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# Neutron Multiplicity Counting with the Analysis of Continuous Detector Signals

PhD Thesis Booklet

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# 1 Background

Neutron multiplicity counting is a widely used non-destructive assay method for measuring the mass of fissile materials (primarily plutonium) [Langner, 1998]. The goal in a measurement is to estimate the singles, doubles and triples detection rates which quantify the intensity of detecting one, two or three neutrons, respectively, from the same emission event. The sample mass can then be obtained from the detection rates with algebraic inversion. Traditionally the measurement is performed with  $^3\text{He}$ -gas-filled detectors and the detection rates are determined from their pulse counting statistics. This approach comes, however, with several practical difficulties. The major problem is caused by dead time originating from the overlapping of pulses, which leads to a loss of counts in the counting electronics. Although various dead-time correction techniques are available [Croft, 2012], their use is limited to moderate count rates; at very high count rates, for example when measuring spent nuclear fuel [Reilly, 1991], they break down completely. Another (purely technical) problem is the global shortage on  $^3\text{He}$ , which is a key component of the neutron detectors used in the measurements [Henzlova, 2015].

## 2 Objectives

To overcome the above problems, an alternative version of neutron multiplicity counting has been developed. Relying on a recently published stochastic model of continuous detector signals [Pál, 2014], the new method is based on the direct analysis of the continuous voltage signals of neutron detectors (primarily fission chambers). In applying the method, the values of the singles, doubles and triples detection rates are obtained from suitably chosen moments (cumulants) of the measured signals. Since this procedure does not rely on counting individual pulses, it is insensitive to their overlapping. As a result, the detection rates with the new method can be determined even at very high count rates, which makes it a viable alternative to traditional multiplicity counting, especially when measuring high intensity samples, like spent nuclear fuel.

## 3 Methods

The theoretical foundation of the proposed method is formed by a set of equations that relate the appropriate moments of the continuous detector signals with the singles, doubles and triples detection rates. The expressions of the moments are derived in a master equation formalism. The derivation starts with a backward master equation for the probability density function of the signal. Through lengthy mathematical transformations, the cumulant generating function is obtained in a closed analytical form. The expressions of the cumulants are then obtained with simple differentiation of this function. The evolution of the theoretical model went through three stages. In the first stage, only the so-called one-point (in time) moments (including the mean, and covariance) of the signals have been calculated by assuming that emitted neutrons are detected instantly. In the second stage, the same moments have been recalculated

by assuming that every neutron is detected with a random time delay with respect to its emission. In the third stage, while maintaining the assumption of the delayed detection, certain two- and three-point (in time) moments (including the covariance and bicovariance functions) of the signals are calculated as well. Finally, formulas are presented that express the singles, doubles and triples detection rates with moments estimated from the measured signals of a large detector array and that can directly be used in analysing measurements.

A computational study has been performed to investigate the properties of the new method. In order to provide a unified way of storing and transferring continuous detector signals (simulated and measured ones as well) on a computer, a dedicated file format has been defined. A program has been written for simulating detector signals in a multiplicity counting measurement. Another program has been created that can estimate the moments of recorded detector signals. The behaviour of the two programs have been verified by simulating a set of signals and comparing the recovered detection rates with analytical reference values. Then a large set of signals have been simulated and analyzed in order to assess the impact of various parameters (the measurement time, the detection efficiency, the electronic noise as well as the non-neutron pulses) on the estimated detection rates. The sensitivity of the traditional and the new method of multiplicity counting to dead time losses is compared by simulating measurements at different sample emission intensities and estimating the detection rates both from the moments of the continuous signals as well as with the pulse counting approach.

In order to demonstrate the practical use of the proposed method a measurement set-up, containing four fission chambers, a  $^{252}\text{Cf}$  source and  $^{235}\text{U}$  plates, has been designed and built at the Kyoto University Critical Assembly (KUCA) facility. Monte Carlo simulations were performed to optimize the arrangement of the elements of the set-up. An FPGA based fast data acquisition system was assembled to record the voltage signals of the detectors. Two sets of 14 hour-long signals have been recorded in two different measurement configurations. The moments of the signals were estimated, from which the singles and doubles detection rates could be retrieved. The triples rates were not determined due to the low detection efficiency of the measurement set-up as well as to the limitations of the data acquisition system. The detection rates were also estimated with the traditional pulse counting approach using a dedicated single channel analyser. The applicability of the newly proposed method was assessed by comparing these two sets of detection rates.

