Budapest University of Technology and Economics
Department of Mechanical Engineering

Stability Analysis of Periodic Delay-Differential Equations Modeling Machine Tool Chatter

PhD Theses

Tamás Insperger

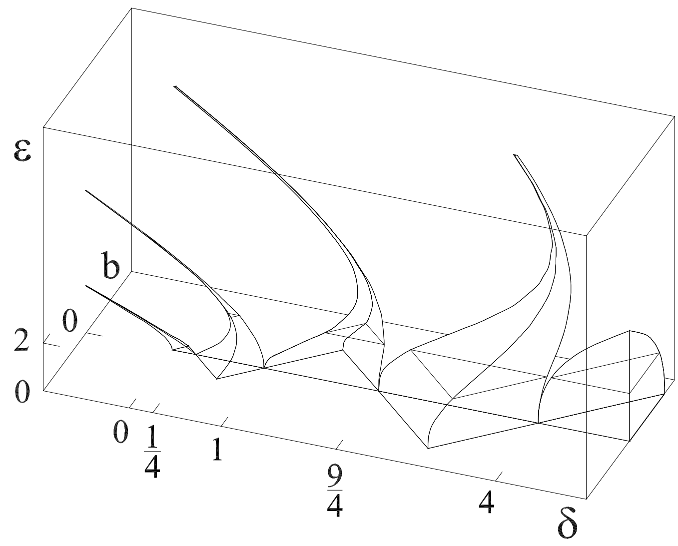
Budapest, 2002

Thesis 1

The closed form 3 dimensional stability chart for the delayed Mathieu equation

$$\ddot{x}(t) + (\delta + \varepsilon \cos t)x(t) = bx(t - 2\pi)$$

was constructed and proved. It was shown analytically, that the boundary curves in the plane $(\delta,\, b)$ are lines for any ε. The number of instabilities was also determined in the domains separated by these lines. At the boundaries with slope $+1$, a characteristic multiplier crosses the unit circle at $+1$, presenting a 2π periodic motion. At the boundaries with slope -1, a characteristic multiplier crosses the unit circle at -1, presenting a 4π periodic motion.



It was also proved, that the damped delayed Mathieu equation also have linear boundary curves in the plane $(\delta,\, b)$ for the period one and period two bifurcations, but secondary Hopf bifurcations may also exist along some non-linear stability boundaries.

Thesis 2

Fargue's theorem (1973) was applied to transform the linear periodic RFDE

$$\ddot{x}(t) + b_0\dot{x}(t) + c_0(t)x(t) = c_1(t)\int_{-\infty}^{0} w_n(\vartheta)x(t+\vartheta)\mathrm{d}\vartheta\,, \quad c_{0,1}(t+T) = c_{0,1}(t)$$

with the weight function

$$w_n(\vartheta) = (-1)^n \frac{n^{n+1}}{\tau^{n+1}n!}\,\vartheta^n\,\mathrm{e}^{n\vartheta/\tau}$$

into the $(n+3)$ dimensional ODE of the form

$$\dot{\mathbf{y}}(t) = \mathbf{A}(t)\mathbf{y}(t)\,.$$

The Floquet transition matrix was determined by piecewise constant approximation of the coefficient matrix $\boldsymbol{A}(t)$. The method was used for approximating RDDEs with finite dimensional ODEs. A time scale transformation was introduced, that make the approximation numerically more effective.

Approximate stability charts for the periodic RDDE (the damped delayed Mathieu equation)

$$\ddot{x}(t) + b_0\dot{x}(t) + c_0(t)x(t) = c_1 x(t-\tau)\,, \quad c_0(t) = c_{0\,\delta} + c_{0\,\varepsilon}\cos(2\pi t/T)$$

were constructed.

Thesis 3

An efficient new method was introduced for the stability investigation of general linear periodic delay-differential equations of the form

$$\dot{\mathbf{x}}(t) = \int_{-\sigma}^{0} \mathrm{d}_\vartheta \boldsymbol{\eta}(\vartheta,t)\mathbf{x}(t+\vartheta), \quad \boldsymbol{\eta}(\vartheta,t+T) = \boldsymbol{\eta}(\vartheta,t)\,.$$

The efficiency of this so-called semi-discretization method was compared to that of full discretization in the time domain. The main steps of the method were listed, an algorithm was presented and also a proof for the convergence was given. The method was applied for the delayed Mathieu equation with various distributed and discrete delays. A range of intriguing stability charts were plotted for parametrically excited delayed oscillators. It was shown by examples, that the semi-discretization method is also more effective than the Fargue-type approximation.

Thesis 4

The dynamic behavior of milling process was investigated. A range of stability charts were constructed that shows the transition between turning and highly interrupted milling through partial immersion milling operations as intermediate levels. Via localization of the relevant characteristic multipliers, the bifurcation types were identified. It was shown, that in addition to secondary Hopf bifurcation, period two bifurcation is also a typical way of stability loss.

The transition between up-milling, full immersion milling and down-milling was investigated. It was shown, that up-milling operations have different stability properties than the down-milling operations with the same immersion. The effect of backward cutting was detected as an explanation for the intriguing stability lobes of the full immersion milling. The results were experimentally verified.

The vibration frequencies during milling operation were identified. In addition to the two types of chatter frequencies (the Hopf type frequencies or the period two type frequencies), the frequencies caused by the tooth pass excitation effect and the natural frequency of the tool were explained according to the theoretical model and also identified experimentally in the corresponding vibration signals.

Thesis 5

The dynamic behavior of turning process with varying spindle speed was investigated. The connection between spindle speed variation and the resulted time varying delay in the governing equation was determined. Three types of spindle modulation was investigated, the cosine, the increasing saw and the decreasing saw.

A range of stability charts were constructed for 10% modulation amplitude. It was shown, that the stability properties improve for low modulation period, while for high modulation period, the system can be considered quasi-autonomous, and the charts converge to the ones of the conventional turning process.

It was shown, that the stability properties improve for low mean spindle speed only, and the spindle speed variation is not an effective way of chatter suppression for high-speed cutting.

It was shown, that in addition to secondary Hopf and period two bifurcations, period one bifurcation is also a typical way of loss of stability.