Nonlinear Phenomena in Piecewise-Linear Nonlinear Mechatronic Systems

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Declaration/Nyilatkozat

I, the undersigned Péter Pál Stumpf, hereby declare that the dissertation submitted contains the results of my own work and that all other results taken from the technical literature or other sources are clearly identified and referred to.

Alulírott Stumpf Péter Pál kijelentem, hogy ezt a doktori értekezést magam készítettem és abban csak a megadott forrásokat használtam fel. Minden olyan részt, amelyet szó szerint, vagy azonos tartalomban, de átfogalmazva más forrásból vettem, egyértelműen, a forrás megadásával megjelöltettem.

Budapest, 2013. december 27.

Stumpf Péter Pál
The scientist does not study nature because it is useful, he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful it would not be worth knowing and if nature were not worth knowing life would not be worth living.

J.H. Poincaré
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Abstract

Many unexpected irregular behaviours caused by nonlinearities arise in mechanical and mechatronics engineering systems. The most common nonlinearities in the engineering practice are the friction, saturation, hysteresis, dead band and the backlash. Another source of nonlinearities caused by the controlled switching actions in power electronics converter. They play important role in up-to-date high-performance digitally controlled engineering systems, from milling machines via robotics through to the utilization of renewable energies, as they enable the efficient conversion of electric power to mechanical power and vice versa, furthermore to convert the electric power from one form to another.

In general, the power electronics converters are variable structure systems generated by a series of periodic switchings. Generally each structures can be modelled by linear time-invariant models therefore the systems are piecewise-linear. The overall systems are nonlinear as the switching times depend on one or more state variables or they are determined by nonlinear Pulse Width Modulation techniques.

One of the main goals of the dissertation was the presentation, comprehension and explanation of some unexpected phenomena caused in piecewise-linear systems. The other goal of my work was to improve the reliability and extend the stability range of variable structure systems by novel auxiliary state vector technique.

The dissertation is divided into three chapters based on the three main research topics.

Chapter 1 presents the DC components and subharmonics generated by the carrier based PWM techniques for high speed or high-pole count motors, where the necessarily high fundamental frequency and the limited carrier frequency result in low frequency ratio. An expression providing the exact value of the DC component when the frequency ratio is integer is derived. By using the expression the effect of the frequency ratio, the amplitude ratio, the phase shifting between the carrier and the reference signal, the modulation technique and the sampling form is shown. Furthermore the significant effect of the subharmonic voltage, current and flux components developed by different carrier based PWM techniques for the operation of induction machine is presented, when the frequency ratio is not integer and small. The theoretical results are verified by simulations and experiments.

Chapter 2 concerns with the stability analysis of two current controlled variable structure piecewise linear systems. One of them is the peak current mode controlled permanent magnet DC drive system and the other one is the digitally implemented average current mode controlled Power Factor Correction (PFC) converter. The stability analysis were carried for both cases by using an earlier published novel method implying the so-called auxiliary state vector. It is demonstrated that the method is capable to determine straightforward the Jacobian matrix without the derivation of the Poincar Map Function (PMF). Based on the eigenvalues of the Jacobian matrix the stability border could be calculated. Furthermore the chapter presents that the method inherently contains the feasibility to extend the stability range by adding a stabilizing signal into the control loop and the calculation of the parameters of the stabilizing signal. The theoretical results of the stability analysis and the effect of the stabilizing signal were verified for both systems by computer simulations and laboratory measurements.

Chapter 3 deals with a speed sensor-less Field Oriented Controlled induction machine drive, when shunt resistors, placed on the bottom of the three phase inverter legs, are applied to measure the stator currents which limits the sampling frequency of the current. It results that the ratio of the sampling to the actual fundamental frequency is low around the maximum speed of a high speed or high-pole count motor. The chapter demonstrates by computer simulation and experimental results that in this case the stability range of the drive can be extended by approximating the rotor flux angle change and applying Double Sampled Space Vector Modulation technique instead of Regular Sampled one.
Kivonat

A gépészeti és mechatronikai rendszerekben fellépő nemlinearitások számos váratlan és rendhagyó jelenséget okoznak. A mérnöki gyakorlatban fellépő nemlineairítások forrása leggyakrabban a süröldás, a telítődés, a histerézis, a holtsáv és a holtjáték. Egy másik típusú nemlinearitást okoznak a kapcsolások a teljesítményelektronikai konverterekben. Ezen berendezések egyre nagyobb szerepet játszanak a korszerű műszaki berendezésekben, a szerszámgépek-től kezdve az ipari robotokon át a megjövő energiaforrásokat hasznosító rendszerekig, mivel velük költséghatékonyan megvalósítható a villamos és mechanikai energia oda-vissza történő átalakítása.

A kapcsolóüzemű konverterek váltakozó struktúrájú rendszerek, hiszen minden kapcsolás után egy másik struktúra valósul meg melyes adott számú kapcsolás után periodikusan követik egymást. Annak ellenére, hogy minden struktúrát jó közelítéssel lineárisnak tekinthetünk a teljes rendszer nemlineáris lesz. A nemlinearítás forrása, hogy a kapcsolási időpontok és így a struktura változások időpontjai belső állapotváltozók értéktől függenek vagy azokat nemlineáris Impulzusszélesség Modulációs (ISZM) algoritmus állítja elő. A disszertáció egyik fő célja szakaszonként lineáris rendszerekben fellépő váratlan irreguláris viselkedésének bemutatása, megértése és magyarázata. A másik fő célja labilis szakaszonként lineáris rendszerek stabilitása tanulmányát kibővítése egy új fajta módszerrel, a virtuális állapotvektorral.

A disszertáció három fő fejezetre tagolódik a három fő kutatási témának megfelelően.

Az első fejezet az ISZM módszerek által generált egyen és szubharmonikus komponenseket vizsgálja nagyfordulatú vagy nagy pólusharmonikus hajtás esetére, ahol a magas alapharmonikus frekvencia és a korlátozott vivő frekvencia miatt a frekvencia aránya alacsony. Levezetésre kerül egy összefüggés, aminek segítségével az egyenkomponek és a kapcsolók közötti szög, a modulációs technika és a mintavétel hatása az egyenkomponek szerint. A dolgozat részletesen tárgyalja a különböző vivőfrekvenciás ISZM technikák által generált szubharmonikus frekvencia és a mágneses egyenáramú hajtás, amikor a frekvencia aránya kis és nem egész szám. Az elméleti eredményeket szimulációs és mérési eredmények igazolják.

A dolgozat második fejezetében bizonyítja a szimulációs és mérési eredményeket az egyenkomponek alapján. Az egyenkomponek és a kapcsolók közötti szög, a modulációs technika és a mintavétel hatásának segítségével a mágneses egyenáramú hajtás stabilizálása megvalósítható. A disszertáció bemutatja a szimulációs és mérési eredmények megfelelőségét és hatékonyságát, hogy ilyen esetekben a szabályozás megfelelőségét növelni lehet a mágneses egyenáramú hajtás stabilizálása.
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Preface

Many unexpected irregular behaviours caused by nonlinearities arise in mechanical and mechatronics engineering systems. The most common nonlinearities in the engineering practice are the friction, saturation, hysteresis, dead band and the backlash. Another source of nonlinearities caused by the controlled switching actions in power electronics converter. They play important role in up-do-date high-performance digitally controlled engineering systems, from milling machines via robotics through to the utilization of renewable energies, as they enable the efficient conversion of electric power to mechanical power and vice versa, furthermore to convert the electric power from one form to another.

In general, the power electronics converters are variable structure systems generated by a series of periodic switchings. Generally each structures can be modelled by linear time-invariant models therefore the systems are piecewise-linear. The overall systems are nonlinear as the switching times depend on one or more state variables or they are determined by nonlinear Pulse Width Modulation techniques.

One of the main goals of my research work was the comprehension and explanation of some unexpected phenomena caused in piecewise-linear systems, like the malfunction and breakdown of a high speed electro-mechanical drive including induction machine fed by voltage source converter or the large oscillations in the current signal of a Power Factor Correction (PFC) Converter. The analysis and understanding of nonlinear phenomena can help to improve the performance of the systems. The other goal of my work was to improve the reliability and extend the stability range of variable structure systems by novel auxiliary state vector technique to avoid their oscillations. Its application is demonstrated in a PFC Converter.

The dissertation is divided into three chapters. Chapter 1 discusses the effect of the carrier based Pulse Width Modulation Techniques on the operation of high speed drives. Chapter 2 presents the application of a novel stability analysis method based on the auxiliary state vector through two practical engineering example, like the DC servo motor drive and a PFC converter. In the third part the a speed sensor-less Field Oriented Controlled high speed drive is studied. It will be shown that by using Double Sampling Space Vector Modulation instead of the generally applied Regular Sampling technique the stability range of the drive system can be extended.

Each chapter follows the same structure. After presenting the main motivation for the research a short review of the literature in the field is given. After that, the theoretic background required to understand the novel scientific contribution of the chapter is discussed. It is followed by the physical and mathematical description of the new contribution verified by simulation and experimental results supporting the theoretical predictions. At the end of each chapter the novel results and achievements are summarized in form of thesis. Furthermore, my related publications and the practical significance of the results are given.

At the end of the dissertation the plans for the future research work will be presented.
Chapter 1

DC Component and Subharmonics Generation of PWM Techniques in Ultrahigh Speed Drives

1.1 Motivation

Failures of ultrahigh speed induction machines (USIM) experienced in the laboratory initiated my research work in the field of Pulse Width Modulation (PWM) techniques applied in three-phase two-level Voltage Source Converters (VSC). A system generating electric power during the process of pressure reduction in steam and gas networks has been developed for utilizing waste energies and renewable energy resources. The electromechanical energy conversion in the system is performed by a turbine-generator set. The system applied USIM, in the speed range of 100 krpm with rated power around 5 kW, to match the speed of the generator to that of the gas turbine. In addition, it resulted in reduced weight and increased efficiency.

The source of difficulties was the interaction of the VSC and the USIM. It is widely known that the higher harmonic contents of the voltages or currents, supplied by the converters, result in a number of undesirable effects, e.g., additional copper losses due to current harmonics, additional iron losses caused by flux harmonics, currents through the ball bearings that can reduce their lifetime, accelerated aging of the insulation due to high $dv/dt$ and long cable connection between VSC and USIM, torque pulsation due to current ripples, etc. A different kind of difficulty is caused by DC components and subharmonics. Both can cause serious malfunction and breakdown in the USIM. To have a deeper insight, I started to investigate the generation and adverse effect of the DC and subharmonic components when different carrier-based PWM techniques are applied.

1.2 Introduction

In the last decade increasing attention has been given to high speed induction and synchronous machine drives to reduce system size and improve power conversion efficiency. In [1] the main design problems of high speed drives are discussed by taking into account both electrical and mechanical aspects. In [2] the rotor dynamics of an ultrahigh speed motor with nominal speed 120 krpm was studied by finite element analysis. Paper [3] introduces the optimal design of a Permanent Magnet Synchronous Machine (PMSM) with nominal speed of 18 krpm and a nominal power of 1.5 MW. A multiphysic analytical model was used in [4] to design a 2-kW slotless PMSM with a rated speed of 200 krpm. Multidomain analysis applied in [5] to design a high speed (75 krpm) high power density (28 MW/m$^3$) laminated-rotor induction machine with rated power of 10 kW for and electrically assisted turbocharger. An analytical approach capable for rapid design of high speed rotor is presented and verified by using a 300 kW induction machine with rated speed 60 krpm used for air compressor.

The high speed drives poses many challenges not only in the field of electric motor design, but also in the field of industrial electronics. The two basic divisions of industrial electronics,
power electronics and digital signal processing play a decisive role in the development and the appearance of high performance ultrahigh speed motor drives [6].

The basic features of a three-phase PWM controlled VSC fed high speed drives are the necessarily high fundamental (synchronous) frequency \( f_1 \) (\( \geq 1 \) kHz) and the limited carrier (switching) frequency \( f_c \) (\( \leq 15\text{–}25 \) kHz). They result in low frequency ratio \( (m_f = f_c/f_1 < 20) \) leading to far more unfavorable stator voltage and current harmonic spectra as compared to those obtained at standard low fundamental frequencies. Few examples are as follows. A sensorless control of a PMSM driven turbo-compressor with nominal speed 72 krpm was introduced in [7]. Here Doublesampled Space Vector Modulation (SVM) technique (see later) is applied to generate the switching signals for the three phase inverter. The switching frequency was selected to be 15 kHz resulting in \( m_f = 12.5 \). In [8] an unmodulated square wave two-level VSC with variable DC-link voltage is applied for a real ultrahigh speed (500 krpm) application. Parallel operation of PWM controlled VSCs is presented to reduce the current ripple in a high speed motor drive in [9]. The approach presented in [10] is based on the application of current source inverters instead of VSC for a 30 krpm induction machine drive. Paper [11] introduces different modulation strategies and control schemes for a PMSM, when \( f_1 = 200 \) Hz and the carrier frequency is only \( f_c = 420 \) Hz.

