MŰEGYETEM 1782

BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS

FACULTY OF MECHANICAL ENGINEERING

THESIS BOOKLET

Róbert Sánta

Master of Science

SIMULATION AND OPTIMIZATION OF COMPRESSOR DRIVEN HEAT PUMPS FOR BUILDING SERVICES

Budapest

February 2014.

1. Introduction

Increasing the efficiency of energetic processes has been in the focus of attention in almost all countries worldwide as modern economies and societies require and use more and more energy. At the same time, problems are accumulating with the use of traditional energy resources, including, primarily, expanding detrimental impacts on the environment. The highly significant role fossil fuels are supposed to play in climate change has been made explicit in the course of the past decade. A tool to firmly restrict this impact is to increase the use of renewable energy resources.

Heat pumps, especially compressor equipment, represent a highly efficient and technologically accomplished means of applying renewable energy resources. The energy efficiency of compressor heat pumps is demonstrated by the so-called power factor.

In order to increase heat pump power factor and improve operation quality, it is essential to make attempts to describe heat pump operations and processes most precisely as well as to develop and refine the physical and mathematical models to provide a basis for them.

Only a mathematical model can provide a satisfying answer to the totality of phenomena within a heat pump as well as to a series of questions on structural solutions and dimensions of the system. A mathematical model serves as a tool to simulate the behavior of a heat pump heating system and to determine its geometric and energetic optimums.

Having reviewed heat pump design, installation and operation problems, I came to the conclusion that a mathematical model fully satisfying requirements is missing from both research and development, which would describe the operation of heat pump heating systems as well as energy and material flows through the system of balance equations.

Since the 1980s to the present day, a number of stationary and instationary mathematical models of concentrated and divided parameters have been issued in the subject of modeling heat pump systems. However, such mathematical models created describe the operating process of heat pumps only partially or subject to certain neglects.

My dissertation discusses the operational design of water-to-water heat pumps with compressors on scientific grounds. In the framework thereof, my main objective is to describe, more accurately than before, the thermodynamical, heat transmission and fluid mechanics processes in the technological equipment and system components realizing the heat pump cycle, as well as to identify changes of state at each and otherwise any discretional point of the cycle.

My dissertation rests upon the following two pillars, constituting the two main directions of my research work:

- Water-side and coolant-side hydraulic and heat engineering modeling of heat exchangers – the evaporator and the condenser. Measurement of pressure losses and heat transfer coefficients and setting up new functions to describe correlations between the thermodynamical and fluid mechanics parameters of mediums.
- If, as a result of the input-output analysis of mechanical engineering components, the functions to describe connections between inputs and outputs are available, then in their possession the entire cycle – operation of the mechanical engineering components interconnected and working together to realize the cycle, including descriptions of the water-side processes of the evaporator and the condenser – can be examined most meticulously and most precisely.

In possession thereof, the main aim of my dissertation was to set up a physical and mathematical model and to develop a solution algorithm by which system operation can be optimized for different heating demands, involving research on how to satisfy a given heat demand with a maximum output coefficient and by using a minimum amount of electric power.

So the point is to maximize the coefficient of performance (COP) as a target function of the system and to examine what decision making parameters and what values thereof are required to set this at the water side of the evaporator and the condenser by taking into consideration the behavior of the compressor and the throttle valve at operating points different from the nominal ones.

2. Methods applied

In the course of my research work I performed laboratory and service tests for the fully comprehensive computerized modeling of heat pump systems, to determine heat transmission between the coolant and the tube wall, coolant pressure loss, as well as heat transfer coefficients and pressure loss on the water side of the evaporator and the condenser.

Experiments were performed at the Department of Thermal Engineering of Subotica Technical College. The working equipment is a water-to-water heat pump shown in Figure 2.1. There are shell and tube heat exchangers in the heat pump system. The coolant is R134a, flowing within the tubes of the evaporator and the condenser, while the cooled/heated medium – that is, water –flows along the outer side of tubes, that is, in the mantle with deflector plates. Deflector plates are placed in the mantle to increase the intensity of heat transmission. The deflector plates of the heat exchangers examined are circular segments. A type L'unite Hermetique (CAJ4511Y, R134a, N214QT-G- ind, Tension G: 208-220V 1-50 Hz) piston compressor was used for measurements at the Department Laboratory.

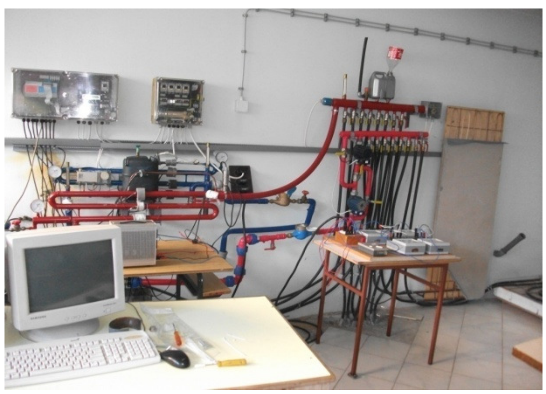

Figure 2.1.: Water-to-water heat pump system

The heat pump system was instrumented as follows:

Measuring instruments were installed along the evaporator and the condenser, the length of which was 3m. Measuring points were installed at 10 discrete points in the heat exchangers. The distance between measuring points was 30 cm. At the measuring points thus installed, the temperature and pressure of the coolant as well as of the cooled and the heated medium were measured as shown in Figures 2.2 and 2.3. Along the branch between the condenser and the throttle valve, a coriolis mass flow meter was installed to measure coolant mass flow, as shown in Figure 2.4. The volumetric flow of the cooled and heated mediums was measured by a water meter, installed in the inlet branch of the heat exchangers shown in Figure 2.5. The piston compressor of the heat pump was also equipped with measuring instruments as displayed in Figure 2.6 in order to measure output and electric power demand.

Table 1.1 below shows the description and accuracy of the measuring instruments applied.

Table 1.1. Measuring instruments applied and their accuracy

Measuring instruments	**Thermometer sensor**	**Pressure gauge and pressure sensor**	**Flow meters**	**Output and electric power meters**
Type:	Dallas, DS18S20.	Mihailo Pupin Transducers MP-1M2.	INSA, BMET, Krohne Optimas 6400.	Iskra Øelo.
Accuracy:	$\pm 0.3\ K$.	1%, 0.5%.	0.6%, 0.2%, 0.1%.	0.5%.

Measurement results were processed by a CSOP2 measuring unit shown in Figure 2.7. The tasks of the information system of the measuring unit included the following: digital temperature measurement, processing and storage of the data measured, display of the data measured, and transmission of the data measured to a personal computer.

