notably, I pointed out the prominent role of single-particle oscillations in the dynamical one-point functions, originating from the particle states in the post-quench spectrum. I demonstrated the validity of this claim both for integrable and non-integrable post-quench dynamics. The results were published in Ref. [4].

3. I derived a second-order perturbative expansion to post-quench overlaps applicable to generic quantum field theories in principle, and to models with at most a small integrability breaking in practice. In comparison with the numerical data I have justified the perturbative approach for quenches of moderate size in the $E_8$ model for one- and two-particle states. Importantly, the presence of pairs composed of different particles indicated that the initial state is not of an integrable form. I applied the expansion to quenches with non-integrable dynamics for the lightest particles and found serious limitations otherwise. The results were published in Ref. [1].

4. I investigated the Kibble–Zurek mechanism in the Ising field theory. Exploring the parameter space, by numerically modelling ramps slowly approaching and crossing the critical point, I verified the distinct scaling predictions corresponding to the two integrable directions. In particular, I found strong evidence for the general arguments behind the

Kibble–Zurek scaling in terms of various quantities from the microscopic level to dynamical scaling functions. The unique spectrum of the $E_8$ model allowed for a nontrivial demonstration of the scaling brought about by single-particle states. Furthermore, I offered a set of numerical and analytical arguments in support of the universal scaling of the full distribution of the excess heat induced by the non-equilibrium protocol. The results were published in Ref. [5].

# Publications

[1] K. Hódsági, M. Kormos, and G. Takács, "Perturbative post-quench overlaps in quantum field theory," *Journal of High Energy Physics* **2019** (2019) 47, `arXiv:1905.05623 [cond-mat.stat-mech]`.

[2] X. Wang, H. Zou, K. Hódsági, M. Kormos, G. Takács, and J. Wu, "Cascade of singularities in the spin dynamics of a perturbed quantum critical Ising chain," *Phys. Rev. B* **103** (2021) 235117, `arXiv:2103.09128 [cond-mat.str-el]`.

[3] H. Zou, Y. Cui, X. Wang, Z. Zhang, J. Yang, G. Xu, A. Okutani, M. Hagiwara, M. Matsuda, G. Wang, G. Mussardo, K. Hódsági, M. Kormos, Z. He, S. Kimura, R. Yu, W. Yu, J. Ma, and J. Wu, "$E_8$ Spectra of Quasi-One-Dimensional Antiferromagnet $BaCo_2V_2O_8$ under Transverse Field," *Phys. Rev. Lett.* **127** (2021) 077201, `arXiv:2005.13302 [cond-mat.str-el]`.

[4] K. Hódsági, M. Kormos, and G. Takács, "Quench dynamics of the Ising field theory in a magnetic field," *SciPost Physics* **5** (2018) 027, `arXiv:1803.01158 [cond-mat.stat-mech]`.

[5] K. Hódsági and M. Kormos, "Kibble–Zurek mechanism in

the Ising Field Theory," *SciPost Physics* **9** (2020) 055, `arXiv:2007.08990 [cond-mat.stat-mech]`.

Further publications not related to the statements:

[P1] G. Szabó and K. Hódsági, "The role of mixed strategies in spatial evolutionary games," *Physica A* **462** (2016) 198.

[P2] K. Hódsági and G. Szabó, "Bursts in three-strategy evolutionary ordinal potential games on a square lattice," *Physica A* **525** (2019) 1379.