Budapest University of Technology and Economics
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Booklet of PhD Theses

Computed Torque Control and Utilization of Parametric Excitation for Underactuated Dynamical Systems

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Introduction

The purpose of the research work presented in this study is the extension of certain control algorithms for underactuated multibody systems. In classical robotics, the number of degrees of freedom is 6 and the number of actuators is also 6. These multibody systems are fully actuated and they can accomplish essential tasks in the 3D space. However, underactuated multibody systems appear in nature and in engineering almost everywhere. Consider, for example, the human grasping, walking and running, the fishes' swimming, the birds' flying and the corresponding engineering structures, like robotic hands, passive walkers, boats, under-water and air vehicles, cranes, see Fig. 1b and c. While the control algorithms of these underactuated systems are much more complicated, their mechanical structures provide energy efficient and agile operation. The elasticity of the mechanical parts of a robotic system like light-weight robots can also be handled as an underactuated system, see Fig. 1a.

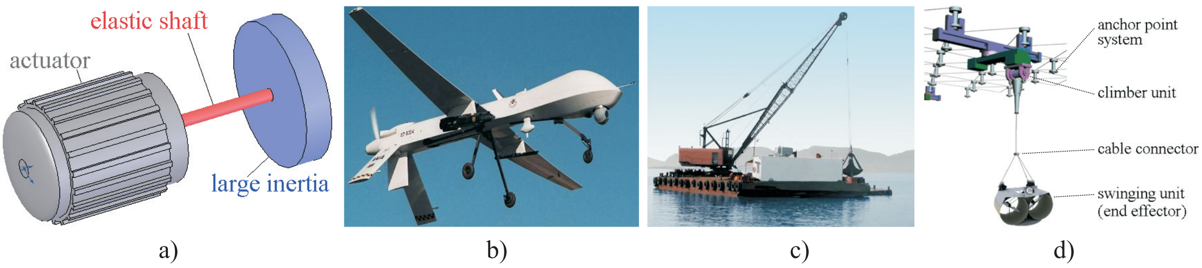

FIGURE 1: Real life examples for underactuated dynamical systems

The following definition of underactuated mechanical systems is general in the literature. Consider a general controlled mechanical system, the mathematical model of which is usually given in the form of a second order ordinary differential equation:

$$\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, t) + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, t)\mathbf{u}\,, \tag{1}$$

where $\mathbf{q}$ is the vector of the generalized coordinates of minimum number, $\mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, t)$ is a vector field that determines the dynamics of the system, including gravitational, spring, damper forces and also centrifugal and Coriolis forces, gyroscopic effects, and so on. $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, t)$ is the control input matrix and $\mathbf{u}$ is the control input vector, which represent the actuator forces and torques. The only assumption in the general equation of motion (1) is that the control input $\mathbf{u}$ appears linearly.

The system is fully actuated, if the rank of the input matrix $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, t)$ equals to the DoF of the system:

$$\mathrm{rank}(\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, t)) = \dim(\mathbf{q})\,. \tag{2}$$

We speak about underactuated systems if the number of the independent control inputs is lower than the DoF of the system or in other words, the rank of $\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, t)$ is smaller than the dimension of $\mathbf{q}$:

$$\mathrm{rank}(\mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, t)) < \dim(\mathbf{q})\,. \tag{3}$$

Overactuated systems also exist, when the number of the independent control inputs is larger than the DoF and more than one actuator can be in connection with one DoF, like in the muscular system of humans and animals. The study of overactuated systems is outside of the focus of the present research work.

Aims of the work

The present research was motivated by the developement of a domestic robot called Acroboter within the European Union 6th Framework Project (IST-2006-045530) coordinated by the Department of Applied Mechanics, Budapest University of Technology and Economics.

The Acroboter hangs down from the ceiling on a suspension cable similarly to cranes (see Fig. 1d), and it is able to utilize the pendulum-like motion efficiently. This specially designed domestic robot has 12 DoF multibody structure and 10 actuators only, consequently it is underactuated, which requires the extension of the existing motion control algorithms. This extension was necessary partly because the robot has some essential singular configurations when minimum set of general coordinates $\mathbf{q}$ are used. This problem can be resolved by means of non-minimum set of the appropriate choice of redundant descriptor coordinates, which are widely used for the description of multibody dynamical systems in the literature. Since the prescribed task related to the position and orientation of a rigid body in 3D space, the task is 6 dimensional only, and consequently, the Acroboter is also a kinematically redundant structure. The corresponding dynamical model is a system of differential algebraic equations. The present work addresses the developement of model based motion control algorithms for underactuated multibody systems, in general.

