Generating Smooth Surfaces from Discrete Pointsets

THESIS BOOKLET

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1. Introduction

The creation of smooth surfaces is a fundamental problem in computer-aided geometric design. While many geometric primitives can be defined implicitly, parametric surfaces are used for the definition of truly free-form shapes. Common parametric representations, such as Bézier or B-spline/NURBS surfaces are defined by a convex combination of discrete control points. Such control point representations serve not only as a convenient interior data structure, but provide also a natural set of handles by means of which the user can modify the surface. It is important then, that these control points are related to the underlying surface shape in some natural way. My work concerns the correspondence between smooth surfaces and their control structure.

The first area of interest in my dissertation is the approximation of given data points with a trimmed tensor product Bézier or B-spline surface. The unknown surface should not only approximate the data up to tolerance, but possess a well-behaved control structure as well, that conforms to the surface geometry, and is free of spurious oscillations and similar artifacts. I presented new techniques that support the fitting of high-quality trimmed tensor product surfaces. One such technique is called labeling, which is helpful for finding a well-suited parameterization for a surface. I also complemented this by extension, which stabilizes a fitting process without applying excessive regularization. My experiments have demonstrated that the combination of the two techniques result in substantial improvements in surface quality over existing methods.

The second area of interest in my dissertation is the control point-based design of multi-sided surface patches of general topology. In this case, a control structure is given and a surface is to be determined that naturally mimics its structure. Genuine multi-sided surfaces are traditionally defined as deformations of a planar polygonal shape. However, parametric domains bounded by straight edges often result in surfaces with unnatural shape and excessive parametric distortion. This served as a motivation for the introduction of curved domains, bounded by general planar curves. I have found that surfaces defined over curved domains are capable of modeling high-quality shapes bounded by complicated curve and cross-derivative configurations, and even interior hole loops. I also developed a new type of curved domain surface, called the Generalized B-Spline patch, by means of which complex B-spline boundary conditions can be combined in the design of challenging shapes.

2. Research goals and methodology

In the early stages of my research, I studied the problem of mesh parameterization under geometric constraints, and developed a general framework for constrained parameterization, which I published in [Vaitkus and Várady, 2014, Vaitkus and Várady, 2015a]. My goal was to apply these new methods to the problem of trimmed tensor product surface fitting. After studying the published literature on surface fitting and mesh parameterization, I concluded that the particular requirements of trimmed surfaces have not been adequately addressed by previous
works in either area. Motivated by this, I have developed the concept of labeling [Vaitkus and Várady, 2015b], and worked out a suitable parameterization method. During experiments, the need for additional constraints quickly arose, and a technique based on the so-called guiding frame was introduced as a solution. Still, weak control points caused difficulties, which required careful parameter tuning to resolve – this served as a motivation for the development of new mesh extension methods. I observed remarkable quality improvements in the output of an existing surface fitting library, after applying labeled parameterization and extension and published my results in [Vaitkus and Várady, 2018a] and [Vaitkus and Várady, 2018b]. Originally, labels had to be chosen manually, but later I used my accumulated experience with labeled fitting to develop an automatic labeling algorithm as well [Vaitkus and Várady, 2019]. As part of my work, I created a pre-processing library for spline fitting, that interfaced with the Sketches program [Sketches, 2018].

Earlier experience with convex and concave Generalized Bézier patches have suggested some inherent limitations of polygonal domains for multi-sided surfacing. This motivated my work on curved domain Bézier surfaces. I developed and implemented a method for the generation of curved, multiply-connected domains and a local parameterization based on harmonic coordinates. This work became part of the publication [Várady et al., 2020]. Curved Domain Bézier patches were proven quite effective at modeling complex shapes, however it was found beneficial to split long, complicated boundary curves into several pieces. This suggested a natural extension to B-spline boundaries, and led to the development Generalized B-spline patches. The local parameterization of periodic hole loops presented a new problem, that I solved using a new method based on harmonic interpolation and a virtual cutting of the domain. These results are going to be published in [Vaitkus et al., 2020]. I developed a large, interactive, graphical prototype system for experimentation with various domain generation and parameterization methods, that has been integrated into the Sketches modeling environment [Sketches, 2018].

