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Parameter Estimation in Electromagnetic Devices by the Multilayered Medium and Model-Based Approach

PhD theses

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1. PREFACE

The sensorless principle is becoming more and more popular for improving the cost effectiveness and reliability of systems which utilize electromagnetic actuators. The idea is that certain physical parameters of the device, which parameters otherwise could not be measured or with dedicated costly sensors, are estimated under normal operation by using a model of the electromagnetic device and measuring its alternative input and output quantities.

The main focus of my research was initially on studying the sensorless principle considering linear electromagnetic actuators, i.e., creating improved methods for estimating the position of its moving part (spool), its external load and thermal state. Typical fields of applications of such devices include switching (contactor) and flow controlling (valve) purposes in embedded systems, which have strict limitations in computational resources. Therefore, the low complexity and the computational load were key requirements along with accuracy and reliability towards the new models and methods.

In the first part of my research, I developed methods for estimating the position of the spool, the external load and the electrical resistance of the coil at a low model complexity and computational load; thus, meeting the special requirements of embedded systems. Compared to using a thermal model, the estimation of the resistance is computationally more economical if it is based on an electrical model. Moreover, the electrical resistance of the coil can provide some information about the internal thermal state of the actuator, i.e., about the average temperature of the coil; however, it is not suitable for in depth thermal analyses. For the estimation of the position of the spool, a variety of approaches already exist in literature; although the simultaneous estimation of the position and the external load is still under research. Considering flow and pressure controlling applications, the actuator can be subject to a time varying load which has an influence on the necessary drive current. Therefore, the estimation of the external load enables to save force or pressure transducers from the system and also enables to reduce the amount of the drive current; thus, improving the cost effectiveness and efficiency of the system. Altogether, the sensorless methods have to be compatible with the PWM (pulse width modulation) technique, which is the most popular methods for driving electromagnetic devices; and have to ensure that the device fulfills its original actuating roles. My corresponding research results can be found in [di1], [di4-5] and in the I. Thesis.

During normal operation the parameters of electromagnetic devices can change. Because the sensorless principle relies on a model of the system, the continuous identification of the parameters of the model is important to achieve accurate and reliable estimation. Therefore, the knowledge of the internal thermal state of the device might be necessary because several parameters of the model, especially the electrical resistance of the coil, significantly depend on the temperature. Additionally, an in depth thermal model is also of great use at the design, optimization and analysis (localization of hot-spots) of electromagnetic devices. Due to the reasons above, the second part of my PhD research aimed at creating an elaborate thermal model of linear electromagnetic actuators, which model could be of a general use and could give a detailed description of the distribution of the thermal field. Therefore, an analytical solution of the thermal field (diffusion equation) was preferred, because analytical solutions give direct insight into the physical processes and show how the behavior of the system depends on the parameters, which properties are advantageous during design and optimization. Moreover, analytical solutions are mostly closed-from, compact solutions therefore preferable in embedded applications which have strict resource limitations. Considering the applied modeling approach, the so called multilayered medium approach was applied for creating the thermal model, because linear electromagnetic devices, especially the winding, can be considered as a multilayered medium that is layered along the radial direction. This part of my research can be succinctly summarized as the analytical solution of the diffusion equation in multilayered media. It has to be highlighted that because of the generality of the studied problem, my results are valid in every field which is governed by the diffusion equation, e.g. to electrostatics and heat conduction. Furthermore, the results are applicable not only to the modeling of electromagnetic actuators but to a wider field of research and application, where the multilayered medium approach holds, e.g., for the analysis of composite materials. My corresponding research results are to be found in [di2-3] and in the Theses II.-III.

2. SCOPES AND APPLIED METHODS

2.1 Operation of a linear electromagnetic actuator

The linear electromagnetic actuator is a single or two phase linear motor, i.e., the moving part (spool) follows a linear, limited motion. Typical fields of its application include switching operation (contactor, relay) [1-2] and flow control (valve) [3-4]. A detailed illustration of a single phase electromagnetic valve is provided in the left side of Fig.1. The operation of an electromagnetic actuator (valve) is briefly as follows.

