Dual-Rotor Vibrotactor

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January 2015
Declaration of Authorship

I, Ákos MIKLÓS, declare that this thesis titled, 'Dual-Rotor Vibrotactor' and the work presented in it are my own. I confirm that:

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- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed: 

Date:
This thesis is about the development of a novel dual-rotor vibrator device—called the Dual Excenter—which is planned to be used primarily for haptic purposes. The novel design consists of two eccentric rotors driven by DC motors, independently, thus with the correct setting of the phase angle between the rotors the amplitude of the generated vibration can be adjusted during operation. With this solution a basic limitation of simple ERM vibrotactors can be eliminated. The new concept enables smart and simple design, while the transferred tactile information density can be increased.

In the first chapter the tactile sense of the human being is investigated by the related literature. Then in the second chapter existing tactile exciters are presented and the concept of the Dual Excenter is introduced with its advantages and disadvantages in relation with the available designs. The third chapter presents the realized Dual Excenter prototype device including main mechanical parts, electric units and basic concepts of the control algorithm. The fourth and fifth chapters contain analytical, numerical and measurement results of the uncontrolled and controlled Dual Excenter, respectively.

The results of the research presented in this work prove that the Dual Excenter concept is feasible, thus it is possible to generate vibrations with independent frequency and amplitude in a wide frequency range with it.
I would like to use this opportunity to express my gratitude to everyone who supported me throughout the course of my PhD research project. I am thankful for their guidance, invaluably constructive criticism and friendly advice during the project work. I am sincerely grateful to them for sharing their truthful and illuminating views on a number of issues related to this project.

I would like especially thank Zsolt Szabó and Gábor Stépán for their guidance and persistent help during the last six years, András Tóth for his constructive criticism on the Dual Excenter prototype, Richárd Wohlfart for his invaluably help on constructing the device and the whole staff of the Department of Applied Mechanics for the number of advices which brought me closer to the solution of many problems. Without their contribution this work might not have been possible. Furthermore, I would like to thank Péter Galambos and Tamás Insperger for their comments and advices regarding the first version of my thesis.

I would also express my warmest thanks to my family, my parents for their steady support and patience during my whole life, and my wife for her love and tireless work which allowed the peaceful and inspiring milieu for my work.

Thank you,
Ákos

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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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</thead>
<tbody>
<tr>
<td>ABB</td>
<td>Automatic Ball Balancer</td>
</tr>
<tr>
<td>CoM</td>
<td>Centre of Mass</td>
</tr>
<tr>
<td>DoF</td>
<td>Degrees of Freedom</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>EoM</td>
<td>Equation of Motion</td>
</tr>
<tr>
<td>ERM</td>
<td>Eccentric Rotating Mass</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
</tr>
<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse Width Modulated</td>
</tr>
<tr>
<td>ZOH</td>
<td>Zero Order Hold</td>
</tr>
</tbody>
</table>
Symbols

Upper case Latin letters

\( A \)  
Dimensionless vibration amplitude of the frame  

\( A_0 \)  
Stationary dimensionless vibration amplitude of the frame  

\( \mathbf{A} \)  
Full \((8 \times 8)\) state matrix of the linearised system  

\( \mathbf{A}_{\text{ctrl}} \)  
\((10 \times 10)\) state matrix of the controlled and linearised system  

\( \tilde{\mathbf{A}} \)  
Simplified \((7 \times 7)\) state matrix of the linearised system  

\( \mathbf{C}_{\text{lin}} \)  
Linearised damping matrix of the system  

\( D_{\Delta} \)  
Dimensionless derivative control gain of the phase angle  

\( D_{\delta} \)  
Derivative control gain of the phase angle \( V \text{s} \text{rad}^{-1} \)  

\( E_{\text{kin}} \)  
Kinetic energy of the system \( J \)  

\( E_{\text{pot}} \)  
Potential energy of the system \( J \)  

\( F_{\text{max}} \)  
Maximal exciting force of the ERM vibrotactor \( N \)  

\( \mathbf{I} \)  
Identity matrix  

\( I_{\lambda} \)  
Dimensionless integral control gain of the frequency ratio  

\( I_{\Delta} \)  
Dimensionless integral control gain of the phase angle  

\( I_{\delta} \)  
Integral control gain of the phase angle \( V \text{s} \text{rad}^{-1} \)  

\( I_{f} \)  
Integral control gain of the frequency \( V \text{s} \)  

\( J \)  
Sum of the principal inertias of a rotor and an armature \( \text{g} \text{mm}^2 \)  

\( \bar{J} \)  
Dimensionless mass moment of inertia  

\( \ddot{J} \)  
Total mass moment of inertia of one rotor-armature system with respect to the axis trough point \( C \) \( \text{g} \text{mm}^2 \)  

\( \mathbf{K}_{\text{lin}} \)  
Linearised stiffness matrix of the system  

\( L \)  
The Lagrangian of the system \( J \)  

\( L_0 \)  
The length scale for the dimensionless amplitude \( \text{mm} \)  

\( L_i \)  
Electric inductance of the motor coil \( \text{mH} \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Mass matrix of the system</td>
<td>SI</td>
</tr>
<tr>
<td>(\tilde{M})</td>
<td>Dimensionless mass matrix of the system</td>
<td>–</td>
</tr>
<tr>
<td>(\tilde{M}_{\text{lin}})</td>
<td>Dimensionless linearised mass matrix of the system</td>
<td>–</td>
</tr>
<tr>
<td>(P)</td>
<td>Power of the generalized forces</td>
<td>W</td>
</tr>
<tr>
<td>(P_\Delta)</td>
<td>Dimensionless proportional control gain of the phase angle</td>
<td>–</td>
</tr>
<tr>
<td>(P_{\delta})</td>
<td>Proportional control gain of the phase angle</td>
<td>V rad(^{-1})</td>
</tr>
<tr>
<td>(P_f)</td>
<td>Proportional control gain of the frequency</td>
<td>V s</td>
</tr>
<tr>
<td>(P_\lambda)</td>
<td>Dimensionless proportional control gain of the frequency</td>
<td>–</td>
</tr>
<tr>
<td>Q</td>
<td>Generalized force vector</td>
<td>SI</td>
</tr>
<tr>
<td>(Q_k)</td>
<td>Generalized force of the (k)th generalized coordinate</td>
<td>SI</td>
</tr>
<tr>
<td>R</td>
<td>Electric resistance of the motor coil</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>T</td>
<td>Torque</td>
<td>N mm</td>
</tr>
<tr>
<td>(T_j)</td>
<td>Torque of the (j)th motor</td>
<td>N mm</td>
</tr>
<tr>
<td>(T_{\text{friction}})</td>
<td>Friction torque of the DC motors</td>
<td>N mm</td>
</tr>
<tr>
<td>(T_{\text{damping}})</td>
<td>Viscous friction torque of the DC motors</td>
<td>N mm</td>
</tr>
<tr>
<td>(U_\Delta)</td>
<td>Dimensionless input voltage difference</td>
<td>–</td>
</tr>
<tr>
<td>(U_{\Delta,0})</td>
<td>Stationary dimensionless input voltage difference</td>
<td>–</td>
</tr>
<tr>
<td>(U_{\Delta,\text{ctrl}})</td>
<td>Dimensionless control voltage difference</td>
<td>–</td>
</tr>
<tr>
<td>(U_\Sigma)</td>
<td>Dimensionless input voltage sum</td>
<td>–</td>
</tr>
<tr>
<td>(U_{\Sigma,0})</td>
<td>Stationary dimensionless input voltage sum</td>
<td>–</td>
</tr>
<tr>
<td>(U_{\Sigma,\text{ctrl}})</td>
<td>Dimensionless control voltage sum</td>
<td>–</td>
</tr>
<tr>
<td>Lower case Latin letters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>Vibration amplitude of the frame</td>
<td>mm</td>
</tr>
<tr>
<td>(a_0)</td>
<td>Vibration amplitude of the frame for ideally soft suspension</td>
<td>mm</td>
</tr>
<tr>
<td>(\tilde{a})</td>
<td>Small perturbation of the dimensionless vibration amplitude</td>
<td>–</td>
</tr>
<tr>
<td>(c)</td>
<td>Damping coefficient of the suspension</td>
<td>N s m(^{-1})</td>
</tr>
<tr>
<td>(c_\omega)</td>
<td>Viscous damping coefficient of the motor</td>
<td>N s m</td>
</tr>
<tr>
<td>(c_u)</td>
<td>Scale factor for the dimensionless voltage</td>
<td>V(^{-1})</td>
</tr>
<tr>
<td>(e)</td>
<td>Eccentricity of a rotor</td>
<td>mm</td>
</tr>
<tr>
<td>(f)</td>
<td>Frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>(f_{\text{desired}})</td>
<td>Desired frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>(h)</td>
<td>Right-hand side of the EoM</td>
<td>SI</td>
</tr>
</tbody>
</table>
Symbols

\*h* Dimensionless right-hand side of the EoM –

\(i\) Electric current of the motor coil \(\text{A}\)

\(k\) Stiffness of the suspension \(\text{N m}^{-1}\)

\(k_e\) Speed constant of the driving motors \(\text{V s rad}^{-1}\)

\(k_t\) Torque constant of the driving motors \(\text{N mm A}^{-1}\)

\(m_C\) Mass moving together with point \(C\) \(\text{g}\)

\(m_0\) Mass of one rotor \(\text{g}\)

\(\dot{m}\) Total moving mass \(\text{g}\)

\(q_k\) The \(k^\text{th}\) generalized coordinate \(\text{SI}\)

\(q\) The vector of the generalized coordinates \(\text{SI}\)

\(\ddot{q}\) The vector of the dimensionless generalized coordinates –

\(\ddot{q}_{\text{lin}}\) Perturbation vector of the generalized coordinates –

\(r\) Position vector \(\text{mm}\)

\(r_{OC}\) Position vector of the frame’s CoM \(\text{mm}\)

\(r_{Cj}\) Position vector of the CoM of the \(j^\text{th}\) rotor \(\text{mm}\)

\(u\) Input voltage \(\text{V}\)

\(u_j\) Input voltage of the \(j^\text{th}\) motor \(\text{V}\)

\(u_{\Delta}\) Input voltage difference \(\text{V}\)

\(u_{\Sigma}\) Input voltage sum \(\text{V}\)

\(v_C\) Magnitude of the translational velocity of the frame \(\text{m s}^{-1}\)

\(v_0\) Velocity magnitude of a rotor’s CoM \(\text{m s}^{-1}\)

\(v_j\) Velocity magnitude of the CoM of the \(j^\text{th}\) rotor \(\text{m s}^{-1}\)

\(x\) Translational coordinate of the frame in \(x\) direction \(\text{mm}\)

\(x\) Vector of the system’s state variables \(\text{SI}\)

\(y\) Translational coordinate of the frame in \(y\) direction \(\text{mm}\)

Upper case Greek letters

\(\Delta\) Dimensionless phase angle –

\(\Delta_0\) Stationary dimensionless phase angle –

\(\Delta_{\text{desired}}\) Desired dimensionless phase angle –

\(\Theta\) Dimensionless phase delay of the frame’s CoM –

\(\Theta_0\) Stationary dimensionless phase delay of the frame’s CoM –

\(\Phi\) Dimensionless angular position of the rotors’ common CoM –
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Natural angular frequency of the suspended frame</td>
</tr>
<tr>
<td>γ</td>
<td>Dimensionless dry friction coefficient of a motor</td>
</tr>
<tr>
<td>δ</td>
<td>The half of the phase angle</td>
</tr>
<tr>
<td>δ̃</td>
<td>Perturbation of the dimensionless phase angle</td>
</tr>
<tr>
<td>δ&lt;sub&gt;desired&lt;/sub&gt;</td>
<td>Desired half phase angle</td>
</tr>
<tr>
<td>ε</td>
<td>Small parameter</td>
</tr>
<tr>
<td>ζ</td>
<td>Damping ratio of the suspended frame</td>
</tr>
<tr>
<td>θ</td>
<td>Phase delay of the frame’s CoM</td>
</tr>
<tr>
<td>θ̃</td>
<td>Perturbation of the phase delay of the frame’s CoM</td>
</tr>
<tr>
<td>κ</td>
<td>Dimensionless speed constant of the motors</td>
</tr>
<tr>
<td>λ</td>
<td>Dimensionless common angular velocity of the rotors</td>
</tr>
<tr>
<td>λ&lt;sub&gt;desired&lt;/sub&gt;</td>
<td>Desired frequency ratio</td>
</tr>
<tr>
<td>τ</td>
<td>Dimensionless time</td>
</tr>
<tr>
<td>φ</td>
<td>Angular position of the rotors’ common CoM</td>
</tr>
<tr>
<td>φ̃</td>
<td>Perturbation of Φ</td>
</tr>
<tr>
<td>φ&lt;sub&gt;1&lt;/sub&gt;</td>
<td>Angular position of the 1&lt;sup&gt;st&lt;/sup&gt; rotor</td>
</tr>
<tr>
<td>φ&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Angular position of the 2&lt;sup&gt;nd&lt;/sup&gt; rotor</td>
</tr>
<tr>
<td>ω</td>
<td>Angular velocity</td>
</tr>
<tr>
<td>ω&lt;sub&gt;0&lt;/sub&gt;</td>
<td>Angular velocity of a rotor</td>
</tr>
<tr>
<td>ω&lt;sub&gt;j&lt;/sub&gt;</td>
<td>Angular velocity of the j&lt;sup&gt;th&lt;/sup&gt; rotor</td>
</tr>
<tr>
<td>ω&lt;sub&gt;arm&lt;/sub&gt;</td>
<td>Angular velocity of a motor armature</td>
</tr>
</tbody>
</table>

Others

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>̇□</td>
<td>Derivative with respect to the time</td>
</tr>
<tr>
<td>̇□′</td>
<td>Derivative with respect to the dimensionless time</td>
</tr>
<tr>
<td>c□</td>
<td>Cosine of an angle</td>
</tr>
<tr>
<td>s□</td>
<td>Sine of an angle</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction to tactile feedback

1.1 Mechanical aspects of the tactile sense

Transmitting information is one of the fundamental needs of our society, and it became even more important with the technological development of the recent decades. Either it comes to the transfer of knowledge or public information, entertainment, warning or aiding any other tasks, the effectiveness of these activities depends strongly on the efficiency of the information transfer.

Human beings accept information via senses. For the purposes mentioned above the most capable are the five main, traditionally recognized senses. Most of the information are gathered by the eyesight (vision) and hearing (audition). These are then completed by the touch (tactition or mechanoreception), the smell (olfaction) and taste (gustation). Further senses like the sense of balance and acceleration, time, thermoreception, proprioception and internal senses cannot or just hardly be used for information transfer.

The principal role of sight and hearing leads to the problem that for the sake of efficiency these channels are used primarily for communication, so they can often be overloaded. That way the efficiency of the information transfer deteriorates. The results of previous researches showed that e.g. the tracking of the numerous displays and instruments in the cockpit of fighter planes often overloads the visual capability of the pilots [Ho et al., 2007, van Erp and Self, 2008]. The same can be observed for car drivers: the simultaneous watching of the traffic environment, the signs and the displays of the car can cause fast exhausting of the car driver, thus it can lead to dangerous traffic situations. Further
major limitation of these senses are that the sight is one-directional—therefore only objects in the range of vision can be perceived, while hearing can be disturbed by noisy environment.

Just as in case of lack of sight or hearing it is evident to use other sensing channels to compensate these limitations and to reach optimal information transfer. The present work focuses on the potential of the tactile sensing, since it has no distinguished direction, and because of the high number of the mechanoreceptors in the skin—especially on the palm and tongue—it is possible to transfer information with a relative high rate. The sensibility of touch exceeds in some aspects even the sensing range of vision. The spatial resolution of the fingertip can be even 0.1 mm, which means that two stimuli in such a small distance can be discriminated. When smoothing a surface even smaller objects can be perceived. In case of a spherical obstacle the sensing threshold is about 2 microns, and in case of sharp protrusions this threshold can be 0.06 microns. Also time dependent tactile stimuli can be sensed like vibrations up to 1 kHz frequency. [Srinivasan and Basdogan, 1997] Furthermore, the cognitive abilities of the man make it possible to identify the shape and dimensions of objects touched by fingers or held in hand with good accuracy and confidence [Klatzky et al., 1985, Nakano, 2008].

Consequently, touch is a very advanced sense; moreover there are sophisticated technical methods to generate tactile stimuli. These solutions differ in the physical matter, how the stimuli are generated. Accordingly, we can distinguish between mechanical, electrical, chemical and thermal solutions. The most common is the mechanical solution, because it has fewer risks as electrical or thermal methods. Electrical stimulation is typically better if more than one point has to be stimulated independently on a small surface, thus high resolution is needed [van Erp and Self, 2008].

