Application of Eötvös torsion balance measurements to refine the Hungarian geoid model

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1 Introduction

The Hungarian torsion balance data set is unique in the sense of the number of torsion balance stations and its coverage its application in geodesy, especially in the field of gravity field modeling therefore it may contribute to the determination of the high frequency variations of the geoid model and to the refinement of the small-scale features of the field. Primary aim of geoid modeling is to establish connection between heights related to the gravity field of the Earth’s and purely geometrical heights obtained by GNSS techniques. On one hand coverage and resolution of terrestrial gravity field observables due to the sampling theorem limits the recoverable spectral bands of the geoid model on the other hand different type of gravity field quantities have their own signal-to-noise ratio in the spectrum which suggest that gravity field functionals have their maximal signal power in different spectral bands.

Geoid model with full spectral content could be determined by means of combination of the available gravity field related measurements in a proper mathematical frame, which may provide a quantitative estimation and determination of the contribution of Eötvös torsion balance measurements to the different spectral bands of the computed geoid model.

2 Background and research objectives

The first Hungarian geoid models (Biró et al. 2013), the astronomical and astro-gravimetical geoids were computed by the geometrical method of geoid determination which is based on the fact that deflections of the vertical characterize the normal vectors of the geoid model, which could be regarded as a purely geometrical surface hence variation of geoid height between close points could be calculated as the line integral of the vertical deflections. Determination of gravimetric geoid models is based on the functional relations between gravity related quantities. Geoid models computed by gravimetric methods allowed the determination of the detailed structure of gravity field since the increased resolution of the gravimetric data, besides the computed geoid models refer to global geocentric reference field. Establishment of the National GPS Network (OGPSH) enabled to determine geoid model independent from the previous solutions and also to fit gravimetric geoid models to the height system defined by GPS and national normal heights. Development of computational methods, increase of computational facilities and renewal and expansion of data sets related to gravity field modelling, such as global geopotential models based on satellite gravity missions, new type of gravity field observables and the availability of high resolution topographical and lithosphere models foster the determination of high precision geoid models and form the basis of combined geoid modelling techniques. Deployment of
Eötvös torsion balance measurements in gravity field modelling and in determination the fine structure of the geoid was investigated by Völgyesi (2005), Völgyesi et al. (2007), Tóth és Völgyesi (2007). Tóth (2003) has developed the mathematical framework to process gradient and curvature torsion balance measurements simultaneously in geoid determination. Tóth (2008) has conducted test computations to determine geoid model with the combination of directly usable gravity related quantities with a selected subset of horizontal gravity gradients.

Previous geoid models focused to resolve the medium wavelength part of the gravity field accordingly to the utilized gravity data sets and their resolution. The available torsion balance measurements could contribute to the determination and refinement of the high frequency variations of the gravity field through the high resolution of the data set and through the complex information related to near-surface mass anomalies represented by gravity gradients. Determination of the high frequency part of gravity field model could not be interpreted and quantitized alone, hence utilization of torsion balance measurements in gravity field modelling is inseparable from determination of a full-spectral geoid model. In this context the impact of torsion balance measurements on geoid model determination could be assessed. Aim of this study is to compute a combined, high precision geoid model by combining the available terrestrial gravity related measurements, global elevation and global geopotential models. Investigation of the contribution of different gravity related measurements, i.e. GPS-leveling data, gravity anomaly, deflections of the vertical and horizontal gravity gradients measured by Eötvös torsion balance, to the medium and high frequency part of gravity field was accomplished by different mathematical methods. Possibility of using the huge amount of archive Eötvös torsion balance data in the framework of regional and local geoid modeling was examined. Regarding the full-spectral geoid determination questions and problems related to the computation of low and ultra-high components of the gravity field are also investigated.
3 Investigations

