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Efficient test of analog to digital converters with parameter estimation of the excitation signal

Ph.D. thesis booklet

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Background and objectives

Digital devices has become part of everyday life in the last few decades. These devices process signals quantized in both amplitude and time domain. This conversion is done by analog to digital converters. Thus, the quality of the converted signal depends strongly on the imperfections of the ADC used. Testing the converters is an important task in measurement technology. For example, correct observation of outer signals in a safety critical system is essential. Several different realizations of A/D converters exist, each has its own advantages and disadvantages.

To compare ADCs of different types an architecture independent description of properties is required. The characteristics of an ideal ADC is a step-wise function: the transition levels are located equidistantly in the full scale range of the converter. A widely used description of the errors of an ADC is its characteristic's deviation (linear and nonlinear) from the ideal quantizer. Such description requires the measurement of transition levels and determination of the ADC characteristics. One of the simplest and mostly used methods is the so-called sine wave histogram test (see [3], [9]). This method estimates the transition levels of the ADC by comparing the histogram of the quantized sine wave to the probability density function of a pure sine. The statistical properties of the estimation depend strongly on the relation between the sampling and the signal frequencies: bad parameter settings leads to incorrect results independently from the ADC's quality. This frequency relation defines the number of periods (J) in the record for a given number of samples (N):

$$\frac{f_x}{f_s} = \frac{J}{N}. \tag{1}$$

The model of the excitation signal used in the test is the following:

$$x(k) = C + A\cos\left(\frac{2\pi Jk}{N}\right) + B\sin\left(\frac{2\pi Jk}{N}\right). \tag{2}$$

The IEEE standard for ADC testing [9] defines strict conditions for the frequency settings: the sampling has to be coherent (thus J has to be an integer value) and the number of periods and number of samples have to be relative primes. Dissatisfaction of the former condition results in biased estimation of the transition levels due to the presence of additive samples from the fraction period which appear in some of the code bins and the histogram is thus distorted. The latter condition ensures uniform distribution of the samples in the phase space which minimizes the distance between two adjacent samples, thus the transition levels can be estimated with the best precision. Equation (1) shows that the relation between J and N depends on the signal and sampling frequencies. The correct nominal value of these quantities usually do not ensure undistorted test results due to the imperfections of the ADC and signal generator: the frequencies might differ from their set value.

Further test methods of ADCs require precise knowledge of all signal parameters. These can be estimated using the quantized data by the four parameters sine wave fit method proposed by the standard [9]. However, the computational costs of the algorithm increases strongly with the number of samples: the iterative Gauss-Newton method requires $20N$ operations in every iteration (thorough testing of high resolution ADCs requires a few million samples). Furthermore, its statistical properties are sensitive to overdrive of the ADC and to the presence of harmonic components in the excitation signal. Some of the disadvantages can be handled by preprocessing the signal (see [8] and [10]), but most of the error sources will inevitably harm the results.

Consequently, users will face some difficulties during the application of standard methods: there are no standard methods to recognize bad parameters settings and correct signal parameters themselves still do not ensure precise estimation of the sine parameters.

The two key questions in this thesis are the following:

- Is it possible to check the satisfaction of the conditions of the histogram test using only the measurement record, without any a priori information?

- Is it possible to estimate the signal parameters with less computational requirements and less sensitivity to harmonic distortion?

Main results

Problems with the estimation of signal parameters presented in the previous section can be avoided if the fit is done in the frequency domain. For this purpose, we first window the data with the three-terms Blackman-Harris window (see [2] and [6]). This windows is known for its high rejection (the level of the highest sidelobe is -68.7 dB), thus the information about the signal parameters is compressed around the sine and DC frequencies. The estimation method minimizes the least-squares cost function:

$$\mathrm{CF} = \sum_{k=1}^{n} |e(k)|^2. \tag{3}$$

The kth element of the vector of residuals is:

$$e(k) = \hat{X}_{\mathrm{BH}}(k) - X_{\mathrm{BH}}(k) \tag{4}$$

In the above expression $X_{\mathrm{BH}(k)}$ is the discrete time Fourier transform of the quantized and windowed signal and $\hat{X}_{\mathrm{BH}}(k)$ is its estimated value with respect to signal parameters $\hat{A}, \hat{B}, \hat{C}$ and $\hat{J}$ (see equation (2)). Since the information is strongly concentrated in the frequency domain, only a few samples are needed for the determination of the parameters (5 samples for the sine part and 3 samples for the DC part of the signal). This results in a significant decrease of computational costs: the FFT of a signal of length N requires $(N \cdot \log_2(N))$ operations and the demand of the LS fit is around 320 operations per iteration. In addition, the solution minimizes the negative effects of harmonic components due to the low sidelobes of the Blackman-Harris window. Conversely, the original method requires $20N$ operations in one iteration and the harmonic components can not be separated from the sine wave in the time domain, thus their negative effect on the result of the estimation is inevitable. The speed of the methods were also compared in measurements, the results confirmed the hypothesis. The statistical properties of the frequency domain least squares estimator were studied both theoretically and by simulations. If the standard deviation of the additive noise is σ_n, the covariance matrix (Σ) of the estimator can be determined using the Jennrich-theorem [1]:

$$\underline{\underline{\Sigma}} = \sigma_n^2 \cdot (\underline{\underline{X}}^T \underline{\underline{X}})^{-1}. \tag{5}$$

where $\underline{\underline{X}}$ is the so-called Jacobian matrix (the matrix of the derivatives):

$$\underline{\underline{X}} = \frac{\partial \underline{e}}{\partial \underline{p}^T}, \text{ ahol } p^T = [A, B, C, J]. \tag{6}$$

