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**MATHEMATICAL MODELLING OF DISTRICT HEATING SYSTEMS,  
WITH PARTICULAR REGARD TO OPIMAL REGULATION OF THE  
HEATING CONSUMER SYSTEMS**

Which above named is submitting to obtain her Ph.D. degree

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## **1. Introduction, brief summary of the subject matter, objectives**

Solving our problems of the present and the future, together with our feeling of responsibility for the future of the following generation, makes it unavoidable that we consider the energy supply situation in increasing depth and thoroughness. The recent social and economic changes in the countries of East and Central Europe raise even more strongly the questions in connection with energy management. On the threshold of the new millennium there is an increasing demand for the solutions ensuring long-term and sustainable development. One such solution could be improving the efficiency of energy utilisation. Among the questions dealing with energy supply and improving the efficiency of energy utilisation, those questions relating to the present situation and future of district heating have an important role, in all those countries where district heating has a significant position and role in the supply of energy.

The aim cannot be elimination of and making operation impossible for the already constructed, high capital demand district heating systems. However for the district heating service to satisfy the market, technical and environment protection requirements of the age, the systems' complete verticum must be thoroughly examined and analysed, then the necessary renovations, reconstructions and modifications must be carried out, and even new developments must be also executed. The most important aims of any development carried out in district heating supply, are economic improvement, while satisfying the consumer demand at an established standard and level of security.

District heating is a complex system of innumerable inter-connecting elements, and its technological process is also very complex. In district heating the heat energy reaches the consumer through a complicated thermal-dynamic process. In this complicated system the opportunities for increasing economic efficiency can be essentially listed in four groups:

- increasing the efficiency of energy production (combined electric energy production, improving the fuelling technological processes, application of the sorption technique,...)
- improving the efficiency of energy utilisation (construction of a triple energy connection, trigenerational system, reduction of conveying loss in the produced heat by reducing the circulation work and heat losses, the design and construction buildings with minimal heating energy demand, and energy conscious renovation of the existing ones with the application of special architectural and building structural implements, together with optimisation of the operating parameters,...)
- development of energy conscious consumer behaviour (the consumer should always consume energy according to his demands), by creating the conditions necessary for this.
- Introduction of organisational and institutional changes.

Among the listed possibilities, the reduction of heat production costs is of outstanding importance, as it can be confirmed by economic calculations that the return on outlay indexes are extremely favourable in this area. Subsequent heat insulation and window replacement on buildings connected to district heating is accompanied by significant

energy saving, but the return on outlay indexes are less favourable than for intervention on the heat production side. In both cases significant capital is required for implementation. There is a possibility for improvement in heat supply efficiency without significant investment, with organisational and institutional changes, by creating interest for the consumers saving energy, and with optimisation of operating parameters.

My object was – making use of the accumulated theoretical knowledge and the results of technical development in the past decades - to:

- discover a suitable method for examination of the district heating system as an extremely complicated complex system,
- putting the last element of this complex system, the consumer system, under the microscope, to seek out those possibilities which improve the efficiency of district heating without significant investment,
- to draft the controlling and regulation objectives for heating consumer systems under the changed, consumer habits and demands, and market conditions.

*In my dissertation I deal firstly with operational questions concerning consumer heating systems using hot water, connected to the district heating network, with the setting up of an optimal regulating model, and with establishment of the optimal operating parameters under various consolidated operating conditions. The aim is satisfaction of probable consumer demands varying as a function of the external temperature and other miscellaneous factors, with specified reliability and with the smallest possible operating costs*

Following a review of the brief history of district heating, I have depicted:

- what significance and role district heating has in the heat supply in some European countries, including Hungary,
- how is a district heating system made,
- what are the characteristics of Hungarian district heating,
- what are those most important endeavours which ensure the future of district heating.

In the dissertation I have formulated:

- the method for solving a several stage, technical, economic, decision making task,
- the process for creating the optimal plant control model,
- the control and regulation objective for heating consumer systems connected to district heating, in the cases of flat-rate and consumption proportional charges, making use of the calculus of probability and the known laws of mathematical statistics and the results of a series of measurements carried out by myself at Nyíregyháza in 1999.

In my dissertation, creation of the optimal control and economic target functions of the consumer system, is preceded by:

- discovering the connection between the system's individual elements, describing the functional connection between exit and entry characteristics of the individual elements, recalling the heat transmission and flow technology properties of the system's individual elements,
- setting up thermal and hydraulic models, with the help of the fundamental known functions,
- thermal analysis of the radiators and heated rooms, examination of shortcomings of local regulation and the effect of the various disturbances exercised on the controlled characteristic.