3. Previous work

Surface fitting and parameterization. The problem of trimmed tensor product surface fitting plays a central role in the area of reverse engineering [Várady and Martin, 2002]. The literature on surface fitting is extremely rich – refer to e.g. [Weiss et al., 2002]. Several methods were published for the parameterization of data points prior to fitting, e.g. [Ma and Kruth, 1995, Piegl and Tiller, 2001, Azariadis, 2004, Lai et al., 2006], but these assumed that all four boundary curves of the fitted surface are present in the data. The parameterization of trimmed regions has been studied in [Weiss et al., 2002], but that paper preceded the development of modern mesh parameterization methods. Mesh parameterization is an extremely rich area of research – see the survey [Hormann et al., 2007] – however, the vast majority of published results concern distortion minimization [Liu et al., 2008, Sawhney and Crane, 2017], or alignment to features, such as sharp edges or principal curvatures [Myles and Zorin, 2013, Campen et al., 2016]. While distortion plays a crucial role in texture mapping, and alignment of the isolines is key when a parameterization is used for quad meshing [Bommes et al., 2013], the
requirements of trimmed tensor product surface fitting are different, and have not been addressed before in the literature. The fitting instability caused by weak control points have been discussed by [Weiss et al., 2002], where a heuristic solution was proposed. Smooth extrapolation of surface meshes beyond their boundaries is a well-researched area as well – the proposed method is most closely related to variational and parameterization-based techniques [Lévy, 2003, Jacobson et al., 2010].

**Multi-sided surfaces.** Traditional parametric surfaces are modeled on rectangular domains [Farin, 2002], so the modeling of multi-sided and multiply-connected regions requires special techniques. The limitations of traditional techniques, such as trimming [Marussig and Hughes, 2017], or macro-patches [Peters, 2019] are well-known. Genuine multi-sided patches can be classified as transfinite interpolation methods [Várady et al., 2011], or control point schemes – my work focuses on the latter category. There exist several classical multi-sided generalization of Bézier patches – see [Goldman, 2004] for a survey – but their control structure are usually either too complicated, or too limited for practical use. An exception is the Generalized Bézier (GB) patch introduced in [Várady et al., 2016], which has also been extended to concave polygonal domains [Salvi and Várady, 2018]. A few surface representations can model multiply-connected surfaces – examples include [Kato, 1991, Sabin, 1998, Schaefer, 2017] – but dealing with complex boundary conditions remained an unsolved problem. Multi-sided B-spline surfaces have been also proposed, e.g. [Pla-Garcia et al., 2006, Hettinga and Kosinka, 2020], but there were limitations on the knot vectors, or the geometrical/topological complexity of the shapes that can be modeled. The domains of multi-sided surfaces have been adapted to the geometry for both convex [Várady et al., 2011] and concave [Salvi and Várady, 2018] polygons, but general curved domains to my knowledge have not been considered previously.

### 4. Research results

My results are organized into two thesis groups: methods supporting trimmed surface fitting, and multi-sided surfaces over curved domains. The employed mathematical apparatus is detailed in the dissertation. The results of my work are illustrated with several figures.
Thesis 1.1:

I developed a novel method for the flattening of triangle meshes, which enforced so-called labeling constraints on its boundary curves. My algorithm has two phases: first, I place the triangles on the plane in a rigid manner, with an orientation that respects the constraints, then in the second step I fit a consistent mesh to the planar triangles, while enforcing the constraints. I verified with experiments that the use of labeled parameterizations results in surface fits of higher quality, compared to projective or purely distortion-minimizing parameterizations.

This thesis is based on Chapter 3.1 of my dissertation, and was published in the journal publication [Vaitkus and Várady, 2018a], and the conference papers [Vaitkus and Várady, 2014, Vaitkus and Várady, 2015a, Vaitkus and Várady, 2015b].
Thesis 1.2:

Enforcing label constraints occasionally results in distorted, overlapping parameterizations. With the aim of alleviating this problem, I proposed the use of a guiding frame, which consists of (at most) 4 Bézier curves, fitted to the labeled boundary curves. Thus, further constraints can be prescribed for the ratios of the flattened boundaries, leading to more natural parameterizations.

