An Application Example of Regularization: Time-varying Operational Modal Analysis

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***Abstract*— This paper presents an efficient nonparametric time-varying time domain system identification method for the Operational Modal Analysis (OMA) framework. OMA tackles industrial measurements of vibrating structures in real-life operating conditions without the exact knowledge of the excitation signal. In this work a method is provided to estimate the time-varying output autocorrelation function of vibrating structures and applied to a measurement of wind tunnel testing of an airplane. Using the proposed methodology, the estimated time-varying two-dimensional output autocorrelation model provides a good data-fit with respect to tracking of varying resonances.**

***Keywords*— Bayesian methods; Nonparametric methods; Time-varying system identification; Operational Modal Analysis**

I. Introduction

The paper deals with the time variations of (vibrating) mechanical and civil structures described by the nonparametric Operational Modal Analysis (OMA) framework. OMA is a special identification technique for estimating the modal properties (e.g. resonance frequencies, damping, mode shapes) of structures based on vibration data collected when the structures are under real operating conditions without having access to the excitation signals. This technique can provide the engineers with useful information to understand the dynamic behavior of the underlying structure in real-life usage scenario, and it can be used to validate and update the numerical models developed in the design phase [1].

The main issue is that the dynamics of underlying systems may vary significantly when operating in real-life conditions. In this case, advanced modelling is needed taking into account the time-varying (TV) behaviour because the unmodelled time variations might lead to instability and structural failures. Contrary to the classical identification frameworks, a further challenge with the OMA framework is that the excitation signal is not known exactly, but it is assumed to be white noise.

A good example of a time-varying mechanical structure can be, for instance, an airplane. The TV behavior originates from the decreasing weight due to the fuel consumption, and from different surface configurations during take-off, cruise and landing Moreover, the resonance frequency and damping of most vibrating parts (for instance the wings) of a plane vary as a function of the flight speed and height. In this paper the wind tunnel testing of an airplane is considered.

In this work the first results of a nonparametric TV OMA method are presented to estimate efficiently the linear time-varying (LTV) output autocorrelation function (ACF) of the observed system. In case of TV systems it is a common practice to use different types of short-time Fourier transform (SFT) techniques that describes the underlying system as a series of linear-time invariant (LTI) systems ([2]-[7]). In case of TV OMA, other nonparametric techniques have a limited applicability due to lack of knowledge of excitation. These SFT based models can describe the time-varying behavior quite well when the time-variations are very smooth. The drawback of these methods is that such an LTI model is not sufficient to describe the system's behavior, if the time variations are fast, or when a significant variation occurs.

The proposed method is intended to overcome the issues with the classical SFT based methods and it is based on the regularization methods that have been developed for impulse response function (IRF) modelling of LTI systems ([7]-[9]). The novelty of this work compared to the IRF regularization is to formulate the regularization for the special case of the estimation of LTV systems in an industrial OMA framework – i.e. unknown input– similarly to the regularized LTV framework with known excitation [3],[4].

The paper is organized as follows: in Section II the basic concepts are summarized and the exact problem formulation is given. Section III provides the model structure and the cost function of the problem. Section IV introduces the regularized LTV OMA estimation method and addresses the implementation procedure. Section V shows a measurement example where the results of the classical framework are shown to compare to the results proposed method. Finally, the conclusions are provided in Section VI.

II. Basics

A. Definitions and Assumptions

Several definitions and assumptions must be addressed prior to carrying out any system identification procedure.

A discrete LTV system is completely characterized by its 2D IRF $h[t,\tau]$. Its steady-state response to an arbitrary signal $u[t]$ is given by ([10],[11]):

$$y[t] = \sum_{\tau=-\infty}^{+\infty} h[t,\tau]u[t-\tau] \qquad (1)$$

where the parameter t is the global time (the time when the system is observed) and τ is the system time (the direction of the impulse responses/lags) [3].

The following assumptions are made in this work:

ASSUMPTION 1 The system output $y[t]$ is disturbed by additive, i.i.d. Gaussian stationary noise with a zero mean and a finite variance.

