

# Development of a transport model to study relativistic heavy-ion reactions

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## Introduction

In high-energy heavy-ion reactions, nuclei deprived of their outer electrons collide with each other on relativistic energies, which can range from a few hundred MeV to the TeV range. During these collisions a very dense and/or high temperature matter could be formed, which can provide valuable information on the properties of fundamental interactions, such as the phase diagram of the strongly interacting matter and the properties of the phase transitions. Such and similar colliding experiments are typically performed in high-energy accelerator complexes, such as the LHC in Switzerland or the RHIC in the United States. The importance of heavy ion research is also indicated by the fact that several heavy-ion colliding experiments are currently under construction, which are expected to become operational in the near future. For us, the most important of these, are the FAIR accelerator complex in Germany, where we will be able to use antiproton beams in addition to the usual heavy-ions, and the NICA complex in Russia, where proton, deuteron and various heavy-ion beams will be available.

In this dissertation, I study the structure of the strongly interacting matter at a few GeV bombarding energy in antiproton-nucleus collisions, in which the magnitude of the gluon condensate at a given density can be deduced from the mass shifts of the charmonium states <sup>1</sup>  $J/\Psi$ ,  $\Psi(3686)$  and  $\Psi(3770)$ , respectively. The vacuum and in-medium values of the different condensates play an important role in the study of certain properties of hadrons, such as their mass or decay width, which can be estimated, for example, by QCD sum rules [Cohen94]. The vacuum values of the most important low dimensional condensates e.g. quark condensate, gluon condensate, are well-known, however at finite density only theoretical estimates are available [Cohen91]. The study of the gluon condensate in the high density region is an important task in modern physics, as in addition to its role in the modification of the masses of some hadrons, it could also play a huge role in the physics of quark confinement as well [Nielsen82]. It is known from theoretical considerations that some vector mesons, such as the aforementioned charmonium states, suffer from mass modifications and changes in their decay widths in the dense medium, where the mass shifts should be proportional to the expected value of the gluon condensate at finite density in the dense medium [Morita12]. This fact provides a good opportunity to estimate the value of the gluon condensate by measuring the mass shifts in high-energy heavy-ion collisions, where a very dense matter could be formed (in some cases greater than the normal nuclear density). The actual density, which is formed during a collisions cannot be determined by direct measurements, but requires theoretical considerations.

A widely used method for modeling heavy-ion collisions is the so-called non-equilibrium transport method [Buss12], in which the time evolution of the system can be investigated, typically by numerical methods. The transport method also makes it possible to study quantities, like particle spectra, momentum- and energy distributions, density profiles, cross-sections, fluctuations, and many other properties of the colliding system. The most well-known transport codes used in heavy-ion physics today, are the Ultrarelati-

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<sup>1</sup>Bound states of  $c$  and  $\bar{c}$  quarks.

vistic Quantum Molecular Dynamics model (UrQMD), the Dubna Cascade model, the Parton Hadron String Dynamic model (PHSD), and the Boltzmann-Uehling-Uhlenbeck model. These models differ from each other in the applied energy ranges, in the used cross-sections, the numerical methods, or in the simulation methods used to solve the transport equations.

Some important input parameters of the transport codes are the elementary hadronic cross-sections, which ideally could be determined from measurements, however in many cases, we are interested in such exotic processes, where the cross-sections are not well-known. In these cases the cross-sections could be determined by using e.g. low-energy effective models, statistical models, partonic models etc. which can be incorporated into transport models to examine the desired processes. Such processes, which we are interested in this dissertation, are the inclusive and exclusive charmonium production processes in proton-proton, pion-proton, and proton-antiproton collisions at  $\sqrt{s} \approx 1 - 10$  GeV energies, or the production of the  $X(3872)$  possible tetraquark state in low energy collisions. In both cases only a few measured points are available so it is important to estimate their cross-sections at the energies, we are interested in.

While for the charmonium processes there are some measured values at  $\sqrt{s} \geq 10$  GeV energies, on the basis of which meaningful low-energy extrapolations can be made [Lynn07], for the tetraquarks, at the time of this dissertation, only very few measurements are available with relatively large uncertainties and at very high energies at the TeV range [Chatrchyan13]. The mentioned  $X(3872)$  state is also of particular interest because it does not fit into the conventional meson/barion picture, based on its measured quantum numbers and other properties determined from the measurements. From theoretical considerations it could be a  $[uc\bar{u}\bar{c}]$  tetraquark state, a loosely bound  $D^0\bar{D}^{*0}$  meson-meson molecule state, a  $[uc][\bar{u}\bar{c}]$  diquark-antidiquark state, or a  $c\bar{c}g$  quark-gluon hybrid state. However, the actual structure is not yet known, as a result the theoretical and experimental study of particles like this, is a very interesting field of research in today's modern physics. One possible way to determine the structure of the  $X(3872)$  particle is to use heavy-ion collisions, in which its composition can be deduced from the behavior of the given state in dense matter. This could be possible due to the fact that the dissociation cross-sections strongly depend on the size of the given state, so that a loosely-bound molecular state may disappear in a greater extent in the dense medium due to scattering with nucleons, than a more tightly bound diquark-anti-quark state, thus their structure could be deduced from the particle multiplicities examined at the end of the collisions [Cho13]. Of course, this requires the creational cross-sections for the tetraquark states in the different configurations, for which, in the absence of measurements, we can currently only make theoretical estimates, thus, in the research of such exotic particles, it is currently particularly important to compare the properties calculated by the different models when designing experiments that may be able to measure the individual properties of such states.

