USING NONSTANDARD BASIS FUNCTIONS IN DESCRIPTION OF SIGNALS AND SYSTEMS

Overview of PhD Thesis

By

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Abstract

The subject of the thesis is the application of nonstandard basis functions in description of signals and systems. After an introduction the general concept of representations, the frequency domain representations of discrete-time signals is elaborated, and a system of functions, called generalized orthogonal basis (GOB) functions, that forms an orthonormal basis in the space $\mathcal{H}^2(\mathbb{D})$ is introduced.

The signal representations based upon this system are applied in the detection and identification field in association with signals and systems emerging in application areas, such as industry, engineering, energy production, vehicles control, etc. The main motivation to apply representations upon nonstandard bases is the ability of these constructions to incorporate a priori information upon the characteristics of the system to be analyzed, which can result in high sensitivity on that characteristics themselves. The a priori information involved into the methods studied in the thesis is the location of system poles, that is a dominant constituent in determination of the system dynamics.

The thesis proposes also methods to realize detection and identification tasks on the basis of nonstandard representations, as well as examines the realization and convergence aspects emerging in association with them.
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Introduction

The subject of this thesis is the use of nonstandard basis functions in the description of signals and systems emerging in many application areas, of the industry, engineering, energy production, vehicles, etc. Description has been used to indicate the variety of the application: description mostly means to use a mathematical language to make possible the recognition of it, hence incorporates the modelling and identification of systems, and can be completed into the direction of the goals of the recognition, the decision making upon its state and operation, i.e. detection of phenomena and control. Within this variety the current thesis focuses mostly on the identification and the detection field. Furthermore the thesis deals with signals and systems: the goal is in most cases the cognition of systems, however this can only be done through the signals belonging the them. Observations, measurements can usually be considered as signals, and signal processing tasks can be used for the purpose to recognize the structure and the behavior of the underlying system. This thesis mostly concentrates upon the signals, however the connection with the systems will always be kept in mind.

Finally the expression nonstandard basis functions will be analyzed: this notion in the title suggest that the nonstandard basis functions are used contrary to standard ones. Standard basis in the mathematics and the systems science usually means a trigonometric basis that is strictly connected to the concept of Fourier series and Fourier transform. The motivation to use something different originated from several sources.

In the mathematics this field began to evolve at the beginning of the 20th century
in connection with the convergence problems of the Fourier series in several function spaces, for example in $L^\infty$. Another motor of the development was the demand for finding several orthogonal systems, which can represent any function belonging to interesting Hilbert spaces — as the spaces $L^2$ and $H^2$ — which has led to the construction of the Schauder, Franklin, Walsh, Haar, and other systems.

Within the systems science system identification has been the area, that has principally acted in the generation and development of the principle what is now called nonstandard representations or nonstandard basis functions. The demand to use of nonstandard bases has arisen because of convergence problems were being emerged in the spaces applied to find robust solutions instead of the classical optimal ones, namely the $L^\infty$ and $H^\infty$ [8], [14], [3], [11], [1].

Besides this the motivation of this thesis to concentrate upon the nonstandard bases contain a new component: the nonstandard bases offer a new opportunity to provide them with characteristics that are in connection with a priori information about the system to be described. Equipped with this information the representations built upon the bases will be sensitive to these characteristics, which can be advantageous in identification and detection problems.
Overview

Here a brief overview is given upon the scientific content of the thesis work.

Organization of the thesis

The material covered by the thesis is divided into five major chapters and a complementary one.

First a general introduction is given to the representations of signals (Chapter 1), than the discussion will be concretized to the frequency domain representation in \( \mathcal{H}^2(\mathbb{D}) \) by using information upon the system poles, and the notion of the generalized orthogonal basis (GOB) is introduced (Chapter 2).

In Chapter 3 the realization and measurement aspects of the GOB representations are discussed.

In Chapter 4 and 5 two areas are investigated, where the GOB representations are to be applied, these are the detection, as well as the identification fields; both can be considered as significant areas within the disciplines of systems science and signal processing — with numerous applications in the industry, energy production, transportation, vehicles control, etc. Chapter 4 is devoted to the detection field, and Chapter 5 deals with selected topics of system identification.

Finally a summary of the scientific results is given (Chapter 6), that is also included in this overview.
Signal representations

The concept of representations of signals in several systems of functions and linear function spaces is introduced. The representations are classified: continuous and discrete, time and frequency domain representations of general, causal, and periodic signals are concerned. The linear function spaces $L^2, L^\infty, H^2, H^\infty$ are considered as the most significant representing spaces due to their physical interpretation.