ASSUMPTION 2 The 2D IRF is smooth i.e. the spectral content of the underlying LTV system is highly concentrated at the low frequencies. A more precise definition can be found in [9].

ASSUMPTION 3 The considered discrete LTV system is causal (i.e. $h[t,\tau]=0$, when $\tau<0$) and stable

ASSUMPTION 4 The length of the IRFs ($L+1$) is shorter than the observation (measurement) window length (N).

ASSUMPTION 5 The excitation signal $u[t]$ is not known exactly but it is assumed to be white noise with a TV finite variance.

B. Problem formulation

In this section, a brief overview of the LTV OMA problem is presented where we explain 1) the issues related to the nonparametric identification of LTV systems in general 2) the issues related to the OMA framework w.r.t. the classical identification framework.

1) Nonparametric Identification of LTV Systems

The challenge with LTV systems is that the TV 2D IRF and FRF are not uniquely determined from a single set of input and output signals – unlike the case of LTI systems.

Consider the model defined in (1). Imposing *Assumptions* 1-4, the following measured output $y_m[t]$ is given at time t:

$$y_m[t] = y[t] + e[t] = \sum_{\tau=0}^{L} h[t,\tau]u[t-\tau] + e[t] \quad (2)$$

where $t = 0 \ldots N-1$.

The problem lies in the fact that there is a nonuniqeness issue. The number of unknown in the TV 2D nonparametric model is NL but there are only N measured points. Hence, the model that relates the input to the output is not unique, because there are only N linear relations (measurement samples) for NL unknowns.

2) Operational Modal Analysis Framework

OMA is a very important tool because in the case of vibrating structures it is common that the real operating conditions differ significantly from dynamic measurements performed in laboratory conditions. In case of industrial measurements of large mechanical and civil structures (e.g. airplanes, bridges, wind tunnels) the excitation signal is most of the time not measurable, or it would be too difficult and complex to measure [1].

Because the excitation signal is not known (see *Assumption* 5) an IRF or FRF cannot be directly estimated. Instead, the output autocorrelation function (ACF) and/or its Fourier transform are used in practice.

III. THE NONPARAMETRIC IDENTIFICATION METHOD

A. The Model Structure

Using the OMA framework, the underlying time-varying systems can be represented by their 2D ACF when *Assumption* 1-5 are satisfied. In this case the 2D TV ACF would provide the convolved LTV IRF in time-domain ($\sum_{\tau=0}^{L} h[t,\tau]h[t+\tau]$). The proof is out of scope of this paper, details for the LTI case can be found in [1]. The TV ACF R_{yy} centered around t, at time lag τ, with a window length of (L+1), is estimated with the measured output signal y_m (see (2)) by smoothing over a window with a length of L+1:

$$\hat{R}_{y_m y_m}[t,\tau] = \frac{1}{L+1}\sum_{k=-\frac{L}{2}}^{\frac{L}{2}} y_m[t+\tau+k]y_m[t+k] \quad (3)$$

Next, the double time indices $[t,\tau]$ will be omitted in order to make the text more accessible. The key idea of this work is to extend the existing nonparametric regularization methods such that some advanced prior information can be taken into account.

B. The Cost Function

The basic idea of the regularization technique is that by using the prior knowledge (formalized later on) on the system dynamics, and by introducing some bias error, the variance can be significantly reduced resulting in a significantly reduced mean square error (MSE) [8]. In order to include the prior knowledge in the nonparametric representation, an extended cost function (V) must be defined.

This cost function (V) consists of the sum of the ordinary least squares cost function (V_{LS}) which is now defined for the 2D LTV ACF case as

$$V_{LS} = \left\| vect(R_{yy}) - vect(\hat{R}_{y_m y_m}) \right\|_2^2 \quad (4)$$

and the regularization cost function (V_r):

$$V_r = \sigma^2 vect(R_{yy}^T) P^{-1} vect(R_{yy}), \quad (5)$$

such that the new, combined cost function is given by

$$V = V_{LS} + V_r \quad (6)$$

where P is a – special covariance – matrix containing the prior information, and σ^2 is the amount of regularization (prior) applied – it is usually proportional to the noise variance of the observation. To simplify the model and computational complexity σ^2 is kept constant but in general TV σ^2 – and observation noise– may be considered.