Figure 2.2. Pressure/temperature measurements

Figure2.3. Pressure measurements

Figure 2.4. Mass flow measurements

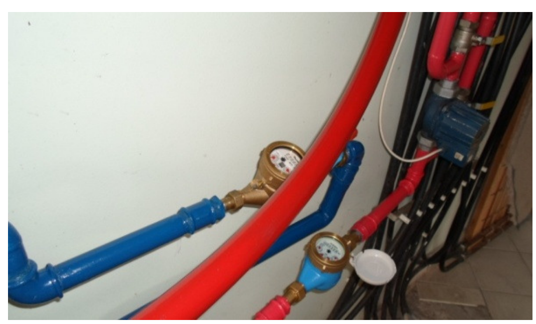

Figure 2.5. Volumetric flow measurements

Figure 2.6. Output measurements

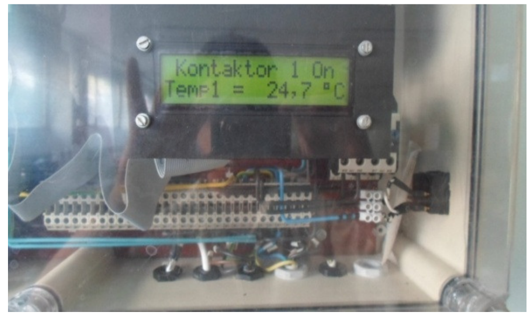

Figure 2.7. CSOP2 measuring unit

By processing the measurement results, I used mathematical statistics tools to set up new and more accurate calculation formulas – compared to those in the literature – for the R134a coolant and shell and tube heat exchangers, to determine heat transfer coefficients and pressure loss for heat transmission and fluid mechanics processes both in heat exchanger tubes and within the mantle, by indicating validity and limitations. In respect of the new correlations set up, I demonstrated the errors and confidence intervals of formulas compared to formulas known and published in the literature, at a reliability level of 95%. I called attention to the fact that while the coefficients of correlations to describe heat transmission and pressure loss are stochastic variables, their accuracy, their limits of applicability and their confidence intervals are not known as they are failed to be published by authors.

3. Literature review

I have established that the structures of the models presented in the literature, as well as results and formulas on process characteristics show extreme degrees of standard deviation; their limits of usability are generally failed to be published and are difficult to compare. I did not get a clear answer to what tube friction and heat transfer coefficient I could take into account for my system engineering tests, for the analysis of the heat pump cycle, particularly in respect of the R134a cooling medium and two-phase flows. The Chen model [12] is most generally applied in the literature to determine the heat transfer coefficient of two-phase coolant mediums; it also forms a basis for the models by Bertsh [15], Kwang [17] and a number of other models in use. Heat transmission in heat pump systems within the mantle – between cooled water and the tubes of the shell and tube heat exchanger – has been anaysed by Kern [8], Bell-Delaware [11], and Taborek [13].

Highlights include the work of **Maiyaleh Tarek** in this field [5], discussing in detail the modeling of heat transmission processes within heat pump condensers; he performed measurements and determined an average condensation coefficient there from. His tests involved R12, R22 and R502 cooling mediums. The model was developed for laminar flow; however, the results are in line with general heat transmission theories, being in the form of formulas built up with dimensionless numbers of similitude.

In descriptions of heat pump system components I came across neglects I would like to step over. In the vast majority of models, researchers consider heat transfer and tube friction coefficients to be constant, failing to take into account their changes in the evaporator and the condenser in function of steam content, or applying inaccurate outdated equations developed for other mediums in earlier decades. In a number of cases, water, pressure and temperature changes are not taken into consideration on the water side of the evaporator and the condenser, either. In the majority of models, researchers deem compression to be isentropic, and the cycle to be generally ideal and loss-free, although this is obviously an approximation only. In his study, **Szabolcs Méhes** provided a systemically most comprehensive analysis of the optimization of the establishment and operation of compressor heat pumps and systems [6]. Méhes worked with global stationary balances. His investigations did not include detailed descriptions (by divided parameters) of the thermal and hydrodynamic behavior of system components.

4. New scientific results

Thesis 1

In order to determine the heat transfer coefficient of a two-phase coolant flowing in the pipes of a shell and tube evaporator, I composed a new formula containing dimensionless numbers for the R134a cooling medium, which is more accurate than the results to be obtained by the models presented in the literature as demonstrated by the measurements I conducted. I disclosed the limits of applicability of the formula and demonstrated the parameter errors of the formula by confidence intervals.

Related publications: [1], [2], [3], [7].

Having evaluated measurement results, I took as a basis the Chen model most widely used in the literature to determine the heat transfer coefficient of the two-phase cooling medium flowing in the evaporator tubes, and I specified correlation [12]

$$\alpha_{kf} = F \cdot \alpha_f + S \cdot \alpha_{nb} \tag{4.0}$$

developed by Chen as follows:

Two-phase correction multiplier factor for convective boiling:

$$\boldsymbol{F = b \cdot \frac{Co^c}{X_{tt}{}^d}}. \tag{4.1}$$

Two-phase correction multiplier factor for bubble boiling:

$$\boldsymbol{S = a \cdot (1 - x)}. \tag{4.2}$$

Auxiliary functions and dimensionless numbers in calculation formulas (4.1) and (4.2):

Martinelli parameter:

$$X_{tt} = \left(\frac{\mu_f}{\mu_g}\right)^{1/8} \cdot \left(\frac{1-x}{x}\right)^{7/8} \cdot \left(\frac{\rho_g}{\rho_f}\right)^{0.5}. \tag{4.3}$$

Convective parameter:

$$Co = \left(\frac{1}{x} - 1\right)^{0.8} \cdot \left(\frac{\rho_g}{\rho_f}\right)^{0.5}. \tag{4.4}$$

Single-phase heat transfer coefficient according to Dittus Boelter [13]:

$$\alpha_f = 0.023 \cdot {Re_f}^{0.8} \cdot {Pr_f}^{0.4} \cdot \frac{\lambda}{d_b}. \tag{4.5}$$

Reynolds number:
$$Re_f = \frac{(1-x) \cdot G \cdot d}{\mu_f}. \tag{4.6}$$

Convection heat transfer coefficient according to Cooper [14]:

$$\alpha_{nb} = 55 \cdot pr^{0.12} \big(-0.4343 \cdot ln(pr)\big)^{-0.55} \cdot M^{-0.5} \cdot q^{0.67}. \tag{4.7}$$

Based on Student's t-distribution, the confidence intervals of coefficients in formulas (4.1) and (4.2) at a 95% reliability level:

$\boldsymbol{a = 0.431 \pm 0.2527}$, $\boldsymbol{b = 12.6 \pm 9.5}$, $\boldsymbol{c = 4.14 \pm 1.2}$ **and** $\boldsymbol{d = 4.80 \pm 1.35}$.

Confidence interval of formula (4.0): $\boldsymbol{\alpha_{kf} - 289, \alpha_{kf} + 289}$.

The formulas (4.1) and (4.2) presented were defined in the following measurement conditions and subject to the following criteria:

Working medium:	R134a.
Mass velocity:	$G = 106, 114, 135 \left[\frac{kg}{m^2 s}\right]$.
Reynolds number range:	$2461 < Re < 3155$ [–].
Input temperature:	$T_{hk} = 4.6, 4.7, 5.4$ [°C].
Steam content:	$x = 0.09 \div 0.98$.
Tube diameter:	$d_b = 6$ $[mm]$.
Number of tubes:	$n = 5$ $[pcs]$.
Heating capacity:	$\dot{q} = 3$ $[kW]$.
Length of evaporator:	$z = 3$ $[m]$.

Figure4.1 below shows the divergence of values of the new two-phase heat transfer coefficient of evaporation from the values measured.