The use of signal representations is advantageous to the description of signals in the respect, that the coefficient space (or the transform) in many cases is significantly simpler than the original one, e.g. contains countable elements, or enhances some characteristics of the signals, that are built in the representing function space.

Representations of signals in $H^2$

The study of signal representations is concretized to the frequency domain representations of band-limited discrete-time signals sampled by satisfying the Shannon-rule, and containing finite energy. The frequency functions corresponding to this representations belong to the space $H^2(D)$. These functions can be represented in the standard trigonometric basis by the commonly used $z$-transform.

In Chapter 2 nonstandard bases are investigated, that have been obtained as the generalization of the Laguerre-functions, and can be built upon a predefined set of finite number of poles associated with the system to be analyzed. These constructions are called generalized orthogonal bases (GOB). The introduction and involving these constructs in the system science, motivated mainly by the demand for representations adequate for the use in robust control, is due to the activity of scientific centers in the past decade, see [8], [14], [11], [12], [3], [17], [13], [17], [40], [15], [16], [5], [1].

In this chapter the characteristics and the convergence properties of the generalized orthogonal bases in the space $H^2(D)$ are studied, as well as a method has been found, which makes possible the approximate computation of the coefficients belonging to the representation of a function in a GOB upon a fixed set of poles, set out from
frequency domain measurement data [25].

**Realizing the GOB representation**

In Chapter 3 the problems of realizing signal representations in generalized orthogonal bases are discussed. The algorithm established theoretically in Chapter 2 is elaborated until realization. Furthermore the characteristics of the argument transform are analyzed, focusing on the interpolation / approximation properties if finite number of samples are used. These properties make these methods suitable to describe systems in general sense, beyond the GOB representations; a measurement scheme will be proposed to support these efforts.

The signal representations upon GOB can be constructed on the basis of frequency domain data arranged in a special non-uniformly spaced scale given by the inverse of the so-called argument transform defined upon the presumed pole structure. This scale is advised to be used as a general framework of frequency domain measurements to be applied also in the classical spectral methods for signals and systems that even approximately satisfy the a priori assumptions. By using this scale more uniform and smoother frequency domain interpretation can be obtained by using smaller number of data in comparison with the conventional descriptions based upon uniform scale.

**Applying GOB representations: detection**

The main motivation to use representations based upon GOBs arises from the opportunity to provide the bases with characteristics corresponding to *a priori* information available of the system to be analyzed. Equipped with this information the representations will be sensitive and selective to the presumed characteristics, resulting in advantages that are not present in the conventional methods operating on the standard basis. The *a priori* assumption contained in a particular GOB is a finite pole set that is associated with the system belonging to the signals to be represented. The
information upon the poles is one of the most important constituent of the system dynamics, hence possesses key significance both in the detection and the identification tasks emerging.

Chapter 4 is devoted to the field of applying signal representations in generalized orthogonal bases: the use of GOB representations in detection, feature enhancement, noise suppression and related problems are discussed.

Detection is one of the most significant part of the signal processing. The goal on this field is to detect phenomena in the dynamics of the system, for example changes in the structure or in the behavior. Detection is a decision problem, the goal is to decide whether the investigated phenomenon is present or no in the signal tested. By using representations upon the GOB concept, detection methods sensitive to the changes in the pole structure — including the number and the multiplicity of them — in comparison with the presumed pole layout are given. Two approaches of these methods have been elaborated: a method based on reasoning upon the behavior of the coefficient values, as well as another one that does not require the computation of the coefficient have been introduced [39].

The first method is based upon the convergence properties of the GOB representations, that have been explored in details within the thesis work. Using this approach the decision can be performed by analyzing the finiteness of the representation, and the number and layout of the coefficients significantly differing from zero. Related problems as noise suppression and feature enhancement can also be interpreted within this framework. The second approach is based upon a measure composed on a discrete scalar product generated by the reproducing kernel belonging to the representing function space, which being used as a residual serves as the base of a decision problem.
Applying GOB representations: identification

In Chapter 5 some aspects of applying generalized orthogonal bases in the identification of systems are presented. After exploring the modelling capabilities of the GOB representations an iterative scheme to improve an initially assumed model is presented, operating within the framework of $H^2$ representations. Than an extension of the GOB representations toward the field of $H^\infty$ spaces is presented, that make these representations suitable to be applied in robust identification problems.

