Budapest University of Technology and Economics
Faculty of Electrical Engineering and Informatics
Department of Measurement and Information Systems

# PhD. Thesis Booklet

# Channel identification methods for complex-valued transmissions

Author:
Barna Csuka
Electrical Engineer M.Sc.

Supervisor:
Dr. Zsolt Kollár
Associate Professor

Budapest
2021

*Budapest University of Technology and Economics*
*Department of Measurement and Information Systems*

*H-1117 Budapest XI., Magyar Tudósok körútja 2., Building I*

*E-mail: csuka{at}mit.bme.hu*

# 1 Background and objectives

Nowadays, in the first quarter of the 21st century, one of the most researched field of the electrical engineering is the wireless communication based on electromagnetic waves. Firstly, during the 20th century, they had been used for audio broadcast, which was later expanded for video transmission as well. Since the Millennium, the telecommunication industry has been developing and growing exponentially, and now the fifth generation technology standard (5G) is the currently introduced technology, but the sixth generation (6G) is already envisioned and designed as well. The 5G networks are able to handle 10 times more devices than the forth generation (4G); therefore, it allows to connect sensors, cars, household devices to the network forming the Internet of Things (IoT). Furthermore, the high number of the connectable sensors and their secured communication protocols enables them to use these networks e.g. in healthcare or in industry leading to the ”Fourth Industrial Revolution” or the so called Industry 4.0.

Unfortunately, the wireless transmissions are less robust than the wired ones. In the later case, as long as the wire remains intact, usually only the superposed noise has to be taken into account during transmission. Controversy to that, if there is wireless connection between the devices, further distortions may appear as indirect transmission paths leading to multipath propagation, which can be modelled as follows. There are many paths between the transmitter and the receiver: a direct one and some others that are reflected. These paths have different lengths; thus, the propagation times of the waves are also different causing time delay between the waves at the receiver side. Furthermore, the attenuation of the paths is different; therefore, the received waves have various magnitudes. These phenomena can be described by an impulse response where the amplitude of the coefficients are proportional to the magnitudes, while the phases of the coefficients are proportional to the time delays.

Considering these circumstances, the transmission channel has to be determined to compensate the effects of the multipath propagation, which requires system identification performed generally as follows. The observed system is excited by a signal, which is commonly a composition of sine waves with random phases (so called multisine signal) or noise. Using these signals and measuring the system response, the frequency response can be estimated. An other approach is the time domain identification: in this case, applying a right exciting signal, the system impulse response can the observed. Having a wireless transmission for which the channel has to be identified, these conventional methods can not be applied any more. The distance between the transmitter and the receiver can grow up to hundreds of kilometers as well; thus, the excitation and the response of the system can not be observed at the same time. In this case, however, if the receiver knows the transmitted signal, then it can identify the transmission channel. Considering that the above mentioned signals have random parameters (noise, multisine signal), such sequences have be transferred, properties of which are similar to the previously mentioned signals. These sequences can be considered as pseudorandom signals, but they are predefined; therefore, the receiver knows the excitation; thus, channel identification can be performed.

In my thesis, two aspects of the channel estimation is investigated: firstly, the Golay sequences are introduced and analysed for telecommunication applications [1], then a suitable method for the channel estimation is presented in details: the effective evaluation of the Wiener filter in sliding window approach based on solution of Kraker [2, 3].

## 1.1 Channel estimation using Golay sequences

If a sequence $\mathcal{G}_A[n] = (\mathcal{G}_A[0], \mathcal{G}_A[1], \ldots, \mathcal{G}_A[N-1])$ is given with length of $N$, then its non-periodic auto-correlation function can be evaluated as follows

$$\mathcal{R}_A[n] = \sum_{i=0}^{N-1-n} \mathcal{G}_A[i] \cdot \mathcal{G}_A[i+n] = \mathcal{G}_A[n] * \mathcal{G}_A[n]. \tag{1}$$

Defining $\mathcal{G}_B$ similarly to (1), $\mathcal{G}_A$ and $\mathcal{G}_B$ are Golay complementary sequences if

$$\mathcal{R}_A[n] + \mathcal{R}_B[n] = \begin{cases} 2N & \text{for } n = 0, \\ 0 & \text{for } 0 < n < N. \end{cases} \tag{2}$$