Determination of the optimal operating parameters, took place according to the controlling, economic target function created to find this out, which, in the knowledge the concrete parameters of the real single and double pipe heating systems, is followed by checking of the calculated optimal operating parameters (temperature, volume flow of heating medium).

The operating costs, besides the operating parameters of the heating medium, are influenced to a large degree by the quantity of heat consumed, the amount of which is dependent on innumerable factors, among others, on the building's geometry and the building's physical characteristics, and on the spatial and temporal heating restrictions. Numerous items of technical literature deal with all these questions, and each could be the topic of an independent dissertation. I close my dissertation with examination of one of these, the spatial heating restrictions, which has been less well known and analysed up to now.

## **2. The relation of the relating technical literature and my own research, to international and domestic research**

In the course of my work I endeavoured to study technical literature compiled in kibernetics - as the scientific field covering control and information processing in natural and artificial systems-. In the technical books published on system theory, control theory and operational research, above all else the methodological descriptions assisted in achieving the set objectives.

Fundamentally two possibilities are offered for setting up the mathematical model. We can set up the model making use of the laws acknowledged in natural sciences and from accumulated data from measurements and experiments. Setting up of the hydraulic, thermal and economic models shown in the paper, took place using mainly laws recognised in the natural sciences. When drafting the simplified, models approaching reality, described for the steady state, it was necessary to utilise technical literature composed in the basic sciences. There was a sufficient amount of domestic and foreign technical literature available to me in this area.

However the number of domestic publications was much smaller which examined heating systems connected to district heating and their operational question, with similar depth, attitude and collection of implements.

I did not meet with the optimisation of temporal heating restrictions, neither in the foreign nor the domestic technical literature.

### **3. The applied methods and calculations carried out**

The essence of the examination method applied during composition of the paper, was analysis of the individual part-systems, acquiring the best possible knowledge of these, and from the parts examined in this way, and by matching them, drawing conclusions relating to the entire system. In the past for a long time, this approach was a successful principle for scientific cognition.

The big question of scientific philosophy at the end of the century however, is whether the school of thought and method – called reductionism – described above, was really as successful as we believed up to now. Recently besides the undoubted splendid results of reductionism – sometimes precisely due to its acknowledged restrictions -, the holistic approach has come to the fore front, according to which the whole is more than just an aggregate of the parts, to understand this, complete, overall recognition of the component parts is not enough, there are some features which can only be successfully analysed by examining the system as a whole. The gothic cathedral is more than just the aggregate of building stones, it bears a new, different quality, for example its intangible content, for the explanation of which, in vain do we know the geometry of every stone, their spatial positioning and the laws of gravity, this is not enough to comprehend the whole.

Thus not refuting the importance of recognising the parts, the results of processes analysed in my dissertation can be used successfully when examining an actual, complete complex and complicated district heating system, at the same time the „personal” features relating to the entire system must also be examined, and in this way the overall picture will be complete.

Another problem similarly of a philosophical character is the question of optimum. The optimums may not be interpreted on their own, considered for example as concentric circuits of the processes built up on each other, and reaching increasingly expanded levels, the optimums can be increasingly different to each other. For example what is optimal from a town district heating system point of view, is not sure to be suitable on a national level, not to speak of regarding the entire World for example. Similarly time has an effect on the optimum, which unmercifully destroys the earlier optimums, compelling their constant revision. Thus the optimum is always a part of a continuously changing system, and as we advance in time and depth, they open up their newer and newer „secrets” before us.

When examining the created models, and when solving the tasks of optimisation, I made use of the program packages listed below, available on the software market:

- with the optimisation of spatial heating restrictions, which was a mixed linear programming task, I used the Student Edition 4.1 version of the program package being the property of the Debrecen University Mathematics Department, entitled MPL which applies the branch-and-bound method, which can work with a maximum of 300 terms and 500 variables.

- I carried out examination of the hydraulic and thermal models with the Bausoft Complex, Heating System heat-technology and hydraulic programme, supplemented by the Microsoft Excel routines.
- the target function analysis took place similarly using the Microsoft Excel routines. A Microsoft Excel Solver uses the General Reduced Gradient (GRG2) non-linear optimisation procedure.

For composition of my thesis I carried out measurements and made heat technological, hydraulic and optimisation calculations with the help of the above mentioned.

