FOUR PARAMETER SINE WAVE ESTIMATION IN FREQUENCY DOMAIN

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I. Introduction

Accurate characterization of analog-to-digital converters (ADC) is an important field of measurement technology. A widely used method for characterizing ADC's is the so called histogram test [1]. In this method first the ADC is excited with a sine wave input, then the integral nonlinearity (INL) and differential nonlinearity (DNL) can be identified from the number of samples in the code bins. Obviously, the parameters of the applied sine wave and the sampling frequency are important. The IEEE standard for ADC testing [2] defines that sampling should be coherent and the number of periods has to be relative prime to the number of samples, in order to achieve the most accurate results. However, the satisfaction of these conditions can only be checked from the measured signal, since both the frequency of the signal generator and of sampling have limited precision. With the increase of the number of samples to obtain more precise measurement of the characteristics of the device under test, even a small deviation from coherence in the sampling causes significant errors in the characterization with false INL and DNL values, and also an increase in the time required to execute the least-squares four parameter sine wave fit algorithm. In this paper an increased-speed algorithm is presented with the capability to decide if the test signal is suitable to test the ADC or not. In some cases the coherence condition can be assured with further preprocessing steps, this will also be studied. [1]

II. Background

A. Histogram Test

In the histogram test the ADC is excited with a sine wave input. The form of the input is

$$\begin{aligned} x[k] = A_1 \sin(\omega k + \varphi) + C = \\ A\cos(\omega k) + Bsin(\omega k) + C, \\ k = 1, 2, \dots N. \end{aligned} \tag{1}$$

Where $A_1 = \sqrt{A^2 + B^2}$ is the amplitude, ω is the angular frequency, C is the DC offset, and $\varphi = \arctan(B, A)$ is the initial phase, and N is the number of samples. Let $H[i]$ be the total number of samples received in the code bin i, and

$$H_c[k] = \sum_{i=0}^{k} H[i]. \tag{2}$$

If we know the parameters A_1, and C (A, B, and C), then the nth transition level can be estimated as

$$T[n] = C - A_1 \cos(\pi H_c[n-1]/N). \tag{3}$$

The kth code bin width is given by

$$W[k] = T[k+1] - T[k], \tag{4}$$

and the INL and DNL values can be estimated.

[1]This work is based on ”Verification of parameter settings in ADC test with sine fitting”, IMTC 2012 abstract

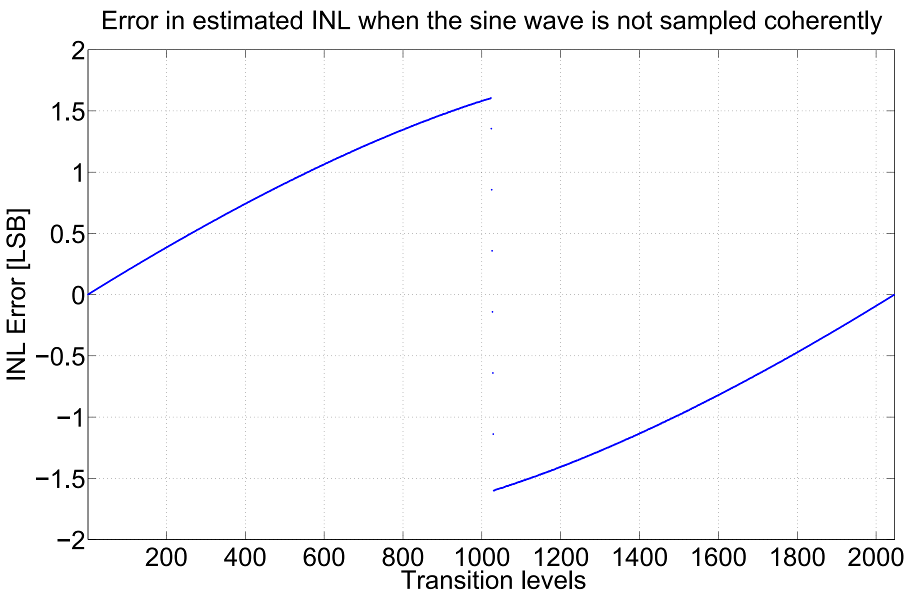

Figure 1: Errors in the INL estimation when not coherently sampled sine wave is applied