In a modern closed loop controlled high speed drive systems, all the signal processes including the speed and current regulation loop and also the PWM block are implemented in the digital domain. Even with the up-to-date digital devices with clock frequency in the range of tens of MHz, the sampling frequency \( (f_{samp}) \) is limited. As a result, the ratio of the sampling frequency and the actual fundamental frequency \( F = f_{samp}/f_1 \) around the maximum speed of the motor is also low, resulting in stability problems and sampling error in the regulation loop. The effect of stability problems caused by the low \( F \) ratio is discussed in more detail in Chapter 3.

It should be noted, the problems encountered previously with the high speed drives appear also in high-pole count motor, used widely for hybrid and electric vehicles. As in this case the number of poles is 20 or higher, the required synchronous frequency \( f_1 \) is higher than 1 kHz similarly to ultrahigh speed drives, while the output speed is few thousand rpm [12]. Furthermore, in some high power application the carrier frequency \( f_c \) is kept at low value in order to reduce the switching losses resulting again low \( m_f \) [13]. These application fields give also practical significance of the research work presented in this chapter.

### 1.3 Overview of PWM controlled VSC

#### 1.3.1 Voltage Source Converter (VSC)

Figure 1.1(a) shows the schematic circuit diagram of the conventional two-level three-phase VSC, which is one of the most common power converter topology in industry similarly to the DC-DC converters. It is composed of a DC link capacitor or voltage source and an arrangement of two power semiconductors per phase. Nowadays in low \( (< 2\text{kw}) \) and medium power \( (2 - 500 \text{kw}) \) level generally Metaloxide-Semiconductor Field-Effect Transistor (MOSFET) or Insulated-Gate Bipolar Transistors (IGBT) are applied as switching device. In high power \( (> 500 \text{kw}) \) drive systems Integrated Gate Commutated Thyristor (IGCT) are one of the most commonly applied power semiconductor [6]. Where bidirectional current flow is required an anti-parallel diode also connected as it can be seen on Fig.1.1(a).

The gating signals of switches in one leg are complementary. Thus, in each switching state of the VSC, three switches are on and the other three are off, connecting the output terminal of the VSC to the positive or to the negative bar generating only two possible output voltage levels (Fig.1.1(b)). It results in eight possible structures shown in Fig.1.2, where 0 and 1 denote the state of the upper switch. Six of them (Fig.1.2(b)-1.2(g)) apply voltages at the output (active switch state), while Fig.1.2(a) and Fig.1.2(h) corresponds the short circuiting the bottom and top switches (inactive switch state), respectively.

Assuming ideal components each structure can be modelled by linear time-invariant equations, but the whole VSC are nonlinear as the switching instants determined by nonlinear PWM techniques. In a closed loop operation, like Field Oriented Control, the switching instants depends on the state variables as well.
Space vector, or in some literature Park vector, technique is a widely applied modelling tool to represent three-phase systems in a stationary $\alpha - \beta$ complex plane. The complex space vector is defined as [14]

$$x(t) = \frac{2}{3}[x_a(t) + ax_b(t) + a^2 x_c(t)]$$  \hspace{1cm} (1.1)

where

$$a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}; \quad a^2 = e^{-j\frac{2\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

Calculating the output space vectors $v_k$ of the VSC belonging to the switching states the peak of the active vectors ($v_k, \ (k=1...6)$) form a hexagon as depicted in Fig.1.1(c). Their length depends only on the $V_{DC}$ voltage. The inactive switch states can be described by two zero vector ($v_0, v_1$).

### 1.3.2 Pulse Width Modulation (PWM)

To obtain the desired output voltage with variable frequency and magnitude the duration of the active and inactive switching states should be varied with a PWM technique. PWM techniques have been a hotspot in controlling of power converters as it is directly related to the efficiency of the overall system affecting the economical profit and performance of the final product [6].
The PWM techniques can be classified in a multiple ways. According to [6] the PWM techniques are divided into four main groups: carrier-based PWM, Space Vector Modulation (SVM), harmonic control modulation, and other variable switching frequency methods, like hysteresis or predictive control.

Carrier-based PWM and SVM are the most successful modulation method among the most commonly used modulation techniques, because of their high performance, simplicity, fixed switching frequency, and easy digital and analog implementation [6].

**Carrier-Based PWM**

The carrier-based PWM techniques apply a triangular carrier signal $v_{\text{car}}$ compared against a reference waveforms $v_{\text{ref}}$ to generate the switching signals.

**SPWM:** One of the most widely applied carrier-based PWM technique, introduced by Schöning in 1964 [15], use sinusoidal (SPWM) signal as reference waveform (Fig.1.3(a)). In the case of three phase VSC three reference signals that are $\frac{2\pi}{3}$ radian out of phase should be compared with a common triangular carrier signal.

The ratio between the peak value of the sinusoidal signal $\hat{V}_{\text{ref}}$ and the maximum value of the triangular carrier signal $\hat{V}_{\text{car}}$ is the amplitude modulation index $m_a$

$$m_a = \frac{\hat{V}_{\text{ref}}}{\hat{V}_{\text{car}}} \quad (1.2)$$

An important feature of the SPWM is that the amplitude of the fundamental component of the output phase voltage $v_{a0}$ denoted by $\hat{V}_{a0}$ (see Fig.1.1) depends in linear fashion on $m_a$ (assuming $m_a \leq 1$)

$$\hat{V}_{a0} = \frac{m_a V_{\text{DC}}}{2} \quad (1.3)$$

In the case of overmodulation ($m_a \geq 1$) the number of pulses is reduced in the output phase voltage resulting in nonlinear relation between $\hat{V}_{a0}$ and $m_a$.

**THI-PWM:** By adding an adequate third harmonic zero sequence component $v_h$ to the sinusoidal reference waveform makes it possible to increase the peak of the output phase voltage up to 15.5%. This technique, introduced by Buja and Indri [16], is called Third Harmonics Injection PWM (THI-PWM) and it is depicted in Fig.1.3(b). The optimal amplitude of the zero sequence component to increase the utilization of $V_{\text{DC}}$ by 15.5% is $\frac{1}{6}$ of the peak value of the sinusoidal reference signal $\hat{V}_{\text{ref}}$ [17]. One disadvantage of the THI-PWM technique is that the injection has to be synchronized in each phase requiring a Phase Locked Loop (PLL) and the reference voltage amplitude must be known for variable speed and closed-loop operation [18, 6].

**Nonsinusoidal PWM:** The idea to use an injected third harmonic zero-sequence signal for a three-phase inverter initiated a research on nonsinusoidal carrier-based PWM techniques [19]. The developed algorithms are better choices for variable speed and closed loop operation as they have the advantage of avoiding the need of a PLL. The reference signal $\tilde{v}_{\text{ref},i}$ in each

![Figure 1.3: Sinusoidal and sinusoidal with third harmonic carrier-based PWM](image-url)
phase can be expressed as a sum of a sinusoidal \( v_{ref,i} \) and the nonsinusoidal zero-sequence component \( v_h \)

\[
\tilde{v}_{ref,i} = v_{ref,i} + v_h \quad (i = a, b, c)
\]  

(1.4)

where \( v_h = 0 \) corresponds to SPWM.

As it was shown in Fig.1.1(c) that the six active voltage space vectors divide the fundamental period into six sectors (Fig.1.4(a)). Along each sector a maximum \( v_M \), a medium \( v_{mid} \) and a minimum \( v_m \) value can be calculated from the three sinusoidal reference signals. For example in the switching interval inside of sector 3 shown in Figure 1.4(a), \( v_{ref,b} \) and \( v_{ref,c} \) have the maximum \( v_M \) and minimum \( v_m \) values, respectively, while \( v_{ref,b} \) has an intermediate value \( v_{mid} \).

A general algorithm allowing to build the nonsinusoidal zero sequence component \( v_h \) as a function of \( v_M \) and \( v_m \) is given as

\[
v_h = -\left\{ (1 - 2\mu) + \mu v_M + (1 - \mu) v_m \right\}
\]  

\[\text{(1.5)}\]

where \( 0 \leq \mu \leq 1 \) is the distribution ratio [19]. By applying \( \mu = 0.5 \), (1.5) becomes

\[
v_h = -\frac{v_m + v_M}{2} = \frac{v_{mid}}{2} = -\frac{\min(v_{ref,a}, v_{ref,b}, v_{ref,c}) + \max(v_{ref,a}, v_{ref,b}, v_{ref,c})}{2}
\]  

(1.6)

It is called min-max sequence injection. Practically it is the carrier-based realization of SVM (Fig.1.4(b)).

So SVM can be realized as a carrier-based modulation with a three phase nonsinusoidal reference signal \( \tilde{v}_{ref,i} = v_{ref,SVM,i} \) \( (i = a, b, c) \) (Fig.1.4(b)) and a common triangular carrier signal \( v_{car} \). The Fourier decomposition of \( v_{ref,SVM,a} \) in phase \( a \) [20]

\[
v_{ref,SVM,a} = m_a \left[ \sin(\omega_1 t) + \sum_{k=0}^{\infty} \frac{3\sqrt{3}(-1)^k}{\pi (3 + 6k)^2 - 1} \sin((3 + 6k)(\omega_1 t)) \right]
\]  

\[\text{(1.7)}\]

The term with the sum sign is the Fourier series of the \( v_h \) function (Fig.1.4(b)).

In most cases the reference wave \( v_{ref,SVM,a} \) is given approximately by the first several terms

\[
v_{ref,SVM,a} \approx m_a \left[ \sin(\omega_1 t) + 0.2067 \sin(3\omega_1 t) - 0.02067 \sin(9\omega_1 t) \
0.0074 \sin(15\omega_1 t) - 0.0038 \sin(21\omega_1 t) \right]
\]  

(1.8)

Depending on the selection of the distribution ratio \( \mu \) in (1.5), large number of possible modified reference signal can be obtained. Depenbrock concluded in 1977 that by applying a discontinuous zero sequence component the number of switchings can be reduced [21]. It initiated a research in the field of Discontinuous PWM (DPWM) [22, 23]. By selecting \( \mu = 1 \) (or \( \mu = 0 \)) one of the output phase voltage is clamped to the positive \( P \) (or negative \( N \)) bar for 120° and there are no switchings during this interval in that phase while the other two phases are modulated. In the literature the selection of \( \mu = 1 \) and \( \mu = 0 \) are often referred to as DPWMMIN and DPWMMAX, respectively. Another implementation form is to change \( \mu \) from 1 to 0 and back, each \( \mu \) value lasts alternatively for 60°. The reference signal of two widely applied methods, DPWM1, or often referred to as Flat-top modulation, and DPWM3 can be seen in Fig. 1.4(c) and 1.4(d), respectively. The main advantage of applying DPWM is that it can reduce the inverter size and cost by simplifying the thermal management issues due to the reduced switching losses [24]. Later on the dissertation focuses only on the continuous PWM techniques.
As it was shown (Fig.1.4(b)) the SVM can be realized as a nonsinusoidal carrier-based PWM technique. Other realization does not use carriers and comparators to generate the switching signals. It is the SVM technique based on the vector representation of the possible 8 output voltage space vectors of the VSC presented in Fig.1.1(c).

The ideal reference voltage space vector $\mathbf{v}_{\text{ref}}$ using (1.1) is rotating with angular speed $\omega_1 = 2\pi f_1 = 2\pi/T_1$ and its amplitude is constant in the $\alpha - \beta$ stationary reference frame. SVM uses the two adjacent active vectors and two zero vectors to approximate $\mathbf{v}_{\text{ref}}$ during one carrier period $T_c = 1/f_c$ [17, 19, 25]

$$\mathbf{v}_{\text{ref}} = \mathbf{v}_- t_- + \mathbf{v}_+ t_+ + \mathbf{v}_0 t_0 + \mathbf{v}_7 t_7$$  \hspace{1cm} (1.9)

where $\mathbf{v}_0 = \mathbf{v}_7 = 0$ and

$$\frac{t_-}{T_c} = \frac{\sqrt{3} m_a}{2} \sin \left[ -\omega_1 t + \frac{s}{3} \pi \right]$$ \hspace{1cm} (1.10)

$$\frac{t_+}{T_c} = \frac{\sqrt{3} m_a}{2} \sin \left[ \omega_1 t - \frac{(s-1)}{3} \pi \right]$$ \hspace{1cm} (1.11)

$$t_0 + t_7 = T_c - t_- - t_+$$ \hspace{1cm} (1.12)

Here $s = 1, 2, ... 6$, the sector number (Fig.1.1(c)). Note that $t_-$ belong to the right adjacent voltage vector, $t_+$ to the left adjacent vector while $t_0$ and $t_7$ to the zero vectors. In sector 1, $s = 1$ and $\mathbf{v}_- = \mathbf{v}_1$; $\mathbf{v}_+ = \mathbf{v}_2$ (Fig.1.5(a)). One possibility by knowing $T_c$, $m_a$, $\omega_1$ and $s$, the three unknown quantities, $t_-$, $t_+$ and $(t_0 + t_7)$ can be calculated at the beginning of each period $T_c$.

