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Wavy Aspects in Open Precessed Cylindrical Channel

*A thesis booklet submitted to Budapest University of
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1. Introduction

The forced oscillations that occurred in open cylindrical channel under precession conditions were discussed in the proposed thesis. Those oscillations are sometimes called the inertial waves which are ubiquitous in rotating flows. Those waves have common features like they have the same speed and they have different wavelengths, those waves appear due to the detuning effect of Coriolis force as long as the driving frequency smaller than twice the base rotation rate. Examples in nature are varied, in external geophysics, atmospheric vortices such as Hurricanes or tornadoes are also subject to precessional forcing, as their axes rotate with the rotation of the Earth around the polar axis, thus, the fluid in precession with an angle α equal to the co-latitude. (**Lagrange *et al.* 2010**). In the previous work all cases focused on those oscillations in closed cavities (cylinders and spheres) because of the corresponding natural cases. The spheres for instance caught the attention because researchers wanted to know more about the dynamo that generates the Earth's rotational motion. Studies showed that this dynamo is because of high temperature inside the core that stimulates the currents inside the core and the mantle, however due the moon and sun gravity forces on the equatorial, the Earth started to get bulged on this area and then a precession of the Earth vertical coordinate from the north pole started to appear, this precession period is about 25800 years. It is suggested thus that precession plays an important role in the inertial waves formed in the core and in the mantle as this precession produces other type of Coriolis force that is called Poincaré force that is not totally balanced with the pressure force thus it causes this motion. (**Malkus 1968**). The cylinder cases were also of high attention as the stability of the fluid contained the spacecraft is of high importance. (**Manasseh 1992**). In open channels, the forced oscillations are caused by navigation by ships and boats, the oldest example is the tide which is forced by the attraction of the sun and the moon, also the solitary wave which was generated in front of a boat that is pulled with horses along one of the Scotch channels. It is noticed that the effect of precession can be harnessed in many engineering and mechanical aspects for instance lately it was noticed that in combustion chambers, and agitators if the container is under weakly precession the turbulent effect can be sustained for long time without impellers. (**Goto *et al.* 2014**). The unsteady and turbulent phenomena in open channel is in particular the scope of this study in this thesis. In channels one may find simple linear waves, and other nonlinear ones this includes the weakly and strongly nonlinear ones. This nonlinearity leads to instability that may result in the interaction between waves and the mean flow, in addition to wave breaking. Those waves

can be generated by navigation, or the operation of hydraulic controls or accidentally. Such phenomena affect the hydraulic conditions in the channel and pose engineering problems. The violent impacts of water waves on walls create velocities and pressures that are bigger in magnitude than the normal ones, which may increase the erosive action and warrant the reinforcement or redesign the channel. Also, one should not forget the dynamics of sediment transport processes generated by the action of the flow phenomena.

In the present thesis the effect of rotation and tilt is included and studied in length in a new case study, which is a closed cylindrical canal, although Airy first used closed canals to study the famous forced oscillation (the tide) in the previous century but most of the later cases ignored this case, or treated the problem in a different way than channel. The precession mechanism in triggering inertial waves is simple and widely used better than using obstacle along the cross section of the channels. Closed canals in nature could be found along rivers if there is a land in the middle of the river where big closed canal can be found. The mathematical treatment is divided into mainly two parts: the first is the linear one, and the second is the nonlinear one. The linear part used in treating the small waves, under irrotational conditions and the field equation solution gave the velocity potential, and new dispersion equation. This solution is compared with real waves in the experiment and their frequencies with good match. Other linear approximation based on the shallow water theory led to solution for the long wave equation numerically using matrix inverse and Jacobi iterative methods. When including the shear into the linear system of equation a new version of Burns condition is derived and solved, the results accord with the cases under normal conditions where two values of the wave velocity can be derived in comparison with the stream and slip velocities. The nonlinear part of the problem focused on the bore and the single Kelvin mode. For the bore new system of characteristics is derived and solved with time and space. A new conjugate depth relationship is derived and a scheme developed to get results from the experiment. Concerning the single Kelvin wave, it was noticed that it has characteristics similar to the solitary wave in open channels so the treatment used the asymptotic methods, for three cases all new: the first by assuming potential flow, which has forcing term includes the tilt, and varied with time and space, its coefficients include the rotation effect, it was solved using the finite difference methods with good fit with the real observed wave. The second model includes the shear effect where other KdV is derived and solved numerically using Fourier transform methods. The final model was a KdV that has coefficients varied with time and space but was solved numerically based on discontinuity as an initial guess to track the bore. The nonlinear shallow water equations also solved numerically using Roe solver in finite volume methods. The instability of the wave discussed only