This thesis is based on Chapter 3.1 of my dissertation, and was published in the journal publication [Vaitkus and Várady, 2018a].
Thesis 1.3:

I developed a method, which alleviates the problem of “weak” control points while fitting trimmed surfaces, by adding auxiliary data points. I extend the original mesh, first in the parameter plane, then I determine the three-dimensional image of said extension by minimizing a suitable smoothness energy. The extension can connect to the original surface with tangent or curvature continuity. My experiments have shown, that fitting the extended data points significantly improves surface quality.

This thesis is based on Chapter 3.2 of my dissertation, and was published in the journal publication [Vaitkus and Várady, 2018a] and the conference paper [Vaitkus and Várady, 2018b].

Thesis 1.4:

I developed an algorithm, which assigns labels to the boundary curves of a surface to be parameterized. I classified the six possible label configurations based on the number of label candidates and the number of corners they form. My algorithm first categorizes boundary curves as either label candidates, or trim curves. Then, I detect which of the six label configuration fits the boundary curves best, by iteratively merging and removing label candidates.
This thesis is based on Chapter 4 of my dissertation, and was published in the journal publication [Vaitkus and Várady, 2019].

Figure 6: Example of automatic labeling.

2nd thesis group: Control point based multi-sided surfaces over curved parametric domains

Figure 7: Modeling with curved domain patches.
Thesis 2.1:

In contrast with traditional straight-edge polygonal parametric domains, I proposed the use of domains bounded by general curves. These allow for the construction of multi-sided surfaces with higher quality and lower parametric distortion. The parametric domain can be multiply connected as well, which opens new possibilities for multi-sided surfacing.

This thesis is based on Chapter 5 of my dissertation and was published in the journal publication [Várady et al., 2020].

![Convex polygonal domain patch](image1) ![Curved domain patch](image2)

Figure 8: Curved domain patches can model more complex surfaces than traditional patches over polygonal domains. Colors indicate mean curvature.

![Curved domain patches](image3)

Figure 9: Curved domain patches have low parametric distortion. Colors indicate symmetric ARAP energy.

Thesis 2.2:

I developed an algorithm, which determines a natural curved parametric domain, based on the boundary curves and normal vectors of a surface patch to be created. My method is based on the preservation of the shapes of the boundary curves as seen from within the surface (i.e. their geodesic curvature). I extended the method to the case of multiply connected domains as well.

This thesis is based on Chapter 7.1 of my dissertation and was published in the journal publication [Várady et al., 2020].

Thesis 2.3:

I developed a new method for the local parameterization of curved parametric domains, based on harmonic interpolation, which can be extended to the periodic pa-
rameterization of interior boundary curves as well. I also proposed alternative parameterization methods, which reduce the distortion manifesting near strongly curved boundaries.

This thesis is based on Chapter 7.2 of my dissertation and was published in the journal publication [Vaitkus et al., 2020].

Figure 10: Comparison of local parameterizations. Top: Earlier parameterizations based on generalized barycentric coordinates. Bottom: New parameterizations based on harmonic functions.

Figure 11: Curved domain patches can model multiply-connected surfaces. Left: ribbons; Middle: slicing; Right: mean curvature map.

Thesis 2.4:

I introduced a novel multi-sided surface representation, called the Generalized B-Spline surface (GBS, for short), in the vein of Generalized Bézier surfaces. GBS surfaces interpolate boundary curves and cross-derivatives defined by B-splines with arbitrary degree and knot vector, over curved, possibly multiply connected parametric domains.

*Currently under revision.
This thesis is based on Chapter 6 of my dissertation and was published in the journal publication [Vaitkus et al., 2020]∗.

(a) Complex patch perpendicular to cylinder.  
(b) Multiply-connected patch smoothly joining extruded surface.  
(c) Multiply-connected patch smoothly joining developable surface.

Figure 12: Examples of GBS patches.

5. Research projects

This project has been supported by the Hungarian Scientific Research Fund: OTKA, No. 124727, Modeling general topology free-form surfaces in 3D, and No. 101845, Fitting multiple surfaces with geometric constraints.
My publications (peer-reviewed journals)


My publications (conference proceedings)


References