The order of voltage vectors applied in one carrier period depends on the particular SVM technique. The most commonly used technique is the center aligned pattern, where the sequence of the voltage space vectors are symmetrical to the half carrier period (Fig.1.5(b)).
1.3.3 Selection of frequency ratio \( m_f \) in ultrahigh speed drives

There are different opinions for the optimal selection of the frequency ratio \( m_f \), when it is a low value [26, 18, 27, 17]. In most of the literature it is suggested to apply synchronous PWM and keep \( m_f \) integer if \( m_f \leq 12 - 15 \) even when \( f_1 \) varies. In spite of this, in most of the commercially available three-phase inverters the switching frequencies can be varied only in discrete steps (e.g. 3-6-12-16 kHz) resulting in asynchronous PWM for variable high speed drives when \( f_1 \geq 1 \) kHz. It gives the practical significance for investigation of the effect of low and non-integer \( m_f \).

In many papers it is suggested to maintain \( m_f \) as multiple of three because of the three-phase symmetry of the machines and the triplen harmonics are of no concern in three wire balanced load [27]. Some other papers state also that the \( m_f \) should be odd number and multiple of three [25, 26]. These requirements give limited choice for \( m_f \) and results in wide variation of the switching frequency in variable high speed drives. Furthermore the sudden change in \( m_f \) can cause current transients. In spite of these according to the book [17], which is often referred as the major reference textbook on PWM theory, the "cancellation of harmonics between phase legs is independent of the frequency ratio between the carrier and the fundamental" and "there seems to be no particular reasons to require an odd carrier/fundamental ratio" and "there is no particular benefit to be gained by a triplen carrier pulse ratio". My finding was that using even and not multiple of three value for \( m_f \) DC components with considerable amplitudes can be generated if \( m_f < 20 \). The DC voltage depends on the \( m_f \) frequency ratio, the amplitude ratio \( m_a \), the phase shifting between the carrier and the reference signal, the modulation technique and the sampling form.

1.4 Sampling Techniques

Based on the sampling pattern of the reference signals three different sampling techniques used to be classified: the Regular Sampling (RS), Natural Sampling (NS) and Oversampling (OS) (Fig.1.6).

1.4.1 Regular Sampling (RS)

In the case of RS the reference signals \( \nu_{r_f,i} \) \( (i = a, b, c) \) are sampled at the beginning of every carrier signal period (Fig.1.6(a)) and keeping this sampled \( \nu_{r_f,i} \) value in one carrier period. This is the most commonly used method of implementation, because it is convenient to implement by Digital Signal Processor (DSP) or by microcontroller [28].

By decreasing the frequency ratio \( m_f = f_c/f_1 \), the accuracy of the RS is deteriorated as well. When \( m_f \) is low, like in the case of ultrahigh speed drive, the stepped approximation
\( \bar{v}_{ref,i} \) of the reference signal are getting less accurate and has delayed response to the reference signal.

### 1.4.2 Natural Sampling (NS)

The most accurate implementation form of a PWM technique if the switching signals are obtained by comparing the continuous \( v_{ref,i} \) signals with the carrier signal. This analog implementation form is also called Natural Sampling (NS).

NS is an asymmetrical form unlike RS, which is a symmetric one, because it takes into consideration the change in the input signals during a carrier period resulting that the switching pattern will not be symmetrical to \( T_c/2 \) as the rising and falling edge of the triangular carrier signal are compared with different values of the reference signals. Of course, when \( m_f \) is a high number this effect is negligible.

NS is the best form of sampling especially at low \( m_f \) as it does not introduce distortion or a delayed response to the reference signal [17, 29]. In the past NS was implemented by using analog devices, like comparators and integrators. Nowadays up-to-date digital devices, like FPGA or DSP, and Oversampling technique (see next section) are used to approach the performance of the NS technique [29, 30, 31, 32, 33]. In 1.4.4 a practical digital implementation form of NS will be presented.

### 1.4.3 Oversampling (OS)

Obviously by increasing the number of samplings of the reference signals during one carrier period, the accuracy of the sampling techniques can be increased. The most common solution is to sample the reference signals twice (Doublesampling (DS) or Resampling, Fig. 1.6(b)) in each \( T_c \) as the registers in PWM peripherals of DSP and microcontrollers can be up-dated twice during a carrier period to avoid glitches in the switching signals.

Utilizing the parallel computing properties of the FPGA the number of samples can further be increased (Fig.1.6(c)) [31, 32, 33].

This technique is the same as the NS, when the number of samples approaches infinity. In practical applications using FPGA with oversampling rate \( n = 4, 8, 16 \) or 32 the difference between NS and OS is getting negligible.

The main problem of OS is the multiple edge generation (Fig. 1.6(d)), because of the stepped nature of the sampled waveform. However this problem can be solved during the implementation by ensuring that the switch can be turned on and off only once during a carrier period [29].

![Sampling techniques](image-url)
The sampling techniques has a great influence on the performance and the harmonic content of the PWM techniques, even when the frequency ratio \( m_f \) is low. Later on the following notation will be used: XS-PWMy, where XS denotes the sampling technique (RS, NS, OS or DS) and PWMy refers to the given PWM technique (SPWM, THI-PWM or SVM).

1.4.4 Digital implementation of NS

In every application, like in ultrahigh speed drives, where the frequency ratio \( m_f \) is low, the NS techniques can be the most favourable implementation form as no distortion or delayed response to the reference signal are introduced when analog technique is used [17, 33, 29, 34]. As analog techniques does not lend itself to up-to-date high performance drive systems, I have been seeking to create a practical digital implementation of NS.

In this section a new method is presented. It is capable to realize the carrier-based PWM techniques using NS with very good accuracy in open loop where the values of the reference signal can be precalculated (see later \( v_{ref}^{**} \) and \( v_{ref}^{***} \)). It has been successfully implemented by me into a PWM peripheral module of a low-cost 16-bit DSC (dsPIC33FJ32MC204) using fixed-point arithmetic in Q15.16 form, in DSP (TMS320F2808) using IQmath library and in an ARM Cortex-M3 processor (STM32F100CB).

Generally the PWM peripheral modules applied in digital devices consist of an up-down counter, a Period Register \((PR)\) and three Compare Registers for each phase \((CR_i, (i = a, b, c))\). The triangular carrier curve is approximated by large number of steps stored in \(PR\) (Fig 1.7). PWM peripherals support RS and DS as they allow to up-date the value of the \(CR\) maximum twice during a carrier period to avoid glitches in the switching signals, however, NS can be also realized by properly approximating the reference signal.

Figure 1.7 presents the calculation method for the determination of the intersection points of the carrier and the reference signal. The function is called at each positive and negative peaks of carrier signal. Assuming that the value of point \( v_{ref,h}^{*} \) (Fig.1.7(a)) (or \( v_{ref,l}^{*} \), Fig.1.7(b)) is known from the previous calculation, the algorithm calculates the point \( v_{ref,h}^{**} \) and \( v_{ref,l}^{**} \) (or \( v_{ref,l}^{***} \) and \( v_{ref,l}^{***} \)) of the known theoretical reference curve (denoted by red line). If the value of point \( v_{ref,h}^{*} \) is larger than \( PR/2 \) then the theoretical curve is approximated by a straight line between \( v_{ref,h}^{*} \) and \( v_{ref,h}^{**} \) (Fig.1.7(a)). Otherwise, when point \( v_{ref,l}^{*} \) is smaller than \( PR/2 \) the theoretical curve is approximated by a straight line between \( v_{ref,l}^{*} \) and \( v_{ref,l}^{***} \) (Fig.1.7(b)).

By simple mathematical relationships the crossing point can be determined and its value can be latched to the \(CR_h\) (or \(CR_l\)) register. The value of point \( v_{ref,h}^{***} \) (or \( v_{ref,l}^{***} \)) can be used in the next calculation as point \( v_{ref,h}^{*} \) (or \( v_{ref,l}^{*} \)).

It should be noted that the calculation takes less time than \( T_c/2 \), however, the microcontroller vendor suggests to update the \(CR_i\) (i = a, b, c) registers of the digital PWM peripheral only in the next half carrier period. It results in a constant \( \frac{T_c}{2} \) time delay. In open-loop this effect can be compensated by phase advancing the angle with \( \Delta \rho = \frac{\pi}{m_f} \). Furthermore the delay can be avoided if the value of the \(CR\) register belonging to the rising ramp are calculated during falling ramp and vice-versa. Figure 1.8 represents the calculation process.

One of the main advantage of the proposed digital NS contrary to OS using sampling rate \( n = 4, 8, 16 \) or 32 presented in [31, 32, 33] is that there is no problem caused by the multiple edge generation. Furthermore it is enough to call the function twice per carrier period.

Figure 1.9 shows the measured time function of the duty ratio of the upper switch in phase \( a \) for SVM applying RS, DS and NS technique when \( m_f = 20 \) and \( m_f = 10 \). The amplitude modulation ratio in all cases is \( m_a = 0.955 \). The duty ratio \( D = t_{on}/T_c \) is the ratio of the duration when the switch conducts \( t_{on} \) to carrier period \( T_c \). For example based on Fig.1.5(b) the duty ratio of the upper switch in phase \( a \) in sector 1 is \( D = \frac{t_{-}+t_{+}+t_{7}}{T_c} = \frac{T_c-t_{0}}{T_c} \). The switching signals are generated by using the dsPIC33FJ32MC202 DSC. The time function of the duty ratio was obtained by using the "Measurement Trend" in-built function of the digital oscilloscope available in the laboratory. This feature is capable to calculate the duty ratio from
a PWM modulated square wave signal. The trigger signal which calls the function to calculate duty ratio were also depicted.

It can be concluded when \( m_f \) is high (\( m_f = 20 \)) the difference between RS, DS and NS are negligible (Fig.1.9(a), 1.9(c), 1.9(e)). By decreasing \( m_f \) the difference between the three sampling form becomes considerable. For the better visualization the time function of both RS and DS are plotted in Fig.1.9(f).

1.5 Harmonic analysis using double Fourier series method

The determination of the harmonic content of the quasi-square wave phase voltage \( v_{ao} \) depicted in Fig.1.1(b) was one of the topics for the researchers in the last decades developing analytical solutions for almost any PWM strategy [17, 35, 36, 37, 38, 39, 40, 41, 42]. One of the standard approaches for calculation of the harmonic analysis of PWM signals is the Double Fourier series expansion method [17, 35, 40, 41, 42]. The following is a concise summary of the method based on [17].

According to the Fourier series expansion the output voltage signal \( f(t) = v_{ao}(t) \) can be

\[ f(t) = \sum_{n=1}^{\infty} \left( a_n \cos(n \omega_0 t) + b_n \sin(n \omega_0 t) \right) \]

\[ a_n = \frac{2}{T} \int_{0}^{T} f(t) \cos(n \omega_0 t) \, dt \]

\[ b_n = \frac{2}{T} \int_{0}^{T} f(t) \sin(n \omega_0 t) \, dt \]

The figures are manipulated using Inkscape® software to improve the quality, however, the results are not modified.
Figure 1.9: Duty ratio versus time. Comparison of digitally implemented NS-SVM with DS and RS SVM, \( f_c = 4 \text{ kHz} \), \( m_a = 0.955 \) \( y \) scale: 20%

Expressed as an infinite series of sinusoidal harmonics [17]

\[
f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega t + b_m \sin m\omega t)
\]  

(1.13)

where

\[
a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos m\omega t \, d(\omega t) \quad m = 0, 1, \ldots, \infty
\]  

(1.14)

\[
b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin m\omega t \, d(\omega t) \quad m = 1, 2, \ldots, \infty
\]  

(1.15)

The waveform \( f(t) = v_{a0}(t) \) varies as the function of two time variables \( f(t) = f[x(t), y(t)] = f[x(t), y(t)] \), where

\[
x(t) = \omega_c t + \varphi_c
\]  

(1.16)

\[
y(t) = \omega_1 t + \varphi_0
\]  

(1.17)
and $\omega_c$ and $\omega_1$ are the angular frequency of the triangular carrier signal $v_{\text{car}}$ and the reference signal $v_{\text{ref}}$, respectively. $\varphi_c$ and $\varphi_0$ are the initial phase shifting of $v_{\text{car}}$ and $v_{\text{ref}}$, respectively.

