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Fermionic Entanglement Theory and the Black Hole/Qubit Correspondence

THESIS BOOKLET

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Introduction

One of the most important features of quantum mechanics is the possibility of entanglement between subsystems. It is a type of nonlocal correlation which is the unifying concept behind various physical phenomena in the microworld. It is also the key resource in the field of quantum computing which realizes exponential speedup in completing certain tasks compared to classical computing. Recently, there has been considerable amount of evidence that the glue holding together classical spacetime is quantum entanglement as well. Therefore, it is an essential task to classify and quantify different types of entanglement possessed by a given quantum state.

Entanglement also plays a key role in the physics of black holes. Indeed, sharp versions of the black hole information paradox invoke properties of certain entanglement measures, such as monogamy inequalities. The research presented in this dissertation was initiated by one of the early attempts to connect the physics of black holes to quantum entanglement. This attempt is based on the observation of Duff, that some charged supersymmetric black hole solutions in a variant of four dimensional $\mathcal{N} = 2$ supergravity have a Bekenstein-Hawking entropy that is the square root of a well-known entanglement measure for three qubits, the three-tangle, provided that we associate a three qubit state to the charges of the black hole [Duff, 2007]. While it turned out that this observation has little to do with entanglement between quantum degrees of freedom of the black hole, still, the analogy can be extended to incorporate more systems and can be fruitful both for classifying stringy black holes and entanglement in some simple multipartite quantum systems.

Goals

The objective of this doctoral research was to understand more about the black hole/qubit correspondence and, if possible, apply the results of each of the fields to the other one to draw new conclusions.

They key mathematical property behind the correspondence is the notion of *prehomogenity* of the action of a group on a vector space. It is a common feature of simple small entangled systems and it is connected to supersymmetric black holes via a certain geometric construction, the Hitchin functionals, where prehomogenity turns into the crucial requirement of *stability*.

An important ingredient of generalized Hitchin functionals is the theory of spinors and the action of the Clifford group on them in higher dimensions. A natural question in light of the black hole/qubit correspondence is whether the Clifford group have significance in the theory of entanglement. This is indeed the case, it provides a natural way of extending the fermionic entanglement classification problem to states not having a definite particle number. This observation led us to a variety of new results in fermionic entanglement theory.

It was also fruitful to apply entanglement related formalism, such as the embedding of qubits into fermionic spaces, to obtain new results in supergravity. Such is the obtained new expression for the entropy of general nonextremal black hole solutions of $\mathcal{N} = 2$ supergravity in terms of duality invariants. This is a small but crucial step on the road to understand the microscopics of asymptotically flat nonextremal black holes.

New scientific results

In this section, we summarize the results in the dissertation which are the authors own work, by collecting them into thesis points. We indicate the connecting publications at the end of each point.

1. We observe that the even Clifford group acting on spinors is a natural extension of the usual SLOCC[1] group acting on fermionic states with a fixed number of particles. The additional transformations in the even Clifford group can be interpreted as Bogoliubov transformations. We propose the classification of spinors under the action of the even Clifford group to be a natural generalization of the fermionic SLOCC classification problem. Therefore, we call the even Clifford group, the extended SLOCC group. Pure spinors, which form the simplest orbit under the Clifford group, are shown to generalize the notion of separable states. We observe that spinor invariants give possible entanglement measures and we show how various known entanglement measures, like the pure state concurrence for two qubits or the three tangle for three qubits originate from known spinor invariants. The connecting publication is [P.2].

2. We identify a real structure which is present on fermionic Fock spaces. We use this to relate SLOCC covariants, used to separate entanglement classes of states, with reduced density matrix elements via spinor Fierz identities. The entanglement monogamy equations for the tangles of three qubits are shown to be a consequence of such a Fierz identity. In fact, we show that these equations are special cases of monogamy equations obeyed by three fermions with six single particle states. As a consequence, we confirm that the quartic SLOCC invariant introduced in [Lévay and Vrana, 2008] indeed measures three body entanglement. We identify the fermionic concurrences measuring two body entanglement. The connecting publication is [P.4].

[1] SLOCC: Stochastic local operations and classical communication.