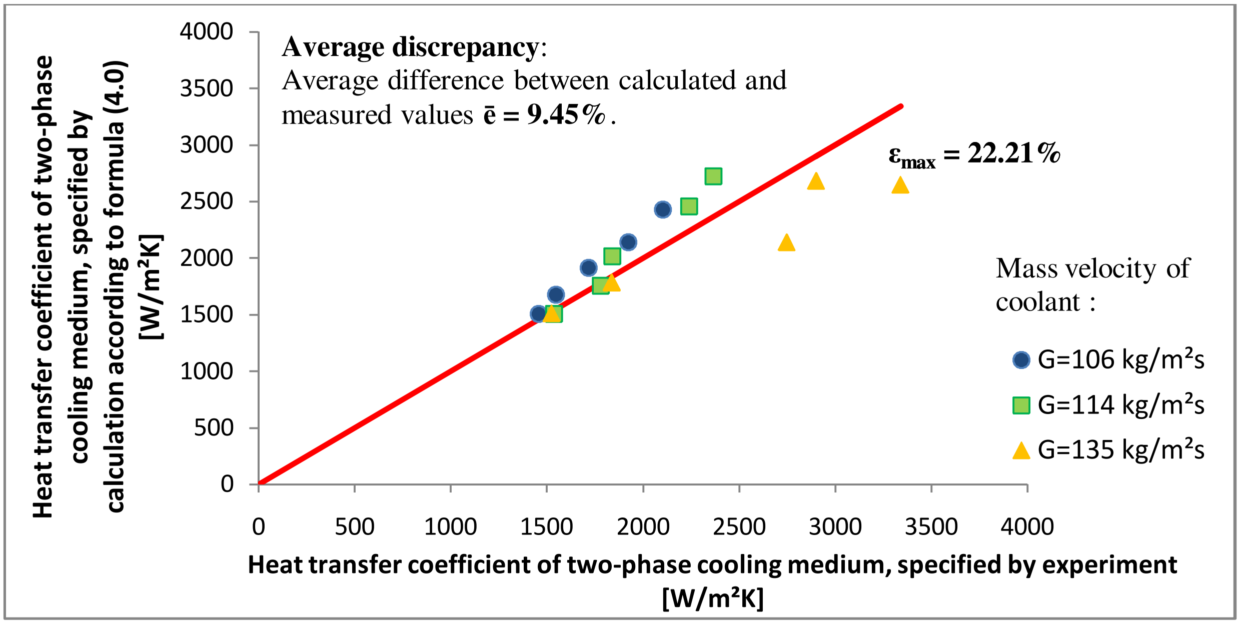

Figure 4.1. Comparison of values obtained by own model (formula 4.0) for the heat transfer coefficient of evaporation with measured values

Figure 4.1. above shows that the maximum discrepancy of the values supplied by the model set up by me is $\varepsilon_{max} = 22.21$ %, while the average discrepancy is only $\bar{e} = 9.45\%$.

Table 4.1. summarizes the discrepancies of values calculated by different models from the measurement results measured by me.

Table 4.1. Comparison of heat transfer coefficients of evaporation applied in the literature

Two-phase heat transfer model	***Average relative error $\bar{e}$***	***Maximum discrepancy ε_{max}***
Chen [12]	30 %	50 %
Bertsh [15]	15 %	75 %
Kattan [16]	9.7 %	24.8 %
Kwang [17]	20 %	55 %
New heat transfer coefficient of evaporation according to equation (4.0)	9.45 %	22.21 %

The values reported in Table 4.1. show that the most favorable results within the range examined are yielded by the formula (4.0) developed by me.

Thesis 2

In order to determine the heat transfer coefficient of a two-phase coolant flowing in the pipes of a shell and tube condenser, I developed a new formula for the R134a cooling medium, which is more accurate than the results to be obtained by the models presented in the literature as demonstrated by the measurements I conducted. I disclosed the limits of applicability of the formula and demonstrated the parameter errors of the formula by confidence intervals.

Related publications: [4], [7].

Having evaluated measurement results, I set up the following correlation to determine the heat transfer coefficient of the two-phase cooling medium flowing in the condenser tubes:

$$\alpha_{kf} = a \cdot e^{Re_e \cdot b} \cdot Pr^c \cdot \frac{\lambda}{d}, \tag{4.8}$$

where

$Re_e = \frac{G_{ekv.} \cdot d}{\mu_f}$ - equivalent Reynolds number,

$Pr = \frac{\mu \cdot c_p}{\lambda}$ - Prandtl number,

$G_{ekv.} = G \cdot \left[(1-x) + x \cdot \left(\frac{\rho_f}{\rho_g} \right)^{0.5} \right]$ - equivalent mass velocity.

Based on Student's t-distribution, the values of constants and confidence intervals in formula (4.8) of the heat transfer coefficient, at a 95% reliability level:

$$\boldsymbol{a = 28.6 \pm 5.16}, \qquad \boldsymbol{b = 3.15 \cdot 10^{-5} \pm 4.48 \cdot 10^{-6}}, \qquad \boldsymbol{c = 1.11 \pm 0.88}.$$

Confidence interval of formula (4.8): $\boldsymbol{\alpha_{kf} - 78, \alpha_{kf} + 78}$.

The formula (4.8) presented was defined in the following measurement conditions and subject to the following criteria:

Working medium: R134a.

Mass velocity: $G = 106, 114, 135 \left[\frac{kg}{m^2 s}\right]$.

Reynolds number range: $\boldsymbol{3500 < Re < 4950}$ $[-]$.

Input temperature:	$T_{hk} = 34.51, \quad 35.49, \quad 38.94$ [°C].
Steam content:	$x = 0.99 \div 0.09$.
Tube diameter:	$d_b = 6\ [mm]$.
Number of tubes:	$n = 5\ [pcs]$.
Length of condenser:	$z = 3\ [m]$.

Figure 4.2 below shows the divergence of measured values of the two-phase condensation heat transfer coefficient from the values yielded by the new heat transfer equation.

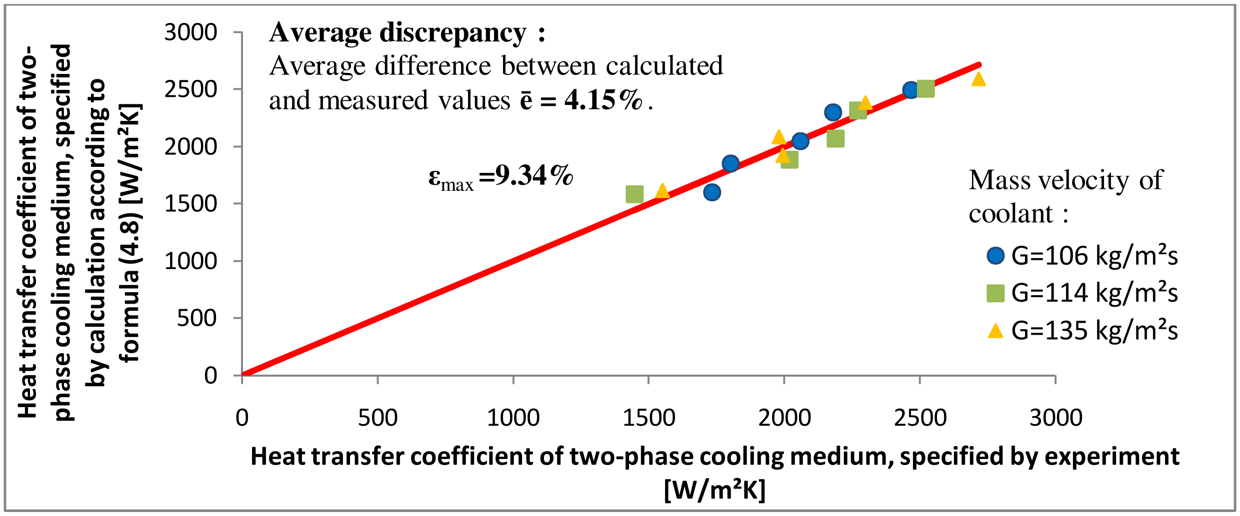

Figure 4.2. Comparison of values obtained from the new model of condensation heat transfer (formula 4.8) by calculation with measured values