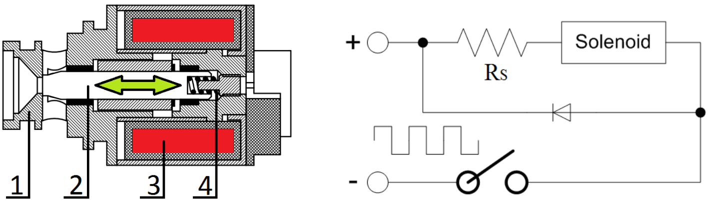

Fig. 1: Structure of an electromagnetic valve (left side) and PWM drive (right side): 1-valve, 2-spool, 3-electric coil, 4-return spring.

The terminal voltage that is applied to the coil forces electrical current through the coil, which current generates a magnetic field that exerts an attractive magnetic force to the ferromagnetic spool. Thus, the spool is displaced (valve opens) enabling the controlled medium (gas, fluid) to flow through the valve. The size of the orifice, i.e., the resistance to the flow is set by the position of the spool. The attractive magnetic force and the external load, latter is caused by the pressure of the controlled medium, is counteracted by the valve return spring which pushes the spool out from the housing and keeps the valve closed. Based on the above description, three unique subsystems can be identified which are the electrical, magnetic (electromagnetic) and mechanical subsystems, respectively. Electromagnetic devices are usually driven by the PWM (pulse width modulation) technique, the simplest configuration of which is illustrated on the right side of Fig. 1.

2.1.1 The sensorless principle

The idea is that a certain output quantity of the system is not measured directly with a dedicated sensor but estimated (if possible) on the basis of an elaborate model of the system and by measuring alternative input and output quantities of the system. Considering electromagnetic valves, the position of the spool is estimated from a model of the electromagnetic subsystem and by measuring its electrical signals, e.g., current and voltage. The main advantage of the sensorless principle is that costly external sensors, e.g. a position sensor, can be saved along with its mechanical and hardware layout.

Due to the fact that linear electromagnetic actuators are most commonly used in an embedded environment, the hardware and software, which are necessary for driving the device and applying the sensorless approach, have to be minimal for the sake of cost effectiveness. Additionally, the sensorless methods have to be compatible with the PWM technique and have to ensure that the actuator fulfills its original actuating roles. The sensorless methods for estimating the mechanical parameters (position, velocity, force) exploit the dependence of the electromagnetic subsystem (electrical impedance) on the position of the spool, e.g., the change in the inductance (electrical impedance) [2-3], [5] flux linkage [4], [6]. For this reason, it is necessary to identify the electromagnetic subsystem and measure and to compute the necessary parameters. Since the inductance is a dynamic quantity and it is related to a specific location in the magnetization curve, the inductance is computed from the system's response that is given to a dedicated scan signal. This scan signal can be, for example, the PWM itself [5] or a sinusoidal component in the input voltage [7]. Contrarily, the flux is an integral quantity and its measurement may require continuous integration and the use of auxiliary windings [4], which are disadvantageous from the perspective of cost effectiveness. The use of complex electromagnetic models and sophisticated control schemes is also an alternative approach [6]; however, these are less compatible with the special requirements of low complexity and computational load.

In flow control applications the flow of the controlled medium can exert an external force on the actuator, which force varies in time. The parallel estimation of the position and, especially the external load, in linear electromagnetic actuators is still an open issue in the technical literature. During my research, the minimization of the resource needs of the developable sensorless methods (optimization to embedded systems) was a key principle; therefore, they had to be usable with the simplest PWM drive hardware, which is illustrated in Fig. 1.

2.1.2 Variations in the parameters of the model

According to section 2.1.1, the sensorless principle is based on a model of the electromagnetic subsystem and on measuring the electrical signals. Therefore, the electrical resistance of the coil (further on resistance) plays an important role and directly influences the accuracy of the estimates of the mechanical parameters. As the resistance can change significantly during normal operation due to internal heat generation and to changes in the ambient temperature (40% for 100 °C change), the estimation of the mechanical quantities becomes erroneous unless the resistance is measured continuously. Besides, the resistance can provide information about the thermal state of the coil, e.g., about its average temperature. Considering the reasons above, the estimation of the resistance is required; and preferred from an electrical model under PWM drive conditions.