Transmitting information via touch is part of the field of haptics, which has many applications nowadays [Murray et al., 2003, Srinivasan and Basdogan, 1997]. Devices in telecommunication warn to a given event. In surgery it is possible to train or carry out operations in small sizes with robots without losing the sense of touch. The same way it is possible to train movement therapy for patients with disabilities [Ding et al., 2013]. It is also possible to create a virtual reality not only with real scene and sounds but with a touchable environment [Galambos, 2012], which soon can have a very high importance in medical, industrial, educational and entertainment applications.
Another application is if the tactile sense has to aid or replace a damaged sensing channel. There are numerous researches where balancing is aided by electric tactile devices placed on the tongue [Vuillerme and Cuisinier, 2008, Vuillerme et al., 2008]. Replacing or complementing sight and hearing is also possible to some extent [Kaczmarek et al., 1997, Segond et al., 2005, Soneda and Nakano, 2010].

In the following, properties of the tactile sense with technical importance are investigated, and the potential of the touch as transfer channel for communication is presented.

1.1.1 Mechanoreception in human skin

Mechanoreception happens almost in every case over the skin, since sensed forces are distributed on the surface of the touching objects. Although tactile sensing is also possible e.g. by the tongue, most of the above mentioned applications use the mechanoreceptors of the skin, within this the receptors of the fingers, palm and arm. Intensive investigation of these receptors has been carried out since the 1970’s. There are four known types of mechanoreceptors which provide the tactile sense, and in [Johnson, 2001, Johnson et al., 2000] detailed description can be found. Here we only mention the facts important from mechanical point of view.

The SA1 (Slowly Adapting) or Merkel nerve endings play significant role in the perception of the texture and curvature of the touched surface. They are more sensible to dynamical excitation as for static stimuli. The sensibility of the receptor varies over the receptive field which makes it possible to resolve the stimulus with a high spatial resolution. From physical point of view Merkel cells sense the local strain energy density of the skin.

The RA (Rapidly Adapting) mechanoreceptors or Meissner afferents are responsible for the feeling of sliding between hand and object held in the hand, thus they make it possible to adjust the grip force. The sensitivity is uniform over their receptive field; therefore the spatial resolution is very poor. They sense the motion of the skin, just like the PC or Pacinian afferents, but the latter senses rather vibrations in higher frequency range. PC receptors have the lowest sensing threshold at about 200 Hz, where vibration amplitudes even about 10 nm can be detected. If the stimulus would be applied directly on the receptor cell, this threshold could be even smaller, 3 nm. The role of this receptor
is to detect transmitted vibrations by objects held in the hand. For low frequency vibrations and for static stimuli it is practically insensitive.

We have the least information about the fourth type of mechanoreceptors. It is the SA2 afferent, and its task may be the detection of the relative motion of objects contacting the skin.

The operation of these four receptors can be derived from some mechanical effects occurred in the skin. Generally, either motion or stretch of the skin is detected, thus numerous researches were conducted to construct mechanical models for simulating these effects for the better understanding of the tactile sense. A plane mechanical model was created by Wu et al. which considers the stress distribution in the skin of the fingertip and the dynamical behaviour of the skin taking the characteristics of the mechanoreceptors into account as well [Wu et al., 2004]. Furthermore, there are researches for better modelling the characteristics of the single receptors [Bensmaïa, 2002, Slavík and Bell, 1995].

1.1.2 Reception of mechanical quantities

The most common way of haptic feedback is to stimulate the mechanoreceptors of the skin by mechanical vibration of contacting surfaces. Since the present work is about a device for this purpose, in the following tactile reception of mechanical vibrations and their parameters will be further investigated.

The two main questions of sensing are: first, in which ranges of mechanical parameters can we percept vibration stimuli and how do these parameters influence the feeling of vibration; and second, is it possible to differentiate between vibrations with different parameters within this range?

To answer the first question the sensation threshold in the present context has to be defined. The sensing threshold means the minimum vibration amplitude which can be just perceived. This threshold is strongly affected by the frequency and the direction of the vibration relative to the skin. The frequency dependency can be described by a "U"-shaped curve like in Figure 1.1, which has its minimum at about 200 Hz for the fingertips and the palm. This characteristic shape of the frequency dependency can be
observed independently from other parameters; however an offset in vertical direction is possible if circumstances change.

!["U"-shaped curve of the sensing threshold for human skin](image)

**Figure 1.1:** "U"-shaped curve of the sensing threshold for human skin [Brisben et al., 1999, Hwang, 2011, Verrillo, 1966b].

The direction of the vibration plays also an important role. Vibrations normal to the skin surface can be perceived at lower thresholds than vibrations tangential to the skin [Brisben et al., 1999, Hwang, 2011]. The sensing threshold depends on the location of the tactile stimulus, of course. Since the density of mechanoreceptors depends strongly on the locus, the fingertips or the foot is much more sensitive than the skin of the abdomen or the back. The hairiness of the skin has an influence on the sensitivity as well [Mahns et al., 2006, Verrillo, 1966b, Whitehouse et al., 2006, Wilska, 1954]. The area of the contacting surfaces is also connected to the location, as if the number of the stimulated receptors increases, thus the contacting surface is large, the sensing threshold gets lower [Brisben et al., 1999, Verrillo, 1963, 1966b]. This implies, that in case of point-wise stimulation the sensation threshold depends more strongly on the location [Whitehouse et al., 2006]. The perception depends furthermore on the shape of the contacting object [Verrillo, 1966a], and on the contacting force between skin and object. In case of higher contacting force the sensitivity gets better [Soneda and Nakano, 2010].

Among the most important factors we have to mention that the sensing threshold could be influenced by previous stimulation of the investigated location as well. Since mechanoreceptors adopt to vibrations, if we stimulate the given location, the sensing threshold increases. The higher is the amplitude and the longer is the duration of the previous vibration, and the nearer the frequency to the later vibration frequency is, the larger is the effect of this adaptation [Bensmaïa, 2005, Verrillo and Gescheider, 1977].

Less important factors are the temperature, the age and the humidity of the skin. The sensing is optimal at around 30°C [Verrillo and Bolanowski, 2003], at lower temperatures
the sensitivity threshold increases steep, while at higher temperatures it increases less steep. The humidity of the skin does not influence the sensing threshold but the feeling of perceived vibrations above the threshold [Verrillo et al., 1998]. This is because the mechanical behaviour of the skin changes. The influence of the age can be also detected, the younger age-class has lower sensing threshold [Cholewiak and Collins, 2003, Verrillo, 1979].

Above we discussed the influence of several parameters on the sensing threshold. The second question, how to differentiate between vibrations with different parameters, is not so well clarified.

The amplitude of vibrations can be certainly detected, since the feeling of vibrations can vary from the sensing threshold to painful, even to intolerable level. However, according to the “U”-shaped sensitivity curve the intensity of vibrations with equal amplitude can be also felt as different, if the frequency changes. This frequency-dependency cannot be compensated neither when we base the sensitivity on another parameter like the velocity or acceleration amplitude of the vibration. Investigating the sensitivity curve, its initial section is proportional to the third derivative of the displacement [Johnson, 2001]. However, it was shown that it is possible to differentiate between vibrations based on the frequency as well [Malns et al., 2006]. Moreover, it is also possible to differentiate vibrations based on their sub-harmonics [Soneda and Nakano, 2010].

A further important mechanical question is if it is possible to identify the direction of the vibration. Based on the work [Olausson et al., 2000] it could be possible, which is understandable based on the knowledge about the mechanoreceptors in the skin, since the roles of the RA and SA2 receptors are related to the direction of motion.

The investigation of the human touch enables us to conclude that we are able to perceive complex information over the skin. However, about the possible information density over the tactile channel we only have indirect knowledge. If we compare the reading speed of printed text and Braille text (writing for visually impaired persons based on touch), we experience a 3-4 times slower speed in case of Braille reading [Veispak et al., 2012]. Thus in this sense the visual and tactile sensing has a comparable performance, so it could be worth to apply more complex information also in case of tactile transmission of information, where not only the fact of vibration has a meaning, but the amplitude,
the frequency and the direction of the vibration carry dense and complex information as well.

### 1.2 Options for generating vibration stimuli

There are several possibilities to generate vibrotactile stimuli in the skin. Every solutions have their own weaknesses and benefits, thus the optimal choice depends on the application. Many solutions are based on electric or most commonly on mechanical effects.

Electrical (electrocutaneous) stimulation can generate the sensation of touch by electric current passing through the skin. The devices are in most cases tiny electrodes which provide localized stimuli, that can be felt as pressure, vibration or pain. The benefits of this solution are that there are no moving parts, so the device can be small, therefore the stimulus can be localized precisely. Furthermore, dense arrays of several tactors can be applied on a small surface of the skin. The technology has some disadvantages as well, so it became widespread only in scientific laboratory researches. The most critical limitation is that the feeling of electrotactile stimuli strongly depends on the condition of the skin and on the contact between the device and the skin. This way the solution needs continuous supervision.

More common is to use mechanical stimulation of the skin. There are various solutions for this purpose, which could be classified by the supply medium. According to that electro-mechanical devices are the most common in hand-held applications. They can be rotary- or linear inertial [Halmai and Lukács, 2007] motors, piezzo-electric or pin-based devices. In applications where compressed air is already there, pneumatic tactors can be used as well. The working method of hydraulic devices is the same, but they need the hydraulic supply, which makes them applicable rather for high power applications.

Haptics is nowadays a very fast growing field because of the many hand-held devices used almost by everybody in developed countries. In 2012 only on the market of mobile phones 1.75 billion units were sold to end users according to the report of research company Gartner [van der Meulen and Rivera, 2013]. If we consider, that almost every mobile phone has a vibration alert function, we can get a picture of the market of vibrotactors. In cell phones and also in other small sized applications eccentric rotating mass (ERM)
vibrotactors are almost the only solution used. The success can be explained by the simple design (they are simple electro-motors with an unbalanced rotor), the robust operation (they need no closed loop control) and the electric supply, which is already the primary option in hand-held devices. From design point of view there exist two types of vibrator motors; coin-shape [Precision Microdrives, 2014a] and cylindrical [Precision Microdrives, 2014b] motors (see Figure 1.2).

![Figure 1.2: Types of ERM vibration motors and their components: (a) coin- or button type [Precision Microdrives, 2014a] and (b) cylindrical [Precision Microdrives, 2014b].](image)

However, common ERM vibrotactors have a remarkable disadvantage, if we want to generate vibrations with arbitrary mechanical parameters. Since the excitation force of a rotating mass depends on its angular velocity, the amplitude and frequency of such devices are not independent. Thus, we only have one channel to transmit information, which is an intensity of the vibration composed by the amplitude and the frequency of the vibration.

Because of the high quantity, ERM vibrotactors can be produced very cost-efficiently. If any other solution has to replace them or share the market with traditional ERM vibrotactors, it should have the same price or lower, or it has to provide major benefits which make the eventually higher price paid off.

The current work will present a possible solution, which has the simplicity of the ERM vibrotactors, but it is possible to generate vibrations with independently adjustable frequency and amplitude with it.
Chapter 2

The Dual Excenter concept

2.1 The dual-rotor design

Our research will focus on the application field of mobile hand-held devices, thus pneumatic and hydraulic solutions can be excluded because of the lack of supply medium. Linear vibroactuators could seem to be a possibility, but their frequency range is limited around a characteristic frequency, which also strongly limits their application in haptics. This way a new design is needed to provide the independent frequency and amplitude at a wide frequency range.

To overcome the problem of the simple ERM vibrotactor there is an obvious solution, namely to change the eccentricity of the rotor during operation. For larger devices like soil compacting vibromotors or other industrial applications it could be an optimal solution, but for mobile devices, where the small size is a primary issue, it cannot be an option to pack a further actuator into the tiny rotor. Therefore we use another solution, which is the dual-rotor design, where we use two eccentric rotors instead of one. This enables us to change the eccentricity of the rotating system by changing the phase angle between the two rotors, and the simplicity of the original design remains. The working method of the dual-rotor concept can be seen in Figure 2.1.

As it is shown in Figure 2.1 if the phase angle between the two rotors of the device is changed at a given frequency, the resulting amplitude of the vibration gets changed. The effect of the phase angle can be implemented as it would change the position of the common centre of mass (CoM) of the rotors, thus the resulting eccentricity can be set...
The Dual Excenter concept

Figure 2.1: Changing the vibration amplitude at given frequency by the dual-rotor concept.

this way. In the first case the rotors are in-phase, so the amplitude is high, while in the last picture in case of anti-phase motion the amplitude is small, and if the eccentricities of the rotors are the same, it could be even zero. That means that at a fixed frequency the amplitude can change between zero and a maximum value which depends on the frequency and the inertial parameters of the rotors.

The design of the dual-rotor vibrotactor can be derived from any of the designs in Figure 1.2, so it could be a double cylindrical or coin-shaped vibrator motor. From the design point of view it is a relatively simple task to double the number of vibrator motors, more difficult issue is to control the phase angle between the two rotors, which problem will be handled later. Later on, the name “Dual Excenter” refers to the dual-rotor vibrotactor concept.

2.2 Working modes and possible parameter regions of a dual-rotor vibrotactor

In addition to the primary working method of the Dual Excenter there are further possible working modes showed in Figure 2.2. In the case considered in the previous section identical rotor speeds and turning directions were assumed. However, if the angular velocities are slightly different, the resulting vibration is a pulsating one, since the common eccentricity changes slowly (see Figure 2.2(a)). Another possibility is to drive the rotors in the opposite direction with identical angular speed, thus the vibration
of the rotors vanishes in one direction (in case of identical eccentricities of the rotors), and the resulting vibration becomes uni-directional (see Figure 2.2(b)). This solution is also used in a US patent for video game control interfaces [Schena and Park, 2007]. Furthermore we can combine the two methods, and have opposite turning rotors with different speeds, this way the direction of the rotation changes as well (Figure 2.2(c)).

![Figure 2.2: Further possible working modes of the Dual Excenter: (a) pulsating, (b) one-directional and (c) direction changing vibration.](image)

The vibrations provided by the methods above have different effects, if we consider the aim to generate vibrotactile stimuli on the human skin. The single-rotor design has only one degree-of-freedom, which means that despite of the changing frequency and amplitude of the vibration, these parameters are limited to a curve in the frequency-amplitude parameter region as shown in Figure 2.3(a). If there are two rotors, the independent speed of them give us two degrees-of-freedom. If the rotors are turning in the same direction showed in Figure 2.3(b), the frequency and the amplitude become independent, and the working points of the device lie in a region of the frequency-amplitude parameter domain. In case of opposite turning rotors we also have two degrees-of-freedom, but beside the intensity (coupled frequency and amplitude) the direction of the vibration can be changed. That can be depicted by a surface in the frequency-amplitude-direction parameter domain as in Figure 2.3(c). For the methods where the rotors are turning in the same direction, the direction of the vibration cannot be identified, since the direction of the excitation force rotates with the rotors, as well.

### 2.3 Multi-rotor solutions

As there are different methods for generating vibrations with adjustable frequency–amplitude or intensity–direction, the idea is obvious to combine these features to have a vibration with all mechanical parameters independent. Of course, this goal can only
achieved by increasing the number of rotors if we insist on using the ERM concept. If two dual-rotor devices are combined, which have adjustable frequency and direction as in Figure 2.4, with the angle between their directions the amplitude of the resulting vibration can be set.

Further possibilities with spatial vibration direction could be imagined as well, but with the increasing number of rotors the initial goal to keep the design as simple as possible could not be met.

Although improved solutions are absolutely feasible, the present work will focus only on the dual-rotor design.

2.4 Potential of the new concept

In this section first advantages then disadvantages will be listed and explained, which were considered already at the beginning of the development of the Dual Excenter device.
+ **Increased information transfer**

The most important advantage is the increased information transfer due to the improved haptic characteristics of the concept. With the primary method we are able to cover a region of the frequency-amplitude domain, which is combined with the sensation threshold of the human skin in Figure 2.5.

![Figure 2.5: Working region of the Dual Excenter combined with the sensation threshold of the human skin in log-log scale.](image)

The possible working region of the device is bounded by the maximum amplitude resulting from the eccentricity of the rotors and the actual frequency, which is a nearly constant curve if the frequency of the vibration is above the natural frequency of the moving system. The stiffness and damping parameters of the human skin were investigated e.g. in [Boyer et al., 2007], and for that parameters the natural frequency of a 50 g device lies between 3-10 Hz. The three other boundaries depend on the performance of the control and on specifications of the driving electro-motors.

+ **Fast response time**

The next major advantage of the device is the very fast response time in the amplitude. With the Dual Excenter we no longer need to speed up and slow down the rotation of the rotor to adjust the amplitude, but we only have to change the phase angle between the rotors. This action can be performed in a very short time interval, since we have to change the rotation speed only to reach the desired phase angle, and then change it back to the common speed to hold the phase angle constantly. In opposite to simple ERM vibrotactors this way we don’t have to sweep trough the frequencies from zero to
the desired one, which is beneficial especially if the resonance had to be avoided (e.g., because of the jamming of the vibrotactor discussed later). With the Dual Excenter the desired frequency can be reached with zero amplitude, and then the desired phase angle can be set.

We could see before, that there are working modes of the Dual Excenter, where we easily can get a pulsating vibration pattern. In addition to that feature the increased response performance makes the device very efficient in providing vibration patterns. It is possible to perform step functions of the amplitude with sharp edges and the amplitude can be varied also at a given frequency.