System identification is a key problem in recognizing the structure and the behavior of systems, studied exhaustively both in theoretical and practical level. The use of representations in GOB does not offer an exclusive alternative to the classic approaches, however equipped with the \textit{a priori} information upon the approximate pole structure — usually obtained by using classical methods — is devoted to refine this, i.e. to find more accurate model, incorporated in the pole layout of the system. The advantage of the new method is that it can bother also with the multiplicity of poles, which is not characteristic in the classical methods.

To realize the improvement of the assumed initial pole layout, an iteration method has been proposed [36], that algorithmically changes the pole positions with the purpose to find a layout best fitted to the function to be identified on the basis of measurement data. The algorithm of the iterative pole placement is based upon a bi-orthogonal system defined upon the GOB with respect to a discrete scalar product.

Finally in connection with the identification problem the GOB representations has been extended from the space $H^2$ to $H^\infty$ by proving a theorem on the convergence in the space $H^\infty$ by using the concept of the $\phi$-summation schemes [25]. Hence the generalized orthogonal bases can also be used in the robust identification field, furthermore, they can be used in all the areas where $H^\infty$ criteria are used, e.g. in robust detection, and noise suppression problems.
New results

The new scientific results achieved during the research — subject of the thesis work — are summarized in the theses numbered from 1 to 6 given as follow.

Thesis 1.

The use of signal representations in function spaces specifically constructed by applying a priori knowledge of the system has been proposed in the identification, detection, noise suppression, and other related areas emerging in association with the application of signal processing and systems science.

If the a priori knowledge is used to define the form of the functions taking place in the representing functions set, the representation will be sensitive to the matching of function analyzed with the assumed characteristics itself. Hence the representation coefficients will be suitable for making selective decisions upon whether the concerned signal possesses the assumed characteristics, or no, as well as based upon quantitative relations the measure of matching with them can also be judged.

This idea has been introduced in the Chapter 1, and elaborated in Sections 4.1 and 4.2 of Chapter 4.

Thesis 2.

A method has been constructed for the purpose to approximately compute the coefficients of a representation of a function belonging to the space $\mathcal{H}^2(\mathbb{D})$ in a generalized
orthonormal basis defined by a set of poles \( \{a_k \in \mathbb{D} \mid k = 0, 1, 2, \ldots, N - 1\} \). The method is based upon a non-uniformly spaced partition of the frequency domain — belonging to the discrete-time form of the underlying band-limited signal, and normalized to cover the interval \([-\pi, \pi]\) — by applying the inverse of the argument function corresponding to the Blaschke-product defined on the poles. The method uses the FFT algorithm to compute the coefficients.

The coefficients \( c_n = c_{N\ell+k} \) of the representation of \( F \in \mathcal{H}^2(\mathbb{D}) \) in the generalized orthonormal basis \( \{\Phi_n\} \) can approximately computed by accomplishing the following steps:

- Deriving a scale containing \( M \) uniformly spaced samples on the interval \([-\pi, \pi]\) of parameter \( s \), i.e. \( \{s_0, s_1, \ldots, s_M\} \) for \( m = 0, 1, \ldots, (M - 1) \).

- Approximately computing the values of the inverse argument function \( \beta_a \), i.e. \( \{t_0, t_1, \ldots, t_M\} \) by \( t_m = \beta_a^{-1}(s_m) \) for \( m = 0, 1, \ldots, (M - 1) \).

- Computing the function

\[
\begin{align*}
  f_k(s) &= F(e^{i\beta_a^{-1}(s)})\overline{\Phi_k(e^{i\beta_a^{-1}(s)})\beta_a'(s)},
\end{align*}
\]

on the points \( s_m \) as well as \( t_m = \beta_a^{-1}(s_m) \) by the use of \( F(t_m) \) measurements for the indices \( k = 0, 1, \ldots, (N - 1) \) and \( m = 0, 1, \ldots, (M - 1) \).

- On the basis of the expression of coefficients with the function \( f_k(s_m) \)

\[
< F, \Phi_{\ell N+k} > = \frac{1}{2\pi} \int_{-\pi}^{\pi} f_k(s)e^{-itNs} \, ds.
\]

\( N \) number of M-point FFTs is performed according to

\[
d_{kj} = \frac{1}{M} \sum_{m=-M/2}^{M/2-1} f_k(s_m)e^{-i2\pi \frac{jm}{M}},
\]

resulting in \( \{d_{kj}\} \) finite sequences of Fourier coefficients, where \( k = 0, 1, \ldots, (N-1) \) and \( j = 0, 1, \ldots, (M - 1) \).
Decimating the sequences of the coefficients by $N$, and ordering the elements of the $N$ number of sequences into a single sequence of the representation coefficients by applying $\ell = j/N$,

$$c_{\ell N + k} = d_{k,\ell N} \quad (k = 0, 1, \ldots, N - 1) \quad \text{and} \quad (\ell = 0, 1, \ldots, M/N - 1)$$

The method has been introduced in the Chapter 2, and elaborated until algorithmic level in the Chapter 3 in the thesis, and has been published in the conference paper [25].