The problem that was considered during my research for estimating the resistance is illustrated in Fig. 2, where the device is represented as R_s and L_s. During the on (left side) and off (middle) periods of the PWM cycles, the current of the coil flows through a different circuit, the overall resistance of which can be different due to the resistance of the not ideal electrical components, e.g., cables, connections. The overall resistances of the different circuits add to the resistance of the coil thus produce an error at the estimation of the resistance, which error depends on the duty ratio. This problem is generalized on the right side of Fig. 2 which illustrates that the system changes its time constant (L/R) during the on and off periods of the PWM cycles. If considering the difference between the overall resistances of the PWM periods, then the bias in the estimate of the resistance is reduced. I studied both the static (steady-state) and transient cases of the PWM, because in certain situations the system undergoes a long transient period due to changes in the duty ratio; however, a refreshed value of the resistance is needed.

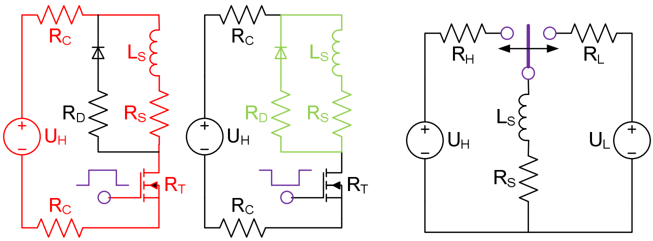

Fig. 2: Non-ideal PWM drive configuration and its generalization.

2.2 Model of multilayered media and an analytical solution to the diffusion equation

The model of the multilayered medium is illustrated in Fig. 3 for slabs and for two dimensional *(x,y)* steady diffusion. The multilayered medium is represented as the group (array) of #*N* linear slabs with equal height *H* but with different *L* length, where the slabs connect to each other at the interfaces (junctions). The layers are piecewise homogeneous, isotropic and are described by the *k* parameter of diffusivity. The diffusion takes place only along the *x* transverse (in the direction perpendicular to the layers) and in the *y* longitudinal (in the direction parallel to the layers) coordinates. The layers are subjected to the homogeneous (excitation is zero) and non-homogeneous (excitation is not zero) boundary conditions for which the following three types are defined:

- Of the first kind: the potential (*T, U*) is prescribed for the respective surface,
- Of the second kind: the flux is prescribed for the respective surface,
- Of the third kind: the flux is proportional to the potential difference between the ambient and the respective surface by the *h* convection coefficient.

According to Fig. 3, the surfaces of the layers are subjected to boundary conditions of the third kind with the respective ambient temperatures and convection coefficients; however, a boundary condition could be also of the first or the second kind with the *h* coefficient set to infinity or zero. The subscripts *u, b, l, r* refer to the upper, bottom, left-hand side and right-hand side surfaces, respectively. The transverse boundary conditions are the boundary conditions that are defined on the surfaces the normal vector of which points into the x direction, e.g., $T_{1l}(y)$. The longitudinal boundary conditions are the boundary conditions that are defined on the surfaces the normal vector of which points into the y direction, e.g., $T_{1u}(x)$. Additionally, the transverse flux inhomogenity is defined at the junction of two layer; and each layer involves the $q(x,y)$ internal energy generation (dissipation).The solution of the potential field in the layers is sought in the steady-state under the prescribed homogeneous, non-homogeneous boundary conditions and internal energy generation.

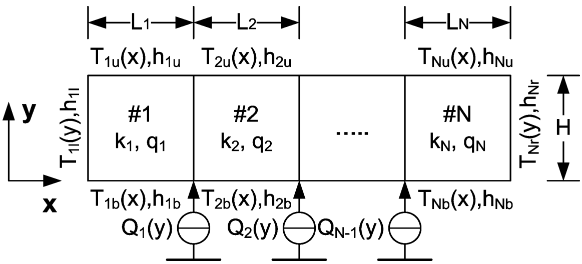

Fig. 3: The model of the multilayered medium.