Minimizing (6) with respect to R_{yy} gives the solution which estimates the 2D IRF of an LTV system:

$$vect(\hat{R}_{yy,reg}) = (I + \sigma^2 P^{-1})^{-1} vect(\hat{R}_{y_m y_m}) \quad (7)$$

where I denotes the identity matrix.

IV. REGULARIZATION

In the section the practical implementation of the inclusion of the prior knowledge into the covariance matrix is addressed.

A. Considered Kernel Functions

The covariance hypermatrix P is constructed by using kernel functions. The specific choice of the kernels has a major effect on the quality of the estimate. Next, the kernels that will be used in this paper are explained.

The smoother Radial Basis Functions (RBF) is defined by

$$P_{RBF}(t_1,t_2) = e^{-\frac{(t_1-t_2)^2}{\gamma}} \qquad (8)$$

where γ is a parameter representing the length scale. The larger the length scale, the smoother the resulting estimated function is.

In case of Diagonal Correlated (DC) kernels next to the smoothness assumption, exponentially decaying envelope can be imposed and is defined as follows:

$$P_{DC}(t_1,t_2) = e^{-\alpha|t_1-t_2|}e^{-\frac{\beta(t_1+t_2)}{2}} \qquad (9)$$

where α quantifies the correlation length between adjacent impulse response coefficients, or in other words it controls the smoothness, and β scales the exponential decaying.

Please note that depending upon the prior knowledge, different shorts of kernel functions may be used.

B. The Covariance Matrix and the Prior Knowledge

In order to overview the properties of the TV ACF an illustration is shown in Fig 1. The first prior knowledge is that the considered ACFs are smooth. This smoothing is applied once over the system time τ which refers to "classical" evolution of IRF/ACF, and once over the global time t which gives a support to handle the time-varying behavior.

In addition to smoothing properties, another assumption can be imposed and incorporated in P by applying a more strict definition of stability: the IRFs and therefore the ACFs are exponentially decaying. Note, that most of the stable mechanical systems exploit this behaviour. When this is not the case other kernels might be used.

In the proposed approach, every point of the 2D ACF surface is connected to each other. It means that all the elements in the covariance matrix are non-zero, therefore the number of constraints is high, and the degrees of freedom of the system of linear equations are significantly decreased resulting in a unique solution. The elements in the τ direction of the autocorrelations (horizontal blue direction) are linked by DC kernels. Elements in the global time t direction (vertical red direction) are linked by RBF kernels. This contraction is formulated as follows:

$$\mathrm{P}_{\{t_1,t_2\},\{\tau_1,\tau_2\}} = \mathrm{P}_{RBF}(t_1,t_2)\cdot \mathrm{P}_{DC}(\tau_1,\tau_2) \qquad (10)$$

for every possible pair of t and τ.

C. Tuning of the Model Complexity and Hyperparamters

$\gamma,\alpha,\ \beta,\sigma^2$ parameters in (5),(8),(9) are the so called hyperparameters and their values are restricted as follows: $\gamma,\sigma^2 > 0$ and $\alpha,\beta \geq 0$. The nonparametric system identification method presented in this work consists essentially of two steps: 1) optimization of the hyperparameters to tune the matrix P, and 2) computation of the model parameters using (7).

All the hyperparameters are tuned with the use of measured output data only by maximizing the marginal likelihood of the observed output ([8],[9]).