**Thesis 3.**

A method has been elaborated to detect phenomena in correspondence with the structure of a system on the basis of the representation of the signals belonging to it in the generalized orthonormal basis generated upon the \textit{a priori} assumed pole set associated with the system. The detection method is based upon the fact, that the representation of a function belonging to the subspace $\mathcal{B}_{a|N} \mathcal{H}^2(\mathbb{D})$ generated by the finite Takenaka-Malmquist system defined upon the presumed poles is finite, i.e. finite number of the representation coefficients differ from zero. In the case of differences on the pole structure, i.e. a deflection occurs on the concerned function from the presumed subspace, the representation becomes infinite.

It has been shown, that the coefficients in an infinite representation beyond the indices corresponding to the multiplicity of the poles exponentially tend to zero with a decay rate proportional to the difference.

A detection scheme has been set up on the basis of the representation coefficients to detect

- changes on the pole placement,
- occurring new poles,
- disappearing existing poles, as well as
changes on the zero structure
of system under analysis. The detection is realized by decision algorithms upon the
values of the coefficients by using deterministic limits or statistical considerations.
The detailed description can be found in the Chapter 4 of the thesis, and
has been published in conference papers [39, 37].

Thesis 4.

In connection with the identification of systems by representing the signals belonging
to them in generalized orthogonal bases a method for the refinement of the a priori
assumed pole structure has been constructed. The method establishes a convergent
iteration scheme upon the modification of the poles — involved also their multiplic-
ities — in the direction determined by a biorthogonal system with the generalized
orthonormal basis in respect to a discrete scalar product. The iteration on the poles
\( a = \{a_0, a_1, \ldots, a_{n-1}\} \) with multiplicities \( m = \{m_0, m_1, \ldots, m_{n-1}\} \) is defined by
\[
a^{\nu+1} := G(a^\nu) \quad (\nu \in \mathbb{N}, \; a^0 \in \mathbb{D}^n)
\]
where
\[
G_k(a) := a_k + \frac{1}{m_k - 1} \frac{F_k(a)}{F_k^{-1}(a)} \quad (k = 1, \ldots, n) \quad G := (G_1, \ldots, G_n).
\]
and
\[
F_k(a) := [\Phi_k^m(m_k^{-1})(\cdot, a), f\hat{a}]_N, \quad F_k^{-1}(a) := [\Phi_k^m(m_k^{-2})(\cdot, a), f\hat{a}]_N.
\]
The functions \( \{\Phi_{k\ell}\} \) form a bi-orthogonal system with the generalized orthogonal
basis \( \varphi_{k\ell} \) with respect to a discrete scalar product \( [\cdot, \cdot]_N \) defined upon \( N \) number of
measurement points, i.e.
\[
[\Phi_{k\ell}, \varphi_{rs}] = \delta_{k\ell} \delta_{rs}
\]
The discrete scalar product has been defined on the basis of the discrete form of the
Cauchy integral. A method for the construction of the functions \( \Phi_{k\ell} \) is also given.

The detailed description can be found in Section 5.4 of the Chapter 5 of the thesis,
and has been published in the conference paper [36].
Thesis 5.

A measurement method has been proposed, based upon non-uniformly spaced frequency values, computed from the inverse argument function — belonging to the Blaschke-product defined upon a predefined pole set — of a uniformly spaced set of values in the interval $[-\pi, \pi]$. This non-uniformly spaced set of frequency-domain measurements forms the starting point of the representation methods in generalized orthonormal bases (see Thesis II), however it has benefits in any other signal processing method connected to spectral functions, since

- for signals belonging to the subspace $B_{a|N} \mathcal{H}^2(\mathbb{D})$ generated by the Blaschke-product this scale results in optimal arrangement of sample points on the spectral function, since these points are reference points of an exact interpolation operator.

- for signals residing near to the subspace $B_{a|N} \mathcal{H}^2(\mathbb{D})$ the scale results in suboptimal — however advantageous — arrangement of sample points on the spectral function.