The long and medium wavelength parts of the gravity field are available as global geopotential models compiled from different type of gravity related quantities with various mathematical approaches from which coefficients arbitrary gravity field functional can be computed by means of spherical harmonic synthesis. Recent, GRACE and GOCE based, combined and satellite-only models were investigated utilizing the available terrestrial data sets to determine whether which model fits the best at the long and medium wavelength of the gravity field in Hungary, which model form the basis for further geoid modelling. In case of low-resolution satellite-only models the high resolution EGM2008 model (Pavlis et al. 2012) and SRTM3 (Jarvis et al. 2008) surface model were applied to reconstruct the medium, high and ultra high spectral components of the gravity field to mitigate the effect of the omission error which results from the finite resolution of the investigated models (Hirt et al. 2011b). Combining low-resolution satellite-only models with the high resolution EGM2008 model gradually, degree by degree enables to determine their spectral content adequately. Figure 1. demonstrates the result based on the comparison of gravity anomaly data. It was revealed that beside the gravity anomaly vertical deflections, which characterize the small-scale features of the gravity field are capable to indicate the improvements of high degree gravity signal determination based on satellite gravity gradiometry measurements. However it was demonstrated that the terrestrial gravity gradiometry measurements feature the local, high frequency variations of the gravity field and

![Figure 1: Fitting of the investigated models to the gravity anomaly data set, \(l_{tr}\) represents the combination degree of the low-degree satellite only models and the high resolution EGM2008 model. Increasing of dispersion curves indicate the maximal spectral content of the models](image-url)
their signal content does not have overlapping spectral intervals even with the highest resolution global models.

The high frequency variations of the gravitational field originate from the topographical masses located in the close vicinity of the observable points (boundary surface). The high resolution SRTM3 surface model has almost global coverage with homogeneous accuracy for that reason it is widespread applied in forward gravity field modelling to determine the high frequency constituents of the gravitational field. SRTM3 is a surface model due to the data acquisition method therefor heights derived from the model are systematically higher that actual topographic heights. The gravitational difference between surface and topographic models, i.e. the changing in gravity field functionals due to the approximation of the topography with surface model was determined in a test area by interpreting the difference between surface and topographic models as mass surpluses and mass deficiencies with geo-realistic constant density which could be regarded representative for the territory of Hungary. Model differences were transformed to gravitational potential by forward modelling (Nagy et al. 2000) using rectangular prism as volume element. It was found that the gravitational difference between surface and topographic model is insignificant in terms of geoid height, however the changes in the investigated second derivatives are remarkable (Table 1.) at least in case of the test area. It is commensurate with the torsion balance measurements itself (Fig. 2.) in case of low signal strength gravity field as in the territory of Hungary.

The previous investigations related to the main and very fine structures

Figure 2: Gravitational difference between SRTM3 surface model and topography in terms of horizontal gradients of the vertical derivative of the potential
Table 1: Statistics of the effect of model differences on various gravity related quantities

<table>
<thead>
<tr>
<th>statistics</th>
<th>$N$ [cm]</th>
<th>$\delta g$ [mGal]</th>
<th>$V_{yz}$ [E]</th>
<th>$V_{xz}$ [E]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.03</td>
<td>−0.10</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>std.</td>
<td>0.02</td>
<td>0.21</td>
<td>3.6</td>
<td>2.9</td>
</tr>
<tr>
<td>minimum</td>
<td>0.00</td>
<td>−0.69</td>
<td>−12.7</td>
<td>−9.4</td>
</tr>
<tr>
<td>maximum</td>
<td>0.07</td>
<td>0.32</td>
<td>12.0</td>
<td>13.0</td>
</tr>
</tbody>
</table>

of the gravity field. After these consideration the study focuses on the determination of the medium and high frequency components of the gravity field based on the available terrestrial measurements.

