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Booklet of PhD Theses

Application of configurational-force-based mesh refinement methods in elastic-plastic problems

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Introduction

Nowadays, the numerical methods became common in the solutions of engineering problems. There are many softwares in every fields of engineering at hand, that are capable to predict the behaviour of a given product by numerical simulation. Finite element method is a widely used tool in the field of the solid mechanics. Using more and more efficient computers, the solutions of very complex problems are available. Therefore, the quality of the discretization parameters is a very important topic in order to achieve reliable results. During the development such a software, it is necessary to work towards the automatization of the geometry discretization process. In this way, the importance of the user experience can be reduced in the numerical analyses. The assesment of the results and the potential changes in the parameters is also an important task. In order to perform this, there are various adaptive methods in finite element analysis, that can help to create a suitable mesh. Configurational force based methods represent a group of them. The configurational force itself originates from the articles by J. D. Eshelby from the 50's. Since then, the development of the computational tools made possible the utilization of his results. Nodal configurational forces in the finite element method allows to find the optimal nodal configuration that belongs to the minimal values of the total potential. This called r-adaptive strategy. Since the configurational forces represent the discretization error in this case, their also can be used as an indicator for h-adaptive mesh refinement. There are several articles in this topic both for linear and finite strain problems. This method can be extended also for elastic-platic problems if we can find a suitable potential function to compute configurational forces.
Beside the conventional finite element method, the concept of isogeometric analysis (IGA) is published a few years ago. This method uses the interpolation functions of the CAD geometry itself for the numerical integration. In the case of IGA the control mesh and the physical mesh are separated. This property allows to use r-adaptivity in a more flexible way, compared to the finite element method. Considering some specific features of IGA, h-adaptivity also can be used.
Other important issue is the testing and the validation of the numerical methods. For this purpose, analytical benchmark problems are essential. These kind of problems have closed form analytical solution. Unfortunately, the number of such a problems and their validity range is limited. This is especially true for elastic-plastic problems. The so called wedge problem offers a bunch of examples in the field of elastic-plastic deformation, where

we can find closed form solution for various types of geometry and loading case. It is also possible to produce solutions that was not examined yet.

Aims of this work

The main goal of this PhD dissertation is the extension of the configurational-force-based adaptive methods for numerical solutions of elastic-plastic problems. For this purpose, a suitable potential function is required. On the basis of this function, a condition can be derived that governs the methods mentioned above.
To validate my numerical results, I prepared a closed form analytical solution for an elastic-plastic wedge problem, that is not examined in the literature. This solution, besides a few other one, was applied for testing my methods.
In addition, I also examined the applicability of the r-adaptive strategy in the context of the isogeometric analysis.

Implementation

Two main issues emerged during the numerical implementation. At first, the numerical solution of the given boundary value problem was necessary. In the case of finite element method, mainly the ANSYS software was applied. For specific problems where the application of Tresca's yield criterion needed, the HYPLAS finite element software package was required.
The second task was the realization of the new methods based on the research. For this purpose, the *Wolfram Mathematica* symbolic algebra system was applied. The additional subroutines for the computation of the configurational forces and the adaptive refinement were developed in this environment. Since the feasibility was more important task in this research, then the effectiveness, this software tool was appropriate. In addition, the comparison of the numerical result with the analytical benchmark problems was also very convenient in this platform.
In the case of isogeometric analysis, commercial software was not available to perform

basic computations. To overcome this issue, it was necessary to develop the whole software in *Wolfram Mathematica* from the model generation to the post-processing of the results.

Thesis 1.

Configurational-force-based r- and h- adaptive strategies are also applicable for elastic-plastic analysis. The additive decomposition of the strain tensor - that is widely accepted in the literature in the case of small deformations - is used. By this, the nodal configurational force can be computed similarly to the elastic case.

An equilibrium equation is deduced for elastic-plastic deformation using the expressions relating to the purely elastic case, with the introduction of an elastic-plastic potential function. Using the additive decomposition of the strain tensor to an elastic- and a plastic contribution, the expression for the computation of configurational force is given. This is used as an indicator of the discretization error and applied for the improvement of the discretization parameters. The condition, that belongs to the minimum of the potential function demands the fullfillment of the

$$\mathrm{div}\left[\frac{1}{2}\left(\boldsymbol{\sigma}:\boldsymbol{\varepsilon}^{e}\right)\boldsymbol{\delta}-\left(\mathbf{grad}^{T}\boldsymbol{u}\right)\cdot\boldsymbol{\sigma}\right]+\boldsymbol{\sigma}:\mathbf{grad}\boldsymbol{\varepsilon}^{p}=\mathbf{0}$$

equilibrium equation in every node. The implementation is carried out by using elastic-ideally plastic material model. After the implementation, the methods are tested on benchmark problems that have analytical solutions. The modification of the nodal configuration of the mesh is governed by the nodal configurational force. The new position of the nodes is given by the direction of configurational force vectors. The results are compared to the analytical solutions of the benchmark problems in order to demonstrate the applicability of the methods. The independence of the yield criterion can be considered as an other advantage of the method.