+ **Size and manufacturing**

Because of the possible small size the device is capable for many hand-held and/or mobile applications. In addition to that it can be used e.g. for experimental purposes, where the precise vibration parameters are essential, and the mass of the vibrotactors has to be as small as possible. Manufacturing the device should be feasible and cost-efficient, since the used technology is very similar to the one of simple ERM vibrotactors.

– **Complex design and the need of control**

One of the most important drawbacks is the increased complexity of the design. Instead of one rotor and driving motor we need two of them, and the electric driving circuits have also to be doubled. To adjust the phase angle between the rotors the control of the device has to be improved as well. For a robust and proper operation of the device a closed-loop control with feedback of the phase angle and angular velocities is needed. However later investigations will show that applying open-loop control would also be possible with some limitations.

– **Energy consumption**

Another issue could be the energy consumption. In case of a square wave like vibration pattern the traditional ERM vibrotactor has to be stopped and started periodically, which implies no energy consumption while stopping the rotors. With the dual-rotor
solution the rotors don’t have to be stopped, but this way the energy consumption remains at a non-zero level. Although the start of the rotor consumes extra energy as well, in most cases the energy saving by the constant velocity covers only a short time period of the energy needed for driving the rotors. However if the period of the square signal is very short, the energy saving could balance the additional consumption.

2.5 New results

**Thesis 1.** A novel concept was developed for generating mechanical vibrations, where frequency and amplitude can be adjusted independently during operation. In this solution two eccentric rotors are rotating with identical angular velocities and at that fixed frequency the amplitude of the vibration can be changed by the phase angle between the two rotors. For the proper setting of the angular velocities and the phase angle a closed-loop control might be necessary. The method is suitable for small sized applications like haptic feedback devices or experimental purposes, where size and proper vibration parameters are essential, no space is available for complicated mechanisms to change the eccentricity and wide frequency range is needed.

Publications in connection with the thesis [Miklós and Szabó, 2013a, 2014] and a patent submitted in connection with the thesis [Miklós et al., 2013].
Chapter 3

Prototype device

Although the prototyping followed the modelling and numerical verification of the concept, the final parameters and physical properties of the device are used in the analytical and numerical investigations. For this reason the present documentation deals with the design of the device first, and for the analysis all characteristic data will be considered as known.

3.1 The mechanical layout of the device

Based on the concept presented in Section 2.1 the mechanical layout of the device can be sketched. In Figure 3.1 the main mechanical components of the dual-rotor vibrotactor are shown. Although for the design of the device many solutions can be imagined, all of them consists of two eccentric rotors, which are driven by two electric motors, independently. These driving motors are mounted on a common frame that is connected to the environment by some kind of suspension, which is not necessarily part of the device. The most critical mechanical parts are the eccentric rotors and the electric motors, which will be investigated in the following sections in details.

3.2 Eccentric rotors

The most important parts of the device are the two eccentric rotors. To achieve a proper operation of the vibrotactor it is essential to investigate the effect of the shape
and position of the rotors.

As we mentioned before, if we want to have the option to change the vibration amplitude to zero at non-zero frequency, the eccentric mass moment \((mass \times distance)\) of both rotors have to be the same. This can be easily ensured if the geometry of the rotors are identical. However, in later investigations we can see that even with identical rotor designs additional excitations can occur. Another option for equal eccentric mass moment is to vary with the mass of the rotor and the distance of the CoM from the axis of rotation such that the product of them remains constant.

First, we consider identical rotor geometry, and the effect of the positioning will be investigated. If the two rotors are similar, we have to deal with the additional and possibly unwanted rotational excitations resulting from the moments of the exciting forces (these effects can also occur with different rotor geometry, but with identical design we are not able to avoid them). In Figure 3.2 the two possible kind of alignments of the rotors can be observed.

In the first case (see Figure 3.2(a)) the two rotors are rotating in the same plane, but there is a distance between the shafts of the rotors, thus the exciting forces have a
periodic moment around the axis parallel with the axis of rotation. The frequency of
this moment is the same as the desired excitation, and the magnitude depends on the
magnitude of the exciting forces and the distance between the shafts. In Figure 3.2(b)
the two rotors are coaxial, but the CoM of each rotor has a different plane of motion
normal to the axis of rotation. This way the rotating system consisting of the eccentric
rotors and the rotors of the electric motors is dynamically unbalanced, which results in
a moment with constant magnitude around an axis, which rotates with the rotors.

Further misalignments like non-parallel shafts can result in other additional translational
and rotational excitations, but these can be avoided by proper design and manufacturing.

If we chose the option of non-identical rotor design, we can get rid of the confusing
rotational excitations, however the geometry of the rotors is more complicated. A pos-
sible solution is shown in Figure 3.3, where the shafts of the rotors are coaxial, and the
CoM of the rotors are moving in the same normal plane as well. This way all unwanted
excitations can be suppressed, and theoretically the zero vibration amplitude can be
realized at non-zero frequency.

![Figure 3.3: A possible solution for an in-plane and coaxial arrangement of the rotors.](image)

Although the second solution seems to solve our problem with additional excitations,
because of the simplicity of the identical rotors the prototype device has been built
according to Figure 3.2(b). This way there is a rotational excitation as well, but with
relative small distance between the rotors the magnitude of the moment is small enough
not to hinder the operation of the device.

All mechanical parameters relevant for modelling and simulation is collected in Ap-
pendix A in Table A.1.
3.3 Driving electric motors

Another step of the designing is to choose the motors, which can provide proper driving performance for the rotors. Important points are the speed range and the control characteristics of the motors. As the field of the application are mobile devices, the choice of electric motors is obvious, however there are different options for that as well. Since in mobile devices direct current is available, the two simple options are DC motors or stepper motors.

In our application we have chosen DC motor because of the more suitable characteristics. Stepper motors are widespread in automation, since it is very easy to realize position control without any closed loop control, however the available speed of these motors is much slower than the speed of DC motors. Another minor issue is the more complex design of the stepper motors, and the need of driving electronics. DC motors in turn are suitable for high speed applications with continuous rotation, their design is very simple, however for position control a closed-loop control is needed. In our case we need to control the rotating speed and the relative position of the rotors as well, thus closed loop control is needed anyway. Our choice can be certified also by the fact, that single-rotor ERM vibrotactors also use DC driving motors.

For the shape of the motors we have two options as shown in Figure 1.2. Although the compact design of coin-shape motors is very attractive in mobile applications, in prototyping we have to use simple solutions to make manufacturing and mounting easy. For this reason the cylindrical motor type has been used.

After the type and shape of the motor has been decided, we have chosen a suitable motor with the requirements that the maximum available frequency should be at least 200 Hz, and the dimensions of the motor should be in the 10–20 mm range. After all, the chosen driving motors are 1.2 W brushed DC motors of the Maxon RE-max program with φ13 mm diameter, 20.5 mm length and a maximal permissible speed of 19000 rpm (order number 203889).

The physical parameters of the driving motors relevant for modelling and simulations are collected in Appendix A in Table A.1.

Finally, with the known main parts of the mechanical layout we can design the prototype device. In Figure 3.4 the CAD model of the Dual Excenter prototype can be seen.
3.4 System layout

To operate the device the control of the electric motors has to be solved as well. Since we use DC motors and position control is needed for the proper setting of the phase angle between the rotors, a closed-loop control has to be applied. The system layout of the Dual Excenter system is presented in Figure 3.5.

Here we can see, that the state of the device is measured by reflective optical sensors, whose signals are captured by edge triggered counters connected to the digital inputs of a PIC microcontroller. The data of the counters can be processed by the microcontroller to frequency and phase angle signal, thus a control algorithm can be applied for the device. The computed driving signals are transmitted to the motor driver electronics as PWM signals by the digital outputs of the microcontroller. At the same time the
computed frequency and phase angle are sent to the user’s PC by serial communication, while the user can send the desired frequency and phase angle as well as the control parameters to the control algorithm.

3.5 Signal feedback

The last important component of the system which was not discussed yet is the signal feedback for the closed-loop control. As it has been mentioned before, for this purpose reflective optical sensors have been used, however, there would have been other options as well. In the CAD drawing of the device (Figure 3.4) the optical sensors are the black parts attached to the printed circuit board (PCB) which is marked with green.

There are numerous options for the measurement of the frequency and the phase angle between the rotors, of course. For measuring angular position of DC motors the most widespread solution is the optical encoder. With such kind of sensor it would be possible to identify the precise position of both rotors, but at the planned high frequency an additional electric circuit would be needed to process the data of the encoders. The used one is also an optical solution, but in a more simple form than common encoders.

Other options could be capacitive or Hall effect sensors, which provide a proportional signal with the angular position of the rotors. This way the frequency of each rotors can be easily measured. The phase angle between the rotors can be calculated as the difference of the positions, which can have significant errors, unless these values are precise enough, or we can measure it directly (e.g. the capacitive measurement between the rotors).

The most important benefit of the chosen reflective optical solution is its simplicity. This feature makes the sampling rate very fast. Furthermore, since we use only a few components and no off-the-shelf “black boxes”, whose characteristics could modelled hardly, the whole system can be directly governed (exact time delays and processing times are known).

The main components of the signal feedback system are the two reflective optical sensors, which are attached to the frame of the device by a PCB next to the rotors. The design of the rotors is such that the sensors give low value signal in one half of a turn, and high
value signal in the other half. Thus the frequency of each rotors can be calculated from the time between the changes of the corresponding signal, while the phase angle can be obtained from the time delay between the signals.

### 3.6 Experimental setup

The complete experimental system of the Dual Excenter can be seen in Figure 3.6, which also shows the electronic components of the system.

![Figure 3.6: The experimental setup of the Dual Excenter.](image)

The signals of the reflective optical sensors are suited for digital processing by a custom made electric circuit (8) and transmitted to the digital input pins of a PIC microcontroller (2), which in turn provides the necessary driving signals for the motor driver electric circuits (7). The PIC microcontroller is attached to a demo board (1), which has all the necessary connectors for communication, programming and driving the Dual Excenter device. The electric components are wired through an experimental breadboard (10).

### 3.7 Physical characteristics of the device

At this point the physical properties of the device are defined by the design, thus the characteristics of the prototype of the Dual Excenter can be summarized.
Frequency range

First of all, the parameters of the generated vibrations have to be considered. With the chosen driving motors the highest frequency of the device is about 300 Hz, however the nominal frequency for which the device was designed is 200 Hz. The lowest frequency is theoretically zero, but at low frequency values the control of the angular speed of the motors loses performance because of the slow sampling, the indeterminate influence of dry friction and the increased effect of gravitational forces on the eccentric rotors. However, from the point of view of tactile stimulation the very low frequencies do not play an important role.

Maximum amplitude

The other parameter of the generated stimuli is the amplitude of the vibrations. Since the eccentricity of the rotors is known, the maximum exciting force can be given as a function of the frequency.

\[ F_{\text{max}} = 2m_0e(2\pi f)^2, \]  

(3.1)

where \( m_0e \) is the eccentricity of one rotor and \( f \) is the frequency of rotation. With the phase angle the force amplitude can be changed between this maximum value and zero, since the rotors have the same design. Figure 3.7 shows the value of the maximal exciting force in case of the known eccentricity parameters of the Dual Excenter.

![Figure 3.7](image_url)  

**Figure 3.7:** The maximum exciting force of the Dual Excenter with \( m_0e = 3.36 \) gmm.

Since the sensing threshold of the human skin is given rather in translational amplitude than in force, it can be useful to investigate the maximum amplitude of the generated vibrations as well. For forced vibrations by rotating masses the frequency response curve
can be found in [Ludvig, 1983],

\[ \frac{a}{a_0} = \frac{\lambda^2}{\sqrt{(1 - \lambda^2)^2 + 4\zeta^2\lambda^2}} \]  

(3.2)

where \( a \) is the amplitude of the vibration, \( a_0 \) is the amplitude of the displacement of the common CoM of the whole system due to the rotation of the rotors, and it can be given as \( 2m_0e/\hat{m} \), where \( \hat{m} \) is the total moving mass of the device. \( \lambda \) and \( \zeta \) are the frequency ratio and the relative damping, respectively. In Figure 3.8 one can see that at frequencies higher than the natural frequency of the suspended system the amplitude tends to the value of \( a_0 \), which can be explained if we consider that the common CoM of the system stays in place at high frequencies. The value of \( a_0 \) is for the presented prototype device about 0.15 mm.

![Figure 3.8: Frequency response curves due to rotating unbalance.](image)

**Size and electric specifications**

The overall dimensions and the holes for fixing the device can be seen in Figure 3.9. In this form the prototype device is small enough to provide haptic feedback, and it is possible to set up laboratory measurements on it (attaching accelerometers, etc.). However, for real hand-held applications further miniaturization is necessary.

The nominal voltage and current of the driving motors is 10 V and maximum 2 A, respectively. The control and logical electronics may need other supply specifications, in our case the microcontroller, the control of the motor driving electronics and the reflective optical sensors need 3.3 V supply.
**Figure 3.9:** Overall dimensions and mechanical connections of the Dual Excenter prototype.
Chapter 4

Mechanical modelling of the Dual Excenter

4.1 Literature of rotating systems

Rotating parts are inevitable in many kind of machines. Since the ancient times we have some typical examples for rotating motion in our machines, like wheels or mills, and with the use of them the man is interested to know about them as much as possible. There are also some interesting behaviour of rotating parts, like the gyroscopic effects or the whirling of rotors mounted on elastic shafts, which drew the attention of many scientists of the past and the present. In my work the aforementioned effects have a rather minor role, however, with the use of two rotors a further interesting phenomenon can be observed, which is typical for multi-rotor systems, but may not obvious for the first look, namely self-synchronization. Furthermore with the use of DC motors as power supply the Sommerfeld effect (or jamming) will also influence the behaviour of our device. In the following the literature of the synchronization will be presented, which significantly influences the characteristics of our system, then some connection with automatic ball balancers will be showed, and finally, results in connection with the Sommerfeld effect will be given.

The self-synchronization was first described by the Dutch scientist Christiaan Huygens in the 17th century, who experienced the anti-phase synchronization of two pendulum clocks [Huygens, 1673]. In his experiment two pendulum clocks were attached to a
wooden beam, which was able to move in the horizontal direction. He observed that after a short period of time independently of the initial conditions the two clocks started to move synchronously with opposite directions. Although Huygens gave a rather right explanation of the phenomenon, precise mathematical description was not possible because of the lack of the differential calculus. Another early observation was made by Lord Rayleigh, who described the synchronization of organ pipes [Rayleigh, 1877]. Of course, not only these interesting examples can be mentioned for synchronization, but before we gather more occurrences of the phenomenon, let us formulate, what is synchronization.

After the aforementioned observations the detailed analysis of synchronization was carried out by Blekhman in the second half of the 20th century, which was enabled by the development of the small parameter and the averaging method in the first half of the 20th century [Poincaré, 1899, van der Pol, 1920]. Blekhman published several books in the field of synchronization [Blekhman, 1971, 1988, 2000] and gave the definitions of synchronization. According to [Blekhman et al., 1997] synchronization in its most general interpretation means correlated or corresponding in-time behaviour of two or more processes, which means, that two precise clocks are also considered synchronized, even if they are not connected in any sense (natural synchronization). However, more interesting is the synchronization of interconnected systems, where some coupling in the system leads to the synchronous behaviour without any artificially introduced external action. This effect is called self-synchronization, which is the case for the observations of Huygens and Rayleigh, and for a lot of further examples like applause in a concert hall, the synchronization of electric generators of power plants in the same electric network, the rotational and orbital motion of the Moon, the walking of people on bridges (the London Millennium Footbridge), etc. In any of these examples it is important, that there is no direct constraint for the position or the phase of the motion, but rather for the velocity (e.g. asynchronous electric motors, drives with slip, etc.). Depending on the sensitivity of the synchronization on the initial conditions we can differentiate between partial and complete synchronization, when synchronous motion occurs for only one set or all initial conditions, respectively [Blekhman et al., 1997, 2002].

If two systems are interconnected, but the conditions for self-synchronization are not fulfilled, the forcing of the system is also possible [Blekhman et al., 1997] (controlled synchronization). This is the case if musicians, walking soldiers or robots have to carry out tasks synchronously, if multiprocessor systems have to communicate, but the day and
night cycle of animals and plants is also forced by daylight (circadian rhythms). Controlled synchronization can be open-loop or closed-loop. In our case of the dual-rotor vibrotactor both controlled- and self-synchronization will be experienced and investigated.

In his later works Blekhman developed further analytical methods to investigate synchronization in dynamical systems. In [Blekhman and Yaroshevich, 2004] the domain of integral stability criterion for oscillator excited by rotating bodies was extended, so the same results as of the small parameter method or direct motion separation method can be obtained more easily. Following the work of Blekhman the state-of-the-art of the synchronization theory was summarized by Leonov, who gave application examples from the mechanics and electronics in [Leonov, 2006].

