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# Nonparametric identification of linear time-varying systems

Summary of main findings of the PhD thesis of

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# 1. INTRODUCTION

Engineers and scientists look for a reliable mathematical model of the observed phenomenon for understanding, design and control. System identification is a tool which allows them to build high quality models of dynamic systems starting from experimental noisy data.

In modeling and measurement techniques it is commonly assumed that the observed systems are linear time-invariant. This point of view is acceptable as long as the time variations and the nonlinearities of the systems are negligible. However, in some cases this assumption is not satisfied and it leads to a low accuracy of the estimates. In this thesis linear time-varying systems are considered.

## 1.1. OBJECTIVES

The time-varying systems are split into two classes. The first class consists of systems which are inherently time-varying. It means that the time-variations are the natural part of the observed phenomenon and in many cases they cannot be (significantly) controlled. A well-known example for this class of systems is aging.

In the second class, time variations depend on one or more special external variables (in most cases they are the scheduling variables). A good example can be for instance a tower crane, where the cable length can vary at any time resulting in a time varying behavior. The length of the cable is set by the operator of the machine.

This thesis mainly focuses on the first class of the systems, but under some conditions the provided methods can be applied for the second class as well.

A further distinction can be made between the cases, where the time variations follow a periodic behavior or there is no periodicity. The systems with no periodic time-varying behavior are the arbitrary time-varying systems. In this thesis the – general – arbitrary time-varying situation is studied.

The common problem in the above-mentioned application examples is that the system dynamics can change during the measurements. Think of the tower crane

in real operating mode: the cable length (and even the weight of the load) can change several times during a measurement. The challenge is to build accurate models which can track the varying dynamics of these systems, while using as few experiments as possible.

In this thesis, nonparametric models are considered. It is already shown that the linear time-varying systems can be nonparametrically described in the time domain with a two dimensional impulse response function. However, due to the high number of parameters and the underdetermined system of linear equations, it is barely used in practice. Let us take a simple example: a measurement of a time-varying system contains $N$ samples, which are (in time) equidistantly collected. But during the measurement – at these sample times – the system can have N different dynamics (in time domain they can be represented by impulse response functions). If we assume that the length of each instantaneous impulse response function is $L$, then we have $NL$ different parameters to be estimated. On the other hand, we have only $N$ equations (measured samples). Using nonparametric modeling, these equations will have very high degrees of freedom. This means that we have infinitely many solutions, which are equally possible.

As a consequence, time-varying systems cannot be uniquely determined from a single set of input and output signals – unlike in the general case of linear time invariant systems. Due to this fact, the number of possible solutions grows quadratically with the number of samples.

To decrease the degrees of freedom, some user-defined adjustable constraints will be imposed. These will be implemented by using two different approaches. First, a special two dimensional regularization technique is applied. The second implementation technique uses generalized two dimensional smoothing B-splines. In addition to the beneficial effects on the degrees of freedom, the effect of the disturbing noise can be decreased and a possible transient elimination technique will be shown.

Using the proposed methods, high quality models can be built.

# 2. SUMMARY OF OWN RESULTS

In this PhD thesis three different but interrelated nonparametric identification methodologies were developed, implemented and statistically analyzed. Using the proposed methods it is possible to obtain a good quality model of a linear time-varying system based on noisy observations.

The first method applies kernel based regularization (see Section 12.1.1).

The second and the third methods (see Section 12.1.2) are based on B-splines techniques, where a frozen (Section 12.1.2.1) and a non-frozen (Section 12.1.2.2) models can be obtained.

All methodologies have been validated for correctness on simulation examples and also on measurements of a time varying device. The problems were formulated mainly in the time domain.

The main contributions of my research are summarized in the present chapter. Because of lack of space, some of the results were not discussed in this thesis but they are mentioned in this chapter in a distingished section. The references – as the outputs of the research – placed in this chapter refer to own works.

## 2.1. MAIN CONTRIBUTIONS OF THIS THESIS AND SCIENTIFIC STATEMENTS

### 2.1.1. REGULARIZATION TECHNIQUE FOR LTV SYSTEMS

Regularization is of particular interest in the bias-variance trade-off that characterizes model estimation processing noisy observations. The main idea in regularization is the introduction of a penalty term to the least squares cost function. By doing this we are able to put a limit to the model complexity.