As an application of the results, the proposed control algorithms are applied for varying topology systems, like fully actuated systems in the presence of actuator saturation. Actuator saturation is a relevant nonlinearity, which is treated here as a decrement in the number of independent control inputs. Another group of varying topology underactuated systems in focus belong to the limbless locomotion.

One of the most intricate problems is when certain tasks are prescribed for the passive DoF of an underactuated system. By augmenting the actuator forces with some periodic excitation for the active DoF, the tasks could be approached even for the passive DoF. Since this periodic excitation at the actuators usually presents some time-periodic parameters in the equations of motion, this kind of forcing is called parametric excitation in classical mechanics. In this sense, parametric excitation could succesfully be used for the control of certain underactuated systems. Case studies of stabilization of water vessels and the control of pendulum-like robots via parametric excitation are presented.

Finally, the motion control of the Acroboter is accomplished, which is partially based on closed form formulae derived from simplified pendulum-like models of the robot. The simplified control appoaches are combined with the general methods derived in the first part of the dissertation. The control approaches are tested and applied in laboratory experiments for the Acroboter prototype.

Thesis 1.

The method of computed torque control was extended for underactuated dynamical systems, where a non-minimum set of descriptor coordinates is used to avoid singularities and nonlinearities in the generalized mass matrix. It was shown that the computed torque control can be realized in real time by means of the backward Euler discretization if the relative degree of the system is two, which is often the case in multibody systems.

Two procedures were developed and compared to each other:

a) In case the controlled and uncontrolled descriptor coordinates can be separated, the calculation of the control torques was carried out in a reduced size algebraic system.

b) In case the separation of the controlled and uncontrolled coordinates is not feasible, the control torque calculation was carried out by means of the coupling of the servo-constraints and the algebraic system resulted by the implicit Euler discretization.

While method b) is more general and easy to implement, the computational demand of method a) is more favourable. The underactuated Aeropendulum rig was used to show experimentally that the CTC can be realized in real time even with the more general control algorithm b).

Related publications: [1, 2, 3, 4, 5, 6, 7, 8]

Thesis 2.

The method of computed torque control was generalized for underactuated dynamical systems where inertial coupling appears between the control input and the desired output, and the desired motion is prescribed by means of servo-constraints in a mathematical structure similar to the geometric constraints. It was shown that the computed torque control can be realized in real time by means of the method of Lagrange multipliers even for some non-collocated mechanical systems.

In the differential-algebraic equation model of the system, two procedures were developed using the Baumgarte stabilization for the servo-constraints only. Both procedures are efficient if they are far enough from singularity configurations of the coordinates and the control actions.

It was demonstrated that the control torque can be expressed in an explicit form by means of the null-space projection and the pseudo-inverse of the input matrix. This procedure is computationally more efficient than the application of the direct inversion of the coefficient matrix in the full linear system, which works for collocated mechanical systems only.

Related publications: [9, 10, 11, 6, 8]

Thesis 3.

The actuator saturation is considered as a change in the topology of the controlled dynamical system: the fully actuated robot becomes underactuated as some of the actuators saturates. A high level algorithm is developed that switches between the computed torque control algorithm applied for the fully actuated system and another applied for the underactuated one.

A servo-constraint $\hat{\boldsymbol{\sigma}}$ of reduced dimension is introduced for the motion design of the underactuted system, which is generated from the original servo-constraint $\boldsymbol{\sigma}$ of the fully-actuated system with the help of the transformation:

$$\hat{\boldsymbol{\sigma}} = \left(-(\boldsymbol{\sigma}_{\mathbf{q}}^{\mathrm{T}})^{\dagger}\mathbf{H}\mathbf{T}\right)^{\dagger} \boldsymbol{\sigma} \,,$$

where $\boldsymbol{\sigma}_{\mathbf{q}}$ is the Jacobian of the servo-constraint, $\mathbf{H}$ is the control input matrix, $\mathbf{T}$ is the selector matrix that identifies the non-saturated control inputs and $\dagger$ refers to the pseudo-inverse of the eventually non-quadratic matrices occuring during the reduction process.