2.2.1 Analytical solution to the heat equation and the problem of non-homogenous longitudinal boundary conditions

If there is internal energy generation, the steady diffusion (conduction of heat in solids) is governed by the Poisson equation which can be expressed as (1) in Descartes coordinates for a homogeneous, isotropic slab. The term $T(x,y)$ denotes the potential field. If there is no internal energy generation, i.e. the term $q(x,y)$ is zero, then (1) turns out to be the Laplace equation [8], [10]

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = -\frac{q(x,y)}{k}. \tag{1}$$

A popular method for deriving an analytical solution to the Laplace equation is the method of separation of variables (SOV), as a result of which, the solution is established as the infinite sum of functions which are the products of the so called eigenfunctions. For a slab-shaped region that is subjected to homogenous longitudinal and non-homogeneous transverse boundary conditions, the analytical solution to (1) can be expressed as (2) by SOV [8]. It has to be noted that the method of SOV is usually not applicable to the Poisson equation because the underlying partial differential equation can not be separated because of the internal energy generating term

$$\begin{aligned} T(x,y) &= \sum_{n=1}^{\infty} X_n(x,\lambda_n) Y_n(y,\lambda_n) = \\ &= \sum_{n=1}^{\infty} \left(a_n e^{\lambda_n x} + b_n e^{-\lambda_n x}\right)\left(c_n \sin(\lambda_n y) + d_n \cos(\lambda_n y)\right). \end{aligned} \tag{2}$$

The functions $X(x)$ and $Y(y)$ are the eigenfunctions which belong to the x and y coordinates, the term λ_n is the eigenvalue which is associated with the n^{th} eigenfunctions and the terms a_n, b_n, c_n, d_n are specific constants of the eigenfunctions. In theory, the solution involves an infinite number of eigenfunctions but in practical applications the series are truncated. In case of (2), the eigenvalues are the solutions to the transcendental equation that is constructed from the homogeneous (in this example, the longitudinal) boundary conditions. The complexity of the transcendental equation thus the necessary computational resources to obtain the eigenvalues depend on the homogeneous boundary conditions. This step of obtaining the eigenvalues is referred to as the eigenvalue problem in the related technical literature. The c_n, d_n coefficients are computed from the longitudinal, and the a_n, b_n coefficients are computed from the non-homogeneous transverse boundary conditions by exploiting the orthogonality of the $Y(y)$ eigenfunctions, which is expressed in (3)

$$\int_H Y_n(y,\lambda_n) Y_m(y,\lambda_m) \cdot dy = \begin{cases} 0 & if \quad n \neq m, \\ N_n^2 & if \quad n = m. \end{cases} \tag{3}$$

The superposition theorem is often exploited to simplify the diffusion problem in case of linear diffusion, as it enables to consider and to study the effects of the excitations of the system separately [8], [10]. Since the diffusivity and the boundary conditions are discontinuous in multilayered media (differ layer by layer), the diffusion problem is usually decomposed into separate single layer problems; and these single layer solutions are then linked to each other through the interface conditions, e.g., the continuity of temperature and flux.

In case of homogeneous longitudinal boundary conditions, the eigenvalue problem that is associated with analytical solutions becomes straightforward in multilayered media [8]. However, if a longitudinal boundary condition is non-homogeneous, the eigenvalue problem complicates [8] because the homogeneous boundary conditions, which are necessary to construct the transcendental equation, are to be derived from the interface conditions, which are non-homogeneous and couple the solutions in layers to each other. A literature survey has shown that the great majority of related publications do not consider the non-homogeneous longitudinal boundary conditions in the analytical solution or consider only under special considerations. Furthermore, the inclusion of internal energy generation is also an open issue.

2.2.3 Numerical difficulties at computing the analytical solution to the diffusion equation in multilayered media

From the homogeneous longitudinal boundary conditions (the example in section 2.2.1) a set of basis functions (kernel) can be defined in each layer, where the eigenfunctions of the layers satisfy the orthogonality condition in (3) within the respective layer. However, the set of eigenvalues and eigenfunctions are not necessarily the same for every layer because the material properties and longitudinal boundary conditions may differ. Therefore, the scalar product of two eigenfunctions, which belong to the different indexes *n,m* and belong to the different layers *j* and *k*, may not be zero but can be expressed as in (4)

$$\int_H Y_{j,n}(y,\lambda_{j,n})Y_{k,m}(y,\lambda_{k,m})dy = N^2_{(j,n),(k,m)} \neq 0 \cdot \tag{4}$$

The $a_{j,n}$ and $b_{j,n}$ coefficients of the $X_{j,n}(x)$ transverse eigenfunction of the $T_j(x,y)$ solution, which solution is defined in layer #*j*, are to be computed from the corresponding transverse and interface conditions of layer #*j* by using (3). If the kernels of the layer #*j* and the adjacent layers' are different, then all the transverse eigenfunctions of the other layers have to be considered when computing a particular $a_{j,n}$ and $b_{j,n}$ coefficients (because of the interface conditions).