Applying this non-uniform scale the spectral function of signals satisfying the conditions of the above cases can be represented better even by using less sample points, than the classic methods applying uniform spacing in sampling.

The detailed description can be found in Sections 3.3 and 3.4 of the Chapter 3 of the thesis, and published in the conference paper [38].


By applying the $\phi$-summation schemes the convergence of the representations based upon generalized orthonormal bases has been extended from the $\mathcal{H}^2$ space to $\mathcal{H}^\infty$. Based upon the definition of the $\phi$-summation operator

$$U_{m}(F) = \sum_{k=0}^{N-1} \sum_{\ell=0}^{\infty} \varphi(\frac{\ell N}{m}) \langle F, \Phi_{\ell N+k} \rangle \Phi_{\ell N+k}$$
where \( \{\Phi_n\} \) are the members of a generalized orthonormal basis, \( \varphi \) is the function of a summation-procedure; and the definition
\[
M_p(\hat{\varphi}) := \int_{-\infty}^{\infty} |t|^p |\hat{\varphi}(t)| \, dt < \infty \quad p \geq 0
\]
the following theorem has been proven:

**Theorem 0.0.1.** Let \( \varphi \) be a compactly supported even continuous function with \( \varphi(0) = 1 \) and \( M_1(\hat{\varphi}) < \infty \). Then there exists a constant \( C > 0 \) depending only on \( a \in D^N \) and \( \varphi \) (and independent on \( m \)) such that
\[
\|U_m^\varphi F\|_{H^\infty} \leq C \|F\|_{H^\infty}
\]
where \( F \in H^\infty, m = 1, 2, \cdots \). Moreover, for any functions \( F \) belonging to the disc algebra we have
\[
\lim_{m \to \infty} \|U_m^\varphi F - F\|_{H^\infty} = 0.
\]

The detailed description can be found in Section 5.5 of the Chapter 5 of the thesis, and has been published in the conference paper [25].
Beyond the elaboration of theory a significant point of view during the thesis work has been the applicability of the theoretical results in practical applications.

The methods worked out within the research has been realized for test and demonstrational purposes in MATLAB® environment of Mathworks. For practical applications a set of procedures have been implemented in ”C” language environment, that are ready to build in several applications on the fields of signal processing, identification, detection and control.

The methods are intended to be applied in the identification and detections tasks emerging in the control and fault diagnostics of vehicles and technological equipment.
Activity

Scientific activity

The results reported in this thesis has been achieved by the author in the course of continuous research activity on the fields of signal processing, systems and control theory, within the scientific community provided by the Systems and Control Laboratory of the Computer and Automation Research Institute of the Hungarian Academy of Sciences. The scientific activity of the author can be represented by the following list of the selected publications, classified according to the underlying fields:

**Failure detection and diagnostics:** Conference papers [34], [33], [31], [26], [4], [2], [9], [24], [22], [6], [7], [21], and periodical papers [35], [23].

**Control theory and applications:** Conference papers [41], [19], [27], [29], and periodical paper [28].

**Vehicle detection and control:** Conference paper [30].

**Signal processing, joint time-frequency, wavelets:** Conference papers [20], [18], and [32].

**Signal processing, systems theory, GOB:** Conference papers [25], [39], [36], [38], and [37].

**Medical applications:** Periodical paper [10].
The present and future scientific activity of the author concentrates on the directions of applying special representations incorporating wider class of a priori knowledge in the detection and identification field.

**Professional activity**

The scientific activity is complemented with industrial application areas is several directions. The most significant ones belong to the field of energy production, as well as to automotive industry.

The signal processing, failure detection and diagnostic activities have been applied in the Nuclear Power Plant Paks, Hungary formerly within the framework of the development of systems for reactor and primary loop noise diagnostics, and later in connection with the design and realization of the reactor protection system.

The signal processing, detection, and control activities have found applications area in the detection of the position, as well as identification and control of the movement of vehicles within the framework of the cooperation with the Budapest Development Center of Knorr Bremse (Germany).

**Educational activity**

Invited lecturer on graduate and postgraduate level subjects "Signal Processing" and "Digital Measurement Technology" at the Faculty of Transportation Engineering of the Budapest University of Technology and Economics.

**Membership**

Member of the Scientific Society for Measurement, Automation and Informatics (Mérés-technikai, Automatizálási és Informatikai Tudományos Egyesület — MATE), Hungary since 1978.

Member of the IEEE (Institute of Electrical and Electronics Engineers), Control Systems and Signal Processing Society since 1997.
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