Figure 4.2. above shows that the maximum discrepancy of the values supplied by the model set up by me is $\varepsilon_{max} = 9.34\ \%$, while the average discrepancy is only $\bar{e} = 4.15\%$. Table 4.2. summarizes the errors of heat transfer coefficients obtained from various models.

Table 4.2. Comparison of condensation heat transfer coefficients applied in the literature

Two-phase heat transfer model	***Average relative error*** $\bar{e}$	***Maximum discrepancy*** ε_{max}
Akers [18]	6.41 %	16.27 %
Shah [19]	26.88 %	58 %
Tang [20]	75.1 %	211 %
Thome [21]	8.7 %	34.62 %
New heat transfer coefficient of condensation according to equation (4.8)	4.15 %	9.34%

Thesis 3

In order to determine the pressure loss of single-phase and two-phase coolants flowing in the pipes of a shell and tube evaporator and condenser, I developed a new model for the R134a cooling medium, providing more accurate results than the results to be obtained by the models presented in the literature.

Related publication: [5].

My objective was to determine frictional and inertial pressure losses – defined separately by certain authors – together in a complex manner. Inertial pressure loss is caused by the convective acceleration of the medium, which only occurs in the compressible steam phase. A complex method for determining pressure loss is provided by the Navier-Stokes equation stated correctly.

A new scientific result is the calculation formula to determine the pressure loss of the two-phase cooling medium and of superheated steam:

$$\frac{\Delta p}{\Delta z} = -\left(\frac{\dot{m}}{A}\right)^2 \cdot v'' \cdot \frac{\Delta x}{\Delta z} - \frac{\lambda}{2 \cdot d} \cdot \left(\frac{\dot{m}}{A}\right)^2 \cdot v'' \cdot x. \tag{4.9}$$

Equation (4.11) derived from the Navier-Stokes equation describes highly precisely the pressure loss occurring while the two-phase coolant flows along the tube. In order to apply this difference equation, it is required to know the values (x) of specific steam content along the tube. In the knowledge thereof, the Δp/Δz gradient of pressure change can be generated numerically. If the trends of development of steam content (x) along tube axis (z) are known by polynomial description, then the difference equation can be solved analytically as well by reformulation thereof into a differential equation.

The formula (4.9) presented was defined in the following measurement conditions and subject to the following criteria:

Working medium:	R134a.
Mass velocity:	$G = 106, 114, 135 \left[\frac{kg}{m^2 s}\right]$.
Reynolds number range:	$3500 < Re < 4950$ [−].
Input temperature:	$T_{hk} = 34.51, \quad 35.49, \quad 38.94$ [°C].
Steam content:	$x = 0.99 \div 0.09$.

Tube diameter: $d_b = 6\ [mm]$.

Number of tubes: $n = 5\ [pcs]$.

Length of evaporator: $z = 3\ [m]$.

Figure 4.3. below shows the divergence of measured two-phase values of pressure loss by evaporation from the values yielded by the new pressure loss model.

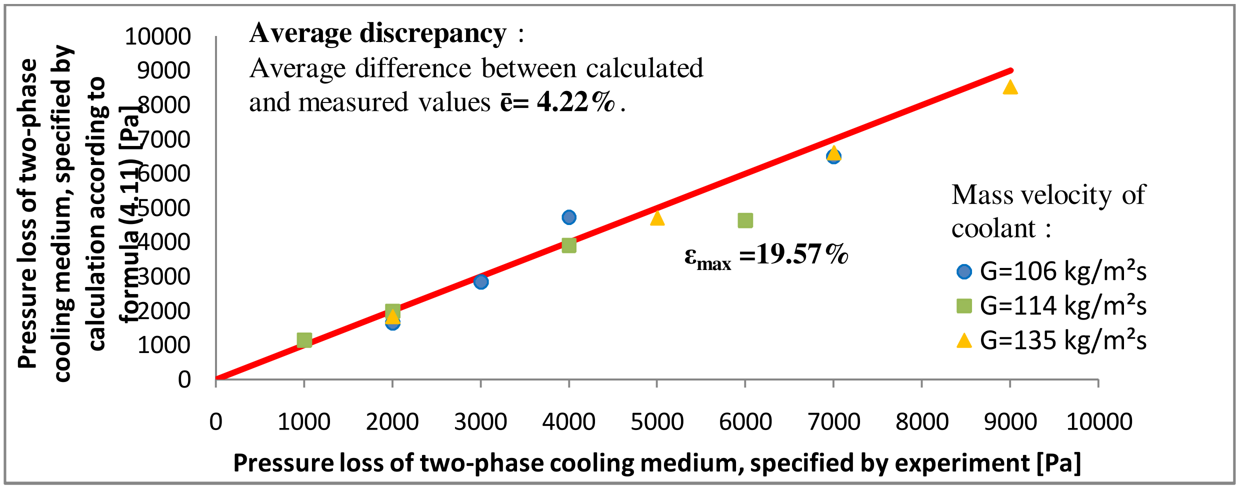

Figure 4.3. Comparison of values yielded by own model (formula 4.9) developed for determining pressure loss in the evaporator with measured values

Figure 4.3. above shows that the maximum discrepancy of the values yielded by the mathematical model developed by me for determining pressure loss of the two-phase cooling medium is $\varepsilon_{max} = 19.57\ \%$ compared to the values measured, while the average discrepancy is$\bar{e} = 4.22\ \%$. Table 4.3. summarizes the discrepancies of values calculated by different models from the measurement results measured by me.

Table 4.3. Comparisons of coolant pressure loss figures in the evaporator

Two-phase pressure loss model	*Average relative error* $\bar{e}$	*Maximum discrepancy* ε_{max}
Wilson [22]	18.6 %	52 %
Friedel [23]	22.44 %	60 %
Lockhart and Martinelli [24]	24.52 %	59 %
Grönnerud [25]	14.56 %	28 %
Coolant pressure loss in the evaporator according to equation (4.9)	4.22 %	19.57%

Figure 4.4. below shows the comparison of values determined by the developed calculation formula (4.9) with measured values.