Because of the interface conditions and of the different kernels, the computation of the transverse parameters of a layer involves solving a system of linear equations which considers all the transverse eigenfunctions of every layer (the *a* and *b* parameters). Formally, the problem is described by a system of linear equations with a coefficient matrix that contains very large and very small elements from the exponential transverse eigenfunctions; therefore, the solution (by inversion) of the system of linear equations leads to an ill-conditioned problem. Implementation and computation have a limited arithmetic precision in practice, which can result in an erroneous solution because of the accumulation of numerical errors (e.g., from truncation and round-off) during the calculation process, during the construction of the so called transfer matrices, etc. The numerical instability is susceptible to the number of the layers, to the transverse extension of the multilayered medium and to the number eigenfunctions that are to be included in the solutions. This phenomenon sets a limit to the practical applicability of the analytical solutions. In technical literature, the majority of the problems consider the "global orthogonality", i.e., the kernels of the layers are the same.

3. NEW SCIENTIFIC RESULTS (THESES)

I. I have developed methods, which are applicable under PWM drive conditions and target embedded applications, for estimating the parameters of linear electromagnetic actuators.

I.1 I have developed a method to simultaneously estimate the position of the moving part and the external load in linear electromagnetic actuators. The estimation is based on an empirically acquired set of position to inductance and position to average current curves, which curves are recorded at different external loads. The inductance is measured by dedicated scan signals (chopping of the PWM and sinusoidal duty ratio component), which are generated with the simplest PWM drive hardware. [di4-5]

I.2 I have developed a set of methods, which have low computational complexity, to estimate the electrical resistance of the coil of linear electromagnetic devices. The difference between the overall resistances during the on and off periods of the PWM cycles is also considered thus the error (bias) at the estimation is reduced. [di1]

I.3 I have proven that under PWM drive conditions with constant supply voltages and with a constant duty ratio, the time evolution of the average of the current, which is computed for the PWM cycle, can be expressed by the general exponential function for a linear L/R circuit, even if the system changes its electrical time constant during the on and off periods of the PWM cycles. [di1]

II. I have shown that the non-homogeneous longitudinal boundary condition can be included in the solution of the steady-state diffusion equation in multilayered media by solving a single layer problem in the corresponding layer and by solving a set of multilayered medium problems which have homogeneous longitudinal boundary conditions. [di3]

II.1 The eigenvalue problem that is associated with the analytical solution of steady diffusion can be simplified in multilayered media having non-homogeneous longitudinal boundary conditions by reducing the transcendental equation. Thus, the computational resources which are necessary for computing the eigenvalues decrease. [di3]

II.2 I have shown that the internal energy generation, which depends only on the transverse coordinate, can be represented as a one dimensional diffusion problem and as non-homogeneous longitudinal boundary conditions if considering the solution to the steady-state diffusion in multilayered media. [di3]

III. I have developed a mesh-free iterative algorithm to resolve the numerical difficulties typically encountered when computing the analytical solution of the steady diffusion equation in multilayered media. [di2]

III.1 The convergence of the iterative solution improves if varying weighting coefficients are used during the iterations. [di2]

III.2 I have provided an estimate of the optimal weighting coefficients, which are to be used during the iterations, from a lumped-impedance-network representation of the layers. [di2]

III.3 I have generalized the iterative solution to multilayered media on the basis of a hierarchical two-layered medium representation. [di2]

III.4 The number of iterations required to reach convergence decreases if the bisection method is used to create the hierarchical two-layered medium representation of the multilayered medium. [di2]

4. HIGHLIGHTS OF THE RESEARCH

In the first part of the research, new sensorless methods were developed for estimating the parameters, i.e., the position of the spool, the external load and the electrical resistance of the coil; of linear electromagnetic devices. Typical fields of application of such devices include embedded systems, e.g., the automotive industry. Compared to already existing solutions, the major contributions of my work are the sensorless methods which have lower model (computational) and hardware complexity, and the capability of the combined estimation of the position and of the external force. Thus, the cost effectiveness and reliability of the systems that utilize solenoid actuators can further improve by saving additional sensors from the systems, e.g., pressure sensors.