**Self-synchronization**

Blekhman and Leonov investigated the synchronization phenomenon rather in general, however, there are a lot of researches, where specific examples of self-synchronization are investigated—mainly with the tools introduced in the works cited before. Inoue investigated different mechanical set-ups (hula-hoop rolling mechanism and automatic balancer) in his early paper [Inoue et al., 1967], and used the small parameter method for analytical investigations. Several works treat the original problem of Huygens, the synchronization of pendula. In [Kuznetsov et al., 2007] the existence of in-phase synchronization of two metronomes has been proven. Czolczynski analysed slowly moving pendulum systems in connection with wave power plants. He showed in [Czolczynski et al., 2012b] that two or more pendula attached to the common base show a robust synchronous behaviour independent from the initial conditions and perturbations of the system parameters. He also showed that the motion of pendula show some special clusters because of the gravitational force. He also investigated the synchronization of a number of pendula rotating in different directions [Czolczynski et al., 2012a].

Further off the original example, synchronization plays an important role in multi-rotor dynamical systems. From the point of view of the self-synchronization it is important to consider the unbalance of the rotors. In [Paz and Cole, 1992] the self-synchronization of a system with two rotors was investigated, where the axes of the rotors could have an arbitrary spatial orientation. Paz proved the existence of two synchronized motions from
which only one is stable. A number of papers were published by Wen et al. In [Han and Wen, 2008] a vibratory screener excited by two eccentric rotors was investigated. The differential phase equations of such system were developed for the first time to get the necessary conditions of synchronization. After getting the equilibrium motions of the system stability and bifurcation characteristics were described via Lyapunov stability theory, and the results have been proven by numerical simulations. Similar analysis has been carried out in [Zhao et al., 2010] for a system with four rotors. In [He et al., 2013] the self-synchronization in a vibratory machine has been investigated. The conditions for stable self-synchronization have been derived with the modified small parameter method and the Routh-Hurwitz criterion. The concept of coupling torque (torque of frequency capture) and difference torque were introduced. In [Zhang et al., 2014] the dynamical coupling characteristics is generalized for multiple rotors. The equation of motion is derived for multiple rotors and the conditions for self-synchronization are given by dividing the motion into fast and slow motion. The quality of synchronization is introduced as an average error on rotational velocity. Kovriguine investigated the effect of damping on synchronization of rotating parts—to reach synchronous motion or destroy it—also with the separation of fast and slow motion [Kovriguine, 2012].

Another interesting example for self-synchronization is the so called automatic ball balancer (ABB) which was first proposed by Thearle [Thearle, 1932] for balancing static and coupled unbalance. In this solution two or more particles move in one or more grooves on the rotor, while they are subject to viscous damping. In the supercritical frequency domain of the rotor (under specific conditions) the particles tend to synchronize their motion in such a manner that they balance the rotor. In [Bövik and Högfors, 1986] and [Sperling et al., 2000] the existence of this stable balanced state for planar and non-planar rotors has been showed. The literature of the ABB is very extensive, however, in this work we only want to indicate the connection of the two research fields.

**Controlled synchronization**

The general concepts of controlled synchronization have been given in [Blekhman et al., 2002] and [Leonov, 2006], but Nijmeijer considered the problem from a rather control point of view [Nijmeijer, 2001]. Achieving synchronous characteristics in a system is often a very important goal, so there can be found a lot of works on this, as well.
An important issue is the speed-up procedure of multi-rotor systems with unbalanced rotors. In this case it can be beneficial to control the motion of the rotors in the resonant frequency range anti-phase to avoid harmful vibrations and decrease energy consumption [Pogromsky et al., 2003, Tomchina et al., 2000]. These works propose also the control design methodology for reaching synchronization. Fradkov uses PI algorithm and speed gradient method for stabilize a three-rotor system with varying payload and velocities [Fradkov et al., 2013]. For controlled synchronization it is also important how to obtain feedback signals for the control algorithm. In [Liu et al., 2010] an observation method is proposed to get phase difference and velocity data in a twin-rotor machine. The phase difference between two eccentric rotors can be observed by using accelerometers on two symmetric points of the machine body. The proposed observer can be used as the feedback signal to realize the control of the phase synchronization for the rotors.

Sommerfeld effect

In case of unbalanced rotors near to the resonance frequency of the machine the jamming of the rotational motion can be observed if the power supply of the driving moment is limited. This is also known as the Sommerfeld effect, named after the German scientist Arnold Sommerfeld who first described it [Sommerfeld, 1902]. The explanation of the phenomenon is the balance between the power consumption of the increased amplitude vibration in resonance and the limited power supply of the driving moment, in other words if the source is non-ideal. A non-ideal source is one that is influenced by the response of the system to which it supplies power. Brushed DC electric motors, induction motors, drives with dissipative couplings or any kind of load dependent slip, etc. are examples of non-ideal sources [Samantaray et al., 2010]. This effect is rather disadvantageous in most cases and can be mixed up with some nonlinear-like behaviour (the form of the resonance curve depends on the direction of the gradual variation of the frequency of the excitation and it is impossible to realize certain motor speeds near the resonance frequency), however, since damping decreases the stability of the jamming motion, vibration absorbers could be a solution for the problem [Kovriguine, 2012]. Leonov proved that Sommerfeld effect could not be a problem while the start-up of synchronous electric motors [Leonov, 2008], which can be also a solution to avoid jamming.
In the next sections we will show how the presented phenomena will influence our dual-rotor vibrotactor.

4.2 Mechanical model of the Dual Excenter

Before we construct a mechanical model of the dual-rotor vibrotactor, we can consider some simplifications. Since the elasticity of the main components of the device have very small influence on the dynamical characteristics, we model the system as a multi-body system, where the three components are the common frame and the two rotors. The frame is connected to the environment via linear springs and dampers, which is a good approximation for a wide range of real applications. In general the motion of the frame could be arbitrary spatial motion with 6 degrees-of-freedom (DoF), but since in ideal case we only have excitations in the directions perpendicular to the axis of the rotors, the motion of the frame will be considered as 2 DoF and planar. Furthermore, the physical properties of the rotors are identical and the suspension of the frame is isotropic. Such simplifications were also made e.g. in [Han and Wen, 2008, Tomchina et al., 2000, Zhang et al., 2014], however, since the rotors in some of these papers are not coaxial, the 3rd planar coordinate is also considered there.

The constructed model according to these simplifications is depicted in Figure 4.1, where $m_C$ is the sum of the masses moving together with point $C$ and $m_0$ is the mass of one rotor. $J$ refers to the sum of the principal mass moment of inertia of one rotor (about its axis normal to the plane of motion) and the armature inertia of the motor. $k$ and $c$ are the spring stiffness and damping coefficient of the suspension (e.g. the clothes, the
skin and tissues between the cell phone and the bones of a user [Lukács and Szabó, 2006]) and $e$ is the eccentricity of the rotors. The driving moment of the DC motors is taken into account with $T_1$ and $T_2$, respectively. The generalized coordinates are $a$ and $\vartheta$, which are the polar coordinates of the rotational—or centre—axis (indicated with $C$) measured from the origin of the global coordinate system $O$. The angular positions of the rotors are described by coordinate $\varphi$ and $\delta$, which are the angular position of the rotors’ common CoM and the half of the angle between the unbalances. The generalized coordinates have a definite physical meaning; $a$ is the vibrational amplitude of the frame, $\vartheta$ is the phase delay of the frame’s vibration to the excitation of the rotors, $\delta$ is the half of the phase angle, which is related to the amplitude of the excitation and $\dot{\varphi}$ is the angular velocity of the common CoM, which in turn gives the frequency of the excitation.

The characteristics of the DC motors can be considered with the linear relation $T = k_t i$, where $T$ is the torque of the motor, $k_t$ is the torque constant and $i$ is the electric current through the coils of the motors. The electric current can be obtained from the differential equation of the DC motor

$$L_i \frac{di}{dt} + R i = u - k_e \omega$$

(4.1)

where $L_i$ and $R$ are the electric inductance and resistance of the motor coil, respectively, $u$ is the driving voltage, $k_e$ is the speed constant and $\omega$ is the angular velocity of the rotor. With this equation we would have one more state variable in our system, but since the dynamics of such electrical systems is typically much faster than that of common mechanical systems (in our case the electrical time constant is $L_i R^{-1} = 16.8 \, \mu s$), it is widespread to use steady characteristics of the motor, where the first term of the motor’s differential equation is neglected. This way the driving torque for each rotor ($j = 1, 2$) looks

$$T_j = \frac{k_t}{R} (u_j - k_e \omega_j)$$

(4.2)

for identical motor parameters.
4.2.1 Equation of motion

The equation of motion (EoM) of the system can be derived with the Lagrangian equation of the second kind:

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = Q_k \quad (k = 1..4),
\]

(4.3)

where the Lagrangian \( L = E_{\text{kin}} - E_{\text{pot}} \) is the difference of the kinetic and potential energy, \( q_k \) are the elements of generalized coordinate vector

\[
q = [a, \vartheta, \phi, \delta]^T
\]

(4.4)

and \( Q_k \) are the generalized forces resulting from the driving torques and damping. In the equations dot denotes derivation with respect to the time (\( \dot{\square} = \frac{d}{dt} \square \)). Now we need the expression of the kinetic and potential energy and the power of the driving torques and the damping.

\[
E_{\text{kin}} = \frac{1}{2} \left( m_C v_C^2 + \sum_{j=1}^{2} \left( m_0 v_j^2 + J \omega_j^2 \right) \right),
\]

(4.5)

where \( v_j \) and \( \omega_j \) are the velocity and angular velocity of the CoM of each rotor. To obtain the kinetic energy as the function of the generalized coordinates, the following position vectors should be defined (in horizontal–vertical coordinate system):

\[
r_{OC} = a \left[ \cos(\phi - \vartheta), \sin(\phi - \vartheta) \right]^T,
\]

(4.6)

\[
r_{Cj} = e \left[ \cos(\phi \pm \delta), \sin(\phi \pm \delta) \right]^T \quad (j = 1, 2),
\]

(4.7)

thus the velocities and angular velocities are

\[
v_{C} = \dot{r}_{OC}, \quad v_j = \dot{r}_{OC} + \dot{r}_{Cj} \quad \text{and} \quad \omega_j = \dot{\phi} \pm \cdot \quad (j = 1, 2).
\]

(4.8)

In the expression of the potential energy we neglect the effect of the gravitational forces, which is acceptable for the typical frequency range of the device, so we only have terms from the springs.

\[
E_{\text{pot}} = \frac{1}{2} k \left( a \cos(\vartheta - \vartheta) \right)^2 + \frac{1}{2} k \left( a \sin(\vartheta - \vartheta) \right) \]

\[
= \frac{1}{2} k \ a^2
\]

(4.9)
The last expression we need before we derive the EoM is the power of the forces not involved in the calculation yet, which are the driving torques and the damping.

\[
P = \sum_{j=1}^{2} T_j \omega_j - c v_C^2, \quad (4.10)
\]

which we have to expand further, if we want to gather the generalized forces for our generalized coordinates:

\[
P = T_1 (\dot{\delta} + \dot{\phi}) + T_2 (\dot{\phi} - \dot{\delta}) - c a^2 - ca^2 (\dot{\theta}^2 - 2 \dot{\theta} \dot{\phi} + \dot{\phi}^2), \quad (4.11)
\]

which has to have the same value for the generalized forces

\[
P = \sum_{k=1}^{4} Q_k \dot{q}_k. \quad (4.12)
\]

The problem with expression (4.11) is that there is a mixed term of the generalized coordinate velocities, thus we cannot clearly identify where to include it. To solve this problem, we can use the original definition of the generalized force

\[
Q_k = \sum_{i=1}^{n} F_i \frac{\partial r_i}{\partial q_k}, \quad (4.13)
\]

which gives for the generalized coordinate \(\dot{\vartheta}\)

\[
Q_2 = -c v_C \frac{\partial r_{OC}}{\partial \vartheta} = -c a^2 \left( \dot{\vartheta} - \dot{\phi} \right), \quad (4.14)
\]

that means that one half of the mixed term belongs to \(\dot{\vartheta}\) and the other half to \(\dot{\varphi}\). After all, the vector of the generalized force can be written as

\[
Q = \begin{bmatrix} -c \dot{a}, -c a^2 \left( \dot{\vartheta} - \dot{\phi} \right), T_1 + T_2 - c a^2 \left( \dot{\varphi} - \dot{\theta} \right), T_1 - T_2 \end{bmatrix}^T. \quad (4.15)
\]

Now we can write the EoM according to the Lagrangian equation of the second kind (4.3) in the form

\[
M (q) \ddot{q} = h (q, \dot{q}). \quad (4.16)
\]

Introducing \(\hat{m} = m_C + 2m_0\) the total moving mass of the system and \(\hat{J} = J + 2e^2 m_0\) the total mass moment of inertia of one rotor and armature system about the centre
axis, furthermore, introducing that the sine and cosine of an angle is denoted by ‘s’ and ‘c’, respectively with the angular coordinate in the subscript (e.g. $s_\vartheta = \sin \vartheta$), the mass matrix $\mathbf{M}$ looks

$$\mathbf{M} = \begin{pmatrix}
\dot{m} & 0 & -2em_0cs_\vartheta & -2em_0sc_\vartheta \\
0 & a^2\dot{m} & -a^2\dot{m} - 2em_0ac_\vartheta & 2em_0csc_\vartheta \\
-2em_0csc_\vartheta & -a^2\dot{m} - 2em_0ac_\vartheta & a^2\dot{m} + 4em_0ac_\vartheta & 2\dot{J} - 2em_0as_\vartheta \\
-2em_0s_\vartheta & 2em_0as_\vartheta & -2em_0as_\vartheta & 2\dot{J}
\end{pmatrix},$$

and the right-hand side $\mathbf{h}$ has the form

$$\mathbf{h} = \begin{pmatrix}
-c_\vartheta a - sa + a\dot{m}\left(\dot{\vartheta} - \dot{\varphi}\right)^2 - 4em_0\dot{\varphi}s_\vartheta c_\vartheta + 2em_0\left(\dot{\vartheta}^2 + \dot{\varphi}^2\right)c_\vartheta c_\vartheta \\
-\dot{c}_\vartheta a^2\left(\dot{\vartheta} - \dot{\varphi}\right) - 2a\dot{m}\dot{\vartheta} - \dot{\vartheta}^2 - 4em_0\dot{\varphi}s_\vartheta c_\vartheta - 2em_0\left(\dot{\vartheta}^2 + \dot{\varphi}^2\right)c_\vartheta c_\vartheta \\
T_1 + T_2 + ca\left(\dot{\vartheta} - \dot{\varphi}\right) + 2a\dot{m}\dot{\vartheta} - \dot{\vartheta}^2 - 2em_0\dot{\vartheta} - \dot{\varphi}^2 c_\vartheta c_\vartheta + 4em_0\dot{\vartheta} c_\vartheta c_\vartheta + 4em_0\dot{\varphi}s_\vartheta c_\vartheta \\
T_1 - T_2 - 4em_0\dot{\vartheta} - \dot{\varphi}^2 s_\vartheta c_\vartheta - 2em_0a\left(\dot{\vartheta} - \dot{\varphi}\right)^2 s_\vartheta c_\vartheta
\end{pmatrix}.$$  

(4.17)

\[4.2.2 \text{ Dimensionless equation of motion}\]

In order to investigate the dynamics of the system it is reasonable to decrease the number of parameters, which can be done by the following substitutions:

$$\alpha = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\alpha m}, \quad \tau = \alpha t, \quad L_0 = \frac{2em_0}{m},$$

$$\dot{J} = \dot{J} \frac{\dot{m}}{2\epsilon^2 m_0^2}, \quad c_a = \frac{k_1 \dot{m}}{4\epsilon^2 m_0^2 \alpha^2 R}, \quad \kappa = \frac{k_1 k_\alpha \dot{m}}{2\epsilon^2 m_0^2 R},$$

(4.19)

(whose numerical values can be found in Appendix A in Table A.2) and introducing the new dimensionless generalized coordinates

$$a(t) = L_0 A(\tau(t)), \quad \vartheta(t) = \Theta(\tau(t)), \quad \varphi(t) = \Phi(\tau(t)), \quad \delta(t) = \Delta(\tau(t)).$$

(4.20)

The new angular coordinates are practically the same as the original ones, however they are now the functions of the dimensionless time $\tau$. Coordinate $a$ is scaled by $L_0$, which is the distance of the common CoM from point $C$ if the phase angle between the
unbalances is zero. In other words if the suspension of the frame would be ideally soft, the amplitude of the vibration would be $L_0$, thus $A$ would be equal to 1.