3. We study the SLOCC and extended SLOCC classes of certain low dimensional fermionic systems where the solution to the classification problems can be extracted from the mathematical literature. Particular emphasis is put on three fermion systems with up to nine single particle modes. We present invariants and covariants of these systems in the unified language of spinor invariants. We give a complete set of (noncontinuous) invariants separating the SLOCC orbits in the case of three fermion systems with six, seven and eight modes. In the nine mode case we give new combinations of the four independent continuous invariants which can be used to separate the semisimple families of Vinberg and Elashvili. We discuss how to embedd the distinguishable system of three qutrits and present how the fermionic invariants reduce to the invariants of three qutrits. As a consequence, we also obtain invariants separating families of three qutrits. The embedding between families is observed to be injective, a property that previously studied fermionic embeddings of three qubits and four qubits also satisfy. We emphasize the connection of the six and seven mode cases to the theory of $SU(3)$ and G_2 special holonomy manifolds via their connection to Hitchin functionals. The connecting publications are [P.1, P.3]

4. We identify the U-duality invariant which is required to express the Bekenstein-Hawking entropy of the most general 4 dimensional, stationary, asymptotically flat, nonextremal STU black holes constructed recently in [Chow and Compère, 2014]. This allows us to write the general nonextremal entropy formula entirely in terms of asymptotic charges for the first time. The expression also involves the "scalar charges" of these black holes which can in principle be solved in terms of the dyonic charges and the mass. We discuss how the formula reduces to some of the known results as the Klauza-Klein black hole and the dilute gas limit of Cvetič and Larsen and how its dependence on the asymptotic scalar moduli can be removed via U-dualities. We discuss the identification of the charges of the black hole with a four qubit state and show that the nonextremal black holes of [Chow and Compère, 2014] are in a *single* semisimple entanglement class, in contrary to the complete identification between extremal black holes and nilpotent classes worked out it [Borsten et al., 2010]. The connecting publication is [P.5].

Papers connecting to thesis points

[P.1] Péter Lévay and Gábor Sárosi. "Hitchin functionals are related to measures of entanglement". In: *Phys. Rev.* D86 (2012),

p. 105038. DOI: 10.1103/PhysRevD.86.105038. arXiv: 1206.5066 [hep-th].

[P.2] Gábor Sárosi and Péter Lévay. "Entanglement in fermionic Fock space". In: *Journal of Physics A: Mathematical and Theoretical* 47.11 (2014), p. 115304. DOI: 10.1088/1751-8113/47/11/115304. arXiv: 1309.4300 [quant-ph].

[P.3] Gábor Sárosi and Péter Lévay. "Entanglement classification of three fermions with up to nine single-particle states". In: *Physical Review A* 89.4 (2014), p. 042310. DOI: 10.1103/PhysRevA.89.042310. arXiv: 1312.2786 [quant-ph].

[P.4] Gábor Sárosi and Péter Lévay. "Coffman-Kundu-Wootters inequality for fermions". In: *Physical Review A* 90.5 (2014), p. 052303. DOI: 10.1103/PhysRevA.90.052303. arXiv: 1408.6735 [quant-ph].

[P.5] Gábor Sárosi. "Entropy of nonextremal STU black holes: The F-invariant unveiled". In: *Phys. Rev.* D93.2 (2016), p. 024036. DOI: 10.1103/PhysRevD.93.024036. arXiv: 1508.06667 [hep-th].

Other papers and preprints by the author

[O.6] Péter Lévay et al. "The coupled cluster method and entanglement". In: (2015). arXiv: 1506.06914 [quant-ph].

[O.7] Alejandra Castro, Diego M. Hofman, and Gábor Sárosi. "Warped Weyl fermion partition functions". In: *JHEP* 11 (2015), p. 129. DOI: 10.1007/JHEP11(2015)129. arXiv: 1508.06302 [hep-th].

[O.8] Gábor Sárosi and Tomonori Ugajin. "Relative entropy of excited states in two dimensional conformal field theories". In: (2016). arXiv: 1603.03057 [hep-th].

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Michael J Duff. "String triality, black hole entropy, and Cayley?s hyperdeterminant". In: *Physical Review D* 76.2 (2007), p. 025017. DOI: 10.1103/PhysRevD.76.025017. arXiv: hep-th/0601134 [hep-th].

Péter Lévay and Péter Vrana. "Three fermions with six single-particle states can be entangled in two inequivalent ways". In: *Physical Review A* 78.2 (2008), p. 022329. DOI: 10.1103/PhysRevA.78.022329. arXiv: 0806.4076 [quant-ph].

David DK Chow and Geoffrey Compère. “Black holes in N= 8 supergravity from SO (4, 4) hidden symmetries”. In: *Physical Review D* 90.2 (2014), p. 025029. DOI: 10.1103/PhysRevD.90.025029. arXiv: 1404.2602 [hep-th].

Leron Borsten et al. “Four-qubit entanglement classification from string theory”. In: *Physical review letters* 105.10 (2010), p. 100507. DOI: 10.1103/PhysRevLett.105.100507. arXiv: 1005.4915 [hep-th].