The main feature of the regularization method is to punish the model complexity in an advanced way. For noisy and short data records, the regularized least squares approach turns out to be more robust and even slightly more accurate than the standard prediction error method/maximum likelihood approach. The underlying reason for such behavior is that it may be beneficial to allow some bias to be able to reduce the variance. Based on a Bayesian interpretation of the problem, one gets insight in the choice of the regularization matrix.

My contribution is to combine the two dimensional impulse response function of the LTV systems with the regularization technique in a special way.

**Statement 1**. I developed a new method to estimate the two dimensional nonparametric impulse response function of linear time-varying systems using a time domain two-dimensional regularization technique. The technique is discussed in details in Chapters 6-7.

As a consequence, the solution for the estimation problem is a unique solution, contrary to the classical maximum likelihood estimator, where infinitely many solutions are equally possible.

Following the general analysis results of regularization, I determined the main statistical properties of the estimator: expected value, bias, variance, and the mean square error. These are computed in Appendix A.3.1

I verified that – by satisfying Assumption 5.1 and Assumption 5.2 – the proposed method boils down to the maximum a posteriori estimation. This is shown in Appendix A.3.1.

In the target application, where LTV systems are considered, the regularization technique plays an essential role. The aim in this particular case is to reduce the degrees of freedom such that a unique, smooth and stable estimate can be obtained.

In the LTI case, when the measurement has a very good quality – or the measurement is sufficiently long – resulting in a lower variance, the regularization term can be neglected. However in the case of LTV systems, the regularization term is always active. It is needed because all the constraints (smoothness of the impulse responses and time variations, decaying of the impulse responses) are defined here

and they are fundamental in the estimation procedure. These constraints are needed to decrease the degrees of freedom.

**Sub statement 1.1** I proved that under certain conditions – by satisfying Assumption 2.2, Assumption 6.1–Assumption 6.6 – the proposed method reduces the degrees of freedom of the system of linear equations which describe a linear time-varying system by its two dimensional impulse response. The problem formulation is detailed in Section 6.1. The essence of the proposed method is explained in Section 6.4. The derivation of the degrees of freedom is shown in Appendix A.5.

A further issue with time varying systems is that in general the measurement cannot be repeated under the same conditions – due to the time-varying behavior – and therefore all the measured samples must be used for the estimation procedure. It has the consequence that the first impulse responses can be corrupted by the transients. To handle this situation I developed a special method.

**Sub statement 1.2** I proved that using an extension of the proposed technique it is possible to eliminate the effect of the transient – if Assumption 7.1 and Assumption 7.2 are satisfied. The method is explained in details in Section 7.2.4.

A further problem with the proposed two-dimensional regularization is that the memory need grows quadratically with the length of the observation. This means that after some hundreds of samples, the computational limits are reached. I designed an extended estimation method which can tackle this problem.

**Sub statement 1.3** I developed a rapid algorithm to handle large datasets. The proposed method uses a special sliding window technique and the advanced transient elimination method (Sub statement 1.2). This is discussed in details in Section 7.3.

This methodologies presented can be found in Chapter 6–7. The research outputs are [J4], [C3], [S1], [S2].

### 2.1.2. B-SPLINE TECHNIQUE USED FOR LINEAR TIME-VARIANT SYSTEMS

The key idea of this method is that generalized B-splines can be used for double smoothing: over the global and system time. If the parameter variation of the observed system is sufficiently slow, with respect to the system dynamics, we will be able to

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- reduce the disturbing noise by additional smoothing,

- reduce the number of model parameters that need to be stored,

- decrease the effect of the undesired transient.

To do this, two different approaches can be used. The first approach assumes that the LTV system can be described with frozen FRFs. The second approach replaces the (non-frozen) impulse response functions by a smooth B-spline representation.

### 2.1.2.1. THE FROZEN FRF AND IRF APPROACH

In the case of linear slowly time-varying systems it is a common practice to describe the system as a series of time invariant systems, one at each measurement time. B-splines are used to smooth either over the "frozen" frequency response functions or over the impulse response functions. Assuming that the parameters are slowly changing it is possible to use a sliding window technique to estimate an LTI model using the data in the window. The output of this sliding window is either a series of FRFs or IRFs. By placing them next to each other, we get a surface that can be modeled with several methods, like for instance with B-spline based spline fitting.

**Statement 2.** I developed a method to estimate the two dimensional nonparametric frozen transfer function/ frozen impulse response of smooth, linear slowly time-varying systems. The method uses a special sliding window technique combining the ideas of the short Fourier transform and a surface description method based on multidimensional complex B-spline basis functions. The problem formulation is discussed in Chapter 10.