Related publications: [1], [2], [4], [5]

Thesis 2.

A closed form analytical solution is presented for a new problem in the context of the elastic-plastic wedges. The solution is valid for 90° wedge angle. The Mises yield criterion and plane stress state are assumed. The loading is a prescribed shear traction force along the horizontal edge of the domain. On the one hand, this solution completes the research related to the elastic-plastic wedge problems. On the other hand, it is applicable for validation of numerical methods.

The elastic-plastic wedge problem originates from the early research of Paul M. Naghdi. He and his co-workers derived several solutions for different conditions. The case presented here is a new analytical solution that cannot be found in the literature before. Plane stress state and Mises yield criterion is used. The angle of the wedge is 90° and the loading is a prescribed shear traction force along the horizontal edge. The solution is valid for an infinite quarter plane. In the case of fully elastic deformation the equilibrium equation, Hooke's law and the compatibility condition are necessary. In the case of elastic-plastic domains these equations are supplemented by the Prandtl-Reuss equations, that makes a relation between the stress and strain rate. This condition allows to determine the stress and strain distribution inside the elastic-plastic domains. The validation of numerical method using analytical solutions found in the literature is complicated in many cases. To avoid this difficulty I was seeking to summarise the results in an accurate, simple form that can be used directly for practical purposes.

Related publications: [3], [4]

Thesis 3.

The concept of configurational-force-based r-adaptive concept is further developed for the application in isogeometric analysis in the case of small deformations. The method can be applied both for the points of the control mesh and the components of the knot vector simultaneously.

Compared to the finite lement method, the concept of r-adaptivity is slightly different in the case of the knot vector components. In the parameter space, only the simultaneous movement of the rows and the columns of the knot-spans is possible. Therefore, the mesh modification is based on the average configurational force computed on each row and column of the parametric mesh. This can be computed by the summation of configurational force vectors. The new value of the knot vector components are computed by this vector. The increment of the basis point values are proportional with the nonzero resultant configurational force components. According to this, the new values are computed by the

$$\Xi_I^{\text{new}} = \Xi_I^{\text{old}} - c \sum_{j=1}^{n_\eta} G_1\left(\xi_I, \eta_j\right),$$
$$\mathcal{H}_J^{\text{new}} = \mathcal{H}_J^{\text{old}} - c \sum_{i=1}^{n_\xi} G_2\left(\xi_i, \eta_J\right)$$

expressions. The boundary of the considered domain must remain intact during the modification of the discretization parameters. This can be achieved by ignoring the first and the last elements of the knot vector during the computation. The r-adaptive method can also be applied for the points of the control mesh. The nodal position of the control points are allowed to be altered arbitrarily, as long as the boundary preserves its original shape. In contrast to the finite method, the merging of the control points and the "element distortion" poses no problem. Therefore the optimization of the discretization parameters is more flexible.

The algorithm, that governs the r-adaptive optimization in the case of the control mesh optimization, similarly to the finite element method works according to

$$\mathbf{B}^{\text{new}} = c \cdot \tilde{\mathbf{G}} + \mathbf{B}^{\text{old}}.$$

The combination of the two methods mentioned above is also applicable. There are two different options in this case. Namely, the two mesh refinement strategy can be applied at the same time or successively.

Related publications: [1]

Publications

[1] G. Hénap, The configurational force and its applications in finite element method (in hungarian), Építés – Építészettudomány 2 (2010) 35–55.

[2] G. Hénap, Configurational-force-based finite element mesh refinement for elastic-plastic problems, Periodica Polytechnica Mechanical Engineering 56 (2012) 23–26.

[3] G. Hénap, L. Szabó, Analytical solution of an elastic-plastic wedge subjected to uniform shear on its face, 39th Solid Mechanics Conference: Book of Abstracts (2014) 39-40.

[4] G. Hénap, L. Szabó, On numerical solution of elastic–plastic problems by using configurational force driven adaptive methods, Finite Elements in Analysis and Design 92 (2014) 50-59.

[5] G. Hénap, Konfigurációs erő analitikus és numerikus számítása kis rugalmas-képlékeny alakváltozás esetén, XI. Hungarian Conference on Theoretical and Applied Mechanics, Miskolc (2011)