Let a transfer channel is given having the following impulse response: $h_{\text{ch}}[n]$. Furthermore, let $x_{\text{tr}}[n]$ be a discrete transmitted signal. In this case, the received signal can be considered as

$$x_{\text{rec}}[n] = h_{\text{ch}}[n] * x_{\text{tr}}[n]. \tag{3}$$

To retrieve the correct transmitted information on the receiver side, the effects of the impulse response have to be estimated and compensated. Let us have a noiseless transmission, and if the transmitted signal $x_{\text{tr}}$ is a $\mathcal{G}_A$ sequence, then the received signal $x_{\text{rec}}$ can be expressed using (3) as follows

$$x_{\text{rec}}[n] = h_{\text{ch}}[n] * \mathcal{G}_A[n]. \tag{4}$$

If this signal is correlated by a Golay filter as (1) shows [4, 5], and thereafter, it is added to a consecutive $x_{\text{rec}}[n]$ when $\mathcal{G}_B$ is sent, the following result is given

$$y[n] = h_{\text{ch}}[n] * (\mathcal{G}_A[n] * \mathcal{G}_A[n] + \mathcal{G}_B[n] * \mathcal{G}_B[n]) = h_{\text{ch}}[n] * (\mathcal{R}_A[n] + \mathcal{R}_B[n]) \tag{5}$$

where the output of the filter is denoted by $y[n]$. Substituting (2) into (5), the impulse response of the transfer channel can be expressed as

$$h_{\text{ch}}[n] = \frac{y[n]}{2N}. \tag{6}$$

Instead of this time domain approach, I showed that this estimation can also be performed in frequency domain by evaluating the channel frequency response. For this calculation, longer blocks ($\mathcal{G}_U$ and $\mathcal{G}_V$) are constructed from $\mathcal{G}_A$ and $\mathcal{G}_B$ according to the directives of the IEEE 802.11ad standard [6]. Using $\mathcal{G}_U$, (4) can be generalized as follows

$$x_{U,rec}[n] = h_{\text{ch}}[n] * \mathcal{G}_U[n]. \tag{7}$$

If (7) is Fourier transformed, then the following equation is given after rearranging

$$H_{ch}[m] = \frac{X_{U,rec}[m]}{\mathcal{G}_U^{DFT}[m]}. \tag{8}$$

After performing the same calculations for $\mathcal{G}_V$ as in (7) and (8), it can be seen that two channel frequency response estimations are already given for $H_{ch}[m]$, and their average can be evaluated. This frequency domain estimation is a suitable solution because Fourier transform is necessary for modulation (e.g., OFDM-transmissions); thus, it is already present in the receiver, while the Golay filter is an extra module without any further usage.

A further advantage of the frequency domain estimation is the following. Similarly to the channel estimations, the compensations can also be performed either in time domain or in frequency domain. The time domain compensation is the backward operation of (3), which is the deconvolution. Considering that this operation suffers from instability issues, and it is currently an open research topic yet, how does it have to be performed to get stable and correct results; therefore, the compensation should be evaluated in frequency domain.

## 1.2 Efficient evaluation of sliding Wiener filter

The idea of the adaptive filtering is to tune the coefficients of a adaptive filter $\mathbf{h}[n]$; thus, the error between the output signal $\mathbf{y}[n]$ and the desired signal can be is minimised. This filter is modelled as a linear time-invariant (LTI) filter having a finite impulse response with $K$ coefficients. As a result, for an input $\mathbf{X}[n]$, the output signal of the adaptive filter can be expressed by a matrix multiplication in a given observation window containing $N$ samples

$$\mathbf{y}[n] = \mathbf{X}[n]\,\mathbf{h}[n]\,. \tag{9}$$

The optimal solution $\widehat{\mathbf{h}}_{\text{opt}}[n]$ for the filter coefficients $\mathbf{h}[n]$ – know as the Wiener-Hopf equation – can be given in least square sense in the following form [7]

$$\widehat{\mathbf{h}}_{\text{opt}}[n] = \mathbf{R}^{-1}[n]\,\mathbf{p}[n] \tag{10}$$

where $\mathbf{R}[n]$ is the auto-correlation matrix of the input signal while $\mathbf{p}[n]$ is the cross-correlation vector of the input and output signals.