B. Standards for choosing signal and sampling frequencies, record length and periods per record

The sine wave input is optimal if the sampling is coherent, so an integer number of periods are sampled, and all the samples represent different phases. If the sampling frequency is f_s, the number of samples is N, then the signal frequency is optimal if

$$f_x = \frac{J}{N} f_s, \tag{5}$$

where J is an integer relative prime to N, and f_x is the signal frequency. Fig. 1 shows why it is important to precisely meet the coherency condition. An ideal ADC was tested with one period of a sine wave, the record length was $N = 2^{20} = 1048576$. To have one period, the optimal signal frequency would be $f_{opt} = f_s/N$, where f_s is the sampling frequency. The applied signal frequency was $f_x = 1.001 f_{opt}$. As Fig. 1 shows, this difference causes significant errors in estimating the transition levels and then in calculating the INL. The second condition ensures that every input sample represents a different phase, with uniform phase distribution. For example, if both the number of samples and number of periods can be divided by 2, then the first and the second half of the sequence are exactly the same (apart from the noise), so the number of useful samples is half the number of total samples.

III. FOUR-PARAMETER SINE WAVE ESTIMATION IN THE FREQUENCY DOMAIN

A. Implementation properties

The goal was to create an algorithm which can identify the parameters of the sine wave much faster than the ordinary least squares method. The sine wave's four parameter form is

$$x(k) = A\cos(2\pi k f) + B\sin(2\pi k f) + C. \tag{6}$$

This form is very advantageous because it is linear in 3 parameters. Due to the nonlinearity in frequency, an iterative numerical method is needed to find the minimum of the least squares cost function as a solution in closed form is not available for nonlinear LS. The output of the ADC is windowed with samples of the four-term Blackman-Harris window, which has side lobes under -91 dB. Then FFT is

Table 1: ”RESULTS FOR ”REAL-LIKE” QUANTIZER”

Estimator	NoB	Added noise	
	Added noise	0	$[-q, q]$
time	μ	$-3.09 \cdot 10^{-8}$	$7.22 \cdot 10^{-8}$
	σ	$6.70 \cdot 10^{-7}$	$2.20 \cdot 10^{-6}$
freq	μ	$-8.28 \cdot 10^{-8}$	$3.41 \cdot 10^{-9}$
	σ	$7.58 \cdot 10^{-7}$	$2.37 \cdot 10^{-6}$

performed, and the cost function is calculated in the frequency domain. After calculating the initial values of the parameters, Gauss-Newton algorithm is used to find the minimum. Since the Blackman-Harris window has small side lobes, the information in the frequency domain is compressed into a few points. In the fitting algorithm we use 30 points to determine the parameters, independently from the length of the time domain signal. The original, time domain algorithm uses every time domain sample in the fit, so for large records the computational burden is high. In this frequency domain fit only the evaluation speed of the FFT depends on N, so it is much faster thus it allows the use of large records.

B. Properties of the estimator

In this section the statistical properties of the estimator will be discussed. We assume that the noise in the frequency domain is approximately normally distributed. This assumption is based on the Central Limit theorem. The noise at the FFT output will be Gaussian independently from its original distribution. The larger the number of samples, the previous approximation is truer. This means that the weighted least squares estimation used is a maximum likelihood estimation, so it has a number of advantageous properties:

- the estimator is asymptotically normally distributed,
- the estimator is asymptotically unbiased,
- the covariance matrix of the estimator asymptotically reaches the Cramer-Rao lower bound.

Due to the first property the estimators can be fully characterized with their mean value (μ) and standard deviation (σ). In the next section the estimator is compared to the result of the ordinary least squares method, described in the standard [2].

C. Comparing the algorithm to the original estimation

In the following tests the standard deviations and the mean values of the estimators were compared between time domain and frequency domain to illustrate the quality of the frequency domain fit of the Blackman-Harris weighted data. In these tests the amplitude, phase, offset and frequency were chosen as random variables uniformly distributed in $[0.5, 1]$, $[0, 2\pi]$, $[-0.5, 0.5]$, $[4f_s/N, 10f_s/N]$, respectively, and the record length was $N = 2^{20}$. To simulate life-like circumstances, a quantizer based on real nonlinear characteristics was created. The number of fraction bits was 10, the full scale range (FS) was 1 V, and the quantizer was bipolar so it worked between -1 V and 1 V, thus it had a sign bit. Fig. 2 shows the INL and DNL characteristics of the quantizer. First we tested with pure sine wave, then the input was disturbed with uniformly distributed noise in $[-q, q]$. The results are shown in Table 1.