experimentally where new schemes are built that take viscosity and rotation effects on the wave, the azimuthal flow also deduced experimentally to check its effect on the wave stability with respect to the three control parameters.

2. Methodology

As mentioned in the introduction that the study will be carried out in a new design, in this section I explain it in details. The structure of the apparatus is clear in Figure (2.1) in the main thesis (Alshoufi 2021). The walls of the circular wave flume are two coaxial cylinders both made of Plexiglas. They are based on a round shaped plastic bottom plate, which is fixed to a rigid wooden round support table of the same radius (60 cm). We refer this as the *tilting table* because the centre of this table is mounted on a support column through a Cardano type universal ball joint. The joint contains two, mutually perpendicular horizontal axles, around which the table can freely tilt in any direction, but it prevents any rotation around the vertical axis. This type of joint is used to transmit torque in between two nonaligned shafts in different machineries (like automobiles). Right below the tilting table there is another horizontally positioned round table of the same size, which is capable of free rotation around the vertical axis. The tilting table is partially supported also by the rotating table on three points arranged in 120° apart from each other. Each of this support consist of a vertical adjustable length spreader screw mounted vertically on the rotating table and a roller on top of the screw, on which the tilting table rests. By carefully adjusting the spreader screws, the tilt angle and the direction of the tilt can be set. When the lower table turns, the tilt direction in which upper table — together with the water flume —tilts also turns around (the vertical axis) without changing its slope or turning around (the vertical axis). This motion is called *precession* in classical mechanics and is well known, e.g., in celestial mechanics. The final bit (not shown in the Figure 1.) of the apparatus is an adjustable speed direct current (DC) electric motor, which drives the rotating table via a belt transmission system and a vertical cylinder shaft turning on two bearings coaxially around the main support column. We note that currently the rotating table can be turned only in the counterclockwise sense.

Experimental methodologies are divided mainly into two parts: the first is by using a CCD camera (JAI PULNiX TM-1405 GE) at rate 30 frames per second to record the wave motion. The resolution of camera's pictures is 1392×1040 active pixels for excellent image quality, $4.65 \times 4.65 \mu\text{m}$, with focal length around 8mm. The camera was mounted on the laboratory floor in front of the channel at a level equals almost to the water level inside it, the camera was connected to the computer by which a *Coyote* program application is downloaded, this program is a property sheet containing two main panels for the device configuration and the acquisition

panel, the laboratory computer is too old and it was not used in the image processing techniques applied to the pictures however another laptop HP ProBook 440 G5 was used, after picture transformation. The illumination in the laboratory was not enough to get clear pictures and external illumination was used, this includes two different lamps were hanged on the outer periphery of the inner cylinder, where they spread their light inside the whole system, sometimes additional big lamp was mounted on the laboratory floor which is adjustable in height to cover the whole channel system with its illumination. It worth to mention that the inner walls of the inner cylinder were covered by blank A3 papers, so does the outer cylinder but only from the back where the camera does not track the motion, this mechanism reduces the light reflection and makes the vision clearer to the motion inside the channel. The extracted pictures suffer from distortion so that the pictures appear as bulged outward radially. A new calibration tool is built by the author in the form of program using Python programming language so that the pictures were calibrated accordingly. The algorithm is based on **Zhang (2000)** closed form method from which one can extract the intrinsic camera parameters, then the extrinsic ones then refine all of them using Levenberg minimization method from which the error between the projected points on the image plane and the real one is minimum. Of course, the remapping process is the next where all the corrected pixels will be saved in a new corrected image form. The visualization processes mentioned above is highly important in deriving results related to waves information like their wavelengths, their amplitudes, etc.