## Objectives

The primary goal of the dissertation is to investigate the behavior of the charmonium states (bound state of a heavy  $c$ -quark and a  $\bar{c}$  antiquark) in antiproton-induced (antiproton + nucleus) reactions. According to theoretical calculations, the mass shift of these states in the medium provides information on the expectation value of the gluon condensate at a given density. Measurements of these kinds are currently do not exist, so my primary goal is to provide an estimate of whether the mass shifts determined from the theory can be measured at the FAIR and NICA accelerators, which are currently under construction. Among them, I am particularly interested in the PANDA experiment at the FAIR complex, where they will collide antiproton beams with heavy nuclei, such as gold or lead, at few GeV center-of-mass energies. To model the collision system, I use an "off-shell" Boltzmann-Uehling-Uhlenbeck type transport code, which we have developed, which is also able to describe the correct dynamics of particles with finite width without compromising energy conservation.

Among the inputs of the transport code, the elementary hadron-hadron collisional cross-sections play a very important role, during which the examined  $J/\Psi$ ,  $\Psi(3686)$  and  $\Psi(3770)$  particles can be formed. Since there are no low energy measurements for these reactions, I have to make the estimations. In order to determine the necessary cross-sections, I aimed to develop a statistical based model, for which it was important to make its uncertainty analysis and determine its possibilities to describe exclusive and inclusive reactions. At low energies, I constructed a Monte-Carlo code that can study processes where many ( $m > 3$ ) particles can appear in the final-state and the analytical calculations would be cumbersome. Such processes are best studied in proton-antiproton annihilation at rest, where there are many measurements available for multi-pion processes, so the model results could be compared to the measurements directly.

For the investigations of the mass modifications in heavy-ion collisions it was important to determine the charmonium cross-sections in proton-proton, pion-proton, and proton-antiproton collisions, where in the first two cases there is a possibility of comparison between the model and the experiments at  $\sqrt{s} \approx 10 - 20$  GeV. Similarly to the charmonium case it was also important to determine the inclusive production cross-sections for the so-called bottomonium states (bound states of  $b$  and  $\bar{b}$  quarks), which will have a greater role in our future research.

In addition to the usual mesonic and baryonic cross-sections, I aimed to estimate the inclusive production cross-sections of the  $X(3872)$  possible tetraquark state in low energy proton-proton, pion-proton, and proton-antiproton reactions, which results will also be of great significance in our future research. With the cross-sections calculated here, we will be able to perform heavy-ion transport simulations, which may be able to provide information about the actual structure of the  $X(3872)$  particle.

After the determination of the elementary cross-sections, it became possible to study the dense medium behavior of the  $J/\Psi$ ,  $\Psi(3686)$  and  $\Psi(3770)$  charmonium states, during heavy-ion collisions with the BUU transport model. The primary aim was to investigate the non-equilibrium dynamical behavior of antiproton-gold collisions at  $E_k = 6 - 9$

GeV laboratory kinetic energies, where I investigated the mass shifts of the charmonium states in the dense medium, and proposed an experimental possibility to measure the mass modifications.

## Examination methods

During the dissertation I used the MATLAB programming language for most of the basic calculations and to create the diagrams. I also used the MATLAB environment to calculate the exclusive-, and inclusive cross-sections, and to create the Monte-Carlo code of the statistical model. The Boltzmann-Uehling-Uhlenbeck non-equilibrium transport code used for transport calculations was written in FORTRAN by György Wolf, in which I participated in the implementation of the code parts required for the charmonium calculations, running the simulations and analyzing the results.

## New scientific results (Theses)

1.

I developed a statistically based model, for the determination of low-energy (few GeV center-of-mass energy) elementary hadron-hadron collisions, which differs from the methods currently used in the literature in several respects based on the applied principles. While the usual statistical models estimate the probabilities of each process from the simple  $n$ -particle phase-space integrals, typically at higher energies, the model proposed here, estimates the exclusive, and inclusive cross-sections with the help of a cascade of fireball processes, the parameters of the resonances, the density of states, and the quark content of the hadrons through quark-combinatorial factors, even at low energies.