With the help of this procedure, the trajectory tracking error of the saturated system is designed and can be optimized.

Related publications: [12, 13]

Thesis 4.

It was proven that the unstable equilibrium position of a floating body on the surface of liquid can be stabilized by means of parametric excitation. The row-vessels, like kayak and canoe, having unstable prescribed equilibrium position, were modelled as underactuated mechanical systems where the athlete can stabilize the vessel only with the periodic vertical motion of his/her centre of gravity, while the vessel is considered as a 3 DoF body moving in the vertical plane. The stability chart of the desired vertical position of the vessel was constructed. By means of this chart, the required rhythm ω of rowing was determined, where the stabilization can already be achived with minimum effort:

$$\omega^* \approx \frac{1}{4}\sqrt{\frac{g}{h}}\sqrt{\frac{m}{J_\mathrm{C}}\frac{6h(2p-h)-a^2}{3(\sqrt{38}-6)}}\,,$$

where a is the characteristic width size of the vessel, p is the height of the common centre of gravity of the vessel and the athlete, h is the diving depth, m is the mass and J_C is the mass moment of inertia with respect to roll axis at the mass centre. The mechanical model and the results were validated by means of the empirical geometric data and the rowing rhythm values in the range of 75-90 strokes/minute. The stabilizability of unstable equilibria of floating bodies by the use of parametric excitation was also demonstrated by a small scale test rig.

The numerical study of the nonlinear system indicated that small amplitude chaotic oscillations arise when the stabilization by parametric excitation is unsuccesful in Ljapunov sense. This means that the system can be considered practically stable even in these cases.

Related publications: [14, 15, 16, 17]

Thesis 5.

The method of Lagrange multipliers was extended for the computed torque control of underactuated robots in cases when the motion of the robot is restricted to a prescribed hypersurface in ith phase-space, where the hypersurface is given as a function of conveniently chosen optional parameters. The number of these parameters is equal to the difference of the degrees of freedom and the number of actuators.

If the prescribed hypersurface can be expressed in implicit form by means of the elimination of the chosen parameters, the corresponding implicit form was defined as a servo-constraint. The computed torque control method was constructed with the corresponding servo-constraint based algorithm, which reduces the size of the numerical task compared to the existing methods in the literature. The size of the task was reduced by the dimension of the eliminated parameters. The elimination of these parameters might be possible, because the time history of the motion on the hypersurface cannot be prescribed in case of underactuated systems.

Related publication: [18]

Thesis 6.

Analytical solution was derived for the inverse dynamics of the Acroboter platform in planar case when the desired orientation of the swinging unit is horizontal during operation. The corresponding CTC algorithms were also tested experimentally on a swinging unit attached to an industrial robot. The inverse dynamical calculation was accomplished also for the spatial multiple mathematical pendulum in closed form, when the trajectory of the lower endpoint is prescribed.

It was shown that the closed form formulae for the desired motion of the actuator and for the desired control forces are identical for the planar Acroboter model and the double pendulum. The results were generalized for the spatial Acroboter model and the simulations indicated that the results coincide with those of the spatial double pendulum. This means that a simplified double pendulum model can be used in the computed torque control of the Acroboter platform. This way, less computational time is needed and the efficiency of the real-time control is improved.

The analytical results can also be used for the validation of numerical results when alternative control methods are tested, like in case of the benchmark problem of underactuated multiple pendulum.

Related publications: [19, 20, 21, 22, 4, 23]

Thesis 7.

In case of underactuated systems, it was shown that the classical definition of kinematic redundancy, which is based on the comparison of the degrees of freedom and the dimension of the prescribed task, is not equivalent to the non-uniqueness of the inverse dynamic problem, while it guarantees the non-uniqueness of the inverse kinematic problem. To clarify this issue, the notion of dynamic redundancy was introduced, which is related to the comparison of the dimension of the prescribed task and the number of the actuators: if the number of degrees of freedom is larger than the number of actuators, and the latter is larger than the dimension of the prescribed task, then the inverse dynamic problem has no unique solution either. Accordingly, the computed torque control method was generalized to dynamically redundant systems: the geometric- and servo-constraints were augmented by the so-called velocity level (non-holonomic) servo-constraints in a way that the inverse dynamic problem was solvable uniquely and the total mechanical energy was also minimized similarly to the pseudo-inverse calculations.

Related publication: [24]

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