Based on synthetic gravity field quantities with different kernel modifications of gradiometric boundary value problems it was investigated that in which frequency band could gravity gradients contribute to gravity field recovery and refinement of gravity field features to assess preliminary the impact of measured horizontal gradients to a combined geoid model in different frequency bands. Investigation concerned integral equations of transforming horizontal gravity gradients to vertical gravity gradient, gravity anomaly and potential. Result of numerical investigations based on synthetic gravity field quantities enables to interpret closed-loop differences between gravity field quantities derived from horizontal gradients via integral transformations and their true value adequately, which were analysed for various wavelength bands both in space and in frequency domain. Computations based on synthetic gradients revealed that gravity gradients could be directly utilized to determine potential by removing the long and medium wavelength parts of the bounded kernel. It was demonstrated that the high frequency component of the geoid model, geoid features up to $\lambda = 30$ km could be determined with sufficient accuracy based on the resolution of the available synthetic gravity field quantities and providing to filter out the low frequency part of the field by kernel modifications.

Convolution integrals arise as the solutions of geodetic boundary value problems result numerical integration over finite number of data cells due to the discreteness of measured gravity field quantities with the kernel function of the convolution represented by a point function referring to the centre of the data cell. In case of singular or rapidly varying kernels the point approximation significantly could influence the computed gravity field quantity. Kernel functions of integral equations transforming horizontal gravity gradients to easily utilizable gravity field quantities, i.e. to anomalous potential,
Figure 3: Transfer function computed from spectra of synthetic geoid heights and geoid height determined from synthetic gravity gradients by integral transform with kernel modifications according to the legend.

to gravity anomaly and to vertical gravity gradient, were investigated in the vicinity of the computational point. Whole-of-cell mean kernel functions were determined by numerical integration based on 2D Gauss–Legendre quadrature (Hirt et al. 2011a). Regarding the investigated kernel functions in case of strongly singular kernel function the difference between whole-of-cell and centre-of-cell kernel could be significant and could reach even 20%. According to the numerical results it is suggested that kernel functions should be computed as whole-of-cell kernels in case of gravity field quantities with low resolution up to $\psi = 1^\circ$ spherical distance (Fig. 4.).

Regional gravity field modelling applies fundamentally two mathematical approaches to transform any measured gravity related parameter to any other gravity field quantity. One of the available combination technique of heterogeneous gravity field quantities is spectral combination method which is based on the spectral forms of integral transformations. The other method applies special base functions to approximate any unknown gravity field parameter from measurements of other gravity field parameters in some optimal way, kernel function produced from the base functions represents the geometrical and analytical structure of the gravity field. These two methods were applied to determine a combined geoid model for Hungary using the available heterogeneous measurements.

Least squares (LS) collocation offers to combine different gravity field functionals in an optimal linear estimation procedure, which was utilized in the remove–compute–restore approach to produce a combined geoid model from the available point gravity field observables. To ensure the minimum mean
Figure 4: Difference between centre-of-cell and whole-of-cell kernel functions which transform horizontal gravity gradient to vertical gravity gradient as a function of the resolution of input data and the distance from computational point