With the model, I determined a number of exclusive cross-sections, for which in many cases there is only a limited number of measurement data. In the case of low final-state particle multiplicities, I gave a method for performing analytical calculations, and also created a numerical Monte-Carlo code, which made possible to describe the probabilities of much more complex, multiparticle final-states as well. I have successfully applied the numerical method to describe the probabilities of multiple pionic final-states and the pion number distribution in proton-antiproton annihilation at rest. The results of the model calculations are within the margin of error compared to the measurement results. A description of the basics of the statistical model with the mentioned calculations is given in publication [P1].

2.

I estimated the relative error distribution of the statistical model, formulated to describe the cross-sections of the elementary hadronic collisions, with the comparison of the model calculations and the measurements. From the known error distribution I determined the uncertainty of the model in the case of exclusive and inclusive reactions. I have given

a method for the easier calculation of the inclusive cross-sections, which avoids the summation of all possible processes, so the calculation of the inclusive cross-sections become numerically manageable. With the method formulated to calculate the inclusive sums I determined the cross-sections of the  $p\pi^- \rightarrow \rho^0 X$ ,  $pp \rightarrow \rho^0 X$ ,  $p\bar{p} \rightarrow \rho^0 X$ ,  $p\pi^- \rightarrow K^0 X$ ,  $p\pi^- \rightarrow K^*(892)^+ X$ , and  $p\pi^- \rightarrow K^*(892)^- X$  inclusive processes at a relatively wide energy range, where only a few measurement points are available. The extended version of the statistical model and the calculations of the inclusive processes, with the model uncertainty estimation is summarized in [P2].

### 3.

I fitted the  $c$  and  $b$  quark-creational probabilities to the few existing low-energy charmonium and bottomonium cross-sections, and estimated several inclusive charmonium and bottomonium production cross-sections in proton-proton, pion-proton, and proton-antiproton collisions from a few GeV center-of-mass energies up to a few ten's of GeV's. For the charmonium states I determined the ratios of direct  $J/\Psi$  production cross-sections to the cross-sections of the higher lying  $\chi_{c1}$ ,  $\chi_{c2}$ , and  $\Psi(3686)$  states, which is also an additional validation of the statistical model. Using the fitted parameters, I made further estimates for the inclusive D-meson production probabilities in proton-proton, pion-proton, and proton-antiproton collisions. An extended version of the statistical model, including  $c$  and  $b$  quarks, with the charmonium and bottomonium cross-section calculations is summarized in [P3].

### 4.

Using the statistical model, I made estimations to the inclusive production cross-sections of the  $X(3872)$  possible tetraquark state in proton-proton, pion-proton, and proton-antiproton collisions at  $\sqrt{s} \approx 1 - 10$  energies, with the assumption that the  $X(3872)$  is a diquark  $[uc]$  - antidiquark  $[\bar{u}\bar{c}]$  bound state in the triplet-antitriplet representation. As a validation step, I compared the model calculations with an available measurement point in proton-proton collisions at  $\sqrt{s} = 7$  TeV energy, in which I determined the high energy ratio of the inclusive production cross-sections of the  $X(3872)$  and  $\Psi(3686)$  particles, with varied triplet-antitriplet, and sextet-antisextet color configuration probabilities. These calculations are described in detail in [P4].

### 5.

With the Boltzmann-Uehling-Uhlenbeck-based transport code, we examined the mass modification of the  $J/\Psi$ ,  $\Psi(3686)$  and  $\Psi(3770)$  vector mesons in the dense matter, formed in  $\bar{p} + Au$  collisions at  $E_k = 6 - 9$  GeV labor kinetic energies, where we proposed the possibility of measuring the mass shifts in the planned PANDA/FAIR experiment. In the calculations, we concluded that in antiproton-induced reactions at few GeV energies, the mass shift of the  $\Psi(3686)$  particle could be detected by examining the final-state dilepton spectrum ( $e^-e^+$ ). As a direct application of the proposed experiment, it will

be possible to determine the expectation value of the gluon condensate at the densities generated in antiproton-induced reactions, which at the proposed energies, corresponds approximately to the normal nuclear density. The results of the transport simulations are summarized in [P5,P6,P7].

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[O1] G. Balassa, P. Rogolino, Á. Rieth, R. Kovács, "New perspectives for modelling ballistic-diffusive heat conduction", *Cont. Mech. Therm.* **33**, 2007 (2021).

[O2] Gy. Wolf, G. Balassa, P. Kovács, M. Zétényi, Su Houng Lee, "Observation of Charmonium Mass Modification in Proton Induced Reactions", *JPS Conf. Proc.* **26**, 024003 (2019).