Now we have to expand the driving torques of the rotors according to Eqn. (4.2)

$$T_1 = k_t \frac{u_1 - k_\psi \dot{\psi} - k_\omega \dot{\delta} \Delta}{R} \quad \text{and} \quad T_2 = k_t \frac{u_2 - k_\psi \dot{\psi} + k_\omega \dot{\delta} \Delta}{R},$$

(4.21)

thus the sum and the difference of the torques have the form

$$T_1 + T_2 = k_t \frac{u_1 + u_2 - 2k_\omega \dot{\delta} \Delta}{R} \quad \text{and} \quad T_1 - T_2 = k_t \frac{u_1 - u_2 - 2k_\omega \dot{\delta} \Delta}{R}.$$  

(4.22)

Finally, introducing the dimensionless sum and difference of the driving voltages

$$U_\Sigma = c_u (u_1 + u_2) \quad \text{and} \quad U_\Delta = c_u (u_1 - u_2)$$

(4.23)

we obtain the dimensionless EoM

$$\ddot{\mathbf{q}} = \mathbf{h},$$

(4.24)

where $\mathbf{q}$ is the vector of the dimensionless generalized coordinates, prime denotes derivative with respect to the dimensionless time ($\Box' = \frac{d}{d\Box}$), $\mathbf{M}$ is the dimensionless mass matrix

$$\mathbf{M} = \begin{pmatrix}
1 & 0 & -c_{\Delta s\Theta} & -s_{\Delta c\Theta} \\
0 & A & -A - c_{\Delta c\Theta} & s_{\Delta c\Theta} \\
-c_{\Delta s\Theta} & -A^2 - Ac_{\Delta c\Theta} & A^2 + 2Ac_{\Delta c\Theta} + \ddot{J} & -As_{\Delta s\Theta} \\
-s_{\Delta c\Theta} & As_{\Delta s\Theta} & -As_{\Delta s\Theta} & \ddot{J}
\end{pmatrix}$$

(4.25)

and $\mathbf{h}$ is the dimensionless right-hand side of the EoM

$$\mathbf{h} = \begin{pmatrix}
-2\zeta A' - A + A (\Phi' - \Theta')^2 - 2\Phi'\Delta' s_{\Delta s\Theta} + (\Phi'^2 + \Delta^2) \ c_{\Delta c\Theta} \\
(2A' + 2\zeta A) (\Phi' - \Theta') - 2\Phi'\Delta' s_{\Delta c\Theta} - (\Phi'^2 + \Delta^2) \ c_{\Delta s\Theta} \\
U_\Sigma - \kappa \Phi' - 2 (\zeta A^2 + AA' + A'c_{\Delta c\Theta}) (\Phi' - \Theta') + 2A\Phi'\Delta' s_{\Delta c\Theta} - A (\Theta'^2 - 2\Theta'\Phi' - \Delta^2) \ c_{\Delta s\Theta} \\
U_\Delta - \kappa \Delta' + 2A' (\Phi' - \Theta') s_{\Delta s\Theta} - A (\Phi' - \Theta')^2 \ s_{\Delta c\Theta}
\end{pmatrix}$$

(4.26)

Now we have only three parameters left characterizing the system, which are the relative
damping $\zeta$ of the suspended system, $\ddot{J}$ contains inertial properties of the rotors and $\kappa$ which comprises the electrical characteristics of the DC motors. In the sequel we will use these equations and parameters for the analysis of the system.

### 4.3 Investigation of the stationary motions

With the derivation of the EoM we have the possibility to investigate how to reach our primary goal, namely the free adjustment of vibration frequency and amplitude. In the most simple approach, if we had two rotors with independent driving motors and without any coupling between them, the solution would be a simple control of the phase angle and rotational velocity. However, from previous researches on such systems we expect the influence of self-synchronization and the Sommerfeld effect, thus it is reasonable to investigate the stationary motions, where

$$A(\tau) = A_0, \quad \Theta(\tau) = \Theta_0, \quad \Delta(\tau) = \Delta_0 \quad \text{and} \quad \Phi'(\tau) = \lambda.$$  

(4.27)

This means that the phase angle between the rotors is constant at a constant angular velocity, thus the amplitude of the vibration is also constant just like the phase delay of the vibration to the excitation. Furthermore, the input voltages have to be constant as well, thus the stationary dimensionless value of the sum and the difference of the input voltages $U_{\Sigma,0}$ and $U_{\Delta,0}$ are also constant. If such solution exists and it is stable, the system is self-synchronized. With the introduced dimensionless time $\lambda$ is practically the frequency ratio of the excited oscillator by rotating unbalance ($\alpha \square' = \square$), while $\Delta_0$ indicates still the half of the phase angle. The higher derivatives of the coordinates (4.27) are all zero in case of stationary motion.

If we put the constant values in the EoM (4.24), we obtain the following equations

$$- A_0 + A_0 \lambda^2 + \lambda^2 \cos \Delta_0 \cos \Theta_0 = 0, \quad (4.28a)$$

$$2\zeta A_0 \lambda - \lambda^2 \cos \Delta_0 \sin \Theta_0 = 0, \quad (4.28b)$$

$$U_{\Sigma,0} - \kappa \lambda - 2\zeta A_0^2 \lambda = 0, \quad (4.28c)$$

$$U_{\Delta,0} - A_0 \lambda^2 \sin \Delta_0 \cos \Theta_0 = 0. \quad (4.28d)$$
From Eqs. (4.28a) and (4.28b) the amplitude and the phase delay can be expressed as
the function of the frequency ratio, the phase angle and the parameter $\zeta$:

$$\tan \Theta_0 = \frac{2\zeta \lambda}{1 - \lambda^2}, \quad (4.29a)$$

$$A_0 = \frac{\lambda^2 \cos \Delta_0}{\sqrt{(1 - \lambda^2)^2 + (2\zeta \lambda)^2}}, \quad (4.29b)$$

which are very similar to the known formulae for the resonance curve and phase delay
of vibrations forced by rotating unbalance (see Eqn. (3.2) and Fig. 3.8). Furthermore,
substituting Eqs. (4.29a) and (4.29b) into (4.28c) and (4.28d) we can derive the equations
for the input voltages, which we need to achieve the stationary motion for given frequency
ratio and phase angle.

$$U_{\Sigma,0} = \frac{2\zeta \lambda^3 \cos^2 \Delta_0}{(1 - \lambda^2)^2 + (2\zeta \lambda)^2} + \kappa \lambda, \quad (4.30a)$$

$$U_{\Delta,0} = \frac{\lambda^4 (1 - \lambda^2) \cos \Delta_0 \sin \Delta_0}{(1 - \lambda^2)^2 + (2\zeta \lambda)^2}. \quad (4.30b)$$

This means that for every $(\lambda, \Delta_0)$ working point we can find the proper values of the
driving voltages. On the other hand from Eqn. (4.30b) one can see that stationary
motions are possible at non-zero voltage difference as well, which refers to the self-
synchronization of the unbalanced rotors. Later we will investigate the stability of these
stationary motions to prove the existence of self-synchronization. The values for $U_{\Sigma,0}$
and $U_{\Delta,0}$ are plotted in Figure 4.2.

![Surface plot of $U_{\Sigma,0}$ and $U_{\Delta,0}$ for the parameters $\zeta = 0.1$ and $\kappa = 1$.](image)

**Figure 4.2**: Surface plot of $U_{\Sigma,0}$ and $U_{\Delta,0}$ for the parameters $\zeta = 0.1$ and $\kappa = 1$.

If we analyse Eqn. (4.30a) further, we can separate the linear term for the electrical
characteristics of the DC motor $\kappa \lambda$ and the nonlinear term originating from the power consumption of supplying the vibration. This first term increases if the phase angle tends to zero which is in-phase motion of the rotors, and it could have a peak at the resonance of the system if the damping has low value, as it can be observed in Figure 4.2(a). This peak can cause the Sommerfeld effect, since for in-phase motion the driving voltage has to overcome the peak before reaching higher frequencies as the resonant one. As it can be seen from Figure 4.2(a) the controlling of the phase angle could be very useful to avoid the Sommerfeld effect, since in case of anti-phase motion the jamming of the DC motors cannot occur (the peak at the resonance will be diminished).

### 4.3.1 Linear stability of the stationary motions

To investigate the stability of the stationary motions given by the frequency ratio and the phase angle we consider small perturbations around the states of Eqn. (4.27) which can be described by the new coordinates:

\[
\begin{align*}
A(\tau) &= A_0 + \varepsilon \tilde{a}(\tau), \quad A'(\tau) = \varepsilon \tilde{a}'(\tau), \quad A''(\tau) = \varepsilon \tilde{a}''(\tau), \\
\Theta(\tau) &= \Theta_0 + \varepsilon \tilde{\theta}(\tau), \quad \Theta'(\tau) = \varepsilon \tilde{\theta}'(\tau), \quad \Theta''(\tau) = \varepsilon \tilde{\theta}''(\tau), \\
\Delta(\tau) &= \Delta_0 + \varepsilon \tilde{\delta}(\tau), \quad \Delta'(\tau) = \varepsilon \tilde{\delta}'(\tau), \quad \Delta''(\tau) = \varepsilon \tilde{\delta}''(\tau), \\
\Phi'(\tau) &= \lambda + \varepsilon \tilde{\phi}'(\tau), \quad \Phi''(\tau) = \varepsilon \tilde{\phi}''(\tau),
\end{align*}
\]  

where $\varepsilon > 0$ is small parameter, thus we can assume that all powers of $\varepsilon$ higher than one can be considered as zero

\[
\varepsilon^2 \approx 0, \quad \varepsilon^3 \approx 0, \quad \varepsilon^4 \approx 0,
\]

and the sine and cosine of small angles can be linearised as

\[
\sin(\varepsilon \vartheta) \approx \varepsilon \vartheta, \quad \cos(\varepsilon \vartheta) \approx 1, \\
\sin(\varepsilon \delta) \approx \varepsilon \delta, \quad \cos(\varepsilon \delta) \approx 1.
\]
Substituting Eqs. (4.31–4.33) into Eqn. (4.24) and considering Eqs. (4.28a–4.28d) we get the following linear equation of motion:

\[ \tilde{M}_{lin} \ddot{\tilde{q}}_{lin} + \tilde{C}_{lin} \dot{\tilde{q}}_{lin} + \tilde{K}_{lin} \tilde{q}_{lin} = 0, \]  

(4.34)

where \( \tilde{q}_{lin} \) is the vector of the perturbations (4.31), \( \tilde{M}_{lin} \) is the linearised mass matrix, \( \tilde{C}_{lin} \) is the linearised damping matrix, and \( \tilde{K}_{lin} \) is the linearised stiffness matrix defined as

\[
\tilde{M}_{lin} = \begin{pmatrix}
1 & 0 & -c_{\Delta_0} \Theta_0 & -s_{\Delta_0} \Theta_0 \\
0 & A_0 & -A_0 - c_{\Delta_0} \Theta_0 & s_{\Delta_0} \Theta_0 \\
-c_{\Delta_0} s_\Theta_0 & -A_0^2 & A_0^2 + 2A_0 c_{\Delta_0} \Theta_0 + \tilde{J} & -A_0 s_{\Delta_0} s_\Theta_0 \\
-s_{\Delta_0} c_\Theta_0 & A_0 s_{\Delta_0} s_\Theta_0 & -A_0 s_{\Delta_0} s_\Theta_0 & \tilde{J}
\end{pmatrix}, \quad (4.35)
\]

\[
\tilde{C}_{lin} = \begin{pmatrix}
2\zeta & 2\lambda A_0 & -2\lambda A_0 - 2\lambda c_{\Delta_0} \Theta_0 & 2\lambda s_{\Delta_0} s_\Theta_0 \\
-2\lambda & 2\zeta A_0 & -2\zeta A_0 - 2\lambda c_{\Delta_0} \Theta_0 & 2\lambda s_{\Delta_0} c_\Theta_0 \\
2\lambda A_0 + 2\lambda c_{\Delta_0} \Theta_0 & -2\zeta A_0^2 & -2\lambda A_0 c_{\Delta_0} s_\Theta_0 + \kappa + 2\zeta A_0^2 & -2\lambda A_0 s_{\Delta_0} c_\Theta_0 \\
-2\lambda s_{\Delta_0} s_\Theta_0 & -2\lambda A_0 s_{\Delta_0} c_\Theta_0 & 2\lambda A_0 s_{\Delta_0} c_\Theta_0 & \kappa
\end{pmatrix}, \quad (4.36)
\]

\[
\tilde{K}_{lin} = \begin{pmatrix}
1 - \lambda^2 & \lambda^2 c_{\Delta_0} \Theta_0 & 0 & \lambda^2 s_{\Delta_0} c_\Theta_0 \\
-2\zeta \lambda & \lambda^2 c_{\Delta_0} \Theta_0 & 0 & -\lambda^2 s_{\Delta_0} s_\Theta_0 \\
4\zeta \lambda A_0 & 0 & 0 & 0 \\
\lambda^2 s_{\Delta_0} c_\Theta_0 & -\lambda^2 A_0 s_{\Delta_0} s_\Theta_0 & 0 & \lambda^2 A_0 c_{\Delta_0} c_\Theta_0
\end{pmatrix}. \quad (4.37)
\]

Rewriting Eqn. (4.34) in a first order linear equation we get

\[
x' = \begin{pmatrix}
-M_{lin}^{-1} C_{lin} & -M_{lin}^{-1} K_{lin} & I & 0 \\
\end{pmatrix} x, \quad (4.38)
\]

where \( x = (\tilde{a}', \tilde{\varphi}', \tilde{\varphi}', \tilde{\delta}', \tilde{\alpha}, \tilde{\varphi}, \tilde{\delta})^T \) is the vector of the state variables of the system. Since Eqn. (4.38) does not depend explicitly on the cyclic coordinate \( \tilde{\varphi} \), the 7th element of \( x \) and the 7th row and column of \( A \) can be removed, leading to \( \tilde{x} \) and \( \tilde{A} \), respectively.

In the sequel the stability of the stationary motions at given \( \lambda \) and \( \Delta_0 \) values can be investigated by the eigenvalues of the \( 7 \times 7 \) matrix \( \tilde{A} \) using the substitutions (4.29a) and
(a) Stability chart of the dual-rotor system with the expected parameter set ($\tilde{J} = 135.3$, $\kappa = 13.92$ and $\zeta = 0.1$). Unstable regions are marked with grey for $\bullet$, $\diamond$ and $\ominus$ critical characteristic roots.

(b) Stability chart with varied inertial parameters; $\tilde{J} = 1000$ and $\tilde{J} = 10$.

(c) Stability chart with varied electrical parameters; $\kappa = 100$ and $\kappa = 1$.

(d) Stability chart with varied damping; $\zeta = 0.5$ and $\zeta = 0.001$.

Figure 4.3: Stability of the working points of the dual-rotor vibrotactor and its parameter dependency.
Numerical simulation of the dual-rotor system

(4.29b). The stationary motion is considered to be stable if and only if the real parts of all eigenvalues are negative.

In Figure 4.3 various stability charts are presented, where the influence of different system parameters can be observed. White coloured regions show stable working points, while different grey areas show how many eigenvalues of the system matrix have positive real parts. Light grey means that one eigenvalue causes instability, darker grey areas show two or three eigenvalues with positive real part.

One can see, that the stability behaviour around the resonance (i.e. at $\lambda = 1$) can be very complicated, while below and above this region the global behaviour of the system is similar; below the resonant frequency the motion is stable if the phase angle $2\Delta_0$ is around 0, which is in-phase motion of the rotors. The stable phase angle domain is about $90^\circ$ wide. Above the resonance it is the opposite case, the anti-phase motion is stable ($2\delta$ is around $180^\circ$, the width of the stable domain is about $90^\circ$), which is analogous to the idea of automatic ball balancers, where eccentric rotors tend to balance each other in the super-resonant case.

The stability can be showed also in the diagram of the region of operation of the Dual Excenter. In Figure 4.4 one can see, which domains of the region of operation can be reached with the direct setting of the driving voltages.

![Figure 4.4](image)

**Figure 4.4:** Working region of the Dual Excenter combined with the stability of the working points and the sensing threshold of the human skin.

A conclusion of the general stability behaviour of the dual-rotor system is that it is possible to adjust the phase angle of the rotors without any active control of the system only with the difference between the motor driving voltages. However, this method is available only in the stable regions, of course.
4.4 Numerical simulation of the dual-rotor system

In the following the results obtained by analytical investigations will be numerically validated. However, the construction of a numerical simulation enables not only the analysis of the stability, but it makes also possible to test further system components like control algorithms and non-ideal effects in later investigations. In this section the device in only its simplest form (see Figure 3.1) will be simulated.

4.4.1 Generalized coordinates and EoM for numerical simulations

Before we start building a numerical model of the system, the mechanical model has to be reconsidered. Since the position of the centre axis $C$ were given by polar coordinates (which were practical in the sense that the stationary motion could be investigated like an equilibrium position), in the static equilibrium state (where $C$ coincides with $O$) the system becomes singular, and the numerical solution of the EoM is not possible. Because of that we use translational coordinates of the centre axis for numerical simulations. Furthermore, the angular position of the rotors will be given by their natural angular coordinates, since the discrimination of frequency- and phase angle related coordinates in the simulation is not interesting. In the following the modified EoM will be derived.

The new model is shown in Figure 4.5 where the only changes are the new generalized coordinates of point $C$, which are translational coordinates $x$ and $y$ instead of $a$ and $\vartheta$, and the generalized coordinates for the rotors $\varphi_1$ and $\varphi_2$ instead of $\varphi$ and $\delta$.