Since the estimator of the number of periods ($\hat{J}$ is quite important in the verification of parameter settings for the ADC test, its variance was studied in further ways. The expression of the variance is the following using the Jennrich theorem:

$$\sigma_{\hat{J}}^2 = \frac{6\sigma^2}{R^2 \cdot \pi^2 \cdot N}. \tag{7}$$

where $R = \sqrt{A^2 + B^2}$ is the amplitude of the sine wave. The above formula was verified in simulations, furthermore the statistical properties of the time and frequency domain least squares methods were compared in many different situations. Results showed that the precision of the two estimators are almost the same (despite the reduction in the number of samples used) when the effect of harmonic distortion is negligible. Moreover, the frequency domain method provided more precise results in the presence of harmonic components.

Using the estimator of the number of periods to verify the test parameters is not possible if the effect of the estimation error is significant in the results of the histogram test. Unbiased, minimum variance estimation of the transition levels requires coherent sampling and relative prime condition between J and N. Carbone and Chiorboli studied the effect of quasi-coherent sampling in [7] and the following bound was defined for the deviation in the number of periods (see also [4]):

$$|\Delta J| \leq \frac{1}{2N}. \tag{8}$$

The estimator, $\hat{J}$ can be used if the effect of the estimation error is negligible. Since the error is normally distributed, this depends on the relation between $|\Delta J|$ and $\sigma_{\hat{J}}$, where the latter is the standard deviation of the error. The value of a Gaussian distributed probability variable is within $\pm 3\sigma$ with 99.7% probability. Assuming the $3\sigma_{\hat{J}}$ error as a worst case, the noise level was determined for which the maximum of the error is greater then the (8) bound:

$$3\sigma_{\hat{J}} \geq \frac{1}{2N}. \tag{9}$$

The expression of the corresponding noise level which would be too high is the following:

$$\sigma_n \geq \frac{R\pi}{\sqrt{24N}}. \tag{10}$$

Assuming a slightly overdriven, bipolar ADC ($2^{b-1} \leq R \leq 1.2 \cdot 2^{b-1}$) and a reasonably chosen record length N with respect to the number of bits of the converter, the above expressed value of the noise level would be too high for histogram testing. For example, testing an ADC of 16 bits with an average 32 samples per code bins the value of σ_n would be 14.51 LSB, so the corresponding value of the signal to noise ratio would be 64 dB. That level of noise is too high for histogram testing, precise estimation of the transition levels would be almost impossible. However, if the noise level is proper for ADC testing (e.g. $\sigma \leq$ LSB), the estimation error is much lower than the bound defined in equation (8). Thus, the algorithm is able to recognize the excitation's bad parameter settings.

When the original record fails to fulfill the conditions of the histogram test, the algorithm can identify the suitable parts of the measurement. In this case the method truncates the record to satisfy coherency and relative prime conditions. Then the histogram test can be performed with the new, truncated record and the statistical properties of the estimator are not harmed. If the length of the coherent part is too short compared to the original measurement, a new signal frequency is proposed with which the test can be repeated.

New scientific results

Statement I.

I have proven that in the case of measuring a sine wave, its parameters can be estimated by applying the three-term Blackman-Harris window function and using only 5+3 samples from its discrete Fourier transform around the sine and DC frequencies. The estimator is much less sensitive to harmonic components than the original, time domain method due to the high rejection of the window function. I developed the DFT based, iterative estimation algorithm. Because of the compression of information in the frequency domain the new method requires only a ($\sim N \cdot \log_2 N$)) total number of operations instead of the original algorithm's $\sim 20N$ operations in every iteration. ([PV1], [PV2], [PV6])

Statement II.

I have proven that in case the SNR of the measurement is adequate for histogram testing (if $\sigma \leq$ LSB, then SNR $> \log_{10} 2^{2b-3}$) the measured sine wave itself provide sufficient information about the fulfillment of the coherency and relative prime conditions, no other source of information is required.

- I developed the method for checking the requirements.
- If the record fails to meet the conditions, the algorithm is able to modify the original record: determine the necessary amount of truncation or adjustment in the value of the signal frequency.

([PV2], [PV3], [PV5])

Utilization of results and outlook

The algorithms are part of an ADC test toolbox developed at the department, implemented in both Matlab and LabVIEW environment. The two platforms have different advantages: in Matlab language algorithm development and testing are well supported, while the LabVIEW system design software makes it easy to collect and process measurement data on the same computer. These toolboxes provide opportunity for processing real measurement data, which were also used for testing the new algorithms. These software are freely available on the internet from the project site [5]. The standard methods for ADC testing are implemented (ADC characterization, sine parameter estimation, spectral analysis of the input signal). Beyond the standard methods the following algorithms based on the results of this thesis are used to support efficient ADC testing:

- Quality analysis of the record: the software is able to check the fulfillment of the coherency and relative prime conditions.
- The algorithm is able to identify the coherent part of a noncoherently sampled sine wave.
- If the coherent part is too short for precise test results, a new signal frequency is proposed with which the measurement can be repeated.

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