IV. Using the estimator to preprocess data

Two important conditions should be met to obtain the best results. These are coherent sampling and the appropriate relation between the number of samples and the number of periods of the sine wave

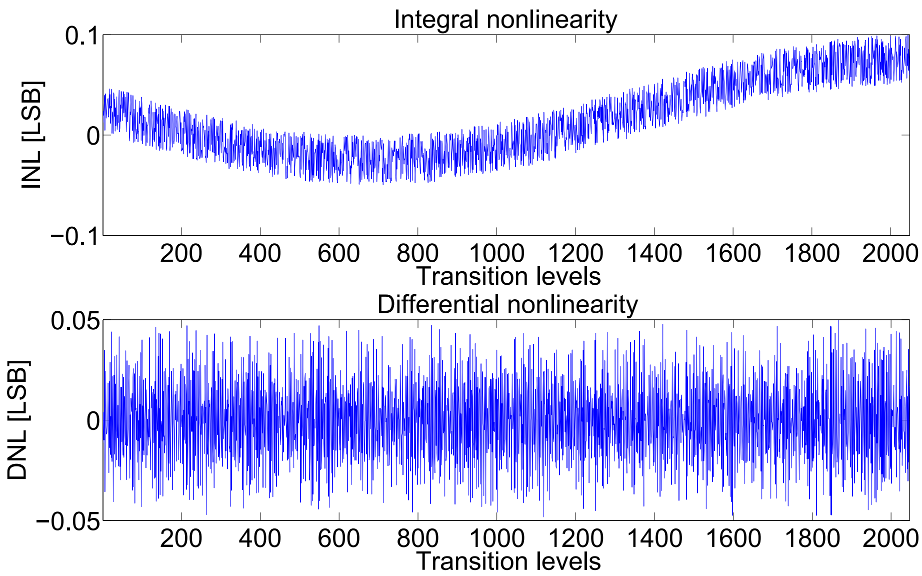

Figure 2: INL and DNL characteristics of the quantizer

applied. Both of the conditions can be met if the input signal's frequency is chosen correctly. So the frequency domain estimator is a feasible tool to check if the input sine wave is appropriate to test the converter. The distribution of the phases depends on the deviation of the wave frequency from coherence. In details, if

$$\frac{f_x}{f_s} = \frac{J}{N} + \Delta \tag{7}$$

then none of the sampling points deviate from the ideal phase from more than $\frac{2\pi}{4N}$ (where $\frac{2\pi}{N}$ is the ideal distance between two adjacent sampling points) if

$$|\Delta| \leq \frac{1}{2JN} \tag{8}$$

is true [3]. So we can check if the input sine wave meets the previous conditions or not, and warn the user. Furthermore, in the case when the sampling is almost coherent, but we have a few additional samples from the beginning of the next period, we can calculate how many samples should be thrown away to reach the best results. If a few samples are missing from the end of the whole last period, we can either throw away the whole period or suggest new sampling.

V. Conclusion

The main advantage of the estimator that it can determine in no time (compared to the original least squares) if the measured signal is applicable to characterize an ADC properly, and in some special cases it can modify the input to ensure better results.

References

[1] J. Blair, "Histogram measurement of ADC nonlinearities using sine waves," *IEEE Transactions on Instrumentation and Measurement*, 43(3):373–383, Jun 1994.

[2] "IEEE standard for terminology and test methods for analog-to-digital converters," *IEEE Std 1241-2010 (Revision of IEEE Std 1241-2000)*, pp. 1 –139, 14 2011.

[3] P. Carbone and G. Chiorboli, "Adc sinewave histogram testing with quasi-coherent sampling," in *Instrumentation and Measurement Technology Conference, 2000. IMTC 2000. Proceedings of the 17th IEEE*, vol. 1, pp. 108 –113 vol.1, 2000.