![Figure 4.5: Modified 4 DoF mechanical model of the Dual Excenter for numerical simulations.](image-url)
The EoM has been derived with the Lagrangian equation of the second kind again. Because of the new coordinates the vector of the generalized coordinates changes to \( \mathbf{q} = [x, y, \varphi_1, \varphi_2]^T \). So the position vectors are

\[
\mathbf{r}_{OC} = [x, y]^T, \quad (4.39)
\]
\[
\mathbf{r}_{Cj} = e [\cos \varphi_j, \sin \varphi_j]^T, \quad (4.40)
\]

and the angular velocities change to \( \omega_j = \dot{\varphi}_j \), while the velocities are the derivatives of the modified position vectors. Then the kinetic energy can be obtained by Eqn. 4.5. The potential energy (Eqn. 4.9) and the expanded form of the power of the generalized forces (Eqn. 4.11) also changes, that is

\[
E_{\text{pot}} = \frac{1}{2} k x^2 + \frac{1}{2} k y^2, \quad (4.41)
\]
\[
P = T_1 \dot{\varphi}_1 + T_2 \dot{\varphi}_2 - c \dot{x}^2 - c \dot{y}^2. \quad (4.42)
\]

This way the vector of the generalized forces becomes

\[
\mathbf{Q} = [-c \dot{x}, -c \dot{y}, T_1, T_2]^T, \quad (4.43)
\]

and now the modified EoM can be written in form of Eqn. 4.24, where the modified mass matrix looks

\[
\mathbf{M} = \begin{pmatrix}
\ddot{m} & 0 & -e m_0 \sin \varphi_1 & -e m_0 \sin \varphi_2 \\
0 & \ddot{m} & e m_0 \cos \varphi_1 & e m_0 \cos \varphi_2 \\
-e m_0 \sin \varphi_1 & e m_0 \cos \varphi_1 & \ddot{j} & 0 \\
-e m_0 \sin \varphi_2 & e m_0 \cos \varphi_2 & 0 & \ddot{j}
\end{pmatrix}, \quad (4.44)
\]

while the modified right-hand side is

\[
\mathbf{h} = \begin{pmatrix}
-k x - c \dot{x} + e m_0 \dot{\varphi}_1^2 \cos \varphi_1 + e m_0 \dot{\varphi}_2^2 \cos \varphi_2 \\
-k y - c \dot{y} + e m_0 \dot{\varphi}_1^2 \sin \varphi_1 + e m_0 \dot{\varphi}_2^2 \sin \varphi_2 \\
T_1 \\
T_2
\end{pmatrix}, \quad (4.45)
\]

where \( T_1 \) and \( T_2 \) have been defined in Eqn. 4.2.
As it can be clearly seen, the obtained terms of the EoM are simpler than those for analytical investigations. Furthermore there isn’t any singular points of the system, so robust numerical solution of the equations is possible.

### 4.4.2 Numerical simulation of the dual-rotor device

The numerical solution of the modified EoM has been carried out with Matlab® Simulink® with the 4th order Runge-Kutta solver (ode4). For the simulations in this section a fixed time step of 0.001 s has been taken.

For the estimation of the system behaviour it is very important to distinguish whether operating below or above the resonant frequency of the suspended frame, since for the sub-resonant case the in-phase motion is stable and for super-resonant case it is the anti-phase motion. With the parameters listed in Table A.1 the resonant frequency is about 52.9 Hz.

In the sequel, simulation results for the characteristic cases of the system behaviour will be showed. First the possibility of the phase angle change at a fixed sub-resonant frequency is showed in Figure 4.6. In this case the desired frequency is constant through the simulation time and the desired phase angle is changed continuously (left side) and step wise (right side). The driving voltage values are computed then as Eqs. 4.30a and 4.30b. From the simulation results it can be seen that for phase angles around 0 this kind of (open-loop) control is possible, but as the desired phase angle increases, we get unstable working points which cannot be reached without closed-loop control, however the motion remains stable at another frequency and phase angle. Furthermore the step-wise results show that the time required to reach the desired states increases as the system gets closer to the unstable regime.

The setting of the phase angle at super-resonant frequency is investigated in the same way and showed in Figure 4.7, however, here is the anti-phase motion of the rotors stable, thus the initial value of the desired phase angle starts at $\pi$.

Comparing the sub- and super-resonant cases one can see that the system reaches the desired state faster at higher frequencies, thus it is feasible to use the open-loop control in the stable region of operation, although near to the stability limit the operation is not robust enough and the phase angle range is limited. This results show the necessity
Figure 4.6: Simulation of the dual-rotor system below the resonant frequency with phase angle changed continuously (left) and in steps (right).

of a closed-loop control to reach robust and wide range operation of the dual-rotor vibrotactor.

To realize the open-loop control, the parameters of the system have to be known. Most of the parameters (like mass and electrical ones) can be identified with good accuracy, but e.g. the stiffness and damping of the suspension cannot be exactly known in most cases, moreover, they may change during operation. If the parameters have significant error, the realized system state can have large errors, and the motion can become unstable.

In Figure 4.8 the voltage difference is changed, while the sum of the driving voltages is held on a constant value below (left) and above (right) the resonant frequency. One can see that first the phase angle changes stably corresponding the voltage difference, but at a value the system gets unstable and the synchronization of the rotors gets lost. This
results in a pulsating vibration of the device, since the phase angle changes continuously.

In the three simulation cases presented in Figure 4.6–4.8 we focused on the change of the phase angle. In case of the frequency our task is much more simple, since the frequency can be set in most cases stably via the sum of the driving voltages. The only problem can be experienced when going through the resonant frequency, which is in connection with the Sommerfeld effect.

4.4.3 The Sommerfeld effect

If both driving motors are driven with the same voltage and the frequency is changed slowly over the resonant value, one experiences strange behaviour which is similar to
Numerical simulation of the dual-rotor system

Figure 4.8: Phase angle change and stability loss due to voltage difference change below (left) and above (right) the resonant frequency.

that of the hardening nonlinear oscillator. Close to the resonance ($\lambda \approx 1$), despite of the increasing voltage the frequency gets stuck, because the increased input power has to cover the dissipated energy due to high amplitudes of the resonant vibration. After reaching a maximum value of the driving voltage the frequency jumps suddenly to the desired value and the resonant vibration disappears. In the opposite way if the frequency is changed from higher to lower values the amplitude peak is smaller. This behaviour is a typical example of the Sommerfeld effect. The results of the simulation are shown in Figure 4.9.

An interesting issue of the simulations in Figure 4.9 is that the anti-phase motion of the rotors is stable in the whole super-resonant frequency domain, however, in Figure 4.3(a) we can see that there is a small unstable patch close to the resonance. If we change the damping ratio $\zeta$ to 0.095 to enlarge the size of the unstable patch, and reduce the rate
of the voltage change, we can observe the unstable behaviour of the system in the signed region. The simulation result is showed in Figure 4.10 where the stationary vibration (belonging to a stable limit cycle) starts at about 10 s and ends at about 30 s. The development of this kind of motion is very slow, that can be the reason for the stable behaviour in Figure 4.9. Another issue is the sensitivity of the system for the damping. As some numerical solver methods can influence the overall damping characteristics of the system, the sampling time and the solver method could cause the deviation between analytical results and simulation.

From the practical point of view this kind of behaviour is not very interesting, since it is limited to the resonant frequency domain and its existence strongly depends on the actual parameter values of the system. However, it could be worth for later analytical investigations.

Figure 4.9: Simulation of the dual-rotor system through the resonant frequency.
4.5 Experimental validation of the analytical and numerical results

The hardware for the measurements has already been presented in Figure 3.6. However, since the behaviour of the device depends on the suspension between the Dual Excenter and the environment, it was advisable to create a suspension which allows the measurements to be reproducible. The suspension has been made relatively soft to have low natural frequencies and isotropic to be as near as possible to the analytical results. For this reason springs have been made of two 0.4 mm thick and 18 mm wide polyethylene stripes as shown in Figure 4.11. This way the natural frequencies of the system in both translational directions are about 4–5 Hz.

The measurements in this section were carried out by the direct setting of the PWM
supply voltages of the DC motors. For this reason a graphical user interface (GUI) has been built in Matlab®. The outlay of the GUI can be seen in Figure 4.12. At the same time with this interface the measured frequency and phase angle signals can be processed, visualized and stored (frequency and phase shift measurement as described in Section 3.5).

4.5.1 Identification of the system parameters

Although most of the system parameters of the prototype device have been considered as known, measurements pointed out some critical deviances between the modelled and real behaviour of the system. The first difference could be found in the voltage–frequency relation of the DC motors. This issue can be easily understood if one considers that no friction and damping of the rotational motion of the rotors has been included in the
Experimental validation of the analytical and numerical results

modelling. After a set of measurements the real performance of the DC motors could be obtained (measurement data can be found in Appendix B). In Figure 4.13 the real and measured performances are plotted, where a linear function could be fitted on the mean voltage of the two motors with a very good correlation of $R^2 = 0.99997$.

\[ f = 30.066 u - 5.3311, \quad (4.46) \]

where $f$ is the frequency in Hz and $u$ is the driving voltage of one DC motor in V. Figure 4.13 shows also that there is some difference between the performances of the two motors, which will have further effects.

Completing Eqn. (4.2) with the torques of dry friction and viscous damping in no load case and assuming that the rotor is rotating with positive angular velocity we obtain

\[ T_{\text{friction}} + T_{\text{damping}} = \frac{k_t}{R} \left( u_j - k_e \omega_j \right). \quad (4.47) \]

Considering that the damping torque is proportional to the angular speed with a coefficient $c_\omega$, after rearranging the equation we get

\[ f = \frac{k_t}{2\pi (c_\omega + k_t k_e)} u - \frac{RT_{\text{friction}}}{2\pi (c_\omega + k_t k_e)}, \quad (4.48) \]

which has the same form as Eqn. 4.46.

In addition to the deviation in the motor performance there was a mismatch between the predicted and measured voltage difference–phase angle relation, namely higher voltage differences were measured for the same phase angle. After investigation of the system
parameters and their effect on the systems behaviour the values of some parameters had to be changed compared to the values of Table A.1. First, the electrical resistance has been changed to 14 Ω, and this has also been validated by measurement. Second, the damping coefficient of the suspension has been changed to 1 Nsm\(^{-1}\), however, no thorough measurements have been carried out on this. Third and least, the overall mass of the system has been increased and the stiffness of the suspension has been decreased to 45 g and 150 Nm\(^{-1}\), respectively, according to the installed suspension.

The modified physical parameters are collected in Appendix A in Table A.3.

**4.5.2 Measurement of the stationary motions**

To validate the results obtained by analytical investigations and numerical simulations a set of measurements were carried out on the stationary motions of the Dual Excenter. In these measurements the relation between driving voltage difference and phase angle were investigated at practically constant frequencies. One of these measurements is presented in Figure 4.14. At a constant voltage sum of 7.32 V (that is 3.66 V for one motor) the difference between the driving voltages is changing slowly and stepwise. After a change in the voltage difference we wait for the system to settle to its stationary motion. The measurement has been carried out between the limits of the synchronization. Higher or lower differences as the ending or beginning cause the loss of synchronization, thus no stationary motion could be observed. Figure 4.14 shows also that the frequency does not change significantly over time.

This method has been repeated for six different \(u_\Sigma\) voltage sums, and the phase angles belonging to each \(u_\Delta\) voltage difference have been registered. The cumulated results of the measurements are presented in Figure 4.15 and they are collected in Appendix B.

One can see, that the peak of the phase angle is not at \(u_\Delta = 0\), which is caused by the slight difference in the performances of the two motors. In the sequel this deviation will be removed from the measured data.

If we want to compare the measurement to the analytical solution, first we have to modify Eqn. 4.30a of the stationary voltage sum according to the measured motor performance:

\[
U_{\Sigma,0} = -\frac{2\zeta \lambda^5 \cos^2 \Delta_0}{(1 - \lambda^2)^2 + (2\zeta \lambda)^2} + \kappa \lambda + \gamma, \quad (4.49)
\]
Experimental validation of the analytical and numerical results

Where $\gamma$ arises from the dry friction of the rotors, and its value can be found among all other identified parameters in Appendix A in Table A.3.

With Eqn. 4.30b and Eqn. 4.49 we can compute the analytical values of the phase angle belonging to each voltage difference value at a given voltage sum. After converting the
Figure 4.16: Calculated and measured voltage difference–phase angle relationship at different $u_\Sigma$ values: (a) 1.46 V, (b) 2.20 V, (c) 2.93 V, (d) 4.39 V, (e) 5.86 V, (f) 7.32 V.

dimensionless results to dimensional ones we can compare the measurements and the analytical results, which is presented in Figure 4.16 for all $u_\Sigma$ values.

The result is convincing. This means that the analytical model works well, however in the stability analysis the modified motor performance was not considered. The reason of this is that the rotational damping of the rotors would not change the virtue of the system, since it could be combined with the speed constant of the motors. However,
the dry friction is not modelled at all. Despite of complex problems resulting from the
effect of the dry friction in many cases, since the rotors do not change the direction of
rotation during operation, the dry friction of the rotors presumably does not change the
systems behaviour much. It only has the effect of a constant moment.

4.6 New results

**Thesis 2.** *A 4Dof mechanical model has been built for the dual-rotor vibroactuator, which makes it possible to identify the self-synchronization and the Sommerfeld effect. The synchronized stationary motions have been studied by means of linear stability analysis and the stationary voltage sum and difference have been expressed in dimensionless form:*

\[
U_{\Sigma,0} = \frac{2\zeta \lambda^5 \cos^2 \Delta_0}{(1-\lambda^2)^2 + (2\zeta \lambda)^2} + \kappa \lambda, \\
U_{\Delta,0} = \frac{\lambda^4 (1-\lambda^2) \cos \Delta_0 \sin \Delta_0}{(1-\lambda^2)^2 + (2\zeta \lambda)^2},
\]

*The linear stability of the system has been numerically determined as a function of the frequency ratio and the phase angle. Furthermore, the linear stability chart has been explored for the region of operation of the device with different sets of system parameters. Generally, the stability of the in-phase motion (2\delta \approx 0 \pm \pi/2) below the resonant frequency and the stability of the anti-phase motion (2\delta \approx \pi \pm \pi/2) above the resonant frequency has been proven. The analytical results have been verified by numerical simulations and measurements.*

Chapter 5

Controlled dual-rotor vibroactuator

In this chapter the main goal is to investigate the possibility of controlling the Dual Excenter to realize the primary operation mode according to Figure 2.1. The control method should be optimized for operating frequencies above the resonant one (considering stiffness and damping characteristics of the human body surface). In other words we focus on the haptic utilization of the Dual Excenter.

As the first step of the controller design we have to consider the possibilities of controlling the Dual Excenter. The most common and widely used is the PD or PID control method for sure, where the control signal of a device consists of a proportional and a differential part or an additional integrator part. This means, that not just the error between the desired and measured signal, but also its derivative is considered, and this may be completed in some cases with a cumulation of the error, which can suppress residual error of the signal.

Obviously, there are other, more sophisticated control methods like model predictive control e.g., but in general these have higher computation demands, and their stability characteristics is more difficult to analyse. A thorough development of an output feedback control of the Dual Excenter via qLPV model and TP model transformation is presented in [Kuti et al., 2014].
5.1 The control method used for the Dual Excenter

In our case the Dual Excenter has two state variables which have to be controlled: the frequency and the phase angle. It has been shown in previous sections that the sum of the motor voltages is in a very strong connection with the frequency of the rotors, while the difference of the voltages has only a weak influence on it. In case of the phase angle, at a certain frequency the voltage difference has the strong influence, however, if the frequency changes, control parameters of the phase angle might also have to be changed for optimal operation.

Control of the frequency

Furthermore, the frequency of the driving motors can be controlled practically very well without a closed-loop control, thus if the motor performance is known (in ideal case), we could set the frequency with a constant driving voltage sum, defined as Eqn. (4.30a). However, because certain parameters of the system can only be estimated, or in some cases parameters can change during operation (e.g. suspension stiffness and damping), it is advisable to apply a proportional type control to reduce the error of the frequency, where the controlled variable is the frequency itself and the control signal is the voltage sum.

In case of higher demands a further integrating part can be applied to fully suppress errors in frequency signal. Usage of a differential part is not advisable, since the differentiation of discrete signals leads to noisy result, furthermore the electric resistance of the motor coil and the self inductance together provide certain damping effect of the frequency.

Thus, there are two possible ways; namely using only a P controller together with predefined motor performance, or using a PI controller with or without predefined motor performance.

Control of the phase angle

The control of the phase angle is a more difficult task than that of the frequency. In contrast to the frequency, which shows a stable behaviour far enough from the resonant
region, the phase angle has stable and unstable domains as well at almost every frequency value. Thus proportional and differential terms of the control signal are essential to exploit the full range of operation of the device. The calculation of the derivative of the phase angle does not limit the performance of the control, since it can be derived from the difference of the motor frequencies, which can be measured with low noise. To decrease the error of the phase angle the predefined voltage difference in Eqn. (4.30b) can be used. In this case the controlled variable is the phase angle, which is related to the vibration amplitude, and the control signal is the difference of the motor voltages.