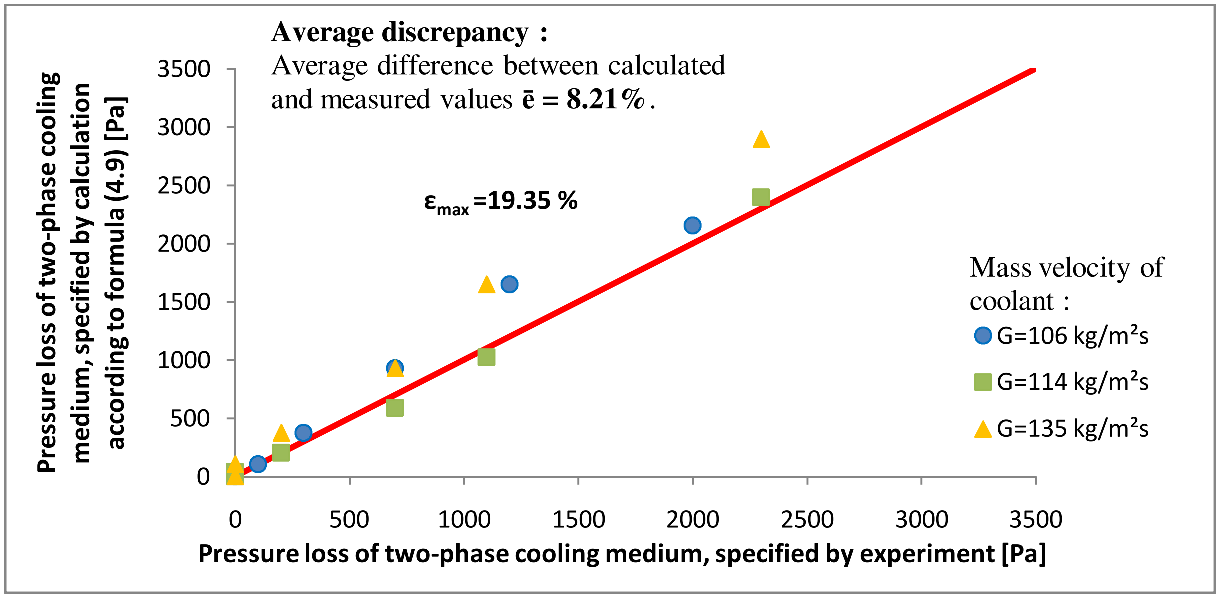

Figure 4.4. Comparison of values yielded by own model (formula 4.9) developed for determining pressure loss in the condenser with measured values

Figure 4.4. above shows that the maximum discrepancy of the values yielded by the mathematical model developed by me for determining pressure loss of the two-phase cooling medium is $\varepsilon_{max} = 19.57$ %, while the average discrepancy is $\bar{e} = 4.22$ %.

Table 4.4. summarizes the discrepancies of values calculated by different models from the measurement results measured by me.

Table 4.4. Comparisons of coolant pressure loss figures in the condenser

Two-phase pressure loss model	***Average relative error*** $\bar{e}$	***Maximum discrepancy*** ε_{max}
Wilson [22]	23.88 %	32.47 %
Friedel [23]	25.98 %	80.02 %
Lockhart and Martinelli [24]	55.74 %	180.28 %
Grönnerud [25]	16.69 %	29.55 %
Coolant pressure loss in the condenser according to equation (4.9)	8.21%	19.35%

Table 4.4. shows that the most favorable figures are yielded by the newly developed formula (4.9).

Thesis 4

In order to determine the heat transfer coefficient of a single-phase cooled or heated medium (water) flowing in the mantle of a shell and tube evaporator and condenser, I developed a new heat transmission model and formula, providing more accurate results than the results to be obtained by the models presented in the literature as demonstrated by the measurements I conducted. I disclosed the limits of applicability of the formula and demonstrated the parameter errors of the formula by confidence intervals.

Related publication: [10].

I used the values yielded by measurements to compose a new calculation formula for determining the value of the heat transfer coefficient. The new formula is based on the general form of the equation for determining the heat transfer coefficient [26], [27]:

$$\alpha = E_1 \cdot Re^{E_2} \cdot Pr^{E_3} \cdot \frac{\lambda}{d} \left[\frac{W}{m^2 K}\right].$$

The new, more precise formula:

$$\alpha_{viz} = a \cdot Re^b \cdot Pr^c \cdot \frac{\lambda}{D_e}, \tag{4.10}$$

where

$Re = \frac{D_e \cdot G}{\mu} [-]$ -Reynolds number,

$G = \frac{\dot{m}}{S} \left[\frac{kg}{m^2 s}\right]$ -mass velocity,

$S = \frac{D_b \cdot P_D \cdot L_b}{P_T} [m^2]$ -flow cross-section,

$D_e = \frac{4 \cdot \left({P_T}^2 - \pi \cdot {d_k}^2/4\right)}{\pi \cdot d_k} [m]$ -equivalent tube diameter,

$\Pr\ [-]$ -Prandtl number,

$\lambda \left[\frac{W}{mK}\right]$ -heat conduction coefficient of water.

Based on Student's t-distribution, the values of constants and confidence intervals in formula (4.10) of the heat transfer coefficient, at a 95% reliability level:

$$\boldsymbol{a = 1.66 \pm 0.66, \qquad b = 0.432 \pm 0.171 \text{ and } c = 0.0382 \pm 0.0151.}$$

Confidence interval of formula (4.10): $\alpha_{viz} - 83, \alpha_{viz} + 83$.

The formula (4.10) presented was developed by using data from the following measurement conditions:

Working medium:	Water
Volumetric flow:	$\dot{V} = 1, 1.5 \text{ és } 2 \left[\frac{m^3}{h}\right]$.
Reynolds number range:	$3800 < Re < 8000$.
Input temperature:	$T_{be} = 13$ [°C].
Internal diameter of mantle:	$d_b = 32\ [mm]$.
Tube diameter:	$d_k = 8\ [mm]$.
Number of tubes:	$n = 5\ [pcs]$.
Position of tubes:	$\theta = 30\ °$.
Distance of deflector plates:	$L_b = 75\ [mm]$.
Portion cut out of deflector plates:	$k_r = 50$ [%].
Length of evaporator:	$z = 3\ [m]$.