The final methodology is the one exploited to extract velocity information using Acoustic Doppler Velocimeter. The vectrino profiler has four reciever arms sorrounded the transmit transducer (the central element). The principle that vectrino depends on finding the velocities is the Doppler effect, which is the change in frequency of a sound wave when a wave source moves with respect to an observer, or when the observer itself moves relative to the wave source. The velocimeter measures velocity by transmitting a pair of short sound pulses of a specific frequency into the water column. Part of the sound waves reflect back to the instrument, where the detected signal undergoes further process based on the Doppler Shift, precisely on the phase difference between two transmit pulses. To read the vectrino signals there is spcific program called *Vectrino Plus*, then the data records can be transformed into simple Excel readable data using other program *WinADV*.

3. Theses

A New KdV Model to Describe the Solitary Wave in the Channel under potential approximation (*Thesis1*)

I have proved the existence of solitary wave in the rotating channel flume theoretically and by comparing it experimentally with the observations in the laboratory. (Chapter 4.2)

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The single Kelvin mode of the flow resembles the famous solitary wave in open flows, it has symmetric form, constant velocity and it somehow preserves its form for relative time before extinction under precession effects. Using the classical perturbation methods and the potential flow approximation (irrotational conditions) a new KdV equation is derived, this equation has novel azimuthal dependency, with coefficients include the rotation effects, and it has forcing term that has tilt effect, this term prohibited me from solving it analytically directly, so I used simple finite difference scheme, and compared it with experimental observations, where very good match noticed.

Publications Related To this Thesis:

1. ALSHOEFI, E. H. 2021. KdV Equation Model in Open Cylindrical Channel under Precession. *Journal of Nonlinear Mathematical Physics*. (**Major Revision/Under Review**).

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A New KdV Model to Describe the Solitary Wave in the Channel Under Shear Conditions, New Rayleigh and Orr-Sommerfeld equations, and New Burns Condition (*Thesis2*)

I have proved the existence of solitary wave in the rotating channel flume theoretically with the existence of shear conditions where it turns out that the shear does not affect the wave form when included, I have proved the instability induced by the shear effect using the new Rayleigh equation. (Chapter 4.3)

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Other version of KdV equation was derived this time under rotational conditions and by including the shear effect into the system of equations, the resultant equation has coefficients that depend on the shear velocity that was extracted experimentally using ADV measurements. Using the perturbation methods, the zeroth order of the problem led to solution of the unknown quantities in terms of the new derived Burns condition, and the nonlinear solution to get the KdV was carried out based on this zeroth order where complex novel KdV equation was

derived. Also, it did not have any analytical solution but this time I used different numerical method which is Fourier Transformation for space discretisation and two step leapfrog method for time. I proved that the shear effect has no impact on the wave form. I solved Burns condition, and it turned out that the disturbance velocity always has two values with respect to the bottom one always bigger than the maximum velocity (at the free surface) and the other is smaller than the minimum one (the slip velocity). I also derived new models for Rayleigh and Orr-Sommerfeld equations, and I discussed the instability criteria using Rayleigh equation, the disturbance either increase or vanish depending on the sign of the exponential function which was the solution for the equation at specific time and distance as both new equations have Coriolis force components that are time-space dependent, thus I fixed the time and space and then solved the problem accordingly.

Publications Related To this Thesis:

1. ALSHOUFI, E. H. 2021. Shear Flow in Cylindrical Open Channel Under Precession. *Journal of Periodica Polytechnica Civil Engineering*. **(Major Revision/Under Review)**

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A New Experimental Tool to Calibrate the Pictures of the CCD Camera Used in the Experiments (*Thesis3*)

I implemented a new program that is able to calibrate the pictures extracted from the CCD camera used in the laboratory. (Chapter 2)

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As I used CCD camera as visualization technique to get informative information of the different waves, I had to calibrate the pictures as they suffer from radial and tangential distortion, so I could not get right geometrical results without calibration. I wrote a new Python program to calibrate the pictures based on Zhang (2000) method, I derived many matrices to include the distortion effects, and solved the problem accordingly. I used object calibration process based on checkerboard with known geometry and conducted several experimental campaigns using two types of checkerboard, to find the proper set of pictures needed for calibration, so that all the pictures from the experiment can be calibrated based on those set of pictures.