I carried out the measurements in November 1999 at Nyíregyháza, on the Örökösöld housing estate, in flats of a 10-story block constructed from panels. I obtained 13600 room temperature values by taking sample measurements in 17 flats.

In the course of my work I relied on:

- the theoretical basics of kibernetik science, the theory of automatics, the theory of information, the theory of systems, the theory of control, the theory of play, the theory of decision and the operation research, together with the methods built up on these,
- the theoretical laws of heat and fluid mechanics,
- the basic theory of heating and district heating technology,
- the science of mathematics, the results of probability calculations, and the mathematical statistics methods.

#### **4. New scientific results and theses**

##### **Thesis 1.**

*I selected the method, which I considered best for creation of the optimal operation control model. In the knowledge and possession of this, I laid out the block diagram in my dissertation for creating the optimal operation control model (Dissertation figure 3.2.), which I utilised for the concrete task, in the interest of achieving the set objective.*

Optimisation of the district heating system's operation is a several staged, technical-economic decision-making task. The method of solving several stage decision-making problems, is the dynamic programming, this is a principle applicable in solving the most variable mathematical exercises, which makes use of the recursive connections of the function equations for the stages following one another. This procedure firstly optimises the last temporal stage decision as a function of the preceding state, then follows back to the earlier optimal stage decisions, right back to the first decision.

##### **Thesis 2.**

*I drafted several different possible supply philosophies, with the help of the probability calculation laws, for the case of consumption proportional charges.*

Definition of the regulational aim relating to building heating systems with consumption proportional charges, connected to the district heating network, took place on the basis of probability calculations, used only relatively in building services up to now, and on the mathematical statistical principles and relations, not applied in any manner up to now.

$$A, \quad P(\xi(t_a, \tau, \dots) \geq 20C^0) \geq 0,99 \quad (2/1)$$

This means the probability that the temperature in the rooms,  $t_i \geq 20^\circ\text{C}$ , must be greater than 99%.

$$B, \quad P(\xi(t_a, \tau, \dots) \geq 24C^0) = 0,9 \quad (2/2)$$

Which means that the heat supplier must operate the system with parameters such that he can ensure a room temperature not less than  $24^\circ\text{C}$  in every room at all moments in time, with 95 % reliability.

$$C, \quad P(t_{i1} < \xi(t_a, \tau, \dots) < t_{i2}) = 0,95 \quad (2/3)$$

The meaning of this mathematical formula: satisfying of the variable, probability character consumer demand, at a designated, specified reliability, such that the required temperature value in the rooms should come within a given interval, with 95% reliability.

$$D, \quad P(t_{i1} < m < t_{i2}) = 0,95 \quad (2/4)$$

Satisfying of the variable, probability character consumer demand, at a designated, specified reliability, such that the required temperature value in the rooms should come within a given interval, with 95% reliability, where  $m$  is none other than the expected value of room temperature,  $t_{i1}$  and  $t_{i2}$  are the bottom and top limits of the interval.

There, where the energy charges are proportional to consumption, and the residents can intervene in the system, that is to say, thermostatic valves are fitted to the radiators, the task of the heat supplier, is to provide the opportunity for satisfying the consumer's optional and variable demands.

Introduction of the conditions for consumption proportional charges is in progress at present. According to the Act concerning district heating, charges based on measurements for every consumer, will be compulsory by 2003. Under the new situation developing in this manner, it must be defined as to what obligations the heat Supply Company undertakes to the consumer, among others, what are the set regulation aims.

### **Thesis 3.**

***In possession of my measurement results, and with application of the mathematical statistical methods, I have proved that it is expedient for the heat supplier to undertake the regulational aims designated in point D.***

With the mathematical statistic implements I established that confidence interval, which contains the unknown characteristic, the multiple expected value, in our case the expected value of internal temperature, with 95% probability.