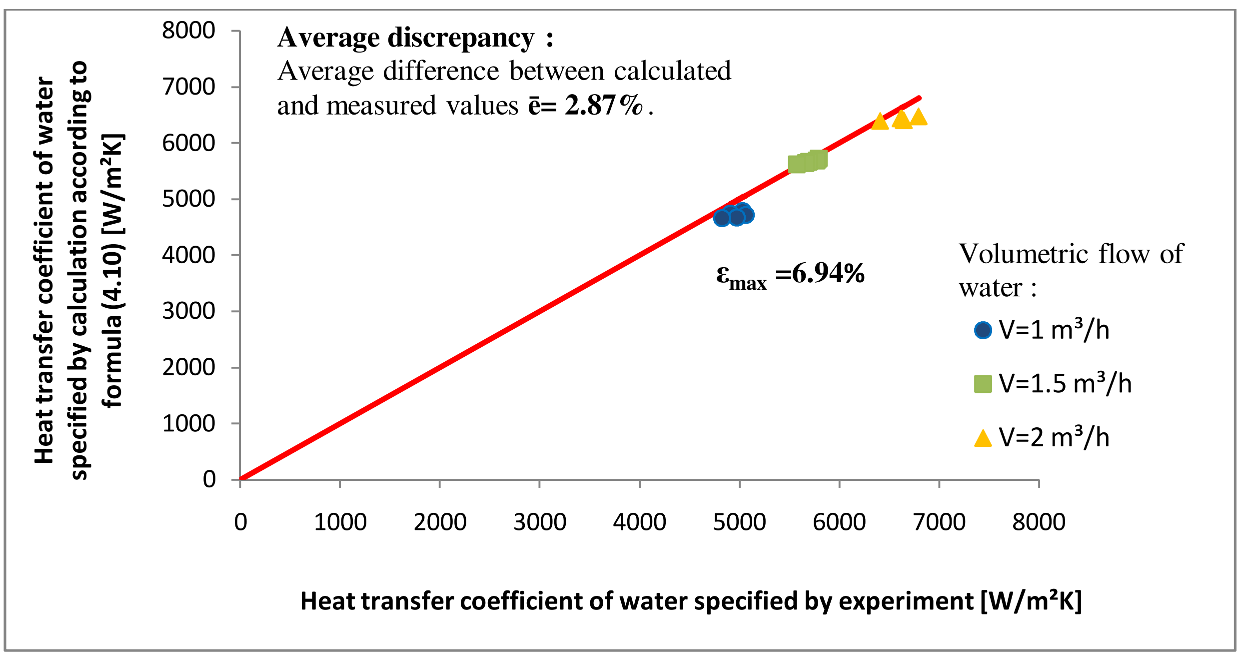

Figure 4.5. Comparison of own model developed for determining heat transfer coefficient values of the mantle (formula 4.10) with measured values

Figure 4.5. shows that the maximum discrepancy of the values supplied by the mantle-side heat transfer coefficient set up by me is $\varepsilon_{max} = 6.94\,\%$, while the average discrepancy from measured values is $\bar{e} = 2.87\%$, representing the most favorable value among the models presented.

The following table summarizes the discrepancy of heat transfer coefficient values for the medium flowing in the mantle space as yielded by different models, compared to the results measured by me.

Table 4.5. Errors of heat transfer coefficients of water within the mantle and their comparisons

Single-phase heat trasfer model	***Average relative error $\bar{e}$***	***Maximum discrepancy ε_{max}***
Kern [28]	8.9 %	14.7 %
Bell-Delaware [29]	4.2 %	7.06 %
Taborek [30]	3.5 %	6.92 %
Heat transfer coefficient in new mantle space (on water side) according to equation (4.10)	2.87 %	6.94 %

Thesis 5

In order to determine the pressure loss of a single-phase medium (water) flowing in the mantle of a shell and tube evaporator and condenser, I developed a new model providing more accurate results than the results to be obtained by the models presented in the literature as demonstrated by the measurements I conducted. An important element of the model is the introduction of a new individual average resistance coefficient to determine deflector plate resistance and to specify its value. I disclosed the value of such resistance coefficient, the limits of its applicability, and demonstrated resistance coefficient errors by confidence intervals.

Related publication:[10].

I used the values yielded by measurements to create a new mathematical model for determining the pressure loss of the water flowing within the mantle of the evaporator and the condenser.

Pressure loss of the medium flowing within the mantle was specified according to the Darcy-Weisbach [31] correlation.

$$\Delta p = \left(\lambda_{cs} \cdot \frac{\Delta z}{D_e} + \xi\right) \cdot \frac{w^2}{2} \cdot \rho, [Pa]. \quad (4.11)$$

Delimiting surfaces within the shell and tube heat exchanger were deemed to be hydraulically smooth surfaces.

Definition of tube friction coefficient by Blasius [31], valid to hydraulically smooth tubes if $Re < 10^5$:

$$\lambda_{cs} = \frac{0.316}{Re^{0.25}} [-], \quad (4.12)$$

where

$Re = \frac{w \cdot D_e}{\nu}$ - Reynolds number,

$D_e = \frac{4 \cdot A}{U} = \frac{\left(D_k{}^2 - n \cdot D^2\right)}{\left(D_k + n \cdot D\right)}$ - equivalent diameter,

$A = \left(D_k{}^2 - z \cdot D^2\right) \cdot \frac{\pi}{4}$ - free cross-section of the mantle,

$U = (D_k + z \cdot D) \cdot \pi$ - perimeter of free cross-section.

So the key to determining pressure loss is to define the average shape resistance coefficient of the mantle space equipped with deflector plates by transforming equation (4.11):

$$\boldsymbol{\xi = \frac{2 \cdot \Delta p}{w^2 \cdot \rho} - \lambda_{cs} \cdot \frac{z}{D_e}} [-]. \quad (4.13)$$

Based on Student's t-distribution, the values of average individual resistance coefficients assessed by mathematical statistical methods and their confidence intervals are as follows, based on pressure loss values measured between 0.15m and 3m at 0.3 m intervals, at a 95% reliability level:

$$\boldsymbol{\xi = 3.09 \pm 0.47.} \quad (4.14)$$

Based on the calculation formula (4.11) presented, the value of individual resistance coefficient $\boldsymbol{\xi}$ was determined by using measurement data arising from the following circumstances of measurement:

Working medium: Water

Volumetric flow: $\dot{V} = 1, 1.5, 2 \left[\frac{m^3}{h}\right]$.

Reynolds number range: $3800 < Re < 8000$.

Input temperature: $T_{be} = 13$ [°C].

Internal diameter of mantle: $d_b = 32$ $[mm]$.

Tube diameter: $d_k = 8$ $[mm]$.

Number of tubes: $n = 5$ $[pcs]$.

Figure 4.6 below shows measured pressure loss differences of the medium (water) flowing within the mantle from the values yielded by the new formula developed by me.