**Sub statement 2.**1 I shown that under certain conditions – by satisfying Assumption 2.1–Assumption 2.2, Assumption 6.1–Assumption 6.6, Assumption 10.1–Assumption 10.3 – the proposed method can estimate the frozen transfer function / frozen impulse response function well. The time domain method to estimate the frozen impulse response function is explained in Section 10.4. The frequency domain method to estimate the frozen transfer function is explained in Section10.3.

**Sub statement 2.2** I developed a two dimensional coefficient reduction method which can be used to improve the signal-to-noise ratio of the obtained frozen estimates. This is discussed in Section 10.3.3 and in Section 10.4.3.

This technique is discussed in Chapter 10. The research outputs are [C1], [C2], [C3], [S1], [S4], [S5].

### 2.1.2.2. THE NON-FROZEN IMPULSE RESPONSE APPROACH

This part presents a new methodology which estimates non-parametrically a two dimensional impulse response function of time-varying systems fully described in the time domain. In this approach we make use of the prior information about the smoothness of the system: the impulse responses and the time variations are smooth. This allows us to use the flexible B-splines such that the classical one dimensional impulse response function is redefined and replaced by a two dimensional B-spline kernel interpretation.

Unlike the frozen impulse response approaches, here smoothed non-frozen impulse responses are considered such that we do not have overlapping effects. Apart from decreasing the effects of the disturbing noise, it is possible to eliminate the effect of the undesired transient terms. Using the proposed method a good model quality can be achieved with a moderated computational time.

**Statement 3.** I developed a new method to describe smooth linear time-varying systems using the over-defined two dimensional nonparametric B-spline impulse responses. The method is discussed in Chapter 9.

In this technique, instead of directly estimating the coefficients of the two dimensional linear time-varying impulse response function, the coefficients of the linear time-varying B-spline kernels are estimated.

**Sub statement 3.1** I proved that under certain conditions – by satisfying Assumption 2.2, Assumption 6.1 – Assumption 6.6 – the proposed method reduces the degrees of freedom of the systems of linear equations describing a linear time-varying system by a two dimensional kernel representation. This is shown in Appendix B.5.

As a consequence, the solution for the estimation problem is a unique solution, contrary to the classical maximum likelihood estimator where infinitely many solutions are equally possible.

**Sub statement 3.2** I proved that using an extension of the proposed technique it is possible to eliminate – if the Assumption 7.1 and Assumption 7.2 are satisfied – the effect of the transient. This is shown in Section 9.6.

**Sub statement 3.3** I determined the main statistical properties of the estimator: expected value, bias, variance, and the mean square error. This is computed in Appendix B.3.

The related publications are [J1] , [C1] , [S1] , [S2]. This technique is discussed in Chapter 9.

# 3.     LIST OF PUBLICATIONS

*Journal papers:*

[J1] Péter Zoltán Csurcsia, Johan Schoukens, István Kollár, John Lataire: "Nonparametric Time Domain Identification of Linear Slowly Time-variant Systems using B-splines," *IEEE Transactions on Instrumentation and Measurement*, volume 64, issue 1, pp. 252-262, 2015

[J2] Péter Zoltán Csurcsia: "Nemlinearitások detektálása multiszinuszos gerjesztéssel," *Elektronet (Hungarian journal)*, volume xxiv, issue 5, pp. 36-40, 2015

[J3] Péter Zoltán Csurcsia: "Static nonlinearity handling using best linear approximation," *Pollack Periodica*, volume 8, issue: 1, pp. 153-165, 2013

[J4] Péter Zoltán Csurcsia, John Lataire: "Nonparametric Estimation of Time-varying Systems Using 2D Regularization," *IEEE Transactions on Instrumentation and Measurement*, 2016, special issue, accepted

*International conference papers:*

[C1] Péter Zoltán Csurcsia, Johan Schoukens, "Nonparametric Estimation of a Time-variant System: An Experimental Study of B-splines and the Regularization Based Smoothing," *IEEE International Instrumentation and Measurement Technology Conference*, pp. 216-221, Pisa, Italy, 2015

[C2] Péter Zoltán Csurcsia, Johan Schoukens, István Kollár: "A first study of using B-splines in nonparametric system identification", *IEEE 8th International Symposium on Intelligent Signal Processing*, pp. 87-92, Funchal, Portugal, 2013