## 4 Results

The new scientific results presented in the thesis can be summarized in the form of the following propositions:

**Proposition 1.** I have upgraded the preliminary theory of a new form of neutron multiplicity counting, which is based on estimating the one-point (in time) moments of continuous detector signals. While the original model assumed that neutrons are detected instantly after their emission, I have introduced a random time delay before the detection to account for the migration of neutrons in the system. I have derived expressions for several one-point moments (including the mean value and the covariance) using a master equation formalism. Based on analytic considerations, I have shown that the singles, doubles and triples rates

in this upgraded model can be recovered from the one-point moments of the signals in fast detection systems, where the average time delay is short compared to the length of a typical voltage impulse, hence pulses generated by the same emission event tend to overlap in time. At the same time I have shown that in thermal detection systems, which are predominantly used for measurements and in which the average time delay is much larger than the length of a typical voltage impulse, the information on the doubles and triples rates vanishes from the one-point moments of the signal, because pulses generated by the same emission event are dispersed in time. As a result, in such systems only the singles rate can be retrieved [P1, P2, P3, P8].

**Proposition 2.** I have further extended the theory described in Proposition 1. Besides using the one-point moments of the detector signals, I have introduced the use of their two- and three-point (in time) moments as well, because they are able describe the temporal correlations between different points of the signals. I have derived expressions for several two- and three-point moments (including the covariance function and the bicovariance function) using a master equation formalism. I have calculated the integrals of these moments with respect to their parameters and I have shown that their values (with one exception) are independent of the time delay, because the integration process brings temporally dispersed pulses generated by the same emission event into overlap. As a consequence, I was able to show based on analytic considerations that – contrary to the one-point moments – the information on the doubles and triples rates does not vanish from the integrals of the two- and three-point moments, even when the average time delay becomes large compared to the length of the voltage pulses. As a result, using the two- and three-point moments the doubles and triples rates can be recovered even in thermal detection systems. [P4, P8]

**Proposition 3.** I have established a practical procedure for extracting the singles, doubles and triples detection rates from the recorded continuous signals of a large array of detectors. I have designed a numerical algorithm for estimating the one-, two- and three-point (in time) moments, described in Propositions 1 and 2, of the recorded continuous signals in an efficient way. Using the expressions of these moments provided by the theoretical models of Propositions 1 and 2, I have derived a set of equations that express the singles, doubles and triples detection rates with the moments estimated from all possible combinations of the signals, thus fully utilizing the information available in them. By using this procedure to analyse simulated detector signals, I have shown that it is capable of producing estimates for the singles and doubles detection rates and that these estimates agree with predictions from the theory. The ability to recover the triples rate could not be confirmed due to the numerical instability of the estimator. [P5, P6, P7, P9, P10, P11]

**Proposition 4.** By analysing a large number of simulated measurements using the procedure described in Proposition 3, I have investigated the effect of the following parameters on the value and/or accuracy of the estimated detection rates: the measurement time, the detector efficiency, the amplitude of an electronic white noise as well as the presence of small-amplitude parasitic pulses induced

by a non-neutron source with Poisson statistics. I have shown that the measurement time and the detection efficiency have a strong impact on both the value and the relative uncertainty of the singles and doubles estimates; the effect of the electronic noise on both parameters is practically negligible; non-neutron pulses do not affect the doubles rate, but introduce a bias into the singles estimate which can, however, be easily corrected for. Finally, I have demonstrated that while the traditional pulse counting approach underestimates the values of the singles and doubles rates at large count rates due to dead time losses, values provided by the new method agree with predictions from the theory. As a result, the new method is a viable alternative of the traditional procedure. [P10, P11]

**Proposition 5.** In order to demonstrate the practical use of the new method of neutron multiplicity counting specified in Propositions 1 and 2, I have designed and analysed a measurement. Among other elements, the measurement set-up contained four thermal fission chambers and a spontaneously fissioning  $^{252}\text{Cf}$  sample. I have performed Monte Carlo simulations to find an optimal arrangement that provides the highest detection rates. Using the procedure described in Proposition 3, I have analysed the measured detector signals to estimate the singles and doubles rates; the triples rate was not determined due to the low detection efficiency of the set-up as well as to the hardware limitations of the data acquisition system. Reference values of the detection rates were obtained from the same measurement using the traditional pulse counting approach. By comparing the two sets of values, I have shown that the singles rate received from the analysis of continuous signals slightly overestimates the reference values due to pulses generated by the inherent alpha background of the fission chambers. On the other hand, the agreement in the case of the doubles rates is satisfactory. [P5, P6, P7, P9, P10, P11]