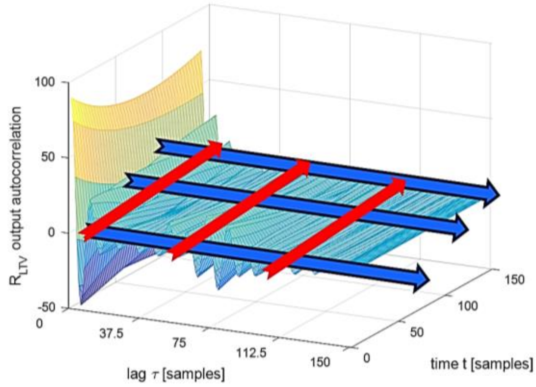

Fig. 1. The TV output ACF of a system. The arrows show the visualization of the possible regularization directions. The blue arrow refers to the τ system time where decaying and smoothness of the adjacent elements can be imposed. The red arrow refers to the direction of time variations (t axis) where the smoothness of the adjacent autocorrelation functions can be imposed.

V. RESULTS

A. Measurement Setup

This section presents an industrial example of the in-line flutter assessment of the wind tunnel testing of a scaled airplane model. The measurement time is 424 sec, the sampling frequency (f_s) is 500 Hz, the number of data samples (N) is 212000. The segmented window size (L+1) is 500 samples (which corresponds to 1 sec, 1 Hz resolution). The Mach number (proportional to the airflow i.e. wind speed w.r.t. sound speed) is varying between 0.07 and 0.79. An illustration of the measurement setup is given in Fig 2.

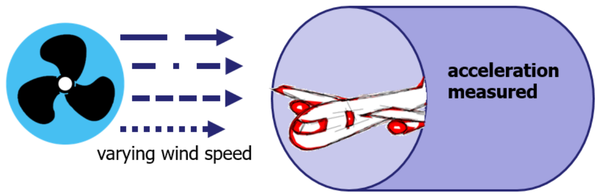

Fig. 2. The measurement setup is shown. The airflow (wind speed) is varying while the airplanes acceleration is measured in the wind tunnel.

In this type of testing, it is desired to carefully verify and track the vibration behavior, since during flutter appearance, system destruction can occur. In this paper, wind tunnel data at various flow conditions are used to validate the approach for tracking the evolution of the resonance frequencies, damping ratios.

Further details on the measurement procedure can be found in [12]. Exact specifications on the airplane are omitted due to the confidentiality of the industrial project.

B. Results

In this section the time and frequency domain results of the traditional SFT approach are compared to results of the 2D regularization method. In case of SFT the output measurement is split into short subrecords, and each time the output ACF is calculated. The result is a series of ACFs which represents the classical TV ACF estimate. The TV collerogram (power spectrum estimate) is then given by the Fourier transform of the TV ACF. When the proposed method is considered, the ACF estimates are normalized and regularized as it is detailed in Section III and IV.

Figure 3. and Figure 4. show the frequency domain results of the SFT and regularized approach. Since the most important part is the evolution the resonances, only the top view of the 2D collerograms is shown. Observe that the traditional approach is very noise where the regularized solution is smooth and more details can be captured.

It is important to remark that the classical SFT results can also be post filtered/smoothened – even with regularization or with other approaches (e.g. B-spline techniques) – but in this case the accuracy of the result will be between the classical and proposed approach.

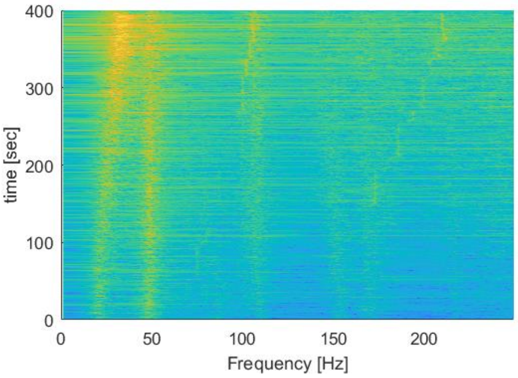

Fig. 3. The time-varying output power spectrum estimate of observed system is shown when the classical SFT method is considered.