In that case where we regard as the initial datum, the top limit of the expected value interval calculated from the temperature measurement data, and we state, the expected value and spread in the living rooms of all similar panel constructed buildings, are the values shown below, determined with the help of the measurement data and the mathematical statistics:

$$m = 22,84^{\circ}C$$

$$\sigma = 1,127$$

then in accordance with the heat supplier philosophy defined in *point C*:

$$\text{the interval's bottom limit } t_{i1} = m - 1,96 \cdot \sigma = 20,63^{\circ}C$$

$$\text{the interval's top limit } t_{i2} = m + 1,96 \cdot \sigma = 25^{\circ}C$$

In buildings with consumption proportional charges and fitted with thermostatic valves, it is in the business policy interest of the heat supplier to operate the system under parameters with which even the 24 C° demand can be satisfied. However this temperature demand does not appear in all flats at the same time. The measurement results demonstrate that the expected value of room temperature comes within an extremely narrow range ( 22,7 - 22,85 C° ) with 95 % reliability. The cause of this narrow range can be found in the large number of measurements and in the fact that in actual fact the expected value of room temperature is in question. If the system operates with parameters sized for this, it will be suitable for satisfying the simultaneously appearing different demands, and also ensures „cover” for the case where a small group of residents (about 10-15%) demand at intervals a temperature of 24 C° or even higher than this

#### **Thesis 4.**

*The method introduced by me, as described in Thesis 3., is also suitable for determination of building heat demand in a manner such, that in the case where a sufficient quantity of measurement data in relation to a building's heat consumption is available, we can proceed in a similar manner as when examining room temperatures. In possession of measured data relating to the building's heat consumption, we can estimate that interval which contains the probable value of the multiple characteristic  $\dot{Q}$  with given reliability. The obtained values give a numerical picture on similar type buildings. When determining the operating parameters, at a given external temperature, the top limit of the confidence interval of expected value estimated on the basis of the measured values, is regarded as the initial datum for all similar panel construction residential buildings.*

The heat demand probability variable for a building depends on numerous factors, which are similarly probability variables. Among these the most important are:

- on the building physical characteristics of building boundary structures (heat storage capacity of the boundary structures, the heat transmission factor), the air transmission of doors and windows, and the volume flow of fresh air entering the

- building,
- on the amount of internal heat gain,
  - on the consumer habits, whether the consumer utilises spatial and temporal heating restrictions, whether the room temperature maintained by the consumer deviates from the planned value, and what is the deviation from the specified value,
  - on the degree of „precision” of the applied regulation, what is the difference between the desired and the set temperature.

Thus the heat demand can be understood as the multiple of various probability variables, or as the sum of these. If on the basis of our experience we can give an estimate for the (k), (n), (t<sub>i</sub>), and the external-internal heat gain range and expected value, then the expected value of the multiple and sum of the probability variables, and their range can be calculated.

Thus we regard the entire building's heat demand as a probability variable, which can be understood as the sum of the heat demands of individual residents, as a large number of probability variables independent of each other. The building's heat demand, as a probability variable, follows a normal distribution. This, in accordance with the central boundary distribution theorem, is true even if the heat demand of individual residents has a non-normal distribution function.

#### **Thesis 5.**

***I have formulated for the consumer system, that control and economic target function, the examination of which resulted in determination of the optimal operating parameters.***

The optimal schedule of heating medium parameters is depicted in figures 6.17/2, 6.18/2, 6.19 and 6.20 of the dissertation.

Operating costs of the consumer system are determined by:

- the consumer heat demand,
- the network's heat loss,
- the circulation capacity.

The invested circulation work heats the heating medium, as a result of the energy dissipation. Taking all this into account, at a given moment in time, assuming the steady state, the target function can be written in the following:

$$K_{\Sigma} = k_{h\bar{o}} \cdot [\dot{Q}_{fo} + \dot{Q}_{cs\bar{o}} - \dot{V}_s \cdot \Delta p_s] + k_{vill} \cdot \dot{V}_s \cdot \Delta p_s \quad (5/1)$$

where

- $k_{h\bar{o}}$  the specific cost of heat energy,
- $\dot{Q}_{fo}$  consumer heat demand,
- $\dot{V}_s$  volume flow of secondary medium,
- $\dot{V}_s \Delta p_s$  pumping capacity, in the case of pump efficiency  $\eta_{sziv} = 100\%$ ,
- $k_{vill}$  specific cost of electrical energy,
- $\dot{Q}_{cs\bar{o}}$  heat loss on secondary side

$$\frac{K_{\Sigma}}{k_{h\delta} \cdot \dot{Q}_o} = \varphi + \frac{\varphi}{M} \cdot k_1 \cdot \frac{I}{e^{\frac{1}{1+m} \cdot k_2} - 1} + \frac{\varphi}{M} \cdot k_3 + k_4 - k_5 \cdot t_{k\delta} + \frac{k_{vill} - k_{h\delta}}{k_{h\delta}} \cdot k_6 \cdot M^3 \quad (5/2)$$

