Phase Transitions in Evolutionary Potential Games

Ph.D. Thesis Booklet

Balázs Király

Supervisor: György Szabó D.Sc.
Consultant: Csaba Tőke D.Sc.

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Introduction

Many research disciplines deal with complex systems composed of numerous interacting units. Due to the similarity of the related research and modeling issues, there are more and more studies that take an interdisciplinary approach to understanding these systems. This is especially true of the applications of the concepts and methods of statistical physics to other disciplines, which, over the last few years, has established new research fields, such as sociophysics [Galam, 2012], econophysics [Mantegna and Stanley, 1999], and network science [Barabási and Pósfai, 2016].

In the study of social phenomena, interactions can typically be reduced to some kinds of decision situations, so such analyses usually have game theoretical aspects [Gintis, 2009]. Pair interactions and matrix games, which are used to describe these interactions when players have a finite number of choices, play an important role in the modeling of these systems. In general, exploring the properties of matrix games, especially when the number of strategies is high, is not a trivial task in itself, and it can be made even more complicated in complex systems by, for example, iterated play, the dynamical rules governing the players' strategy choices, or the topology of their interactions.

Modeled on the expansion of vectors in bases, payoff matrices defining matrix games can be decomposed as linear superpositions of elementary matrices, and furthermore, this decomposition can be performed in such a way that the elementary matrices describe just four different fundamental interaction situations [Szabó and Borsos, 2016]: players unilaterally setting their own or their opponent's payoff, pure coordination, and games of cyclic dominance. Although individual elementary games can be analyzed straightforwardly on their own, the possible regularities of their interplay, for the most part, have yet to be clearly understood. A known advantage of the decomposition is that it gives a simple condition for the existence of a potential, a quantity that simplifies many aspects of a game's analysis: Only those games admit a potential that do not contain any cyclic dominance components.

Aims and objectives

The overarching aim of our research is to explore how much meaning can be given to the above-mentioned anatomy of matrix games, especially in relation to the modeling of complex systems. To lay a foundation for this effort, we first investigated the effects of combining elementary games by studying the behavior of a handful of simple games in arrangements that allows for the direct application of well-known concepts and methods of statistical physics. As a starting point, we chose logit-rule-driven many-player iterated potential games, since these are equivalent to classical spin models. The thesis deals with the square-lattice elementary coordination game, its extension with self-dependent components, and certain symmetrical combinations of more elementary coordinations, with specific attention to the properties of their phase transitions. In the elementary coordination game, the two coordinated strategies are coupled by an Ising-type interaction, which equally rewards or penalizes players by the same amount of payoff for choosing matching or opposing strategies, while the remaining strategies are neutral, that is, both players receive zero payoff whenever they are chosen.

Methods

We employed the mean-field and pair approximation methods to qualitatively explore the general features of the models in question, and we used Monte Carlo simulations to carry out a more detailed qualitative analysis of their phase transitions. We augmented these results with analytical calculations, the visualization of the simulations, and the estimation of invasion velocities along the separating interfaces of the domain structures that emerge in them.

New scientific contributions

My main scientific contributions resulting from the above outlined research work can be summarized in the following thesis statements:

1. I have explored the properties of the square lattice, logit-rule-driven elementary coordination game. I have established that as a result of changing the noise level

parameter, which is analogous to temperature, the system may undergo an order–disorder phase transition whose order depends on the defining parameter of the model, the number of available neutral strategies. As long as this number remains below a threshold value, the transition is continuous and belongs to the universality class of the two-dimensional Ising model; its is of the first order otherwise. I have determined the threshold value and estimated the critical temperature of the transition [P1].

2. I have extended the model mentioned in thesis statement 1 with a self-dependent component that retains the symmetry of its coordinated strategies. I have established that, in the resulting model, the critical point and the order of the original phase transition may both be changed, or the transition may even be abolished altogether depending on the strength of the self-dependent component [P2].

3. By consistently bunching the neutral strategies, I have mapped the extended model mentioned in thesis statement 2 onto the Blume–Capel model, thereby verifying the accuracy of my findings on the properties of the model. I have shown that the same mapping can also be used to replace an arbitrary number of neutral strategies in games defined on regular graphs with a single neutral strategy and an additional self-dependent component whose strength depends on the temperature.

4. I have introduced the concept of maximally nonover-

lapping coordination games as the family of games that have an even number of available strategies and are made up of a maximal number of elementary coordinations that share none of their coordinated strategies. I have explored the properties of these games in a square-lattice, logit-rule-driven setup. At the mean-field approximation level, a general maximally nonoverlapping coordination game is equivalent to the elementary coordination game with the same number of available strategies, so its ordered phase breaks the symmetry of just one of its coordinated pairs. I have demonstrated this property for the four- and six-strategy versions of the game using Monte Carlo simulations. I have assigned two independent order parameters to their phase transitions and determined their critical exponents. While in the four-strategy model one of the order parameters exhibits Ising-type critical behavior just like the elementary coordination game, the six-strategy game is characterized by different critical exponents because of the different permutation symmetry that connects its coordinated pairs. I have identified the four-strategy model as a special case of the Ashkin–Teller model, the clock model. This correspondence provides analytical underpinning for the model's apparent Ising-type behavior, and it can be used to exactly determine the critical temperature of the model's phase transition via a duality relation [P3].

5. I have studied a square-lattice, logit-rule-driven model of competing Ising- and Potts-type subgame compo-

nents. I have shown that even though the system generally exhibits a single order-disorder phase transition, in the vicinity of which the system's critical behavior corresponds to that of the subgame that provides higher payoffs, Ising-type behavior can still be stabilized by entropy effects close to the critical point when the Potts-type subgame is only slightly stronger. In this case an additional, first-order transition can also be observed between the two competing ordered phases [P4].

Research articles related to the thesis statements

[P1] G. Szabó and B. Király, "Extension of a spatial evolutionary coordination game with neutral options," Phys. Rev. E **93**, 052108 (2016).

[P2] B. Király and G. Szabó, "Evolutionary games with coordination and self-dependent interactions," Phys. Rev. E **95**, 012303 (2017).

[P3] B. Király and G. Szabó, "Evolutionary games combining two or three pair coordinations on a square lattice," Phys. Rev. E **96**, 042101 (2017).

[P4] B. Király and G. Szabó, "Entropy affects the competition of ordered phases," Entropy **20(2)**, 115 (2018).

References

[1] S. Galam, *Sociophysics: A Physicist's Modeling of Psycho-political Phenomena*, Understanding Complex Systems (Springer, New York, NY, 2012).

[2] R. N. Mantegna and H. E. Stanley, *Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, Cambridge, UK, 1999).

[3] A.-L. Barabási and M. Pósfai, *Network Science* (Cambridge University Press, Cambridge, UK, 2016).

[4] H. Gintis, *The Bounds of Reason: Game Theory and the Unification of the Behavioral Sciences* (Princeton University Press, Princeton, NJ, 2009).

[5] G. Szabó and I. Borsos, "Evolutionary potential games on lattices," Phys. Rep. **624**, 1 (2016).