The sliding window evaluation bases on an efficient algorithm for the calculation of the inverse of an updated auto-correlation matrix, $\mathbf{R}^{-1}[n+1]$. This method was firstly presented by Kraker. The submatrices of the correlation matrices $\mathbf{R}$ and their inverses $\mathbf{R}^{-1}$ have the following supplementary notations as Fig. 1 shows. Using them, the updated inverse of a auto-correlation matrix can be expressed as follows [2]

$$\mathbf{R}^{-1}[n+1] = \begin{pmatrix} b_{11,r}^{-1} & -b_{11,r}^{-1}\mathbf{b}_{21}^{T}\mathbf{B}_{22}^{-1} \\ -\mathbf{B}_{22}^{-1}\mathbf{b}_{21}b_{11,r}^{-1} & \mathbf{B}_{22}^{-1} + \mathbf{B}_{22}^{-1}\mathbf{b}_{21}b_{11,r}^{-1}\mathbf{b}_{12}^{T}\mathbf{B}_{22}^{-1} \end{pmatrix}. \tag{11}$$

Originally, the classic Gauss–Jordan elimination method is used for inversion having $\mathcal{O}(K^3)$ complexity. Currently, the Coppersmith–Winograd-algorithm – and its slightly optimised versions – gives the lowest complexity for inversion of an $K \times K$ matrix with complexity $\mathcal{O}(K^{2.376})$. Controversy to that, the complexity of this sliding window evaluation is deduced, and it is only squarely proportional to $K$; thus, $\mathcal{O}(K^2)$ complexity can be reached. Furthermore, the exact computational complexity of the algorithm can be further reduced by using the fact that an auto-correlation matrix is symmetric; therefore, it can be satisfactory to perform the calculations only over on the half of the matrix.

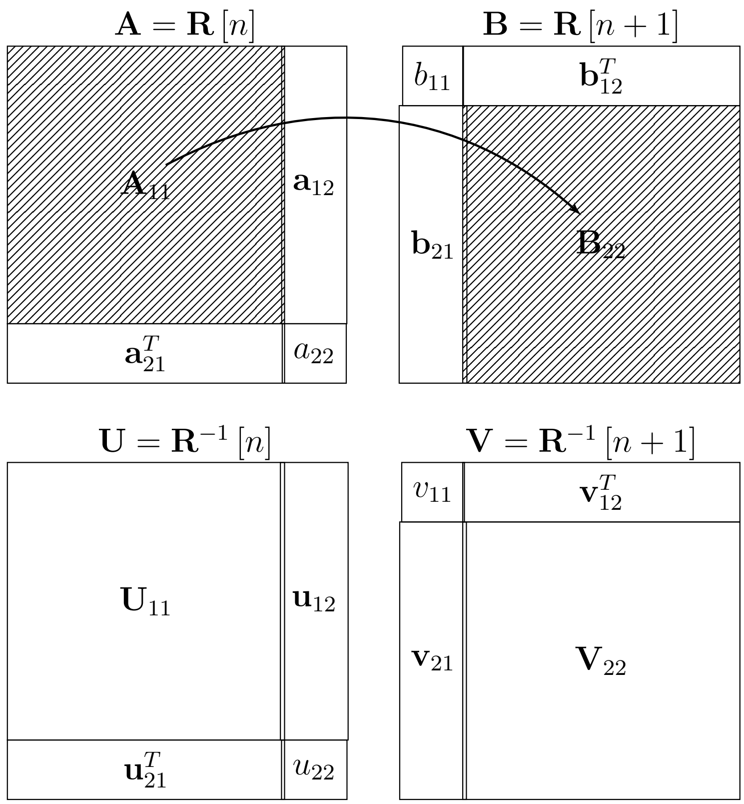


Figure 1: Structure of the correlation matrices $\mathbf{R}$ and their inverses $\mathbf{R}^{-1}$

## 1.3 Complex-valued algorithms for identification

Nowadays, the significance of the IQ modulation increases due to the demand for reaching higher data rate and better spectral efficiency. Having two independent signal sources, they can be superposed into a single wave using cosine and sine wave as carriers. If the signal sources are considered as real and imaginary part of a complex number, then this model can be described by complex-valued arithmetic. In this thesis, the conventional channel identification methods are extended to handle complex-valued signals as well.

Applying these extended algorithms, the computational time becomes more crucial because the complex-valued computations have to be evaluated by real-valued arithmetic units, which requires longer runtime. Therefore, the increased complexity is also analysed; furthermore it is showed that they have the same order of complexity like the real-valued computations.

# 2 Investigation methods

The prime objective of my research was to investigate and implement the procedures that were presented in the previous chapter and to elaborate the related digital signal processing methods.