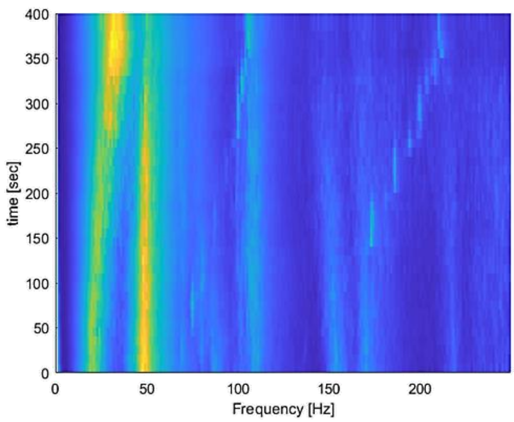

Fig. 4. The time-varying output power spectrum estimate of observed system is shown when the proposed method is considered.

VI. CONCLUSIONS

The main concept of the nonparametric TV OMA framework is to provide more accurate and smoother models which are suitable for simulation, design and – indirectly – control. This will improve the overall quality and safety of products, and speed up the design process. Using the proposed advanced 2D regularization techniques it is possible:

- to reduce the noise influence such that the measurement quality will be significantly better
- to gain a better insight into the details of the time-variations

Further, a nonparametric TV OMA model can be used:

- to capture and to track the time-varying resonance frequencies and damping ratios
- to simulate and validate during the design phases

When it is necessary, the nonparametric TV OMA model can be used to estimate a parametric OMA model by cleaning up the data and allowing direct access to the mode shapes and making the control possible.

The drawback of the proposed method is that the computational load and memory need are significantly higher but this is negligible in the applications where 1) the preparation of the measurement setup takes days, and 2) the safety of the product and its intended users is concerned.

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REFERENCES

[1] B. Peeters and G. De Roeck., “Stochastic system identification for operational modal analysis: a review”, ASME Journal of Dynamic Systems, Measurement, and Control, 123(4), pp. 659-667, 2001

[2] A. B. Jont, “Short Time Spectral Analysis, Synthesis, and Modification by Discrete Fourier Transform”, IEEE Transactions on Acoustics, Speech, and Signal Processing. Vol. 25, issue 3, pp. 235–238, 1977

[3] P. Z. Csurcsia, “Nonparametric identification of linear time-varying systems”. Phd thesis. University Press, Zelzate, 2015

[4] P. Z. Csurcsia. and J. Lataire, “Nonparametric Estimation of Time-varying Systems Using 2D Regularization”, IEEE Transactions on Instrumentation and Measurement, 2016

[5] P. Z. Csurcsia and J. Schoukens, “Nonparametric Estimation of a Time-variant System: An Experimental Study of B-splines and the Regularization Based Smoothing”, IEEE International Instrumentation and Measurement Technology Conference, pp. 216-221, Pisa, Italy, 2015

[6] M Freedman and G. Zames, “Logarithmic variation criteria for the stability of systems with time-varying gains”, In SIAM Journal on Control and Optimization, Pp. 487-507, vol. 6, no.3, 1968

[7] G. Pillonetto and A. Aravkin, “A new kernel-based approach for identification of time-varying linear systems”, IEEE International Workshop on Machine Learning for Signal Processing. pp. 1-6, Reims, 2014

[8] G- Pillonett, F. Dinuzzo, T. Chen, G. De Nicolao and L. Ljung, “Kernel methods in system identification, machine learning and function estimation: A survey”, In Automatica, 50(3), Pp. 657-682, 2014

[9] C.E. Rasmussen and C.K.I. Williams, “Gaussian Processes for Machine Learning”. MIT press, Cambridge, 2006

[10] L. A. Zadeh L.A. “A general theory of linear signal transmission systems”, Journal of the Franklin Institute. Vol. 253, no. 4, p. 293–312, 1952

[11] L.A. Zadeh, “Time-Varying Networks I”, Proceedings of the IRE, 1961

[12] B. Peeters, P. Karkle, M. Pronin and R. Van der Vorst R, “Operational Modal Analysis for in-line flutter assessment during wind tunnel testing”, 15th International Forum on Aeroelasticity and Structural Dynamic, 2011