## 5 References

- [Langner, 1998] D. G. Langner, J. E. Stewart, M. M. Pickrell, M. S. Krick, N. Ensslin, and W. C. Harker. “Application guide to neutron multiplicity counting”. No. LA-13422-M. Los Alamos National Laboratory, Los Alamos, NM, 1998.
- [Croft, 2012] S. Croft and A. Favalli. “Dead time corrections for neutron multiplicity counting”. *Nuclear Inst. and Methods in Physics Research*, A 686 (2012): 115-116.
- [Reilly, 1991] D. Reilly, N. Ensslin, H. Smith Jr, and S. Kreiner. “Passive nondestructive assay of nuclear materials”. No. NUREG/CR-5550. Nuclear Regulatory Commission, 1991.
- [Henzlova, 2015] D. Henzlova, R. Kouzes, R. McElroy, P. Peerani, M. Aspinall, K. Baird, A. Bakel et al. “Current status of helium-3 alternative technologies for nuclear safeguards”. No. LA-UR-15-21201. Los Alamos National Lab.(LANL), Los Alamos, NM (United States), 2015.
- [Pál, 2014] L. Pál, I. Pázsit, and Zs. Elter. “Comments on the stochastic characteristics of fission chamber signals”. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 763 (2014): 44-52.

## 6 List of Publications

The work presented in the thesis is based on the following publications:

- [P1] I. Pázsit, L. Pál, and L. Nagy. “Multiplicity counting from fission chamber signals in the current mode”. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **839** (2016): 92-101.
- [P2] L. Nagy, I. Pázsit and L. Pál, “An extended theory of multiplicity counting from fission chamber signals in the current mode”. In: *International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering*, Jeju, Korea, April 16–20, 2017.
- [P3] L. Nagy, I. Pázsit and L. Pál. “Multiplicity counting from fission detector signals with time delay effects”. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **884** (2018): 119-127.
- [P4] L. Nagy, I. Pázsit and L. Pál. “Two- and three-point (in time) statistics of fission chamber signals for multiplicity counting with thermal neutrons”. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **929** (2019): 148-155.
- [P5] L. Nagy, I. Pázsit, L. Pál, G. Klujber and M. Szieberth, “Multiplicity counting using the two- and three point statistics of fission chamber signals – theory and experimental demonstration”. In: *International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering*, Portland, Oregon, USA, August 25–29, 2019.
- [P6] L. Nagy, I. Pázsit, L. Pál, G. Klujber and M. Szieberth, “Measurements and simulations to investigate the feasibility of neutron multiplicity counting in the current mode of fission chambers”. *EPJ Web of Conferences*. Vol. 225. EDP Sciences, 2020.
- [P7] L. Nagy, G. Klujber, I. Pázsit, I. Barth, Y. Kitamura, T. Misawa and M. Szieberth, “Experimental Demonstration of Neutron Multiplicity Counting in the Current Mode of Fission Chambers”. *INMM 61st Annual Meeting*, (Online), July 12–16, 2020.
- [P8] L. Nagy, Y. Kitamura, I. Pázsit and M. Szieberth, “New Paradigm in Neutron Fluctuation Analysis: Extracting the Statistics of Discrete Detection Events from Time-resolved Signals of Fission Chambers”. In: *International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering*, Raleigh, North Carolina, USA, October 3–7, 2021. (submitted).
- [P9] M. Szieberth, L. Nagy, G. Klujber, Y. Kitamura, T. Misawa, I. Barth and I. Pázsit, “Experimental Demonstration of Neutron Fluctuation Analysis Based on the Continuous Signal of Fission Chambers: Neutron Multiplicity and Reactor Noise Measurements”. In: *International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering*, Raleigh, North Carolina, USA, October 3–7, 2021. (submitted).

- [P10] L. Nagy, G. Klujber, Y. Kitamura, T. Misawa, I. Barth, I. Pázsit and M. Szieberth. “Computational Investigation and Experimental Verification of Multiplicity Counting from the Continuous Signals of Fission Chambers”. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*. (to be submitted)
- [P11] L. Nagy, G. Klujber, Y. Kitamura, T. Misawa, I. Barth, I. Pázsit and M. Szieberth, “A Computational and Experimental Investigation of Multiplicity Counting with Continuous Fission Chamber Signals”. *INMM and ESARDA Joint Annual Meeting*, (Online), August 21–26, 2021. (to be submitted)