## 2.1 Simulation Environment

During my research years, I created a software packet in MATLAB containing a simulation environment to investigate all of the algorithms and solutions that were used (or at least mentioned) in my thesis. This packet was built up from different modules to demonstrate the operation of a transmission chain from the data generation to the last data processing on the receiver side. Compared the sent and the received data and signals, the different quality indicators can also be computed and investigated. The functionality of my software can be separated into three different main parts as the following listing presents.

- To investigate the Golay sequences, I implemented the single carrier mode of the IEEE 802.11ad standard with a transmitter, a transfer channel simulator, and a receiver.

- I implemented the design of an adaptive filter based on the Wiener-Hopf equation using both the conventional methods and the sliding window evaluation.

- However, it is not introduced in the thesis, for future research, I started to implement a quantisation tool to simulate the behaviour of e.g., 8-bits, 16-bits architectures. By using this part, it can be investigated, how do the different algorithms work if only a limited numeric precision is applied.

# 3 New scientific results – Theses

**Thesis I: Extension of the channel estimation algorithms based on Golay sequences**

I investigated the Golay sequences, which are special sequences used for channel estimation in data transmission systems. I proved that the original time domain method can be used for complex-valued channels as well by modifying the coefficients of the Golay filter. Furthermore, I showed that this approach can be extended for four blocks in the case of the IEEE 802.11ad channel estimation field. As a new method, I introduced a frequency domain approach for channel estimation based on Golay sequences using the Fourier transform. I also showed that the proposed method achieves similar bit error ratio as the conventional, time domain approach if it is applied in continuous transmissions.

**Sub–thesis I–1:** I specified complex-valued coefficients for Golay filter to perform the time domain channel estimation in the case of IQ signals.

**Sub–thesis I–2:** I showed that the channel estimation for 802.11ad standard can be evaluated as the average of four pairs of blocks if the channel impulse response is shorter than the applied Golay sequence.

**Sub–thesis I–3:** I introduced a frequency domain approach for channel estimation. I also showed that it achieves the same performance as the time domain method if continuous transmission is applied.

(Please refer to the following sections in the thesis: Section 2.3.2, Section 2.3.3, and Section 2.4.)

The results presented in this thesis were published in: J1, J2, C1, C2, C3, C4.

**Thesis II: Optimisation of the RSC4BI-algorithm**

I investigated the RSC4BI-algorithm presented by A. Kraker, which is used to evaluate the Wiener-Hopf equation in a sliding window manner. I proved that this new method has a lower computational complexity than the conventional inversion calculation; therefore, it is a preferable solution for a sliding Wiener filter. I derived and compared the two calculation methods for RSC4BI based on the dyadic product and based on the permutation. Furthermore, I decreased the computational complexity to its half by using the fact that the input matrix is always symmetric; therefore, it is satisfactory to evaluate only the upper triangle parts.

**Sub–thesis II–1:** I specified, which calculation method – dyadic or permutation – is more preferable for updating the auto-correlation matrix and the cross-correlation vector during evaluation of Wiener-Hopf equation. The preferable algorithm depends on the filter length $K$ and the sliding window size $N$.

**Sub–thesis II–2:** I proved that the computational complexity of the RSC4BI-algorithm is $\mathcal{O}\left(K^2\right)$, while the conventional inversion methods require at least $\mathcal{O}\left(K^{2.37286}\right)$ operations. I also implemented and verified the proposed algorithm; furthermore I compared its performance to the conventional methods using MATLAB and embedded hardware.

**Sub–thesis II–3:** I improved the RSC4BI-algorithm, and I showed that only the upper triangle parts are required for the calculations because the input matrix is symmetric.

Furthermore, I presented an improved algorithm, where the reuse of the already calculated variables can reduce the number of the required calculations.

(Please refer to the following sections in the thesis: Section 3.3 and Section 3.4.)

The results presented in this thesis were published in: J3.

**Thesis III: Extension of the RSC4BI-algorithm for complex-valued signals**

I generalized the RSC4BI-algorithm presented by A. Kraker to generate the Wiener Filter in a sliding window manner if complex-valued signals are applied. As it is known, an auto-correlation matrix and its inverse are always Hermitian. Using this fact, I removed the imaginary parts of the elements in the main diagonal of the inverse of the auto-correlation matrix as the last step of the inversion. This step stabilizes the algorithm, and it protects that against the accumulation of round-off errors of the imaginary parts. Furthermore, I investigated the complexity of the extended algorithm. I showed that it is squarely proportional to the filter length, even though the addition and the multiplication of two complex numbers require additional computations.