[C3] Péter Zoltán Csurcsia, Johan Schoukens, István Kollár: "Identification of Time-varying Systems using a Two-dimensional B-spline Algorithm," *IEEE International Instrumentation and Measurement Technology Conference*, pp. 1056-1061, Graz, Austria, 2012

[C4] Péter Zoltán Csurcsia, Antal Bánovics, István Kollár: "Digital oscilloscope displays results together with confidence bounds," *IEEE International Symposium on Intelligent Signal Processing*, pp. 81-86, Budapest, Hungary, 2009

### PhD symposium papers:

[P1] Péter Zoltán Csurcsia, István Kollár, "Verification of biographical records", *8th International PhD & DLA Symposium,* Pécs, Hungary, 2012

[P2] Péter Zoltán Csurcsia: "Approximation of Nonlinear Systems using the Best Linear Approximation," *Seventh International PhD & DLA Symposium*, Pécs, Hungary, 2012

[P3] Péter Zoltán Csurcsia: "Basics of Best Linear Approximation," *19th PhD Minisymposium*, Budapest, Hungary, 2012

[P4] Péter Zoltán Csurcsia: "Bias of model fitting by Least Squares Estimation," *18th PhD Minisymposium*, Budapest, Hungary, 2011

### Short communications

These publications have been used for professional research workshops – as supplements for the oral presentations/posters. These European (European Research Network on System Identification) and Belgian-Dutch (BENELUX) workshops are important for the system identification society.

[S1] Péter Zoltán Csurcsia, Johan Schoukens, John Lataire: "Estimation of Linear Time Varying Systems using 2D Regularization and B-splines," *Workshop of the European Research Network on System Identification*, Varberg, Sweden, 2015

[S2] Péter Zoltán Csurcsia, Johan Schoukens: "Identification of Linear Time Varying Systems using 2D Regularization", *34th Benelux Meeting on Systems and Control*, Lommel, Belgium, 2015

[S3] Péter Zoltán Csurcsia, Johan Schoukens, István Kollár: "Identification of Linear Time Varying Systems in the Time Domain using B-Splines," *33rd Benelux Meeting on Systems and Control*, Heijden, The Netherlands, 2014

[S4] Péter Zoltán Csurcsia, Johan Schoukens, István Kollár: "An application example for the B-spline based nonparametric system identification," *32nd Benelux Meeting on Systems and Control*, Houffalize, Belgium, 2013

[S5] Péter Zoltán Csurcsia, Johan Schoukens, István Kollár: "Two dimensional spline interpolation for slowly time-varying systems," *31st Benelux Meeting on Systems and Control*, Delft, The Netherlands, 2012

[S6] Péter Zoltán Csurcsia, Johan Schoukens, István Kollár: "B-spline based nonparametric system identification," *Workshop of the European Research Network on System Identification*, 2012

***Miscellaneous***

[M1]Péter Zoltán Csurcsia (*editor*), Tamás Demián, Róbert Galambos, Péter Marx, Vilmos Pálfi, *Proceedings of the 19$^{th}$ PhD Minisymposium*, Budapest, Hungary, 2012

[M2]Péter Zoltán Csurcsia: "Bibliográfiai adatok hitelesítése a Magyar Tudományos Művek Tárában," *MSc thesis*, 2010

[M3]Péter Zoltán Csurcsia: "A BMF villamosmérnöki hallgatók eredményei," MTA Pedagógiai Bizottság (Hungarian Academy of Sciences),  a "Múlt-jelen-jövő a szakmódszertanok tükrében" *conference presentation*, 2009

[M4]Péter Zoltán Csurcsia: "A Budapesti Műszaki Főiskola elsőgenerációs kredites képzésben résztvevő villamosmérnök hallgatók eredményeinek analizálása," *Technical teacher's thesis*, 2008

[M5]Péter Zoltán Csurcsia: "Digitális függvénygenerátor megvalósítása PXI buszrendszeren," *BEng thesis*, 2008

[M6]Péter Zoltán Csurcsia: "AD2175 Digitális jelfeldolgozó processzor alkalmazása a gyakorlatban," *lecture booklet*, 2008

[M7]Péter Zoltán Csurcsia: "A BMF villamosmérnök hallgatók eredményeinek analizálása," *long paper*, first price at the National Students' Scholarly Circles, 2007