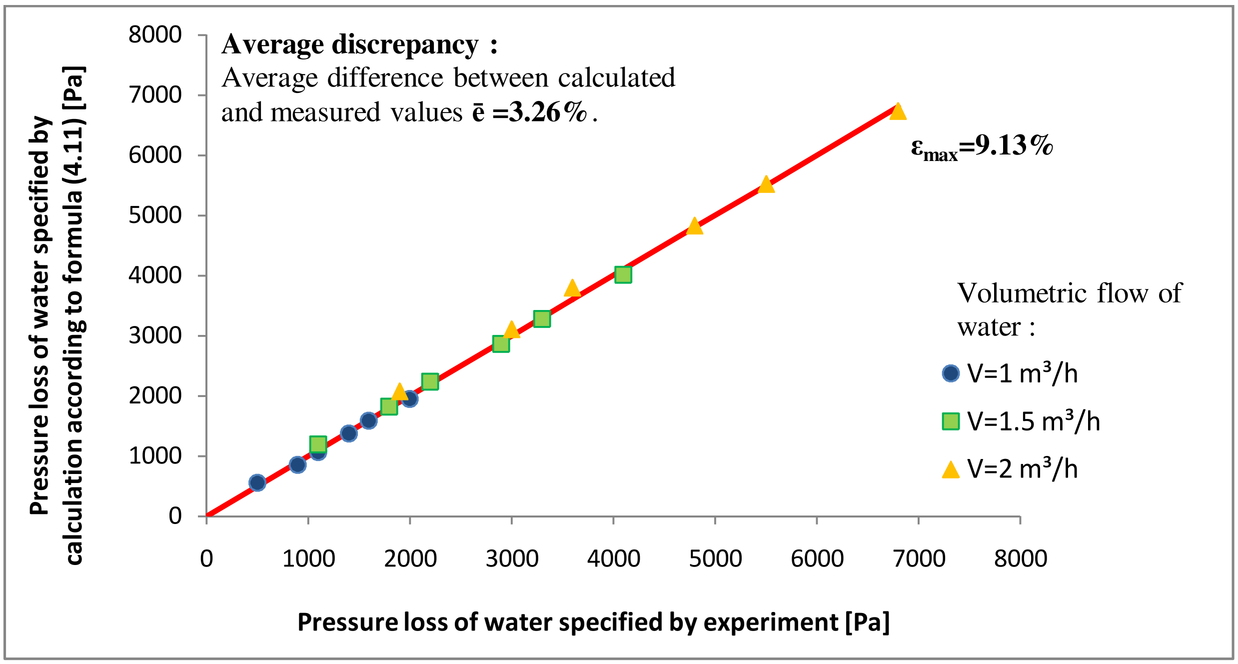

Figure 4.6. Comparison of values obtained from the new pressure loss model (formula 4.11) with measured values

The figure above shows that the maximum discrepancy of values yielded by the formula set up is $\varepsilon_{max} = 9.13$ % compared to measured values, while the average discrepancy is $\bar{e} = 3.26\%$. The following table summarizes the discrepancies of values calculated by different models from the measurement results.

Table 4.6. Comparison of models to determine pressure loss

Pressure loss model	***Average relative error*** $\bar{e}$	***Maximum discrepancy*** ε_{max}
Kern [28]	13.51 %	17.45 %
Bell-Delaware [29]	5.29 %	11.75 %
J. Taburek [30]	5.42 %	8.68 %
New model developed according to equation (4.11)	3.26 %	9.13 %

Thesis 6

I developed a mathematical model to describe the operation of water-to-water heat pump heating systems for steady-state condition. The model can be used to specify the optimal operating point of the heat pump system, including the maximum value of the coefficient of performance, in respect of any given heat demand for heating. In order to work out the mathematical model, I used the Runge-Kutta and the Adams-Moulton methods and wrote a computer software in C++ language for it.

Related publications: [1], [3], [6], [8], [9], [11].

The mathematical model I set up for heat pumps is deterministic, of divided parameters and stationary, meaning that connections between variables can be defined explicitly, are independent of time, and parameters have been taken into consideration by their values according to locations. The mathematical models of heat exchangers were described by coupled differential equations, while the models of the compressor and the throttle valve were described by algebraic equations of concentrated parameters. The mathematical model consists of basic and auxiliary equations.

Basic equations for the evaporator:

- **Differential balance equation to express mass conservation of the flowing coolant:**

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \cdot w)}{\partial z} = 0.$$

  - **For steady-state condition:**

$$\frac{\partial (\rho \cdot w)}{\partial z} = 0.$$

$$\rho \cdot w = G = constans. \tag{4.15}$$

- **Motion equation (dynamic equation):**

$$\frac{\partial(\rho \cdot w)}{\partial t} + \frac{\partial(\rho \cdot w \cdot w)}{\partial z} = -\frac{\partial p}{\partial z} - \frac{\lambda_{cs}}{2 \cdot d} \cdot w^2 \cdot \rho.$$

  - **For steady-state condition:**

$$\frac{\partial(\rho \cdot w^2)}{\partial z} + \frac{\partial p}{\partial z} + \frac{\lambda_{cs}}{2d} \cdot w^2 \cdot \rho = 0. \tag{4.16}$$

- **Energy balance equation of flowing coolant:**

$$\frac{\partial(\rho \cdot h_0)}{\partial t} + \frac{\partial(\rho \cdot w \cdot h_0)}{\partial z} = \frac{\partial p}{\partial t} + \dot{q}_b \cdot \frac{K}{A}.$$

$$h_o = h + \frac{1}{2}\mathrm{w}^2.$$

$$\dot{q}_b = \alpha_{hk} \cdot (T_{cső} - T_{HK}).$$

  - **For steady-state condition:**

$$\frac{\partial\left(G \cdot (h + w^2/2)\right)}{\partial z} - \dot{q}_b \cdot \frac{K}{A} = 0. \tag{4.16}$$

- **Balance equation of heat transmission:**

$$\rho_{cső} \cdot cp_{cső} \cdot A_{cso} \cdot \frac{\partial T_{cső}}{\partial t} = -\dot{q}_b + \dot{q}_k\ .$$

$$\dot{q}_k = \alpha_{viz} \cdot (T_{viz} - T_{cső}).$$

  - **For steady-state condition:**

$$\alpha_{viz} \cdot (T_{viz} - T_{cső}) - \alpha_{hk} \cdot (T_{cső} - T_{HK}) = 0. \tag{4.17}$$

- **Energy balance equation between cooling medium and water:**

$$-\dot{m}_{viz} \cdot cp_{viz} \cdot \frac{\partial T_{viz}}{\partial z} + \rho_{viz} \cdot cp_{viz} \cdot A_{viz} \cdot \frac{\partial T_{viz}}{\partial t} + q_k = 0.$$

$$\dot{m}_{viz} \cdot c_{viz} \cdot \left(T_{viz,be} - T_{viz,ki}\right) - \alpha_{viz} \cdot \left(\overline{T_{viz}} - \overline{\overline{T_{cső}}}\right) = 0. \tag{4.18}$$

Forms of the basic equations specified for the evaporator and the condenser were shown in the dissertation by taking into consideration coolant enthalpy dependence on status flags.

Auxiliary equations include known correlations to determine heat transmission, heat conduction and pressure loss.

Basic and auxiliary equations have been stated for each system component; they constitute a system of coupled equations; by taking into account boundary conditions and equations of state for the coolant, the Runge-Kutta and Adams-Moulton methods specified for the problem can be used for determining the operating points of the system for any discretionary consumer heat demand by fixing the values of the decision making parameters outlined above.

In order to adjust an optimal operating point - the maximum coefficient of performance (COP) - , the intervening features / decision making parameters required include the following:

- Mass flow of cold water,
- Mass flow of cooling medium,
- Mass flow of heated water.

Optimization matrices are used to find optimal operating points. Matrix elements within optimization matrices include the coefficient of performance (COP) values generated at different values of intervening features.

5. Applicability of results; further tasks

The results yielded by the steady-state mathematical model of divided parameters drawn up by me can be used for designing, measuring and dimensioning water-to-water heat pumps, for designing existing systems, as well as for preparing and supporting operating or other decision making processes.

In addition to values for science, innovation and education, the significance of this research lies in the fact that heat pump applications represent a high-priority area in the New Hungarian Energy Strategy (Energy Policy 2010 – 2030).

In spite of useful results, my research work cannot be considered as completed; the following research tasks are required to be performed:

- Instead of the shell and tube heat exchanger model applied, to model the thermodynamic behavior of lamellar heat exchangers; manufacturer’s information.