Furthermore, the problem of the uncertain parameters is also valid for the phase angle, thus the predefined voltage difference may contain significant error. Using an integrating term in the control signal can be a solution in this case, too.

This means that we have the possibility to use a PD controller with predefined voltage difference, if the system parameters are known with good accuracy, or a PID controller to avoid error due to uncertain parameters.

5.2 Stability of the closed-loop controlled system

The most important property of a control algorithm is its stability in the region of operation. To have an overview we can investigate the linear stability of the controlled system analytically in every working point of the region of operation with given parameters. For the analytical investigation we consider the ideal case, where every physical parameters of the system are exactly known, thus the voltage values for the stationary motion can be obtained from Eqs. (4.30b) and (4.30b). Furthermore, the control is also ideal in the sense that the feedback variables are exactly and instantly known, thus the control signal has no delay.

In ideal case the residual error in the controlled signal can be eliminated with the predefined voltage sum and difference, thus in case of stationary motion the integral term can be divided into a constant part resulting from the stationary voltages and an additional varying part. This way, and with the considerations of Section 5.1 we have to investigate a PI controller for the frequency and a PID controller for the phase angle, where the
controlling input voltages have the form

\[ U_{\Sigma,\text{ctrl}} = U_{\Sigma,0} - I_\lambda \int (\lambda - \lambda_{\text{desired}}) \, d\tau - P_\lambda (\lambda - \lambda_{\text{desired}}) \quad \text{and} \quad (5.1a) \]

\[ U_{\Delta,\text{ctrl}} = U_{\Delta,0} - I_\Delta \int (\Delta - \Delta_{\text{desired}}) \, d\tau - P_\Delta (\Delta - \Delta_{\text{desired}}) - D_\Delta \Delta', \quad (5.1b) \]

where \( U_{\Sigma,\text{ctrl}}, U_{\Delta,\text{ctrl}}, U_{\Sigma,0} \) and \( U_{\Delta,0} \) are the controlling and the predefined voltage sums and differences, respectively. The target working point is given by \( \lambda_{\text{desired}} \) and \( \Delta_{\text{desired}} \), while \( P_\lambda, P_\Delta, I_\lambda, I_\Delta \) and \( D_\Delta \) are control parameters.

To be able to handle the integrating term of the control voltages in the EoM we have to extend the vector of the state variables:

\[ x_{\text{ctrl}} = \left( \bar{a}', \bar{a}, \bar{\varphi}', \bar{\varphi}, \bar{\psi}, \bar{\delta}, \int \bar{\varphi}, \int \bar{\delta} \right)^T, \quad (5.2) \]

and the linearised EoM rewritten in first order form looks:

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
x_{\text{ctrl}}' \\
\end{pmatrix}
\]

where \( A_{\text{ctrl}} \) is the state matrix of the linearised and controlled system and \( C_{\text{lin,ctrl}} \) and \( K_{\text{lin,ctrl}} \) are identical with \( C_{\text{lin}} \) and \( K_{\text{lin}} \) except for the elements

\[ \tilde{C}_{\text{lin,ctrl},3,3} = \kappa + 2 \zeta A_0^2 + P_\lambda, \quad (5.4a) \]

\[ \tilde{C}_{\text{lin,ctrl},4,4} = \kappa + D_\Delta \quad \text{and} \quad (5.4b) \]

\[ \tilde{K}_{\text{lin,ctrl},4,4} = \lambda^2 A_0 c_\Delta c_0 + P_\Delta. \quad (5.4c) \]
All other elements of the matrices of the linearised controlled EoM are the same as in the uncontrolled case. Now the stability of the modified linear EoM can be investigated the same way as before, and the results can be seen in Figures 5.1 and 5.2.

Figure 5.1: Stability of the controlled dual-rotor vibrotactor and its dependency on the control parameters.

Figure 5.1(a) shows the system without control as a reference, and the other diagrams of Figure 5.1 show the effect of the five control parameters. One can see that the
parameters $P_\lambda$, $P_\Delta$ and $D_\Delta$ are related to different unstable patches in the stability diagram. Furthermore, the effect of the change in the control parameters can also be observed. In Figure 5.1(b) the effect of the proportional gain of the frequency can be observed. The influenced patch is the one at the resonance at $2\Delta_{\text{desired}}$ near to 0 and $2\pi$, which is in-phase motion of the rotors, where the unstable behaviour results from the Sommerfeld-effect. From Section 4.4.3 it is clear that $P_\lambda$ can counteract this effect.

Figure 5.1(c) shows the effect of the proportional gain of the phase angle. Here we can see a slight difference in the unstable patch at resonance for anti-phase motion, but the obvious effect of $P_\Delta$ is that it makes anti-phase motion and in-phase motion stable at frequencies below and above the resonance, respectively, so globally, the proportional control of the phase angle can counteract the mechanical self-synchronization. Thus it is possible to reach every phase angle value also at higher frequencies via closed-loop control. From Figure 5.2 it can also be seen that the in-phase motion cannot be made globally stable above the resonance, since there is always a frequency where the unstable region appears, it can only be pushed to higher frequencies.

In Figure 5.1(d) the effect of $D_\Delta$ can be seen. This parameter influences the unstable behaviour explained in Figure 4.10, where probably a limit cycle appears. For the present investigations we used a different parameter set of the system as expected for the real device to be able to show the influence of all control parameters. Depending on the system parameters the limit cycle at the resonance frequency for anti-phase motions does not appear, thus the $D_\Delta$ is not always needed for the stability, however it can make the settling time of the phase angle shorter.

Figure 5.1(e) shows the influence of the integral gain of $\lambda$ which is limited to the in-phase motion at the resonance. Other stability limits are only weakly influenced, but the unstable behaviour because of the Sommerfeld-effect disappears even for the lowest frequency ratio.
value of $I_\lambda$. Integral gains lower than 0.001 and higher than 0.1 result practically in the same stability chart as those at the 0.001 and 0.1, respectively.

The integral gain for $\Delta$ has its effect on all other unstable regions. For all positive values of $I_\Delta$ the unstable regions resulting from the mechanical synchronization disappear, and the limit cycle near to the resonance contracts also for higher gains, and it can be even diminished.

So far we used the dimensionless equations and generalized coordinates. It could be beneficial to derive the control parameters also in form with dimension. Using relations (4.19), (4.20) and (4.23) the expressions of the control voltages Eqs. (5.1) can be rewritten in form

$$u_{\Sigma, \text{ctrl}} = u_{\Sigma, 0} - \frac{2\pi I_\lambda}{c_u I_f} \int (f - f_{\text{desired}}) \, dt - \frac{2\pi P_\lambda}{c_u P_f} (f - f_{\text{desired}}) \quad \text{and} \quad (5.5a)$$

$$u_{\Delta, \text{ctrl}} = u_{\Delta, 0} - \frac{\alpha I_\Delta}{c_u I_\delta} \int (\delta - \delta_{\text{desired}}) \, dt - \frac{P_\Delta}{c_u \alpha P_\delta} (\delta - \delta_{\text{desired}}) - \frac{D_\Delta}{c_u \alpha D_\delta} \dot{\delta}. \quad (5.5b)$$

This way we get control parameters with dimension which can be compared to the values used in simulations or experiments.

## 5.3 Modelling the Dual Excenter with digital control

### 5.3.1 Numerical simulation of the controlled system

As it has been mentioned, the linear stability analysis is only valid for the ideal case where system parameters are exactly known, and the control has neither disturbance nor delay. Since this idealised model is very far from the real case, it is advisable to investigate the feasibility of the closed-loop control via numerical simulations. This way the non-ideal effects can be taken into account, which will be counted below.

- The ideal controller used the predefined voltage values for the desired state, thus the residual error of the controlled signal could be eliminated. In the real case
the predefined voltage values can not be exactly calculated, since there can be slight differences in the physical parameters of the prototype device compared to the expected ones, furthermore, the stiffness and damping of the suspension can vary in a wide range. So the predefined voltage values has to be replaced with an integrating term, which is much less sensitive for the variation of the parameters, however, it can destabilize the controller if the integrating gain is too high. In the simulations the integrating term is implemented for both the frequency and the phase angle, and the physical parameters can also be set to any arbitrary value.

- The ideal controller counteracts the errors in the controlled signal instantaneous and with no error. However, in the real case there is always a time period which is needed for the calculation of the control signal, and the controller has also a sampling period, thus a control delay always appears. Furthermore, the exact signal could not be obtained because of the digital units used in the control electronics. This means that the calculation is realized using floating point numbers, and the control voltages can also have only discrete values, depending on the resolution of the PWM module of the controller (10 bit in our case). In the simulations we use a sampling time for the control algorithm, and the discreteness of the control signal has been also considered.

- In the real case the feedback signal is never exactly known because of the measurement method and the digital control unit. First of all, the measurement method described in Section 3.5 uses digital counters to measure the time intervals of one half turn of the rotors. Thus the calculated frequencies and phase angle can also have only discrete values, and the feedback signals are only updated if one or the other rotor completes a half revolution. This can be considered as a zero order hold (ZOH). The discreteness results in significant error in the high frequency range, since there the time period of one revolution is short, thus the discrete representation of the time causes high relative error. On the other hand, the ZOH causes trouble at low frequencies, since the sampling time increases. The effect of these can be observed in Figure 5.3. Other minor effects are the manufacturing errors of the rotors (the shape of the rotor indicates the half turns), and the disturbances of the time period of the controller oscillator. In the simulations the measuring of the feedback signals is precisely modelled, thus the discreteness and the ZOH are considered, furthermore the manufacturing error can also be set.
The last non-ideal effect taken into account is the real behaviour of the DC motor. In our models we used the steady performance of the DC motor, and we have neglected the electric dynamics. In the simulations we introduced further state variables for the motor coil currents, and we also modelled the commutation of the brushes of the motors with a periodic disturbance in the motor torques.

The rotor shaft bearing of the prototype device has been solved with an additional bearing, since the original bearing of the DC motor has a lower limit for radial force than necessary. Thus the bearing is redundant, and unfortunately a slight misalignment of the motor and the additional bearing can be experienced in the prototype, which results in a varying friction force during one revolution. This effect has not been implemented in the simulation yet, although this effect reduces the performance of the controller because of the periodic variation of the frequency.

The mechanical model used in this thesis is a plane model, that is no 3D dynamics has been considered in the simulations. Under conditions this can also cause errors between measurements and simulations, but the obtained results proved that this is not a major limitation of the model.

Rather at low frequencies the effect of the gravitation can be observed in the frequency signal, if the axis of rotation is not vertical. This effect has also not been implemented in the simulations.
Since the feasibility of the closed loop control of the system has only been proven for the ideal case, it is useful to test the control method also for the non-ideal case by numerical simulations. Although the ideal case shows always that higher proportional and derivative gains make the settling of the controlled signal faster and more accurate, in the real case high gains make the system unstable because of the time delay in the control loop. Thus a lower and an upper limit for the control gains can be expected for stable closed-loop control. Furthermore, as we could see in the analytical investigations, there is always an upper limit frequency for every proportional gain of the phase angle where the system becomes unstable.

If we now look at Figure 5.4 where simulation results with a given parameter set at different frequencies are showed, we can see that the control performance is only acceptable in a specific frequency range.

In Figure 5.4 the control parameters are optimized for frequencies around 80 Hz. Thus until 3 s the control method is stable. At 3 s the desired frequency is reduced to a very low value, where the delay of the signal feedback is too high for the chosen control parameters, and the system becomes unstable. If we increase the frequency to 100 Hz or above, the control algorithm cannot counteract the synchronization effect of the mechanical coupling, so, although the motion remains stationary, the desired phase
angle cannot be reached, and so the control performance cannot be considered to be satisfactory.

At higher frequencies the saturation of the integrating term is also considered, which is the case for digital controllers, as well. The saturation has positive effects on the stability of the control, since it removes the effect of old data, on the other hand the residual error cannot be diminished.

Of course, for other frequencies than 80 Hz other control parameter set could be found where the control could work optimally. This work was performed by Kuti in [Kuti et al., 2014]. However, simulations including the presented one showed a high chance to be able to adjust frequency and phase angle independently with a PID controller, which is the primary working mode of the Dual Excenter.

5.4 Experimental investigation of the controlled Dual Excenter

To prove the results of the analytical investigations and numerical simulations a control algorithm has been implemented for the Dual Excenter prototype device. The realized control cannot be considered as an optimal solution. It is rather a qualitative proof that the Dual Excenter concept is feasible. Further optimization of the control can be the subject of later development.

A number of non-ideal effects have been considered in the numerical simulation, nevertheless in the real controller further modifications have been performed. First of all no derivative gain has been used for the phase angle control, since it seemed not to help the control. In the analytical investigations we could see that the derivative term only makes the control faster, but it does not influence the stability too much. Second, the integration of the error between the desired and measured signal has changed to an incrementation carried out at every change of the measured signal by the reflective optical sensors (more detailed in Section 3.5). This made the syntax of the algorithm simpler, but its influence is unfortunately not predictable, however, at approximately constant frequencies the incrementation works like a real integrating term.
As it can be seen in Figure 5.5 the independent control of the frequency and phase angle can be realized with constant control parameters in a quite wide frequency range (30–100 Hz). The performance of the control is satisfactory also for changing phase angle at constant frequency and for changing frequency at constant phase angle. The reason of the peaks in the measured signal is that at every update of the desired values the controller resets the actual value of the integrating term, thus the voltage difference required for a given phase angle has to be gathered by the integrator again.

For lower frequencies another control parameter set has been used. The measurement is showed in Figure 5.6. In the first interval of the measurement the previous parameters were active, so as the frequency was decreased to 20 Hz, the system became unstable. Then the parameters have been updated to their new values (indicated with the red line), and the control stabilized the system again. Even with changed parameters the control method can not be considered to be optimal (in the sense that parameters are optimized to reach the fastest and most robust system behaviour which is possible with the used control algorithm) which can be also seen on the instability at zero phase angle.

For higher frequencies as 100 Hz no parameter set could be found with the simple try-and-error method which would have made the system stable and controllable. In [Kuti et al., 2014] it has been showed that stable behaviour at higher frequencies could be reached e.g. with more accurate signal measurement method.

From the experiments it is obvious that the presented control method can be used for the Dual Excenter, but for optimal operation further developments have to performed.
to reach fast and robust behaviour in the whole frequency range of the human tactile sensation.

Figure 5.6: Measurement of the frequency and phase angle of the Dual Excenter with closed-loop control for low frequency $P_f=0.0293 \, \text{V} \cdot \text{s}$, $I_f \approx 0.0234 \, \text{V}$, $P_\delta=0.233 \, \text{V} \cdot \text{rad}^{-1}$, $I_\delta \approx 0.373 \, \text{V} \cdot \text{s}^{-1} \cdot \text{rad}^{-1}$ and $D_\delta=0 \, \text{V} \cdot \text{s} \cdot \text{rad}^{-1}$, typical frequency considered in integrating terms 20 Hz.
5.5 New results

**Thesis 3.** The linear stability analysis of the ideally controlled dual-rotor device has been performed analytically for the dimensionless control voltage sum and difference:

\[ U_{\Sigma,\text{ctrl}} = U_{\Sigma,0} - I_{\lambda} \int (\lambda - \lambda_{\text{desired}}) \, d\tau - P_{\lambda} (\lambda - \lambda_{\text{desired}}), \]
\[ U_{\Delta,\text{ctrl}} = U_{\Delta,0} - I_{\Delta} \int (\Delta - \Delta_{\text{desired}}) \, d\tau - P_{\Delta} (\Delta - \Delta_{\text{desired}}) - D_{\Delta} \Delta'. \]

Stability charts for various control parameters have been explored over the region of operation of the device. The influence of the control parameters has been separated for the different domains of the region of operation. The effect of the proportional gain \( P_{\lambda} \) of the frequency ratio is mainly limited to the unstable patch of the in-phase motion at the resonance frequency, which is related to the Sommerfeld effect. The proportional gain \( P_{\Delta} \) of the phase angle counteracts the mechanical self-synchronization, thus it has major role making anti-phase motion in sub-resonant and in-phase motion in super-resonant region stable, respectively. The derivative gain \( D_{\Delta} \) of the phase angle has only influence on unstable anti-phase motion close to the resonant frequency. With the smallest value of the integrating gain \( I_{\lambda} \) the unstable region connected to the Sommerfeld effect can be stabilized. Finally, for all positive value of \( I_{\Delta} \) unstable anti-phase motion below the resonance and unstable in-phase motion above the resonance can be stabilized, furthermore the unstable anti-phase region close to the resonant frequency can be reduced or removed.

Publications in connection with the results [Miklós and Szabó, 2013b].
Thesis 4. The non-ideal effects resulting from the digital virtue of the control (e.g. sampling time, discrete control signals, floating point representation) and feedback signal measurement (e.g. zero-order-hold, discreteness) were investigated by means of numerical simulations, which proved the feasibility of closed-loop control of dual rotor vibroactuators.