**Sub–thesis III–1:** I extended the algorithm of A. Kraker for complex-valued signals to evaluate the inverse of the auto-correlation matrix in a sliding window manner. I showed that the elements of the main diagonal in the inverse of the auto-correlation matrix are not purely real because of round-off errors of the algorithm. Therefore, these imaginary parts have to be removed to stabilize the algorithm.

**Sub–thesis III–2:** I proved that the complexity of the algorithm is squarely proportional to the filter length if complex-valued signals are considered.

(Please refer to the following sections in the thesis: Section 4.2 and Section 4.3.)

# 4 Utilisation of the results and outlook

It is very advantageous in 3G or 4G systems to apply Golay sequences for different parameter estimation purposes; furthermore, they can be a very promising candidates for 5G – and even for 6G – networks. During the design of network concepts, the following aspects can be taken into account if Golay sequences are used: easy generation, excellent auto- and cross-correlation properties, signal processing in time or in frequency domain, and opportunity to use time-distributed algorithms. The channel estimation based on complementary Golay sequences can be performed both in time domain and in frequency domain; furthermore, the incoming signals do not have to be limited to be real-valued because complex-valued data – IQ-transmissions – can be handled as well. Using Fourier transform for estimation, the already given hardware is applied for calculations; therefore, there is no need for further arithmetical units as in case of time domain estimation based on Golay filter.

Assuming that the transfer channel and its paths can be modelled by a FIR filter, the channel impulse response can be estimated by Wiener-Hopf equation as well. This solution requires the inversion of the auto-correlation matrix that is a crucial step of the procedure. For case of continuous evaluation, Kraker's method, the RSC4BI-algorithm can be efficiently applied in sliding window manner resulting $\mathcal{O}\left(K^2\right)$ complexity. Extending this algorithm to handle complex-valued signals, this procedure is a suitable and effective alternative for channel estimation of IQ-transmissions.

The investigations – that were performed in this thesis – are not complete, one major aspect was already not studied: the effects of the quantisation. Considering that our systems are almost always digital, the incoming signals have to be digitised by ADCs; therefore, they contain only certain values depending on the resolution. In the current available ADCs, the resolution is reciprocally proportional to the conversion time leading to that the high frequency signals can be digitised only by 8, 10 bits ADCs. Thus, it is going to be a very important study in the future, how high is the quantisation fault tolerance of the algorithms. If this tolerance is high enough, then the number of bits can be reduced until reaching an optimised value. This decreased resolution allows to increase the conversion speed while lower resolutions lead to cheaper ADCs.

# 5 Publications related to the PhD. thesis

## 5.1 Journal papers


J1 B. Csuka and Zs. Kollár. Software and Hardware Solutions for Channel Estimation based on Cyclic Golay Sequences. *Radioengineering*, 25(4):801–807, December 2016. Impact factor: 0.944

J2 B. Csuka and Zs. Kollár. R-DFT-based Parameter Estimation for WiGig. *Periodica Polytechnica - Electrical Engineering and Computer Science*, 61(2):224–230, May 2017

J3 A. Kraker, B. Csuka, and Zs. Kollár. Sliding Window Evaluation of the Wiener-Hopf Equation. *Radioengineering*, 29(2):365–375, June 2020. Impact factor: 1.077


## 5.2 Conference papers


C1 B. Csuka and Zs. Kollár. R-DFT-based Channel Estimation in 802.11ad Systems (Original title in Hungarian: R-DFT alapú csatornabecslés a 802.11ad rendszerekben). In *Mesterpróba 2015*, pages 8–14, 2015

C2 B. Csuka and Zs. Kollár. Parameter Estimation in 802.11ad Systems (Original title in Hungarian: Paraméterbecslés 802.11ad rendszerekben). In *HTE MediaNet 2015 Konferencia szemle: Diákszekció*, pages 8–14, 2015

C3 B. Csuka, I. Kollár, Zs. Kollár, and M. Kovács. Comparison of Signal Processing Methods for Calculating Point-by-point Discrete Fourier Transforms. In *26th International Conference Radioelektronika*, pages 52–55, 2017

C4 M. Kovács, B. Csuka, and Zs. Kollár. Effects of Quantization on Golay Sequence based Channel Estimation. In *27th International Conference Radioelektronika*, pages 52–55, 2017

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