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The Linear Irrotational Theory is Proved (*Thesis4*)

I proved that the linear irrotational theory is valid and applicable in systems that suffer from strong rotation. (Chapter 3)

Although the system under study suffers from rotation but the treatment of the linear part followed the classical irrotational assumption, this was based on the fact that the channel under

atmospheric pressure where the pressure is assumed negligible, and the velocity components where replaced by the potential function. A new dispersion equation was derived to compute the frequencies according to the corresponding wavenumber and the results showed good match for the lowest order of resonance. I conducted many experiments to get information about the azimuthal flow using the Acoustic Doppler Velocimeter, with respect to the control parameters, where I compared the extracted results with the results extracted under closed conditions and it turns that whether free or closed surface the rotating effect is the same on the velocity components, and the free surface results pour in favor of the closed cases. I studied the instability effect on the waves based on the experimental observations only where the nonlinear ones suffered from breaking and dispersion, and the linear ones suffered from bore formation. I prepared two schemes on the instability based on Rossby, Reynolds, and Strouhal numbers.

Publications Related To this Thesis:

1. ALSHOUFI, H. E. 2021. On the forced oscillations in a precessing open cylindrical channel. *Journal of AIP Advances*. vol. **11**, pp. 3-23. **(ACCEPTED)**.
2. ALSHOUFI, E. HAJAR. 2021. Fluid Contained Open Cylindrical Channel Under Precession. *Journal of Fluid Mechanics*. **(Under Review)**.
3. ALSHOUFI, HAJAR. Solution for Shallow Inertial Waves in Open Cylindrical Channel Under Precession. *Journal of Hidrológiai Közlöny*. **(ACCEPTED)**

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Bore Problem (New Characteristics and New KdV model) (Thesis5)

I derived the new characteristics and the new conjugate depth relationships under precession conditions, I also solved the undular bore problem by introducing new KdV model (Chapter 5).

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The system under study has the tilt mechanism that provides the level difference along the channel, if the tilt is big enough the free surface appears as there is a level difference or threshold, the connection between those two levels takes many forms, one of them is totally breaking area in the turbulent plain bore form, but when this tilt level is sensible it appears as smooth undulations in the form of undular bore, and finally it can be breaking undular one. Based on these observations I proposed the corresponding shock model and new characteristic form is derived, the relationship that connects between the conjugate depths is derived as well. Finally, I solved the case of undular bore numerically using the finite volume method.

Publications Related To this Thesis:

2. ALSHOUFI, E. HAJAR. Bore Problem in Open Cylindrical Channel Under Precession. *Journal of Fluid Mechanics*. **(Under Review)**.

Conference Abstract.

1. ALSHOUFI, H. E. 2021. Wavy Aspects in a Precessing Open Cylindrical Channel. *14th Chaotic Modeling and Simulation International Conference*. Greece, Athens, 8 - 11 June 2021.

4. Directions for Future Research

All the study in the present work can be used by the hydraulic engineers in many aspects like studying the ship waves under precession and studying shock waves problems generated by floods and the hydraulic jump for instance. The precession mechanism as mentioned is desirable in wave triggering whether in open or closed flows, and it has many mechanical applications like in combustion chambers and reactors. The work until hour yet has not been completed, and many issues related to the work should be fulfilled, precisely the instability problem, which is related to the turbulent effects and the breaking of Kelvin modes extracted from the theory, this need understanding to vorticity and solving Navier Stokes equations. The irrotational case has proved its validity, but one should study the problem based on the rotational case, by including the effect of Coriolis, Centrifugal and Euler forces into consideration. Other thing related to the nonlinear part particularly the Solitary wave and Boussinesq models which are derived already but I did not solve them yet, in order to learn more numerical methods and apply them like the spectral methods. In general, I can say that this new idea and new channel system is very rich and full of different topics that mix between the open channel flow and forced conditions which is interesting and deserves further work and time to be fulfilled.

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