square error approximation the kernel function of the estimation is to be chosen as the covariane function which could be approximated by planar logarithmic covariance model (Forsberg 1987) since the reduction of the measurements with the high degree reference field. It was demonstrated by means of the spectral analysis of variance-covariance matrix (Fig. 5.) that the analytical covariance model fitted to the empirical covariances does not give the adequate description of the analitical-geometrical structure of the field during the practical evaluation of LS collocation in case of horizontal gravity gradients in their original point distribution. The variance-covariance matrix which is influenced by the parameters of the logarithmic covariance model and the spatial distribution of the available measurements is not a positive definite matrix, which violates the assumption of LS collocation on covariance functions. Therefore Eötvös torsion balance measurements in their original, high resolution point distribution can not be utilized in gravity field modelling in the framework of LS collocation, unless a point selection method is applied by such an extent that due to the sampling theorem the measured gravity gradients could not serve to determine the high frequency variations of the gravity field. Combined GPS-leveling–astrogravimetric geoid model was computed from the other available, self-consistent gravity field parameters (GPS-leveling, gravimetric and vertical deflections data sets) which is based on the high resolution EGM2008 gravity field model and the SRTM3 surface model. Spectral analysis of the variance-covariance matrix revealed that gravity anomaly dominates particularly in the combined geoid solution whereas the contribution of GPS-leveling data is not significant.
Spectra of different measured gravity field functionals could be overlapping hence the aim of full-spectral geoid modelling is not only to combine the available gravity field quantities in a common mathematical approach but to estimate a geoid model that takes into consideration the overlap and complement nature of the spectra of the gravity field parameters. Spectral sensitivity of the available gravity field quantities was determined, namely in which spectral band the measurements have their maximal signal power. The estimated spectra of the gravity field observables were controlled in the frequency domain by comparison with the high resolution EGM2008 model as well as with analytical models. Based on covariance analysis of the investigated gravity related quantities in the space domain the dominant wavelength of the data sets as well as the common wavelength bands in which global models and measured terrestrial gravity field quantities describe mutual gravity field structure were determined. Regarding the resolution and coverage of the available gravity field measurements gravity and horizontal gradient data sets could be utilized in the determination of gravity field by convolution integrals computed with Fourier method. Relative weighting of the different data sets for the determination of the combined geoid model, i.e. weights of global gravitational model, gravity and Eötvös torsion balance data were estimated in the spectrum by taking into account the spectral dominance (Kern et al. 2003) of each gravity parameter. Contribution of the different data sets to the combined model could be determined by integrating the weighting function to the kernel function of the respective geodetic boundary value problem. According to the weighting scheme (Fig. 6.), geoid structures from gravity anomalies incorporate features with wavelengths $\lambda = 20\text{ km} - 10\text{ km}$.

Figure 5: Variance-covariance matrix of the available gravity field measurements (order: $N, \Delta g, \xi, \eta, T_{xz}, T_{yz}$). Size of the matrix: $51742 \times 51742$, number of nonzero elements: 5.2%
<table>
<thead>
<tr>
<th>parameter</th>
<th>mean</th>
<th>std.</th>
<th>minimum</th>
<th>maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(EGM2008)</td>
<td>43.465</td>
<td>1.704</td>
<td>39.086</td>
<td>46.728</td>
</tr>
<tr>
<td>N(∆g)</td>
<td>0.001</td>
<td>0.015</td>
<td>−0.062</td>
<td>0.105</td>
</tr>
<tr>
<td>N(Txz, Tyz)</td>
<td>0.000</td>
<td>0.017</td>
<td>−0.054</td>
<td>0.134</td>
</tr>
<tr>
<td>N(RTM)</td>
<td>−0.003</td>
<td>0.011</td>
<td>−0.047</td>
<td>0.111</td>
</tr>
<tr>
<td>N(comb)</td>
<td>43.464</td>
<td>1.708</td>
<td>39.104</td>
<td>46.838</td>
</tr>
</tbody>
</table>

Table 2: Descriptive statistics of the contribution of the different data sets to the combined model in the spectral combination method as well as statistics of the combined gradio-gravimetric model. Units [m]

Figure 6: Spectral weights for spectral combination based on the spectral dominance of the respective data sets

height computed from gravity anomaly varies in a [−6.2 cm, 10.5 cm] interval with a standard deviation of ±1.5 cm. Regarding the weighting function of horizontal gravity gradient these measurements describe the small scale features of the gravity field within wavelengths $\lambda = 10$ km − 2 km. Contribution of gravity gradients to the combined geoid model in areas covered by densely torsion balance sites is merely 1 cm − 2 cm, statistics of gradient contribution for the whole country is summarized in Tabl.2.
4 Summary, original scientific results

Main results of the PhD thesis based on the numerical results conducted for high precision geoid determination, especially the investigations focusing on the utilization of Eötvös torsion balance measurements in the methods of regional gravity field modelling and results concerning the determination and refinement of the high frequency variations of the gravity field from torsion balance measurements are as follows:

