Linear Distributed RC Network Analysis

Ph.D. Thesis Booklet

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I  Introduction

In the last two decades, due to the thermal problems of integrated circuits, the analysis of RC networks became an essential research area. An often encountered problem is to identify the structure of a network based on measurement or simulation, that is, to determine the pole-zero representation or transfer characteristics of a one-port, creating an equivalent circuit from a time-domain measurement. In most cases the purpose of these calculations is to create an accurate model or to identify the real physical structure from measurements.

II  Aims and Objectives

In an earlier paper [1] it has been shown that the distributed RC networks in many cases cannot be described by the classic network concepts (pole-zero representation, time-constants), they have to be extended to the distributed network domain. In case of RC networks these extensions manifest in real-to-real functions on the negative real axis of the complex plane. In the following I call these functions representative functions.

In network theory the exact relation of the representative functions of different domains (frequency, time, etc.) is a hard expectation, thus it is a natural claim to clarify the relation of these functions. In my first thesis I inspect this area. I formalized the relation of the time-constant spectrum, the dipole-intensity function and the relation of the real and imaginary part of a general network descriptive function with convolution type equations.

The mathematical toolset of network theory is mostly impedance based, it characterizes the examined systems with their impedance, it operates with its excitations and responses. Beyond the pretension of completeness in certain cases it has practical benefits as well to redefine some relations based on admittance. In my second thesis I define the admittance based pairs of the time-constant spectrum and dipole-intensity functions and express the core equation of the so called NID\textsuperscript{1} method in the admittance domain which is used for transient measurement evaluation.

\textsuperscript{1}Network Identification by Deconvolution
The measurement technique based on the NID method is a standardized method to identify the junction-to-case thermal resistance of integrated circuit packages. [2] [3] This thermal resistance is especially important for application developers. They use this value to design the necessary cooling facility for the particular device. The device manufacturers evaluate the thermal resistance based on the thermal transient measurement for the particular device, and they publish these values on the datasheet of the circuit. This evaluation is a network identification method, which results in a discretized RC network in Cauer canonical form with the behavior of the distributed thermal system. The T3Ster-Master commercial software by Mentor Graphics implements such a method based on the publication [4]. Currently this is the most widely used solution to evaluate a thermal transient measurement.

In my third thesis I extend this measurement technique by practical additions. Using the NID method we assume that the thermal response function is exact, which is of course a false assumption if we obtain this function by measurement, we have to deal with deviations.

The physical sources of the deviations are:

- the onset of the step-function excitation does not occur exactly at the $t = 0$ instant (per definition $t = 0$ is the time instant corresponding to the 0 point of the time scale assigned to the measured response),
- the rise time of the excitation is finite,
- the cut-off frequency of the used measurement instrument is finite.

It is highly needed to clarify the effect of these imperfections, in order to have an image about the accuracy of the measurement and identification process currently used and to correct the systematic errors.

Because the standard does not determine the tools for measurement and identification, one can use custom algorithms and implementations to solve these problems. If this custom implementation is inaccurate, despite of standardized methods, the published thermal resistance values can be inaccurate – in extreme situation completely false. To prevent this situation I developed a verification method which can determine the accuracy of the implementation of the evaluation process independently from the measurement setup.
For some operations, the convolution based mathematical toolset on which the NID method is based introduces regulated divergent operator functions. In my fourth thesis I investigate the effect of the regularization as a function of the half-value width of an applied pulse regulator.

III Applied Tools and Inspection Methodologies

The viability of objectives, the applicability of the theoretical results and the practical feedback is of great importance. Therefore, I have implemented every theoretical result in the C programming language. With these implementations all of the behaviors of the new relations became testable in a direct way.

Theoretical research techniques are primarily determined by the mathematical formalism and inspection methodologies of the particular research area. In all of my thesis points the mathematical toolset of calculus has been used, especially the convolution based integral equations. The connection between the descriptive functions of the distributed RC network is defined by general integral equations.

Scaling the variables into the logarithmic domain these equations simplify into convolution type integral equations, which makes further investigations possible. I use this method in the first and second thesis, where I defined new relations between the descriptive functions of distributed RC networks.