Based on [17] (1.13) can be rewritten for the double variable controlled waveform $f(t) = f[x(t), y(t)]$ by using the double Fourier series expansion.

$$f(t) = \frac{A_{00}}{2} + \sum_{n=1}^{\infty} \left( A_{0n} \cos(n[\omega_1 t + \varphi_0]) + B_{0n} \sin(n[\omega_1 t + \varphi_0]) \right)$$

$$+ \sum_{m=1}^{\infty} \left( A_{m0} \cos(m[\omega_c t + \varphi_c]) + B_{m0} \sin(m[\omega_c t + \varphi_c]) \right)$$

$$+ \sum_{m=1}^{\infty} \sum_{n=-\infty, (n\neq0)}^{\infty} \left[ A_{mn} \cos(m[\omega_c t + \varphi_c] + (n[\omega_1 t + \varphi_0])) + B_{mn} \sin(m[\omega_c t + \varphi_c] + (n[\omega_1 t + \varphi_0])) \right]$$

(1.18)

where

$$A_{mn} = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \cos(mx + nx) dx dy$$

(1.19)

$$B_{mn} = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) \sin(mx + nx) dx dy$$

(1.20)

or in complex form

$$C_{mn} = A_{mn} + jB_{mn} = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x, y) e^{j(mx+ny)} dx dy$$

(1.21)

and $m$ is the carrier index variable and $n$ is the baseband index variable. The first term of (1.18) $(m = 0, n = 0)$ is the DC component of the output voltage. The first summation term $(m = 0)$ defines the output fundamental $(n = 1)$ and its baseband harmonics. The second summation term $(n = 0)$ contains the carrier harmonics and the last summation term defines all possible frequencies formed by taking the sum and difference between the carrier signal harmonics and the reference waveform and its associated baseband harmonics. They are called sideband harmonics.

To calculate the value of the harmonic components the $f(x, y)$ function should be determined. Figure 1.10 shows the switching instants during one carrier signal period assuming NS. $f(x, y)$ has only two values $V_{\text{DC}}/2$ or $-V_{\text{DC}}/2$. It changes from $-V_{\text{DC}}/2$ to $V_{\text{DC}}/2$ (rising edge) when

$$x_r = \frac{-\pi}{2} (1 + v_{\text{ref}}) + 2\pi p; \quad p = 0, 1, 2...$$

(1.22)

and $f(x, y)$ changes from $V_{\text{DC}}/2$ to $-V_{\text{DC}}/2$ (falling edge) when

$$x_f = \frac{\pi}{2} (1 + v_{\text{ref}}) + 2\pi p; \quad p = 0, 1, 2...$$

(1.23)

By substituting (1.22) and (1.23) in to (1.21) the amplitude of the fundamental component $(m = 0, n = 1)$ for NS-SPWM ($v_{\text{ref}} = m_a \sin y$)

$$A_{01} + jB_{01} = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \left( \int_{-\pi}^{x_r} - \frac{V_{\text{DC}}}{2} e^{jy} dx + \int_{x_r}^{x_f} \frac{V_{\text{DC}}}{2} e^{jy} dx + \int_{x_f}^{\pi} - \frac{V_{\text{DC}}}{2} e^{jy} dx \right) dy =$$

$$= \frac{V_{\text{DC}}}{2} \int_{-\pi}^{\pi} j \frac{m_a}{2} dy = j \frac{m_a V_{\text{DC}}}{2}$$

(1.24)
which equals the target reference defined in (1.3).

The amplitude of the higher harmonics can be calculated similarly for different carrier-based PWM techniques and sampling form as it was done in [17].

1.6 Calculation of DC component

In this section integer \( m_f \geq 3 \) is assumed\(^2\).

1.6.1 Double Fourier series method

Similar to the fundamental component the DC component of the output phase voltage can be determined for PWM by substituting (1.22) and (1.23) to (1.21)

\[
A_{00} = \frac{1}{2} C \int_{-\pi}^{\pi} \left( \int_{-\pi}^{x_r} - \frac{V_{DC}}{2} \cos(0) dx + \int_{x_r}^{x_f} \frac{V_{DC}}{2} \cos(0) dx + \int_{x_f}^{\pi} - \frac{V_{DC}}{2} \cos(0) dx \right) dy = \frac{V_{DC}}{4\pi} \int_{-\pi}^{\pi} v_{ref} dy \tag{1.25}
\]

It is evident the integral of the reference signal of SPWM, THI-PWM and SVM between \(-\pi\) and \(\pi\) is zero. \( \frac{A_{00}}{2} \) will be also zero according to (1.25). The same holds when RS or OS is assumed [17].

The result gives the impression that according to the double Fourier series expansion \( v_{a0} \) has zero DC component independently of frequency ratio \( m_f \), the amplitude ratio \( m_a \), the phase shifting between the carrier and the reference signal and the sampling form. The same conclusion can be found in [17] and in other papers dealing with the harmonic content of the output voltage of the PWM modulated VSCs [39, 35, 36, 41].

My finding was just the opposite. The carrier-based NS-PWM techniques are prone to generate DC components in the output phase voltage. Furthermore the RS-THI-PWM and RS-SVM also generates DC component in \( v_{a0} \). It should be noted that the source of the DC component is the lower sideband harmonics around the first carrier harmonic group \( (m = 1) \) in (1.18). All the other sideband harmonics are by far negligible. As only even sideband harmonics appear around the first carrier frequency, DC component exists only for even \( m_f \)\(^3\). In this way the magnitude of the DC component can be determined by using the double Fourier series by calculating the amplitude \( A_{mn} \) and \( B_{mn} \) of the lower sideband harmonics. However for the better understanding and to derive a more simple and accurate expression which valid independently of the frequency ratio \( m_f \), the amplitude ratio \( m_a \), the phase shifting between

\(^2\)It should be noted DC component can be generated when \( m_f \) is not integer as well [20]

\(^3\)It will be shown DC component exists for odd \( m_f \) when RS-THI-PWM or RS-SVM is applied
the carrier and the reference signal and the sampling form a novel expression is derived as it will be shown later.

The currents generated by the DC voltage components in the phase voltages of USIM are limited only by the stator resistance, which is generally small, therefore the problem caused by them can be serious.

### 1.6.2 Other methods

In the literature some papers have already reported that the NS-SPWM modulation has non-zero DC component when the \( m_f \) frequency ratio is low. In [43] the DC component for SPWM is given as

\[
V_{DC,a0} = \frac{4V_{DC}}{\pi} \sum_{k=1}^{\infty} \left( \frac{1}{k} J_{k,m_f} \left( \frac{k \pi m_a}{2} \right) \left[ 1 - (-1)^{k(1+m_f)} \right] \sin(k \varphi_c) \right) \tag{1.26}
\]

where \( J \) is the Bessel-function and \( \varphi_c \) is the phase shift between the triangular and the reference signal measured in the period of the carrier signal (assuming \( \varphi_0 = 0 \) (see later Fig.1.11(a)).

In [44] the solution of the Keplers problem was applied to obtain the switching instants of the NS-SPWM in terms of Kapteyn series and to find the explicit expression of the harmonic content. From here the magnitude of the DC component is

\[
V_{DC,a0} = \frac{2V_{DC}}{m_f} \sum_{q=0}^{m_f-1} \left( \tau_1 q - \tau_2 q + \frac{1}{2} \right) \tag{1.27}
\]

where

\[
\tau_1 q = \frac{4q + 1}{4} \frac{m_f}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} J_n \left( \frac{n \pi m_a}{2m_f} \right) \sin \left[ \frac{n \pi}{2m_f} (4q + 1) + \frac{n \varphi_c}{m_f} \right] \tag{1.28}
\]

\[
\tau_2 q = \frac{4q + 3}{4} \frac{m_f}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} J_n \left( \frac{n \pi m_a}{2m_f} \right) \sin \left[ \frac{n \pi}{2m_f} (4q + 3) + \frac{n \varphi_c}{m_f} \right] \tag{1.29}
\]

Based on my numerical calculations\(^4\) (1.26) and (1.27) give the same DC component for the same frequency ratio \( m_f \), amplitude modulation index \( m_a \) and initial phase shifting \( \varphi_c \). However in the case of SPWM, based on their results, the DC voltage component can be ignored when \( m_f > 4 \), that is, practically always.

### 1.6.3 Calculation of DC component by the sum of reference signal values at switching instants

In the consideration of this section integer frequency ratio \( m_f \) is assumed.

Figure 1.11(a) shows the carrier-based modulation process with three different \( v_{ref} \) reference signals and the resulted output voltage \( v_{a0} \) (Fig.1.1(a)) for SVM when \( m_f = 8 \).

The time axis are represented by two different angular abscissas \( \phi = \omega_1 t \) and \( \varphi = \omega_c t \). Angle \( 2\pi \) measured in the period of reference signals results in angle \( 2\pi m_f \) measured in the period of the carrier (triangular) signal, where \( m_f = \omega_c / \omega_1 = f_c / f_1 \). \( \varphi_c = \varphi_{c,a} \) denotes the phase angle measured in the period of the carrier signal between the carrier signal and the reference signals (Fig.2.2(c)).

Unless stating otherwise we assume that the reference and the carrier signal start from zero value at \( t = 0 \) in phase \( a \), i.e. \( \varphi_{c,a} = 0 \). The definition for \( \varphi_{c,b} \) and \( \varphi_{c,c} \) is the same for phase \( b \) and \( c \) as for phase \( a \) in Fig.1.11. It is obvious if \( m_f \) is not multiple of 3 and \( \varphi_{c,a} = 0 \) then the phase angle between the carrier signal and the reference signal in the other two phases are not zero (\( \varphi_{c,b} \neq 0, \varphi_{c,c} \neq 0 \)).

\(^4\)The value of the DC component (when \( m_f = 4, m_a = 0.955 \) and \( \varphi_c = \pi/2 \)) for NS-SPWM is 0.0074\(V_{DC} \) both by using (1.26) and (1.27). The maximum value of both \( \delta \) in (1.26) and \( n \) in (1.27) were selected to be 10. It should be noted the same result can be obtained using (1.36).
Angles $\alpha_i$ ($i = 1, 2, ..., 2m_f$) determined from $v_{ref} - v_{car} = 0$ in Fig.1.11(a) and measured in the period of the fundamental component of $v_{ref}$ and carrier signal $v_{car}$. The instantaneous value of $v_{ref}$ and the intersection points depends on the sampling technique. $\alpha_i$ can be measured in the period of the carrier signal as $m_f \alpha_i$ (Fig1.11(a)).

The DC voltage component in one phase during one fundamental period can be calculated according to Fig.1.11(a)

$$V_{DC,x} = \left(\frac{m_f}{2\pi} \sum_{i=1}^{m_f} \alpha_{2i} - \alpha_{2i-1}\right) \frac{V_{DC}}{2\pi} - \frac{V_{DC}}{2}$$

(1.31)

Figure 1.11(b) shows a period of the carrier signal representing it with two straight lines. $\gamma$ is the angle measured from the last maximum point of the carrier signal.

At switching instant $\alpha_1$, when the output voltage changes from $-V_{DC}/2$ to $V_{DC}/2$ (Fig.1.11(a))

$$v_{ref,x}(\alpha_1) = v_{car}(m_f \alpha_1) = 1 - \gamma \frac{2}{\pi} = 1 - (\varphi_c - \frac{\pi}{4} + m_f \alpha_1 - (j-1)2\pi) \frac{2}{\pi} ; \quad j = 1$$

(1.32)

Angle $\varphi_c - \frac{\pi}{4}$ represents the phase difference between $\varphi = 0$ and the last maximum point of carrier signal.

At switching instant $\alpha_2$, when the output voltage changes from $V_{DC}/2$ to $-V_{DC}/2$

$$v_{ref,x}(\alpha_2) = v_{car}(m_f \alpha_2) = -1 + (\gamma - \frac{\pi}{2}) \frac{2}{\pi} = -1 +$$

$$\left(\varphi_0 - \frac{\pi}{4} + m_f \alpha_2 - (j-1)2\pi - \frac{\pi}{2}\right) \frac{2}{\pi} ; \quad j = 1$$

(1.33)

Figure 1.11: Carrier-based modulation process ($m_f = 8$) and the representation of carrier signal.
The width of the first positive voltage pulse from the above equations is

\[ \alpha_2 - \alpha_1 = \frac{\pi}{m_f} (v_{ref,x}(\alpha_1) + v_{ref,x}(\alpha_2)) + \pi \] (1.34)

In general

\[ \alpha_{2j} - \alpha_{2j-1} = \frac{\pi}{m_f} (v_{ref,x}(\alpha_{2j-1}) + v_{ref,x}(\alpha_{2j})) + \pi \] (1.35)

Substituting (1.35) to (1.31) the final result is

\[ V_{DC,x} = \frac{1}{4m_f} \left( \sum_{i=1}^{2m_f} v_{ref,x}(\alpha_i) \right) V_{DC} \] (1.36)

The DC component generated by all PWM discussed here depends on the values of the reference signal at the intersection points. Equation (1.36) holds for RS, NS and OS as well. By increasing \( m_f \) the term \( 1/4m_f \) approaches zero and the term \( \sum_{i=1}^{2m_f} v_{ref,x}(\alpha_i) \) also converges to \( \int_0^{2\pi} v_{ref,x}(\alpha) d\alpha = 0 \). Depending on the PWM technique, \( m_f \) and \( m_a \) value, \( \varphi_c \) and the sampling techniques, the value of DC component given by (1.36) can be significant at lower \( m_f \).