1. It was investigated which one of the recent GRACE- and GOCE-based satellite-only as well as combined model fits to the long and medium wavelength part of the gravity field in the territory of Hungary based on the available gravity field quantities. It was demonstrated that besides gravity anomaly even deflections of the vertical which characterize the small-scale features of the gravity field are capable to indicate the improvements of the new-type satellite gravity gradiometry measurements in gravity field modeling. Investigations regarding terrestrial gravity gradients revealed that the second derivatives of the gravitational potential are local gravity field quantities therefore their spectra have no overlapping even with the highest resolution global model.
   Related publication: [2]

2. Gravitational effect of the geometric difference between surface and terrain model was investigated by forward modelling method in a test area. It was shown that the gravitational effect of the model differences are negligible in case of potential, however regarding the second derivatives the changes in the investigated quantities are significant, which are commensurable with these type of measurements.
   Related publication: [1]

3. Based on synthetic gravity field quantities with different kernel modification of gradiometric boundary value problems it was investigated in which frequency band could gravity gradients measured by Eötvös torsion balance contribute to gravity field recovery and refinement of gravity field features. Intercomparison in the spectral domain of analytically and with kernel modification numerically computed gravity field quantities proved that gravity gradients could be utilized directly to determine anomalous potential and by filtering out the low frequency part of the field, geoid features up to $\lambda = 30$ km can be computed with sufficient accuracy taking into account the resolution of the synthetic input gravity data sets.
   Related publication: [5]

4. A combined GPS-leveling–astro-gravimetric geoid model was determined
by LS collocation with planar logarithmic covariance model based on the recent, high resolution EGM2008 gravity field model and SRTM3 digital surface model. It was revealed by means of spectral decomposition of the variance-covariance matrix which was computed from the analytical covariance model fitted to the available gravity field quantities that horizontal gravity gradients in their original, high resolution point distribution can not be utilized in gravity field determination by LS collocation. However reconciliation of the resolution of torsion balance data set with the point distribution of the other available gravity field quantities does not allow the determination of the high frequency part of the gravity field.
Related publication: [4]

5. Spectral dominancy of the available gravity field observables was determined. A combined gradio-gravimetric geoid model was computed covering the full spectrum of the gravity field, which combines different gravity field quantities in a complementary way by utilizing their spectral sensitivity. It was proved that terrestrial horizontal gravity gradients according to the spectral content of the measurements contribute to the determination and refinement of the small-scale features of the gravity field.
Related publication: [3]

Numerical investigations related to the above results were carried out by self-developed MATLAB codes. Gravitational forward modelling in case of huge number of volume elements, i.e. computing RTM gravity field functionals was accomplished by the script TC.f written by R. Forsberg, which cuts down PC evaluation time significantly by using different resolution mass models and applying approximation formulas for the gravitational effect of a volume element.
5 List of publications

SCI publication


Independent citation:


Independent citations:


Independent citation:


Publications in Hungarian


Presentations at international conferences
6th Autumn Seminar on Geodesy for PhD Students, Sopron, 24 October 2011

Szűcs E, Papp G (2012): Effect of the difference between surface and terrain models on gravity field related quantities. EGU2012-2346


Szűcs (2012): Preliminary results on the application of $T_{xz}, T_{yz}$ horizontal gradients in gravity field modeling.
7th Autumn Seminar on Geodesy for PhD Students, Sopron, 12 October 2012


Szűcs E (2013): A combined geoid model for Hungary based on the spectral properties of gravity related data.
EUREF Symposium, 29 - 31 May, 2013, Budapest, Hungary

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VII. Geomatika Szeminárium, Sopron 2010. november 4-5.


Szűcs E, Tóth Gy (2012): A nehézségi erőtér spektrumának vizsgálata különböző típusú adatok felhasználásával.

Szűcs E, Papp G, Benedek J (2014): Földfelszíni mért gradiensek alkalmazhatóságának lehetősége a geoid rövid hullámú összetevőjének pontosításában
References


Völgyesi L, Tóth Gy, Csapó G (2007): Determination of gravity field from