A PI controller for the frequency and a PID controller for the phase angle has also been implemented for the digital controller of the realized prototype device. Its performance was confirmed by measurements in various frequency ranges which showed that the developed prototype device with the used control algorithm is able to generate vibrations of independent frequency and amplitude.

Publications in connection with the results [Kuti et al., 2014, Miklós and Szabó, 2015].
Chapter 6

Summary

This thesis presented the Dual Excenter vibrotactor, which is a novel dual-rotor ERM exciter device for generating of vibrations with independent frequency and amplitude.

The literature survey of the human tactile sense in the first chapter is encouraging that with independent frequency and amplitude the information density of the tactile stimuli can be increased. Related researches show that the mechanoreceptors of the skin makes humans able to differentiate between vibrations with different amplitudes or frequencies. For this reason the concept of a dual-rotor ERM vibrotactor has been investigated analytically, with numerical simulations and with measurements, too. In the second chapter the dual-rotor design has been presented and possible advantages and disadvantages have been listed. We showed that the advantage of the independently adjustable frequency and amplitude can be realized with some increase of the complexity of the design. Furthermore, since nowadays mobile devices use tactile feedback very often, the industrial potential of the concept is also quite high. These results have been concluded in Thesis 1.

For analytical and numerical investigations a simple but satisfactory planar mechanical model has been built. The stationary motions of the primary working mode of the Dual Excenter has been investigated by linear stability analysis which showed that there are stable operating regions of the device also without a closed-loop control. This is caused by the interesting mechanical phenomenon of the self-synchronization.
Based on the mechanical model numerical simulations in Simulink® have been performed, where transient motion of the system could also be investigated. The simulations showed very good agreement with the analytical results, and made it possible to design a prototype device which has also been built. This made us possible to compare analytical and numerical results with measurements. These experiments were very satisfactory, the simulations showed good qualitative and quantitative agreement with the behaviour of the realized prototype device. These results have been concluded in Thesis 2.

Because of the unstable operating regions of the uncontrolled Dual Excenter device it was necessary to apply a closed loop control, which was investigated analytically. The stability charts have been explored for the region of operation, and the effect of the control parameters has been separated. The results have been summarized in Thesis 3.

The closed-loop control has also been tested by simulations and by measurements. In the simulations we were able to model non-ideal effects of the digital controller and DC motors and to consider uncertain physical parameters of the device. The numerical and experimental results of the Dual Excenter with closed loop control have been concluded in Thesis 4.

Concluding the work included in this thesis we can state that we have reached our goal. A concept and a prototype device have been build which make it possible to generate vibrations with independent amplitude and frequency, furthermore, the simplicity of the design makes it possible to use the device in small mobile applications.

**Outlook**

Although the primary goal has been reached, the presented prototype device has to be further refined and miniaturized for real tactile applications in mobile devices.

The performance of the presented control algorithm has also to be further optimized. Stable and robust behaviour in the whole range of the operation frequency should be reached, therefore more sophisticated setting of the control parameters, other control method or the improvement of the hardware could be considered.
If these tasks can be successfully done, the device could be tested for tactile applications. From academic point of view it would make it very easy to test amplitude and frequency dependency of the human tactile sense, and the industrial potential of such device is also very promising.
Appendix A

Physical parameters

Table A.1: Parameters of the Dual Excenter prototype used for mechanical modelling and numerical simulations.

<table>
<thead>
<tr>
<th>Name</th>
<th>Sign</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass of the frame</td>
<td>$m$</td>
<td>42.1</td>
<td>g</td>
</tr>
<tr>
<td>eccentric mass of one rotor</td>
<td>$m_0$</td>
<td>1.6</td>
<td>g</td>
</tr>
<tr>
<td>inertia of one rotor</td>
<td>$J$</td>
<td>59.8</td>
<td>g mm$^2$</td>
</tr>
<tr>
<td>eccentricity</td>
<td>$e$</td>
<td>2.1</td>
<td>mm</td>
</tr>
<tr>
<td>spring stiffness</td>
<td>$k$</td>
<td>5000</td>
<td>N m$^{-1}$</td>
</tr>
<tr>
<td>damping coefficient</td>
<td>$c$</td>
<td>3</td>
<td>N s m$^{-1}$</td>
</tr>
<tr>
<td>torque constant of one motor [Farnell, 2014]</td>
<td>$k_t$</td>
<td>5.08</td>
<td>N mm A$^{-1}$</td>
</tr>
<tr>
<td>speed constant of one motor</td>
<td>$k_e$</td>
<td>5.08</td>
<td>mV s rad$^{-1}$</td>
</tr>
<tr>
<td>electric resistance of one motor</td>
<td>$R$</td>
<td>11.3</td>
<td>Ω</td>
</tr>
<tr>
<td>electric inductance of one motor</td>
<td>$L_i$</td>
<td>0.19</td>
<td>mH</td>
</tr>
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</table>

Table A.2: Numerical values of the non-dimensioning parameters.

<table>
<thead>
<tr>
<th>Name</th>
<th>Sign</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>natural angular velocity of the suspended frame</td>
<td>$\alpha$</td>
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<td>rad s$^{-1}$</td>
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<tr>
<td>damping coefficient of the suspended frame</td>
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<td>–</td>
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<td>amplitude of the non-suspended frame</td>
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<td>mm</td>
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<tr>
<td>inertia coefficient</td>
<td>$\bar{J}$</td>
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<td>–</td>
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<tr>
<td>voltage coefficient</td>
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<td>V$^{-1}$</td>
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<tr>
<td>self inductance coefficient</td>
<td>$\kappa$</td>
<td>13.92</td>
<td>–</td>
</tr>
</tbody>
</table>
## Appendix A. Physical parameters

### Table A.3: Identified parameters of the Dual Excenter prototype used for comparison with measurements in Section 4.5.2.

<table>
<thead>
<tr>
<th>Name</th>
<th>Sign</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
<tr>
<td>mass of the frame</td>
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<td>g</td>
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<tr>
<td>eccentric mass of one rotor</td>
<td>$m_0$</td>
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<td>g</td>
</tr>
<tr>
<td>inertia of one rotor</td>
<td>$J$</td>
<td>59.8</td>
<td>g mm$^2$</td>
</tr>
<tr>
<td>eccentricity</td>
<td>$e$</td>
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<td>mm</td>
</tr>
<tr>
<td>spring stiffness</td>
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<td>N m$^{-1}$</td>
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<td>damping coefficient</td>
<td>$c$</td>
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<td>N s m$^{-1}$</td>
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<td>torque constant of one motor</td>
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<td>N mm A$^{-1}$</td>
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<tr>
<td>speed constant of one motor</td>
<td>$k_e$</td>
<td>5.08</td>
<td>V s rad$^{-1}$</td>
</tr>
<tr>
<td>electric resistance of one motor</td>
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<td>Ω</td>
</tr>
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<td>rotational damping coefficient</td>
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<td>N s m</td>
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<tr>
<td>friction torque</td>
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<td>N mm</td>
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<td>voltage coefficient</td>
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<td>V$^{-1}$</td>
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<td>self inductance coefficient</td>
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<td>–</td>
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<tr>
<td>dimensionless friction parameter</td>
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# Appendix B

## Measurement data

### Motor performance

**Table B.1**: Motor characteristics measurement.

<table>
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<th>Sample number</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td><strong>PWM duty A ([1024^{-1}])</strong></td>
<td>63</td>
<td>83</td>
<td>102</td>
<td>127</td>
<td>151</td>
<td>176</td>
<td>200</td>
<td>226</td>
<td>250</td>
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<tr>
<td><strong>PWM duty B ([1024^{-1}])</strong></td>
<td>58</td>
<td>78</td>
<td>98</td>
<td>124</td>
<td>149</td>
<td>174</td>
<td>200</td>
<td>225</td>
<td>251</td>
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<tr>
<td><strong>Motor A voltage ([V])</strong></td>
<td>0.46</td>
<td>0.61</td>
<td>0.75</td>
<td>0.93</td>
<td>1.11</td>
<td>1.29</td>
<td>1.46</td>
<td>1.66</td>
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<tr>
<td><strong>Motor B voltage ([V])</strong></td>
<td>0.42</td>
<td>0.57</td>
<td>0.72</td>
<td>0.91</td>
<td>1.09</td>
<td>1.27</td>
<td>1.46</td>
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<tr>
<td><strong>Frequency ([Hz])</strong></td>
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<td>12.7</td>
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<td>2.91</td>
<td>3.63</td>
<td>4.35</td>
<td>5.43</td>
</tr>
<tr>
<td><strong>Motor B voltage ([V])</strong></td>
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<td>2.21</td>
<td>2.59</td>
<td>2.95</td>
<td>3.71</td>
<td>4.44</td>
<td>5.55</td>
</tr>
<tr>
<td><strong>Frequency ([Hz])</strong></td>
<td>55.3</td>
<td>60.9</td>
<td>71.8</td>
<td>82.5</td>
<td>105.2</td>
<td>127.2</td>
<td>159.4</td>
</tr>
</tbody>
</table>
# Appendix B. Measurement data

## Voltage difference–phase angle

Table B.2: Voltage difference–phase angle measurement 200 frq=16.8 Hz.

<table>
<thead>
<tr>
<th>Sample number</th>
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<tbody>
<tr>
<td>PWM diff. [1024⁻¹]</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>voltage diff. [V]</td>
<td>0.059</td>
<td>0.051</td>
<td>0.044</td>
<td>0.037</td>
<td>0.029</td>
<td>0.022</td>
<td>0.015</td>
</tr>
<tr>
<td>phase angle [°]</td>
<td>n.A.</td>
<td>127</td>
<td>155</td>
<td>177</td>
<td>155</td>
<td>122</td>
<td>n.A.</td>
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</tbody>
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Table B.3: Voltage difference–phase angle measurement 300 frq=28 Hz.

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<tbody>
<tr>
<td>PWM diff. [1024⁻¹]</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
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<tr>
<td>voltage diff. [V]</td>
<td>0.059</td>
<td>0.051</td>
<td>0.044</td>
<td>0.037</td>
<td>0.029</td>
<td>0.022</td>
<td>0.015</td>
<td>0.007</td>
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<tr>
<td>phase angle [°]</td>
<td>n.A.</td>
<td>106</td>
<td>129</td>
<td>143</td>
<td>155</td>
<td>165</td>
<td>177</td>
<td>172</td>
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Table B.4: Voltage difference–phase angle measurement 400 frq=39 Hz.

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<tbody>
<tr>
<td>PWM diff. [1024⁻¹]</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>8</td>
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<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>voltage diff. [V]</td>
<td>0.081</td>
<td>0.073</td>
<td>0.066</td>
<td>0.059</td>
<td>0.051</td>
<td>0.044</td>
<td>0.029</td>
<td>0.015</td>
</tr>
<tr>
<td>phase angle [°]</td>
<td>n.A.</td>
<td>112</td>
<td>127</td>
<td>133</td>
<td>142</td>
<td>147</td>
<td>157</td>
<td>172</td>
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Table B.5: Voltage difference–phase angle measurement 600 frq=61.9 Hz.

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<tbody>
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<td>PWM diff. [1024⁻¹]</td>
<td>-32</td>
<td>-31</td>
<td>-30</td>
<td>-28</td>
<td>-26</td>
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<td>voltage diff. [V]</td>
<td>-0.234</td>
<td>-0.227</td>
<td>-0.220</td>
<td>-0.205</td>
<td>-0.190</td>
<td>-0.176</td>
<td>-0.161</td>
<td>-0.146</td>
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<td>phase angle [°]</td>
<td>n.A.</td>
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<td>114</td>
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<td>131</td>
<td>137</td>
<td>143</td>
<td>147</td>
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<th>13</th>
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<th>16</th>
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<tr>
<td>voltage diff. [V]</td>
<td>0</td>
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<td>-0.029</td>
<td>-0.044</td>
<td>-0.059</td>
<td>-0.066</td>
<td>-0.073</td>
<td>-0.081</td>
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<tr>
<td>phase angle [°]</td>
<td>177</td>
<td>167</td>
<td>157</td>
<td>145</td>
<td>131</td>
<td>123</td>
<td>115</td>
<td>n.A.</td>
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### Table B.6: Voltage difference–phase angle measurement 800 frq=84.3 Hz.

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<tbody>
<tr>
<td>PWM diff. [1024⁻¹]</td>
<td>-56</td>
<td>-55</td>
<td>-54</td>
<td>-52</td>
<td>-50</td>
<td>-47</td>
<td>-44</td>
<td>-40</td>
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<tr>
<td>voltage diff. [V]</td>
<td>-0.410</td>
<td>-0.403</td>
<td>-0.396</td>
<td>-0.381</td>
<td>-0.366</td>
<td>-0.344</td>
<td>-0.322</td>
<td>-0.293</td>
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<tr>
<td>phase angle [°]</td>
<td>n.A.</td>
<td>102</td>
<td>108</td>
<td>115</td>
<td>121</td>
<td>127</td>
<td>133</td>
<td>139</td>
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<th>16</th>
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<tbody>
<tr>
<td>PWM diff. [1024⁻¹]</td>
<td>-35</td>
<td>-30</td>
<td>-25</td>
<td>-20</td>
<td>-15</td>
<td>-10</td>
<td>-5</td>
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<tr>
<td>voltage diff. [V]</td>
<td>-0.256</td>
<td>-0.220</td>
<td>-0.183</td>
<td>-0.146</td>
<td>-0.110</td>
<td>-0.073</td>
<td>-0.037</td>
<td>0</td>
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<tr>
<td>phase angle [°]</td>
<td>147</td>
<td>154</td>
<td>160</td>
<td>167</td>
<td>173</td>
<td>179</td>
<td>175</td>
<td>169</td>
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<th>22</th>
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<th>24</th>
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<tbody>
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<td>PWM diff. [1024⁻¹]</td>
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<td>25</td>
<td>30</td>
<td>33</td>
<td>36</td>
</tr>
<tr>
<td>voltage diff. [V]</td>
<td>0.037</td>
<td>0.073</td>
<td>0.110</td>
<td>0.146</td>
<td>0.183</td>
<td>0.220</td>
<td>0.242</td>
<td>0.264</td>
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<td>phase angle [°]</td>
<td>163</td>
<td>156</td>
<td>150</td>
<td>143</td>
<td>135</td>
<td>127</td>
<td>120</td>
<td>112</td>
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</thead>
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<td>PWM diff. [1024⁻¹]</td>
<td>37</td>
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<td>voltage diff. [V]</td>
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<td>0.278</td>
<td>0.286</td>
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<td>phase angle [°]</td>
<td>109</td>
<td>106</td>
<td>101</td>
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### Table B.7: Voltage difference–phase angle measurement 1000 frq=106.6 Hz.

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<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWM diff. [1024⁻¹]</td>
<td>-91</td>
<td>-90</td>
<td>-89</td>
<td>-88</td>
<td>-86</td>
<td>-84</td>
<td>-82</td>
<td>-80</td>
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<td>voltage diff. [V]</td>
<td>-0.667</td>
<td>-0.659</td>
<td>-0.652</td>
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<td>-0.615</td>
<td>-0.601</td>
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<td>phase angle [°]</td>
<td>n.A.</td>
<td>95</td>
<td>101</td>
<td>105</td>
<td>110</td>
<td>115</td>
<td>118</td>
<td>121</td>
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<th>13</th>
<th>14</th>
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<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWM diff. [1024⁻¹]</td>
<td>-75</td>
<td>-70</td>
<td>-65</td>
<td>-60</td>
<td>-50</td>
<td>-40</td>
<td>-30</td>
<td>-20</td>
</tr>
<tr>
<td>voltage diff. [V]</td>
<td>-0.549</td>
<td>-0.513</td>
<td>-0.476</td>
<td>-0.439</td>
<td>-0.436</td>
<td>-0.366</td>
<td>-0.293</td>
<td>-0.220</td>
</tr>
<tr>
<td>phase angle [°]</td>
<td>128</td>
<td>133</td>
<td>139</td>
<td>143</td>
<td>152</td>
<td>152</td>
<td>160</td>
<td>168</td>
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<th>19</th>
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<th>21</th>
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<th>23</th>
<th>24</th>
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<tr>
<td>PWM diff. [1024⁻¹]</td>
<td>-10</td>
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<td>30</td>
<td>40</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>voltage diff. [V]</td>
<td>-0.073</td>
<td>0</td>
<td>0.073</td>
<td>0.146</td>
<td>0.220</td>
<td>0.293</td>
<td>0.330</td>
<td>0.366</td>
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<td>phase angle [°]</td>
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<td>162</td>
<td>154</td>
<td>145</td>
<td>136</td>
<td>129</td>
<td>124</td>
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<th>29</th>
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<td>63</td>
<td>64</td>
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<td>voltage diff. [V]</td>
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<td>0.439</td>
<td>0.454</td>
<td>0.461</td>
<td>0.469</td>
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<tr>
<td>phase angle [°]</td>
<td>118</td>
<td>109</td>
<td>103</td>
<td>100</td>
<td>n.A.</td>
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Bibliography