![Diagram](image.png)

Figure 1.12: Generation of DC component in phase a, SVM, \( \varphi_c = \pi/4 \), \( m_f = 4 \), \( m_a = 0.955 \)

For better understanding an illustrative example for an extreme case when \( m_f = 4 \) can be seen for the generation of the DC component for SVM in phase a when \( \varphi_{c,a} = \pi/4 \) (Fig.2.5). The DC component with negative sign can clearly be seen from the area of the rectangulars obtained by simulation and checked by calculation.

**Dependence on \( m_f \)**

As it was mentioned previously the reference signals \( v_{ref,x} \) of SPWM, THI-PWM and SVM can be approximated by the sum of different sinusoidal terms. The sine function is an odd function thus \( \sin k\varphi = -\sin k(\varphi + \pi) \), where \( k = 1, 3, 5 \ldots \) odd integer. Thus when the distance between intersection points \( \alpha_j \) and \( \alpha_{j+m_f} \) \( (j = 1, 2, \ldots m_f) \) is \( \pi \) measured in scale \( \phi \) independently of the value of \( \varphi_0 \) the sum term in (1.36) will be zero and no DC component is generated in the output phase voltage. The distance between intersection points \( \alpha_j \) and \( \alpha_{j+m_f} \) is \( \pi \) if the functions determining the intersection points are also odd functions.

The Fourier series of the triangular signal with unit amplitude is [45]

\[ v_{car} = \frac{8}{\pi^2} \sum_{q=1,3,5,..}^{\infty} \frac{(-1)^{(q-1)/2}}{q^2} \sin(q\omega_c t) \] (1.37)
By approximating the triangular carrier signal with its Fourier series the zero crossing of the following trigonometric equation gives the $\alpha_i$ intersection points for example for SPWM

$$f(\phi, m_f \phi, \varphi_{c,a}) = v_{ref} - v_{car} = m_a \sin \phi - (a_1 \sin(m_f \phi + \varphi_{c,a}) + a_3 \sin(3(m_f \phi + \varphi_{c,a})) + a_5 \sin(5(m_f \phi + \varphi_{c,a})) \pm ...)$$ (1.38)

where the $a_q (q = 1, 3, 5, \ldots)$ parameters can be calculated from (1.37). $V_{DC,x}$ is always zero for odd $m_f$ values because

$$f(\phi, m_f \phi, \varphi_{c,a}) = -f((\phi + \pi), m_f (\phi + \pi), \varphi_{c,a})$$ (1.39)

relation is met. As $q$ is odd number it has no effect on relation (1.39). All discrete function values of $v_{ref}$ calculated from (1.38) in the positive half cycle of $v_{ref}$ can be found in the negative half cycle of $v_{ref}$ with negative sign and they cancel each other. The function

$$f(\phi, \varphi_{c,a}) = -f((\phi + \pi), \varphi_{c,a})$$ (1.40)

holds not only for the reference signal of SPWM but for those of THI-PWM and SVM. Consequently $V_{DC,x} = 0$ for all three PWM provided that $m_f$ is odd and triangular carrier signal is used. On the other hand, when $m_f$ is even number then (1.39) does not hold and $V_{DC,x} \neq 0$. DC component is generated by all three PWM.

For better visualization of the dependence of $V_{DC,x}$ existence on $m_f$, the Lissajou curves are plotted on the plane of the carrier versus reference signal in Fig.1.13 when $\varphi_{c,a} = \pi/4$ and $m_f = 5$ and $m_f = 4$.

The $y$ values of the intersection points of the Lissajou curve with the $x = y$ line denoted by red circles give the $v_{ref}(\alpha_i)$ ($i = 1, 2, \ldots 2m_f$) values. When $m_f$ is odd (Fig.1.13(a)) the Lissajou curve is symmetrical both to the $x$ and $y$ axis. This symmetry results in zero sum of $v_{ref}(\alpha_i)$ in (1.36). On the other hand, when $m_f$ is even (Fig.1.13(b)) the symmetry is lost in the Lissajou curve. Now the sum of $v_{ref}(\alpha_i)$ is not zero.

**Dependence on $\varphi_c$.**

The function of the generated $V_{DC,x}$ versus $\varphi_c$ is calculated for different $m_f$ and $m_a$ values from (1.36). Figure 1.14 shows the per unit value of the DC component as a function of $\varphi_c = \varphi_{c,a}$ in the three phases for NS-SVM. Given $\varphi_{c,a}$ determines the angle $\varphi_{c,b}$ and $\varphi_{c,c}$. E.g. when $\varphi_{c,a} = 0$ then $\varphi_{c,b} = -2\pi/3$, $\varphi_{c,c} = -4\pi/3$. Here $m_f = 10$ and $m_a = 0.955$ are assumed. According to the numerical results the peak value of $V_{DC,x}$ develops at $\pm \pi/2$ around the zero crossing (Fig.1.14). Similar curves can be obtained for NS-SPWM and NS-THI-PWM as well.
Figure 1.14: $V_{DC,SVM}$ vs. $\varphi_c$, NS-SVM, $m_f = 8$, $m_a = 0.955$

Dependence on $m_a$ and $m_f$

Figure 1.15 shows $\dot{V}_{DC,x}/V_{DC}$ as a function of $m_f$ and $m_a$, respectively ($\dot{V}_{DC,x}$ is the peak value of $V_{DC,x}$ which develops at $\pm \pi/2$). By increasing $m_f$, the DC component calculated from (1.36) is diminishing rapidly. By changing the amplitude modulation ratio $m_a$ upward, the DC component increases.

(a) NS-SPWM  
(b) NS-THI-PWM
(c) NS-SVM

Figure 1.15: $\dot{V}_{DC,x}/V_{DC}$ versus $m_f$ and $m_a$, NS, $\varphi_c = \pm \pi/2$

Dependence on sampling techniques

So far NS sampling technique was assumed, however, as it was told (1.36) holds for RS and OS sampling techniques as well\(^5\). We iterate here the conclusion written right after (1.40): $V_{DC,x} = 0$ for all three PWM provided that $m_f$ is odd and triangular carrier signal is used. On the other hand, when $m_f$ is even number then (1.39) does not hold and $V_{DC,x} \neq 0$. DC component is generated by all three PWM.

Using RS sampling technique and assuming that the samples are taken at the minimum of

\(^5\)NS sampling technique means that the switching angels are determined accurately
the triangular signal (Fig.1.6(a)), DC component are prone to be generated at particular odd \(m_f\) both for THI-PWM and SVM (see the proof soon). Using double Fourier series method, zero DC component is obtained for RS-SPWM (page 132 [17]), for RS-THI-PWM (page 236 [17]) and for SVM (page 283 [17]).

First even \(m_f\) is assumed. In this case the phase difference measured in the period of the fundamental component between two samplings is \(\Delta \phi = \frac{2\pi}{m_f}\). Furthermore, the value of the reference signal is constant during a carrier period and equals to the value of the reference signal at the minimum of the triangular signal. For the better visualization the modulation process for SPWM is depicted in Fig.1.16 for an extreme case when \(m_f = 4\). Due to the sampling process the initial phase shift measured in the period of fundamental component between the carrier and the reference signal is \(\psi_{c,a}^{*} = \frac{\psi_{c,a}}{m_f} + \frac{\pi}{2m_f}\).

![Figure 1.16: Calculation of DC component, RS-SPWM, \(m_f = 4\)](image)

The value of the DC component for RS-PWM will be

\[
V_{DC,x} = \frac{1}{4m_f} \left( \sum_{i=1}^{2m_f} v_{ref,x}(\alpha_i) \right) V_{DC} = \frac{1}{2m_f} \left( \sum_{i=1}^{m_f} v_{ref,x}(-\frac{\psi_{c,a}}{m_f} + (i - 1) \frac{2\pi}{m_f}) \right) V_{DC}
\tag{1.41}
\]

For any \(\phi\) angle \(v_{ref}(\phi) = -v_{ref}(\phi + \pi)\) in SPWM, THI-PWM and SVM (see (1.40)). By using the notations of Fig.1.16 \(v_{ref}(\alpha_1) = -v_{ref}(\alpha_4)\) and \(v_{ref}(\alpha_2) = -v_{ref}(\alpha_6)\) as in both equations the phase shift is \(\pi\). Furthermore \(v_{ref}(\alpha_1) = v_{ref}(\alpha_8) = v_{ref}(-\frac{\psi_{c,a}}{m_f})\), \(v_{ref}(\alpha_4) = v_{ref}(\alpha_8) = v_{ref}(\alpha_4 + \pi)\). Consequently \(\sum_{i=1}^{2m_f} v_{ref}(\alpha_1) = 0\). The number of red points is \(m_f\) in one period of \(v_{ref}\) and the distance between neighbouring points is \(2\pi/m_f\). Half of the points are on the positive period of \(v_{ref}\) and the other half are on the negative period of \(v_{ref}\). All points on the positive period have their pairs with equal absolute value. Conclusion: When \(m_f\) is even and \(RS\) is used then \(V_{DC,x}\) will be zero independently of the value \(m_a\) and \(\psi_{c,a}\) for SPWM, THI-PWM and SVM.

Second, odd \(m_f\) is assumed (Fig.1.17(a)). Unlike even \(m_f\), DC component can appear in the output voltage \(v_{\phi}\) for THI-PWM and SVM. For the better visualization the modulation process for SPWM and THI-PWM are depicted in Fig.1.17(a) and 1.17(b) for \(m_f = 3\), respectively.

By using sinusoidal reference signal, it is evident that the sum of the reference signal at the intersection points for any odd \(m_f\) is

\[
V_{DC,SPWM} = \frac{1}{2m_f} \left( \sum_{i=1}^{m_f} m_a \sin(-\frac{\psi_{c,a}}{m_f} + (i - 1) \frac{2\pi}{m_f}) \right) V_{DC} = 0
\tag{1.42}
\]

because in any symmetrical \(m_f\) phase sinusoidal systems \((m_f \geq 3)\) the sum of the complex phase vectors are zero at any time.
By adding a third harmonic zero sequence component to the sinusoidal waveform, and using (1.36) and (1.41) the DC component can be calculated (see Fig.1.17(b))

\[
V_{DC,THI-PWM} = \frac{1}{2m_f} \left( \sum_{i=1}^{m_f} m_a \sin(-\frac{\varphi_{c,a}^*}{m_f} + (i - 1) \frac{2\pi}{m_f}) + \sum_{i=1}^{m_f} \frac{m_a}{6} \sin(-3 \frac{\varphi_{c,a}^*}{m_f} + 3(i - 1) \frac{2\pi}{m_f}) \right) V_{DC}
\]

(1.43)

It is clear, the first summation term is zero for any odd \(m_f\) (see (1.42)). If the \(m_f\) (or in other words the sampling rate) is 3, the second term will be (see Fig.1.17(b))

\[
V_{DC,THI-PWM} = \frac{1}{6} \left( \sum_{i=1}^{3} \frac{m_a}{6} \sin(-\frac{3 \varphi_{c,a}^*}{m_f}) \right) V_{DC} = \frac{m_a V_{DC}}{12} \sin(-\varphi_{c,a}^*)
\]

(1.44)

So \(V_{DC,THI-PWM}\) varies sinusoidally for \(m_f = 3\) with amplitude \(\frac{m_a V_{DC}}{12}\). When \(m_f\) is odd and \(m_f > 3\) \(V_{DC,THI-PWM}\) will be zero. For example the second term in (1.43) for \(m_f = 5\) is

\[
V_{DC,THI-PWM} = \frac{1}{10} \left( \sum_{i=1}^{5} \frac{m_a}{6} \sin(-3 \frac{\varphi_{c,a}^*}{m_f} + 3(i - 1) \frac{2\pi}{5}) \right) V_{DC}
\]

(1.45)

It is evident that the value of DC component for \(m_f = 5\) is zero (see (1.42)). It holds for higher odd \(m_f\) value, as well.