In the third thesis I used calculus as well for formalizing the systematic errors of a real measurement setup. This formal description made possible the analytical derivation of the necessary correction functions.

The fourth thesis utilizes the noise concept of linear systems, modeling the signal-to-noise ratio and resolution change in operator functions caused by a convolution operation.
IV New Scientific Achievements

Thesis 1

In the convolution based formalization of distributed RC network theory I achieved the following results.

1.1 I determined the transformation equations between the time-constant spectrum and dipol-intensity function [JN1]

\[ I_d(\Sigma) = \frac{1}{\pi} \arcsin \left( R_M(x) \otimes \frac{1}{1 - \exp(x)} \right) \]

\[ R(\zeta) = \frac{1}{\pi} R_0 \cdot \text{Im} \left( \exp \left( I_d(x) \otimes \frac{\exp(x)}{1 - \exp(x)} \right) \right) \]

These equations in addition to convolution contain non-linear operations as well, the relation of the to representative function is non-linear.

1.2 I determined the Bode integral which defines the relations between network descriptive functions’ real and imaginary part. This can be inserted into the convolution based toolset of distributed network theory by reformulation.

I derived the necessary operator functions. [J1]

\[ \text{Im} \{ Z(\Omega) \} = W_{\text{ReIm}} \otimes \text{Re} \{ Z(\Omega) \} \]

where

\[ W_{\text{ReIm}}(x) = -\frac{1}{\pi} \frac{1}{\text{sh}(x)} \]

and

\[ \text{Re} \{ Z(\Omega) \} = W_{\text{ImRe}}(\Omega) \otimes \text{Im} \{ Z(\Omega) \} \]

where

\[ W_{\text{ImRe}}(x) = \frac{1}{\pi} \frac{\exp(-x)}{\text{sh}(x)} \]
I reformulated some impedance based relations of the convolution based toolset of distributed RC network theory to admittance based ones.

2.1 I defined the complex admittance based pairs of the time-constant spectrum and dipole-intensity function. I derived the calculation methods of the two representative functions.[JN1]

The current response of a voltage unit-step excitation of an RC one-port has many exponential components with different time-constants and magnitudes. The admittance based $G(\zeta)$ time-constant spectrum can be defined as an integral of the amplitudes of an infinitesimally small $\zeta$ section. An admittance based dipole-intensity function of a distributed one-port can be defined by the relative distance of an infinite number of pole-zero pairs (dipoles) along the negative real axis.

They can be calculated based on the $Y(s)$ complex admittance:

$$G(\zeta = -x) = \frac{1}{\pi} \text{Im} \{Y(s = -\exp(x))\}$$

$$I_d Y(\Sigma) = -\frac{1}{\pi} \text{Im} \{\ln Y(s = -\exp(\Sigma))\}$$

Based on the definition of the dipole-intensity function and the reciprocal relation of the impedance-admittance, the two dipole-intensity functions differ only in their signs.

2.2 The NID method used to evaluate the thermal transient measurements operates in the impedance domain, where we use the voltage response of the current unit-step excitation of the inspected system. I developed the complementer admittance based method, where the $i(t)$ current response to voltage unit-step excitation is the starting point of the calculations. [JN1]

The core equation of the complementer method:

$$\frac{di}{dz} = -G(-z) \otimes \exp(z - \exp(z))$$
where $z = \ln(t)$ the logarithmical time variable and $G(z)$ the admittance based time-constant spectrum.

**Thesis 3**

I extended the convolution based toolset of the distributed network theory and the NID measurement method with practically important additions.

3.1 I developed a method to correct the systematic errors of the measurement of time-constant spectra.\[C1][J2]

In the framework of my investigation the time-constant spectrum is regarded as the primary description function of the thermal one-port. I intended to treat the effect of the imperfections as some characteristic distortion of the time-constant spectrum.

I derived the analytical form of distortion of the linear scaled $D(\tau)$ time-constant spectrum caused by a non-ideal $E(t)$ unit-step excitation.

$$K(\tau) = \int_{t_{0}}^{t_{1}} \frac{dE(x)}{dx} \cdot \exp(x/\tau) dx.$$ 

With this correction function it is possible to correct the distorted $D_m(\tau)$ measured time-constant spectrum.