where

$$k_6 = \frac{R_H \cdot \dot{m}_o^3}{\dot{Q}_o \rho^3}$$

$$\varphi = \frac{\dot{Q}_{fo}}{\dot{Q}_o}$$

$$M = \frac{\dot{m}}{\dot{m}_o}$$

$$k_3 = \frac{I}{c \cdot \dot{m}_o \cdot R_{te}}$$

$$k_1 = \frac{I}{c \dot{m}_o} \cdot \left( \frac{I}{R_{te}} + \frac{I}{R_{tv}} \right)$$

$$k_2 = \frac{\dot{Q}_o^{1-\frac{1}{1+m}} \cdot (k \cdot A)_{rad}^{\frac{1}{1+m}}}{c \cdot \dot{m}_o}$$

$$k_4 = \left( \frac{I}{R_{tv}} + \frac{I}{R_{te}} \right) \cdot \frac{t_i}{\dot{Q}_o}$$

$$k_5 \cdot t_{k\delta} = \left( \frac{I}{R_{tv}} + \frac{I}{R_{te}} \right) \cdot \frac{t_{k\delta}}{\dot{Q}_o}$$

$\dot{Q}_o$  the heat demand in measuring state

$\dot{m}_o$  the volume flow in measuring state

$R_{tv}$  heat resistance in the return pipes

$\dot{Q}_{fo}$  momentary heat demand of consumer

$m$  heat transmission exponent, dependent on heat transmitting structure

$R_H$  hydraulic resistance factor of the system

$R_{te}$  heat resistance in the advance pipe

$t_{k\delta}$  temperature of surroundings

$\dot{m}$  momentary volume flow

### Thesis 6.

*With the calculations carried out using the constructed heat technology and hydraulic models, I have proved that:*

*The two pipe heating system operated with thermostatic valves, can only be operated with determined, optimal operating parameters at all moments in time, with the help of the target function, if the heat demand for heating only changes as a result of changes in external temperature, and if not only the advancing water temperature is altered as a function of external temperature, but the volume flow of the medium changes as well. In that case where the heat demand changes under constant external temperature, the optimal medium parameters can only be approximated, and we can use the help of pump regulation for this. Under constant external temperature, it is not expedient to follow the probability type variable and the heat demand by changing the advance water temperature, neither in the single nor the two pipe heating systems.*

The details are contained in the dissertation.

### Thesis 7.

*With calculation carried out using heat technology and hydraulic models constructed for single pipe, cross connection section heating systems, I have proved:*

*In single pipe, cross connection section heating systems fitted with thermostatic valves, it is worth while following changes in heat demand under a given constant external*

*temperature, by changing the volume flow of the heating medium, taking into account that the effect of change in volume flow on the output of the radiators, is different at different floor levels, and the reduction in volume flow is not justified at every radiator.*

The details are contained in the dissertation.

#### **Thesis 8.**

*I have proved that under given marginal conditions a spatial heating restriction configuration exists which results in a minimum energy utilisation for the building.*

This fact can be utilised consciously by a community. In the case of flats, only examined with technical methods and treated as independent units up until now, it is possible and worth while for their residents to „associate”, co-operate in the interest of achieving minimum energy utilisation.

The formulated thesis can be generalised, can also be applied elsewhere under other economic and different circumstances, and is also valid in the case of buildings for other functions, not supplied from the district heating system.

In that case where the examined building is a block of rented flats, or block of rented offices, guest-house or hotel, the lessor or proprietor can save energy not only with temporal, but also with spatial heating restrictions, the value of which depends on the configuration of the already rented and the still vacant flats or offices, or of the rented and the vacant rooms, how these are positioned in relation to each other. This in actual fact is independent of whether the building is heated by an individual boiler-house, or is connected to the district heating.

Construction of the model can be studied in the dissertation.

#### **Thesis 9.**

*a, I have proved that with modification, reduction of the loss factors, in other words, with architectural reconstruction, subsequent heat insulation and replacement of windows, the configurations providing optimal spatial heating restriction, do not change. The proportion of yearly heating energies utilised with the different variations did not change considerably as compared to the energy proportions obtained for the building prior to the reconstruction.*

*b, I have proved that in the case of increasing the number of stories, the effect of spatial heating restriction did not result in change in respect to the consumption proportions, the measure of saving was reduced, that is to say, the optimal configuration is independent of change in number of stories, merely the measure of saving is less if the number of stories is greater.*

## 5. Publications relating to the subject matter

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