As the reference signal of SVM according to (1.8) has a 3\(^{\text{rd}}\), 9\(^{\text{th}},15\(^{\text{th}}\) and a 21\(^{\text{th}}\) harmonics the DC component can be calculated from (1.36) similarly as for THI-PWM

\[
V_{DC,SVM} = \frac{m_a}{2m_f} \left( \sum_{i=1}^{m_f} m_a \sin(-\frac{\varphi_{c,a}^*}{m_f} + (i - 1) \frac{2\pi}{m_f}) + \sum_{i=1}^{m_f} \frac{m_a}{6} \sin(-3 \frac{\varphi_{c,a}^*}{m_f} + 3(i - 1) \frac{2\pi}{m_f}) + \sum_{i=1}^{m_f} a_9 \sin(-9 \frac{\varphi_{c,a}^*}{m_f} + 9(i - 1) \frac{2\pi}{m_f}) + \sum_{i=1}^{m_f} a_{15} \sin(-15 \frac{\varphi_{c,a}^*}{m_f} + 15(i - 1) \frac{2\pi}{m_f}) + \sum_{i=1}^{m_f} a_{21} \sin(-21 \frac{\varphi_{c,a}^*}{m_f} + 21(i - 1) \frac{2\pi}{m_f}) \right) V_{DC}
\]

(1.46)

where \(a_k\) is the amplitude of the harmonic component in the reference signal (see (1.8)). By applying the same considerations as for RS-THI-PWM, RS-SVM has a DC component when \(m_f = 3, 5, 7, 9, 15\) and 21
\[ m_f = 3 \quad V_{DC, SVM} = \frac{m_a}{6} \left( \sum_{i=1}^{3} a_3 \sin(-\varphi_{c,a}^{*}) \right) V_{DC} = \frac{m_a a_3 V_{DC}}{2} \sin(-\varphi_{c,a}^{*}) \]

\[ m_f = 5 \quad V_{DC, SVM} = \frac{m_a}{10} \left( \sum_{i=1}^{5} a_{15} \sin(-3\varphi_{c,a}^{*}) \right) V_{DC} = \frac{m_a a_{15} V_{DC}}{2} \sin(-3\varphi_{c,a}^{*}) \]

\[ m_f = 7 \quad V_{DC, SVM} = \frac{m_a}{14} \left( \sum_{i=1}^{7} a_{21} \sin(-3\varphi_{c,a}^{*}) \right) V_{DC} = \frac{m_a a_{21} V_{DC}}{2} \sin(-3\varphi_{c,a}^{*}) \]

\[ m_f = 9 \quad V_{DC, SVM} = \frac{m_a}{18} \left( \sum_{i=1}^{9} a_9 \sin(-\varphi_{c,a}^{*}) \right) V_{DC} = \frac{m_a a_9 V_{DC}}{2} \sin(-\varphi_{c,a}^{*}) \]

\[ m_f = 15 \quad V_{DC, SVM} = \frac{m_a}{30} \left( \sum_{i=1}^{15} a_{15} \sin(-\varphi_{c,a}^{*}) \right) V_{DC} = \frac{m_a a_{15} V_{DC}}{2} \sin(-\varphi_{c,a}^{*}) \]

\[ m_f = 21 \quad V_{DC, SVM} = \frac{m_a}{42} \left( \sum_{i=1}^{21} a_{21} \sin(-\varphi_{c,a}^{*}) \right) V_{DC} = \frac{m_a a_{21} V_{DC}}{2} \sin(-\varphi_{c,a}^{*}) \]  \hfill (1.47)

**OS sampling technique** Assuming for the number of samples \( n = 2 \) (Doublesampling), furthermore assuming that the samples are taken at all peaks of the triangular signal, the phase difference is \( \Delta \phi = \frac{2\pi}{2m_f} = \frac{\pi}{m_f} \) measured in the period of the fundamental component of \( v_{ref} \) between two samplings. Furthermore, the values of the reference signal approximated by the OS technique at the crossing points with the triangular signal are the value of the reference signal \( v_{ref,x} \) at the peaks of the triangular signal. Thus, the value of the DC component for Doublesampled PWM will be

\[ V_{DC,x} = \frac{1}{4m_f} \left( \sum_{i=1}^{2m_f} v_{ref,x} \left( -\frac{\varphi_{c,a}^{*}}{m_f} + (i - 1) \frac{\pi}{m_f} \right) \right) V_{DC} \]  \hfill (1.48)

As it was mentioned previously the reference signals \( v_{ref,x} \) of SPWM, THI-PWM and SVM are met (1.40). As the number of samples is even both for odd and even \( m_f \) the half of the sampled values of the reference signal are on the positive half period of \( v_{ref} \) and the other half are on the negative half period of \( v_{ref} \) similarly as in the case of RS when \( m_f \) is even.

**Conclusion:** \( V_{DC,x} \) will be zero for DS in all three PWM techniques independently of the value of \( m_f, m_a \) and \( \varphi_{c,a} \).

Next the assumption is OS, \( n > 2 \) and even \( m_f \). By increasing the number of samples OS approaches the performance of NS. It was shown previously when \( m_f \) is even and NS is used then DC component is generated. The numerical calculations based on (1.36) showed that DC component is generated for OS, when the number of samples is \( n = 4 \) and \( m_f \) is even, however, the dependence of \( V_{DC,x} \) on \( \varphi_{c,a} \) is different from the curves seen in Fig.1.14. When \( n \geq 8 \) a somewhat similar curve as shown in Fig.1.14 is obtained. Figure 1.18 presents the numerically calculated \( V_{DC, SVM} \) for \( n = 4 \) and \( n = 8 \). For better comparison the curve of NS-SVM is also depicted.

Finally the assumption is OS, \( n > 2 \) and odd \( m_f \). Based on the numerical calculation the result is that no DC component is generated.

Figure 1.19(a) summarizes the DC component generation in the output phase voltage \( v_{a0} \) for single phase. The green box refers to there is no DC component generated, while red box denotes that DC component can be generated for particular \( m_f \) values.

**Three-phase system**

The phase voltage \( v_{ph, h} \) \((h = a, b, c)\) measured between the output terminal of the VSC and the star point of the load is not the same as the voltage between the output terminal and the
middle point of the DC link seen in Fig.1.1(b) for phase $a$ and denoted by $v_{a0}$. The relation among $v_{ph,h}$ and $v_{h0}$ is

$$v_{ph,h} = v_{h0} - \left(\frac{v_{a0} + v_{b0} + v_{c0}}{3}\right)$$

(1.49)

DC voltage cannot be found in $v_{ph,h}$, if no $V_{DC,x}$ develops in $v_{h0}$.

The next discussion is organized according to $m_f$ and the sampling techniques. The results are summarized in Fig.1.19(b).

Assuming even $m_f$, $v_{ph,h}$ cannot contain DC component when RS and DS is applied and $v_{ph,h}$ may contain it when OS and NS is used.

Assuming odd $m_f$, we have to divide it into two cases.

a. When $m_f$ is multiple of 3, the phase angle between the carrier signal and the reference signal is the same in all the three phases ($\varphi_{c,a} = \varphi_{c,b} = \varphi_{c,c}$) resulting in the same DC component in voltages $v_{a0}$, $v_{b0}$ and $v_{c0}$ (Fig.1.14). They are cancelled in each stator phase voltage of the motor in $v_{ph,h}$. There is no DC voltage component in $v_{ph,h}$, no adverse effect is done in the operation of the motor.

b. When $m_f$ is not multiple of 3 due to the three-phase symmetry $\varphi_{c,a} = \varphi_{c,b} + 2\pi/3 = \varphi_{c,c} + 4\pi/3$. By applying NS sampling technique each phase has its own different $V_{DC,x}$ component but due to the three-phase symmetry $V_{DC,x,a} + V_{DC,x,b} + V_{DC,x,c} = 0$ therefore $V_{DC,x,h}$ are not cancelled in the phase voltages (see (1.49)).

It was shown by using RS-SVM, DC component can be generated in $v_{a0}$ for $m_f = 5$ and $m_f = 7$, however, no adverse effect is done as $\sin(-3\varphi_{c,a}) = \sin(-3\varphi_{c,b}) = \sin(-3\varphi_{c,c})$ and the DC voltage components are cancelled in each stator phase voltage of the motor.

Similar conclusions can be done for OS as have been made for NS if $n$ is large.
Figure 1.19(b) summarizes the DC component generation for three-phase system. It can be concluded that DC component is generated only for NS and OS \((n \geq 4)\) when \(m_f\) is even and not multiple of 3.

**Effect of dead-time**

So far it was assumed that we have ideal switches in the converter. They turn on or off exactly at the intersection points of \(v_{ref}\) and \(v_{car}\). In reality, however, the finite switching time of the switches may cause a short circuit of the DC link. Thus, it is essential to insert a time delay (dead or blanking time) \(T_d\) in their control signals in order to avoid the conduction overlap of the switches [46]. Otherwise I still assume ideal switches.

![Figure 1.20: Effect of dead-time](image)

Figure 1.20 shows the theoretical switching signals \(s_{a+}\) and \(s_{a-}\) controlling the upper and the lower switches in phase \(a\). By neglecting the reverse recovery storage time of the switches the output voltage deviates from the ideal theoretic PWM waveform by the amount of shaded areas in Fig. 1.20(a) depending on the direction of the current flow.

Assuming positive DC voltage component in the output phase voltage \(a\) the resulting DC current will be positive as well meaning that in larger part of the fundamental period the current flows toward the load \((i > 0, \text{Fig.1.20(b)})\). The dead-time \(T_d\) will shrink the duration of the positive and stretch that of the negative output voltage resulting in the reduction of the DC voltage component in the output phase voltage. Similar phenomenon occurs when the DC voltage component and the resulted DC current is negative.

### 1.7 Subharmonics

The output voltage signal \(v_{a0}\) contains a single fundamental component with frequency \(f_1\) and the groups of sideband harmonics around the carrier and multiple carrier harmonics. The frequencies of the sideband harmonics grouped around the multiples of the carrier frequency \(f_c\) is

\[
f_{harm} = \pm (m \cdot m_f \pm n) f_1 > 0 \tag{1.50}
\]

where \(m = 1, 2, \ldots\) and \(n = 1, 2, \ldots\). One constraint is that when \(m\) is odd then \(n = 2, 4, \ldots\) and when \(m\) is even then \(n = 1, 3, \ldots\). The \(V_{nn}\) amplitude of the harmonics for SPWM, THIPWM and SVM as well can be calculated using the double Fourier series method presented previously [17].

\(^6\)This statement is not valid for RS.
How the generation of subharmonics can be understood? There are two ways to explain the development of subharmonics. The first explanation states that if the frequency ratio $m_f$ is not integer subharmonics are generated by the lower sideband harmonics of the first carrier harmonic group ($m = 1$) intruding below the fundamental. For example in the vicinity of $m_f = 8$ the fourth lower sideband harmonics ($n = 8$) will be the subharmonic voltage components. The subharmonic frequencies can be calculated from (1.50) ($m = 1, n = m_{f,int}$) as $f_{sub} = \lvert f_c / f_1 - m_{f,int} \rvert \cdot f_1$, where $m_{f,int}$ is the closest even integer frequency ratio belonging to $f_1$. For example $f_{sub} = |12 \text{ kHz}/1499.5 \text{ Hz} - 8| \cdot 1499.5 = 4 \text{ Hz}$. The second explanation does not need any sophisticated theory and it is applicable when $m_f$ is not integer but rational number e.g. $m_f = 7.02$. The reason is that now $m_f$ can be written always as

$$m_f = N/D = N_s/D_s$$

(1.51)

where $N, D, N_s$ and $D_s$ are integers. $N_s/D_s$ is the simplest form, that is, any common factors in the ratio have been removed (e.g. $m_f = 7.02 = 702/100 = 351/50$). Taking into account that $m_f = f_c / f_1 = T_1 / T_c = N_s / D_s$, it results in $D_s T_1 = N_s T_c$. It can be concluded that in $D_s$ number of reference period the number of carrier period is exactly $N_s$. Consequently now the subharmonic period is $T_{sub} = D_s$ and it is integer assuming $T_1 = 1 \text{ pu}$. Being both $N_s$ and $D_s$ integers, the voltage $v_{a0}$ is in frequency-locked state. When $m_f$ is irrational, we have quasi-periodic state.

Table 1.1 shows the calculated\(^7\) amplitude of the subharmonic voltage components ($m = 1$) in $\text{pu}$ in the vicinity of different integer $m_f$ values for each Natural Sampled PWM techniques when $m_a = 0.955$ and the base value is $V_{ref}$ [17] (for $m_a = 0.955 \cdot V_{ref} = 0.4775 V_{DC}$ see (1.3)).