- I applied the equation of state of the heating medium R134a in the mathematical model of the heat pump. The mathematical model enables the integration of the equation of state and thermodynamic properties of other cooling mediums into the model.

6. List of own publications

[1] Garbai László, Sánta Róbert: A hőszivattyús rendszerek elpárologtatójának vizsgálata állandósult állapotban [Study of heat pump system evaporators in a steady-state condition], Magyar Épületgépészet [Hungarian Building Engineering],Vol.LX, No. 2011/12 szám, HU ISSN 1215 9913, pp.11-16, Budapest, Hungary.

[2] Róbert Sánta, József M. Nyers: Csõköteges elpárologtató hõátadási tényezõjének matematikai modelljei kétfázisú hûtõközegre [Mathematical models of the heat transfer coefficient of shell and tube evaporators for two-phase coolants], Magyar Épületgépészet [Hungarian Building Engineering], Vol. LIX, No. 2010/6, HU ISSN 1215 9913, pp.18-22, Budapest, Hungary.

[3] Jozsef M. Nyers, Robert Santa: Mathematical model of the heat pump coaxial evaporator with distributed steady state parameters, 41. KGH Congress, ISBN 978-86-81505-55-7, pp. 69-79, 1-3.XII, Belgrad, Serbia.

[4] Robert Santa: The Analysis of Two-phase Condensation Heat Transfer Models Based on the Comparison of Boundary Condition, Acta Polytechnika Hungarica, Vol. 9, No. 6, 2012.

[5] Róbert Sánta: Pressure Drop During Condensation of Refrigerant R134a Inside Horizontal Tubes, 3rd IEEE International Symposium on Exploitation of Renewable Energy Sources, EXPRESS 2011, IEEE number: CFP1188N-PRT, ISBN: 978-1-4577-0095-8, PP.117-122, 11-12 March, Subotica, Serbia.

[6] R. Santa, L. Garbai: The mathematical model and numerical simulation of the heat pump system. Annals of Faculty Engineering, Hunedoara – International Journal of Engineering Tome XI 2013, Fascicule 4, pp.271-280, ISSN 1584-2673.

[7] László Garbai, Róbert Sánta: Flow pattern map for in tube evaporation and condensation, 4th International Symposium on Exploitation of Renewable Energy Sources, EXPRESS 2012, ISBN: 978-86-85409-70-7, pp.125-130, 9-10 March, Subotica, Serbia.

[8] Jozsef M. Nyers, Robert Santa: Stationary mathematical model of heating system with heat pump, 22th Internationale Konferenz "Science in Practice", pp.59-66, 18 - 20. Mai, Schweinfurt, Deutsland.

[9] Robert Santa: Investigation of the refrigerants characteristics in vapor compression systems, Acta Technica Corviniensis – Bulletin of Engineering, Tome V (2012), No. ACTA-07/2012-3, Fascicule 3/2012, July-September.

[10] Róbert Sánta, László Garbai: A new heat transfer and pressure drop correlation of single phase flow on the shell side of heat exchanger, 6th International Symposium on Exploitation of Renewable Energy Sources, EXPRESS 2014, 27-29 March, Subotica, Serbia.

[11] J. Nyers, R. Santa: Energy optimum of heating system with heat pump**,** 6th International multidisciplinary conference, 27-28 May, Scientific Bulletin, Serie C, XIX, ISSN-1224-3264, ISBN 973-87237-1-X, 2nd Volume, pp:545-551, Baia Mare, Romania.

6. References

[12] Chen, J. C 1966, A Correlation for Boiling Heat Transfer to Saturated Fluids in Convective Flow, Industrial and Engineering Chemistry, Process Design and Development, Vol. 5, No. 3, pp. 322-329.

[13] Dittus FW, Boelter LMK. University of California (Berkley) publications on engineering, vol. 2. Berkley (CA): University of California, 1930. p. 443.

[14] Cooper, M.G. (1984). Heat flow rates in saturated nucleate pool boiling – a wide ranging examination using reduced properties. Adv. Heat Transfer, 16, 157-239.

[15] Bertsch, S.S., Groll, E.A., Garimella, S.V., 2009. A composite heat transfer correlation for saturated flow boiling in small channels. Int. J. Heat Mass Transfer 52, 2110–2118.

[16] N. Kattan, J.R. Thome, D. Favrat, Flow boiling in horizontal tubes: part 2—new heat transfer data for five refrigerants, J. Heat Transfer 120 (1998) 148–155.

[17] Kwang-Il Choi, A.S. Pamitran, Chun-Young Oh, Jong-Taek Oh: Boiling heat transfer of R-22, R-134a, and CO2 in horizontal smooth minichannels, International Journal of Refrigeration 30 (2007) 1336e1346.

[18] W.W. Akers, H.A. Deans, O.K. Crosser, Condensing heat transfer within horizontal tubes, Chem. Eng. Progr. Symp. Series 55 (1959) 171–176.

[19] Shah MM. Chart correlation for saturated boiling heat transfer: equation and further study. ASHRAE Trans 1982; 88: 185–96.

[20] Tang L. Empirical study of new refrigerant flow condensation inside horizontal smooth and micro-fin tubes.PhD thesis, University of Maryland at College Park, 1997.

[21] N. Kattan, J.R. Thome, D. Favrat, Flow boiling in horizontal tubes: part 3—development of a new heat transfer model based on flow pattern, J. Heat Transfer 120 (1998) 156–165.

[22] M.J. Wilson, T.A. Newell, J.C. Chato, C.A. Infante Ferreira, Refrigerant charge, pressure drop, and condensation heat transfer in flattened tubes, International Journal of Refrigeration 26 (2003) 442–451.

[23] L. Friedel, Improved friction pressure drop correlations for horizontal and vertical two-phase pipe flow, European Two-phase Flow Group Meeting, Ispra, Italy, 1979, Paper E2.

[24] R.W. Lockhart, R.C. Martinelli, Proposed correlation of data for isothermal twophase, two-component flow in pipes, Chem. Eng. Prog. 45 (1949) 39–48.

[25] Grönnerud R. 1979. Investigation of liquid hold-up, flow-resistance and heat transfer in circulation type of evaporators, part IV: two-phase flow resistance in boiling refrigerants. In: Annexe1972-1, Bull. De l'Inst. du Froid.

[26] H. Faltin, Műszaki Hőtan [Thermal engineering], Műszaki Könyvkiadó [Technical Publishing House], 1970, Budapest, Hungary.

[27] Környei Tamás, Hőátvitel [Heat transmission], Műegyetemi Kiadó [Technical University Publishing House], 1999, Budapest, Hungary.

[28] Kern, D.Q. (1950) Process Heat Transfer (McGraw Hill).

[29] K.J. Bell, Delware method for shell-side design, in: R.K. Shah, E.C. Subbarao, R.A. Mashelkar (Eds.), Heat Transfer Equipment Design, Hemisphere Publishing Corporation, 1988, pp. 145–166.

[30] J. Taborek, Shell-and-tube heat exchangers: single-phase flow 1983 Chapter 3.3, HEDH, Hemisphere P. Corporation.

[31] Darcy, H. 1857. Recherches Experimentales Relatives au Mouvement de L'Eau dans les Tuyaux, 2 volumes, Mallet-Bachelier, Paris. 268 pages and atlas.