The second part of my research was dedicated to the analytical solutions to the conduction of heat in solids in multilayered media. The original motivation for this research was to create an in depth thermal model of solenoid actuators; however, multilayered media have a far wider field of industrial and engineering application. As the result of my research, I have developed a general method with which the associated eigenvalue problem can be simplified and non-homogeneous longitudinal boundary conditions and forms of internal energy generation can be comfortably included in the analytical solution to the steady state of the heat equation. Furthermore, I have also developed and optimized an iterative method with which the numerical difficulties at computing the analytical solution are resolved. My results extend the applicability of the analytical solutions to the diffusion equation in multilayered media to a wider field of application; and improve their computational effectiveness.

5. Publications

Peer-reviewed journal articles

[di1] Ivor Dülk, Tamás Kovácsházy
Resistance Estimation in Solenoid Actuators by Considering Different Resistances in the PWM Paths.
PERIODICA POLYTECHNICA: ELECTRICAL ENGINEERING AND COMPUTER SCIENCE 58: (3) pp. 109-120. (2014)
DOI: 10.3311/PPee.7353

[di2] Ivor Dülk, Tamás Kovácsházy
A Method for Computing the Analytical Solution of the Steady-State Heat Equation in Multi-Layered Media.
JOURNAL OF HEAT TRANSFER-TRANSACTIONS OF THE ASME 136, (2014)
DOI: 10.1115/1.4027838

[di3] Ivor Dülk, Tamás Kovácsházy
Steady-state heat conduction in multilayer bodies: An analytical solution and simplification of the eigenvalue problem.
INTERNATIONAL JOURNAL OF HEAT AND MASS TRANSFER 67: pp. 784-797. (2013)
DOI: 10.1016/j.ijheatmasstransfer.2013.08.070

[di4] I. Dülk, T. Kovácsházy
A Sensorless Method for Detecting Spool Position in Solenoid Actuators.
CARPATHIAN JOURNAL OF ELECTRONIC AND COMPUTER ENGINEERING 6:(1) pp. 36-43. (2013)

Conference proceedings

[di5] Dülk Ivor, Kovácsházy Tamás
Sensorless position estimation in solenoid actuators with load compensation.
In: IEEE (szerk.) 2012 IEEE International Instrumentation and Measurement Technology Conference (I2MTC 2012) Proceedings.

Graz, Ausztria, 2012.05.13-2012.05.16. (IEEE IMS) New York: IEEE, pp. 268-273. ISBN: 978-1-4577-1772-7

[di6] I. Dülk, T. Kovácsházy
A Novel Experimental Setup for Solenoid Actuators.
In: IEEE (szerk.)Proceedings of the IECON 2013 - 39th Annual Conference of the IEEE Industrial Electronics Society.
Bécs, Ausztria, 2013.11.10-2013.11.13. New York: IEEE Press, pp. 3862-3867. ISBN: 978-1-4799-0223-1

[di7] Dülk Ivor, Kovácsházy Tamás
Thermal Analysis of Solenoid Actuators.
In: Petras I, Podlubny I, Kacur J, Nawrocka A, Sapinski B (szerk.) Proceedings of the 14th IEEE International Carpathian Control Conference (ICCC 2013). Rytro, Lengyelország, 2013.05.26-2013.05.29. New York: IEEE, pp. 53-56. ISBN: 978-1-4673-4489-0

[di8] Dülk Ivor, Kovácsházy Tamás
Modelling of a linear proportional electromagnetic actuator and possibilities of sensorless plunger position estimation.
In: IEEE (szerk.) ICCC 2011 (International Carpathian Control Conference). Velke Karlovice, Csehország, 2011.05.25-2011.05.28. New York: IEEE Press, pp. 89-93. ISBN: 978-1-61284-360-5
DOI: 10.1109/CarpathianCC.2011.5945822

[di9] Dülk Ivor, Kovácsházy Tamás
A Computationally Effective Method for Calculating the Exponential Fit
In: IEEE ICCC 2014 (International Carpathian Control Conference). Velke Karlovice, Czech republic, 2014.05.28-2014.05.30., pp. 101-104. ISBN: 978-1-4799-3527-7
DOI: 10.1109/CarpathianCC.2014.6843577

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