<table>
<thead>
<tr>
<th>$n$</th>
<th>$m_f \approx$</th>
<th>$V_{sub,SPWM}$</th>
<th>$V_{sub,THIPWM}$</th>
<th>$V_{sub,SVM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>0.0157$\text{pu}$</td>
<td>0.0898$\text{pu}$</td>
<td>0.133$\text{pu}$</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>3.03$\cdot 10^{-4} \text{pu}$</td>
<td>0.0023$\text{pu}$</td>
<td>0.0155$\text{pu}$</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>3.10$\cdot 10^{-6} \text{pu}$</td>
<td>1.46$\cdot 10^{-4} \text{pu}$</td>
<td>0.0115$\text{pu}$</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>$\approx 0$</td>
<td>2.45$\cdot 10^{-6} \text{pu}$</td>
<td>0.0048$\text{pu}$</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
<td>0.004$\text{pu}$</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>$\approx 0$</td>
<td>$\approx 0$</td>
<td>0.0023$\text{pu}$</td>
</tr>
</tbody>
</table>

It is generally considered that the impact of $\hat{V}_{sub,x}$ is negligible, because it is too small (Table 1.1). Even though $\hat{V}_{sub}$ is small indeed but being $f_{sub}$ very small as well, high level of subharmonic flux components can be generated.

Paper [47] deals with the influence of voltage subharmonics on temperature rise distribution in an induction machine with standard rated frequency $f_1 = 50 \text{ Hz}$.

### 1.7.1 Subharmonic flux space vector

To investigate the existence and the value of subharmonic components in $v_{a0}$, an efficient and especially convenient approach is to study the time integral of space vector $v_k$, because its fundamental and harmonic components are suppressed compared to the subharmonics due to their much higher frequencies. Its dimension is $V_{sec}$, thus it is called flux space vector and denoted by $\Psi$. Its dimension is $V_{sec}$, thus it is called flux space vector and denoted by $\Psi$.\(^7\)To calculate the amplitude of the subharmonic components the following equations derived in [17] were used: (3.38) on page 118 (NS-SPWM), (5.43) on page 233 (NS-THI-PWM) and (6.63) on page 285 (NS-SVM).
Zero stator resistance \( R_s = 0 \)

If the stator resistance \( R_s \) could be neglected, \( \Psi \) were the stator flux linkage space vector \( \Psi_s \).

Figure 1.21 shows the subharmonic flux \( \Psi_{sub} = \Psi_1 \cdot e^{j\omega_{sub}t} \) and the fundamental component of flux \( \Psi_1 = \Psi_1 \cdot e^{j\omega_1 t} \) where \( \omega_1 \) and \( \omega_{sub} \) are the fundamental and the subharmonic angular frequency, respectively. Furthermore \( \omega_1 = 2\pi f_1 \gg \omega_{sub} = 2\pi f_{sub} \).

In Fig.1.21(a) \( \Psi_1 > \Psi_{sub} \) while in Fig.1.21(b) \( \Psi_1 < \Psi_{sub} \). As long as \( \Psi_{sub} \) completes one full turn, \( \Psi_1 \) makes large number of turns. The absolute value of the subharmonic flux is equal to the half of the thickness of the ring in Fig.1.21(a) and it equals the mean radius of the inner and outer circle in Fig.1.21(b) (see later the simulation results in Fig.1.28 and 1.29).

![Diagram](image)

Figure 1.21: Fundamental and subharmonic component of the flux space vector \( \Psi \)

As it was shown in Table 1.1 the subharmonic component of the output voltage \( v_{sd} \) of VSC \( v_{sub} = V_{sub} \cdot e^{j\omega_{sub}t} \) is much smaller than its fundamental component \( v_1 = V_1 \cdot e^{j\omega_1 t} \), that is \( V_{sub}/V_1 \) is very small. But after calculating the respective flux components the ratio of \( \Psi_{sub}/\Psi_1 \) is greatly enhanced due to the integration of time function \( V_{sub} \) and \( V_1 \). The integration equals the multiplication of the voltage space vector by \( 1/j\omega \):

\[
\Psi_{sub} = -j(V_{sub}/\omega_{sub}) \cdot e^{j\omega_{sub}t} \\
\Psi_1 = -j(V_1/\omega_1) \cdot e^{j\omega_1 t}
\]

The ratio of the peak value of flux components is

\[
\frac{\Psi_{sub}}{\Psi_1} = \frac{V_{sub}/\omega_1}{V_1/\omega_{sub}}
\]

If \( \omega_{sub} \) is small enough \( \Psi_{sub} > \Psi_1 \) can be the result. In ultrahigh speed drives, the frequency of the significant subharmonic component is 2 to 3 orders of magnitude lower as compared to the fundamental component, thus a subharmonic component with an amplitude of \( 10^{-3} \) pu magnitude results in a subharmonic flux comparable to the fundamental component.

Nonzero Stator resistance \( R_s \neq 0 \)

The ultrahigh speed induction machines has a relative low stator resistance, the accurate amplitude of the stator flux \( \hat{\Psi}_{s,sub} \) taking into account \( R_s \) too is significantly smaller than that of \( \hat{\Psi}_{sub} \) as it will be shown later on in the simulation and laboratory results. The reasons are as follows:

- the small subharmonic frequency \( f_{sub} \) of the subharmonic voltage components results in very small magnetizing and leakage reactances at \( f_{sub} \) accentuating \( R_s \)
- the angular frequency of the subharmonic rotor currents (assuming USIM has one pole-pair) is \( \omega_{r,sub} = \omega - \omega_{sub} \). The slip belonging to the subharmonic voltage components is \( s_{sub} = \omega_{r,sub}/\omega_{sub} \), which is a high value for small \( f_{sub} \), resulting that \( R_r/s_{sub} \) is almost negligible.

\( ^8 \)the direction of rotation of \( \Psi_{sub} \) can be the opposite as well
They cause that the per-phase equivalent impedance of the USIM at the subharmonic frequency is very small and the value of stator resistance, which is also small value, dominates in it. The $V_{\text{sub},\Psi}$ flux producing voltage component is much smaller than $V_{\text{sub}}$ (Fig. 1.22). Even the much smaller subharmonic flux $\Psi_{s,\text{sub}}$ can be dangerous in the operation of the USIM. Furthermore the voltage $V_{\text{sub}}$ can result in high subharmonic stator current $I_{s,\text{sub}}$.

\[ V_{\text{sub},\Psi} = \omega_{s,\text{sub}} I_{s,\text{sub}} X_r = \omega_{s,\text{sub}} I_{l_r} X_{\text{sub},s}. \]

\[ R_s \gg X_{\text{sub},s} \Rightarrow X_{\text{sub},s} = \omega_{s,\text{sub}} (I_{l_s} + I_{l_r}) \]

Figure 1.22: Subharmonic equivalent circuit of USIM

### 1.7.2 Additional losses due to subharmonics

The subharmonic current components with high amplitude results in additional loss on the stator resistance

\[ P_{s,\text{sub}} = \frac{3}{2} I_{s,\text{sub}}^2 R_s \quad (1.55) \]

Furthermore the subharmonic flux components results additional rotor copper loss. To calculate the rotor copper losses the operation of the machine is assumed to be at the rated working point $P_n$ on the rated torque-speed characteristics (blue curve) of the machine (Fig. 1.23). For the sake of illustration, the slip values in Fig. 1.23 are exaggerated.

The refined form of the Kloss formula can be used to determine the working points on the torque-slip characteristics that were used to calculate losses. In general the torque $\tau$ form the Kloss formula

\[ \tau = \frac{2\tau_0 s_0 s_b (1 + \epsilon)}{s^2 + s_b^2 + 2\epsilon s_0 s_b} \quad (1.56) \]

where $s$ is the slip, $\tau_0$ and $s_0$ is the breaking torque and breaking slip, respectively, and

\[ \tau_0 = \frac{3 V_{\text{ph}}^2}{2\Omega_1 (1 + \sigma)^2 (R_s + \sqrt{R_s^2 + X_r^2})}, \quad \sigma = \frac{X_{l_s}}{X_m}, \quad \epsilon = \frac{R_s}{\sqrt{R_s^2 + X_r^2}} \]

$V_{\text{ph}}$ is the rms phase voltage. The other relations are in Fig. 1.23. Equation (1.56) was used to calculate $\Omega(\tau)$ belonging to the fundamental (blue curve) and to the subharmonic (red curve) voltage.

The torque generated by the subharmonic component is $\tau_{s,\text{sub}}$ in the working point ($WP_{s,\text{sub}}$) at the rated speed $\Omega_n$. The speed difference $\Delta \Omega_{s,\text{sub}} = \Omega_n - \Omega_{1,\text{sub}}$ is the same as the angular frequency of the rotor current (as $p = 1$) generated by the subharmonics.

The power absorbed by the machine through the shaft is $P_{s,\text{sub}} = \tau_{s,\text{sub}} \Delta \Omega_{s,\text{sub}}$.

It is assumed that this power is completely turned into copper loss in the rotor. The rated rotor copper loss resulting from the fundamental component is $P_r = \tau_1 \Omega_1 s_n$ and the ratio of the two copper losses is

\[ \lambda = \frac{P_{s,\text{sub}}}{P_r} \quad (1.57) \]

Later on $\lambda$ will be calculated for particular cases to demonstrate the effect of the subharmonic voltage component.
1.8 Simulation results

A complete USIM model together with PWM controlled VSC in Matlab/Simulink environment was implemented. The USIM was modelled by using its direct- and quadrature axis (dq) representation. During the simulation two USIMs, Motor A with rated speed $n = 90$ krpm and Motor B with rated speed $n = 18$ krpm (with field-weakening up to 24 krpm), were investigated. The rated data and main parameters of the machines can be found in Table 1.2\(^9\). Later on in some cases pu system is used in the figures, where the reference values are the rated data of the machines.

During the simulation the VSC keeps the $V/f$ ratio constant by changing the amplitude modulation index depending on the actual reference frequency.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Motor A</th>
<th>Motor B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_n$</td>
<td>Nominal power</td>
<td>4.5kW</td>
<td>3 kW</td>
</tr>
<tr>
<td>$n_n$</td>
<td>Nominal rated speed</td>
<td>90 krpm</td>
<td>18 krpm</td>
</tr>
<tr>
<td>$f_{1n}$</td>
<td>Rated frequency</td>
<td>1500 Hz</td>
<td>300 Hz</td>
</tr>
<tr>
<td>$s_n$</td>
<td>Rated slip</td>
<td>0.0064</td>
<td>0.017</td>
</tr>
<tr>
<td>$V_{LL,\text{rms}}$</td>
<td>Nominal voltage</td>
<td>380 V</td>
<td>380 V</td>
</tr>
<tr>
<td>$I_{\text{ph,\text{rms}}}$</td>
<td>Nominal current</td>
<td>9 A</td>
<td>7.7 A</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>Rated torque</td>
<td>0.48 Nm</td>
<td>1.59 Nm</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Stator resist.</td>
<td>0.21 Ω</td>
<td>1.125 Ω</td>
</tr>
<tr>
<td>$R_r$</td>
<td>Rotor resist.</td>
<td>0.16 Ω</td>
<td>0.85 Ω</td>
</tr>
<tr>
<td>$X_{ls}$</td>
<td>Stator leak. react.</td>
<td>2.93 Ω</td>
<td>4.71 Ω</td>
</tr>
<tr>
<td>$X_{lr}$</td>
<td>Rotor leak. react.</td>
<td>2.93 Ω</td>
<td>2.63 Ω</td>
</tr>
<tr>
<td>$X_m$</td>
<td>Magnetizing reactance</td>
<td>44.59 Ω</td>
<td>84.82 Ω</td>
</tr>
<tr>
<td>$J$</td>
<td>Inertia</td>
<td>3.8 kgcm$^2$</td>
<td>5 kgcm$^2$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Efficiency</td>
<td>95 %</td>
<td>75 %</td>
</tr>
<tr>
<td>$V_{\text{DC}}$</td>
<td>DC link voltage</td>
<td>650 V</td>
<td>540 V</td>
</tr>
</tbody>
</table>

\(^9\)The parameters were given by the manufacturers and all reactances are at rated frequency
Motor A

First integer $m_f$ is assumed. The effect of the blanking time is neglected ($T_d = 0\mu s$) and NS is assumed. The Motor A is investigated by applying different $f_c$ carrier frequencies to obtain different $m_f$ values ($f_c = 12\text{kHz}$ for $m_f = 8$, $f_c = 15\text{kHz}$ for $m_f = 10$, $f_c = 21\text{kHz}$ for $m_f = 14$, $f_c = 24\text{kHz}$ for $m_f = 16$ and $f_c = 30\text{kHz}$ for $m_f = 20$).

The motor is loaded with its rated torque. Later on pu system is used in the figures, where $\Psi_s = \frac{V_{ph}}{2\pi f_1} = 0.0328$ Vsec $= 1\text{ pu}$ and $\sqrt{2}I_{n,\text{rms}} = 13.15\text{ A} = 1\text{ pu}$.

Figure 1.24 shows the time function of the stator phase currents when the carrier frequency is $f_c = 12$ kHz, thus $m_f = 8$. As it can be seen DC currents with considerable magnitude are generated in phase $b$ and $c$ of the USIM by applying THI-PWM and SVM. There is no DC component in phase $a$ as $\varphi_{c,a} = 0$ (see Fig.1.14). By applying SPWM, the value of the DC voltage and current is negligible.