I derived the analytical form of distortion of the linear scaled $D(\tau)$ time-constant spectrum caused by a non-ideal measurement amplifier described by its $w(t)$ weight function.

$$K(\tau) = \int_{t_{0}}^{t_{1}} w(x) \cdot \exp(x/\tau) dx$$

The combined effect of the non-ideal excitation and amplifier can be formulated by convolution equation where the characteristic behavior of the non-ideal components can be taken into account.

Such a combined correction function can be seen in Fig. 1. The relation between the correction function and the measurement error is reciprocal,
thus it can be seen that the systematic errors effect the small time-constants more heavily. Towards the larger time-constants the error diminishes.

3.2 I developed a verification method for qualification of the identification methods of distributed RC networks.[C2][J3]

I have developed this new procedure by examining the NID method. I defined a multilayer reference structure and derived its analytical unit-step response, time-constant spectrum and cumulative structure function. I used the analytical unit-step response as the input of the inspected implementation and examined the deviation of the calculated time-constant spectrum and structure function from the reference functions. The direct comparison of these functions is not informative; in the time-domain the peaks of the time-constant spectrum are smeared. To identify a network in contrast of the absolute values of the time-constant spectrum it is more relevant to examine its integral function. The process of the examination can be seen in Fig. 2. The error of the integral functions give valuable information about
the usability of the implementation and we have the opportunity to define a tolerance band (see Fig. 3). Comparing the structure functions a tolerance diagram can be defined as well. In this case, manufacturing deviations can be taken into account as well, – because the Cauer network defining the structure function describes the 1D heatpath’s material variances. Tightening the tolerance diagram better control can be gained over the variance of the manufacturing parameters.

**Thesis 4**

The paper [5] derives divergent operator functions for some convolution operations (eg. transformation between time- and frequency-domain). It makes these operator functions useful by regulating with a convolution by a pulse function.
Figure 3: The cumulative integral function of the time-constant spectrum and its tolerance band

I found that the half value width of the pulse function and the resolution loss are nearly direct and the signal-to-noise ratio change are inverse proportional.[J1]

This can be considered as a linear system, where the input is the divergent operator function \( Y(x) \), the output of the system is \( U(x) \) and the transfer function of the system is the pulse function \( W(x) \). I derived the signal-to-noise ratio of the output (\( \text{SNR}_y \)) if the signal-to-noise ratio of the input is known (\( \text{SNR}_u \)), taking into account the autocorrelation of the input (\( r_u \)):

\[
\text{SNR}_y = \text{SNR}_u \frac{\left( \int_{-\infty}^{\infty} W(x)dx \right)^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(\theta)W(\theta)r_u(\theta - \vartheta)d\theta d\vartheta}
\]
V Application of Results

In engineering sciences the practical applicability of new results is especially important.

With the results of my first thesis the network identification methods can be extended. Because of these extensions during the identification if we can determine the time-constant spectrum or the dipole-intensity function, the other one can be calculated directly. The extended methods gain another useful property: the total complex impedance function must not be necessarily known. It is sufficient to only know the real or imaginary part of the complex function and each representative function can be calculated accordingly. This is especially useful if the complex impedance of the inspected system can not be measured accurately but only the real or imaginary part can.

The introduced admittance based representations in my second thesis allows the application domain of the NID method to be extended to a wider spectrum. This extension is especially useful in case of clearly electrical network measurement.

The results of my third thesis has been motivated by problems with practical applications. The results presented here make it possible to compensate three major systematic errors after characterization during time-constant measurement. The implementation of the network identification algorithm can also be verified independently from the measurement.

With the results of the fourth thesis the regularization of the operator functions used to transform between descriptive functions can be optimized. These operator functions are used by many methods introduced in the previous theses, thus their accuracies and signal-to-noise ratios are directly influenced by the results of this thesis.
Journal Papers


Journal Papers Under Review


Conference Proceedings


Unrelated Publications


References