According to (1.36) THI-PWM generates DC voltage in phase $b$ and $c$ with opposite sign with value $V_{DC,\text{THI-PWM}} = 0.001V_{DC} = 0.65V$ resulting in $I_{DC,\text{THI-PWM},b} = -I_{DC,\text{THI-PWM},c} = 3.09A \approx 0.23\text{pu}$. When SVM is used the DC voltage component is $V_{DC,\text{SVM}} = 0.00639V_{DC} = 4.15V$ resulting in $I_{DC,\text{SVM},b} = -I_{DC,\text{SVM},c} = 19.76A \approx 1.5\text{pu}$. The same can be read from Fig.1.24(b) and Fig.1.24(c) verifying the expression (1.36).

The high level DC current components result in additional loss in the stator resistance ($P_{s,DC} = R_s(I_{DC,\text{SVM},a}^2 + I_{DC,\text{SVM},b}^2 + I_{DC,\text{SVM},c}^2)$). Based on the numerical values just given in connection of Fig.1.24(c), $P_{s,DC} = 0.21(2 \cdot 19.76^2) = 164\text{ W}$. The total loss in rated operation is $(1 - \eta)P_n = 0.05 \cdot 4.5\text{ kW} = 225\text{W}$. The loss generated by the DC current resulting in serious overheating of USIM.

As it was explained previously the dead-time reduces the DC current in the phase windings. Figure 1.25 shows the generated DC current for NS-SVM in phase $b$ assuming $\varphi_{c,a} = 0$ for different $T_d$ dead-time and $m_f$ values obtained by simulation. The DC current rapidly decreases by increasing the dead-time, but its value can still be considerable by applying NS-SVM.

Figure 1.26 shows the stator currents for RS-SVM and OS-SVM. As it was explained previously no DC component develops for RS (Fig.1.26(a)) and OS, when the the number of samplings is $n = 2$ (Fig.1.26(b)). When the number of samples is increased further the difference between the OS and NS diminishes and almost the same results can be obtained as previously. Comparing the results of Fig.1.26(c) ($n = 8$)to Fig.1.24(c) it can be concluded that
similar results are obtained, however the value of the DC current in phase $b$ and $c$ is slightly less for OS ($I_{DC,SVM,b} = -I_{DC,SVM,c} = V_{DC,SVM,b}/R_s = 17.2A = 1.3pu$).

Second, turning to non-integer frequency ratio, unlike integer $m_f$, subharmonics with considerable amplitudes are generated near the even integer frequency ratios using NS and OS, excluding the $m_f$ values of multiple of 3. The amplitudes of the subharmonic voltages are negligible near odd $m_f$ and when RS or Doublesampled PWM is applied.

Figures 1.27-1.29 show the trace of space vector $\Psi$ obtained by integrating the output voltage space vector $v_k$, $\Psi_s$ and $I_s$ when $f_1 = 1499.5$ Hz ($m_f = 8.00267$) for the three different NS PWM techniques. In all three cases $f_{sub} = 4$ Hz.

The amplitude of the subharmonic voltage component near $m_f = 8$ is almost zero for SPWM (see Table 1.1) resulting in negligible subharmonic flux and current (Fig.1.27). Subharmonic voltage with amplitude $\hat{V}_{sub} = 0.715V$ and $\hat{V}_{sub} = 4.81V$ is generated by applying THI-PWM and SVM, respectively. By neglecting the stator resistance, these subharmonic voltage components result in flux components with amplitude $\hat{\Psi}_{sub} = 0.87\,\text{pu}$ and $\hat{\Psi}_{sub} = 5.83\,\text{pu}$, respectively (Fig.a). Taking into account $R_s$ the amplitude of the stator flux vector $\hat{\Psi}_{s,sub}$ is considerably smaller than that of $\Psi_{sub}$ clearly indicating that the stator resistance is by far not negligible at $f_{sub}$ (Fig.1.28 and 1.29). Based on Fig.1.22 and on the remarks in the previous section the per-phase equivalent impedance of the USIM for the subharmonic component is
$Z_{sub} = 0.2104 + j 0.0151 \Omega \ (\omega_{sub} = 2 \pi f_{sub} = 25.13 \text{ rad/s})$. As it was mentioned previously the stator resistance dominates in the equivalent impedance. Theoretically the subharmonic voltage components for THI-PWM and SVM result in subharmonic current with amplitude $I_{s,sub} = 0.715/|Z_{sub}| = 3.38 \text{ A} = 0.25$ and $I_{s,sub} = 4.8/|Z_{sub}| = 22.38 \text{ A} = 1.7 \text{ pu}$, respectively. The subharmonic flux producing voltage component $V_{sub,\Phi} = \sqrt{V_{sub}^2 - (I_{s,sub} R_s)^2} = 0.086 \text{ V}$ and $V_{sub,\Psi} = \sqrt{V_{sub}^2 - (I_{s,sub} R_s)^2} = 0.3451 \text{ V}$. Theoretically they result in $\psi_{s,sub} = V_{sub,\Phi} / (2 \pi f_{sub}) = 0.00342 \text{ Vsec} = 0.01373 \text{ Vsec} = 0.41 \text{ pu}$. The same values can be read from Fig.1.28 and Fig.1.29.

It should be noted that even the much smaller $\psi_{s,sub}$ than $\psi_{sub}$ is also very dangerous for the operation of the USIM. Suprisingly, the subharmonic voltage component with amplitude only 0.001 pu can results in flux component with comparable magnitude to the fundamental one.

Figure 1.27: Trajectory of $\Psi$, $\Psi_s$ and $I_s$, NS-SPWM, $f_1 = 1499.5$, $m_f = 8.00267$, $m_a = 0.955$. Simulation

Figure 1.28: Trajectory of $\Psi$, $\Psi_s$ and $I_s$, NS-THI-PWM, $f_1 = 1499.5$, $m_f = 8.00267$, $m_a = 0.955$. Simulation

Figure 1.30 shows for NS-THI-PWM and NS-SVM the amplitude of the subharmonic flux and current components versus $\Delta m_f$ in the vicinity of $m_f = 8, 10, 14, 16$ and 20 when the fundamental frequency is varied around its rated value ($f_{1n} = 1500 \text{ Hz}$).

By applying SVM, even at very high carrier frequency ($f_c = 30 \text{ kHz}$, $m_f = 20$), the amplitude of the subharmonic flux and currents is considerable when the USIM is operated near its rated frequency (Fig.1.30). THI-PWM and SPWM generates considerably lower subharmonic flux and current components, when $m_f > 8$.

The subharmonic current and flux components results in additional loss in the stator resistance and copper loss, respectively. Based on the numerical values just given for NS-SVM in connection of Fig.1.29 $P_{s,sub} = 157 \text{ W}$ (see (1.55)). The subharmonic torque at rated speed
Figure 1.29: Trajectory of $\Psi$, $\Psi_s$ and $I_s$, NS-SVM, $f_1 = 1499.5$, $m_f = 8.00267$, $m_a = 0.955$.

Simulation

Figure 1.30: Amplitude of subharmonic flux $\hat{\Psi}_{s,\text{sub}}$ and that of current $\hat{I}_{s,\text{sub}}$ for NS-THI-PWM (a) and NS-SVM (b). Simulation

caused by the subharmonic voltage component is $\tau_{\text{sub}} = 0.011\text{Nm}$ resulting in rotor copper loss $P_{r_{\text{sub}}} = 107\text{ W}$ (see (1.35)-(1.36)), which is approximately three times higher than the rated copper loss ($\lambda = 3.1$, (1.57)). Both can lead to the overheating of the USIM.

Similarly to the generation of DC component, the dead-time reduces the amplitude of the subharmonic flux and current components. Figure 1.31 shows the trace of $\Psi$, $\Psi_s$ and $I_s$ for SVM when the dead time is $T_d = 1\mu\text{s}$ assuming the newest high-performance switches, like MOSFETs. Comparing Fig.1.29 with 1.31 it can be seen that the amplitude both of the flux and current subharmonic are reduced. Now the amplitude of the subharmonic flux and current components are $\hat{\Psi}_{s,\text{sub}} = 0.128\text{ pu}$ and $\hat{I}_{s,\text{sub}} = 0.51\text{ pu}$. Similar values can be obtained as presented in Fig.1.30 by a proper dead-time compensation method [46].

Figure 1.32 shows the path of $\Psi_s$ for RS- and OS-SVM. Subharmonic stator flux component with negligible amplitude is generated for RS (Fig.1.32(a)) and OS, when the the number of samplings is $n = 2$ (Fig.1.32(b)). When the number of samples is increased further the
difference between the OS and NS diminishes and almost the same results can be obtained as previously. Comparing the results of Fig.1.32(c) ($n = 8$) to Fig.1.29(b) it can be concluded that practically the same results are obtained.

The effect of the blanking time was taken into consideration ($T_d = 3 \mu s$) and NS-SVM is assumed. The carrier frequency was selected to be $f_c = 2$ kHz. The load torque is $0.9$ Nm. Later on pu system is used in some cases, where $\Psi_s = \frac{V_{ph}}{2\pi f_1} = 0.16$ Vsec $= 1$ pu.

In Figure 1.33 the time function of the simulated stator phase current in phase $b$ can be seen, when $f_1 = 250$ Hz ($m_f = 8$) and $m_a = 0.955$. The theoretical magnitude of the DC voltage component (when $T_d = 0 \mu s$) is according to (1.36) $V_{DC,SVM} = 0.0064 V_{DC} = 3.5V$, which results in DC current $I_{DC} = V_{DC,SVM}/R_s = 3.1$ A. In Fig.1.33 the DC current component with a magnitude of $I_{DC} \approx 2.8$ A is clearly visible. The main reason for the small difference is the non-zero blanking time.

In Figure 1.34 and 1.35 the time function of the simulated stator phase current and the space vector of stator flux $\Psi_s$ can be seen when $f_1 = 249.8$Hz ($m_f \approx 8, m_a = 0.957$) and $f_1 = 199.9$Hz ($m_f \approx 10, m_a = 0.766$), respectively. The frequency of the generated subharmonic components are $f_{sub} = 1.6$Hz and $f_{sub} = 1$Hz. For $m_f \approx 8$ the theoretic amplitude of the subharmonic voltage component, when the blanking time is zero, is $\hat{V}_{sub} = 4V$ (see Table 1.1). The per-phase equivalent impedance at $f_{sub}$ is $Z_{sub} = 1.127+j0.042\Omega$ (see Fig.1.22 and Table 1.2). It results in
subharmonic current component with amplitude $\hat{I}_{\text{sub}} = \hat{V}_{\text{sub}}/Z_{\text{sub}} = 3.54$ A. The flux producing voltage component is only $\hat{V}_{\text{sub,}\psi} = 0.1$ V, but it generates a subharmonic flux component with amplitude $\hat{\Psi}_{s,\text{sub}} = \hat{V}_{\text{sub,}\psi}/(2\pi f_{\text{sub}}) = 0.01$ V sec = 0.062 pu. Theoretically when $m_f \approx 10$ $\hat{V}_{\text{sub}} = 2.14$V. The per-phase equivalent impedance at $f_{\text{sub}}$ is $Z_{\text{sub}} = 1.129 + j0.024\Omega$. It results in subharmonic current component with amplitude $\hat{I}_{\text{sub}} = 1.9$ A. The flux producing voltage component is $\hat{V}_{\text{sub,}\psi} = 0.046$ V and it generates a subharmonic flux component with amplitude $\hat{\Psi}_{s,\text{sub}} = \hat{V}_{\text{sub,}\psi}/(2\pi f_{\text{sub}}) = 0.0073$ V sec = 0.046 pu.

Similar values can be read from Fig. 1.34 and 1.35. The small differences are caused by the non-zero blanking time.

Figure 1.34: Time function of phase current and the measured trajectory of $\Psi_s$, NS-SVM, $f_1 = 249.8$ Hz, ($m_f \approx 8$). Simulation

Figure 1.35: Time function of phase current and the measured trajectory of $\Psi_s$, NS-SVM, $f_1 = 199.9$ Hz, ($m_f \approx 10$). Simulation
1.9 Laboratory tests

During the laboratory tests two converters, Conv A and Conv B are used to investigate the effect of different PWM strategies. In Conv A the NS-SVM technique while in Conv B both the NS-SPWM, NS-THI-PWM and NS-SVM were implemented by me according to method introduced in 1.4.4. The main parameters of the converters can be found in Table 1.3.

Table 1.3: Rated parameters of Converters

<table>
<thead>
<tr>
<th></th>
<th>Conv A</th>
<th>Conv B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal power</td>
<td>5.5kW</td>
<td>1 kW</td>
</tr>
<tr>
<td>Maximum output frequency</td>
<td>1050 Hz</td>
<td>1000 Hz</td>
</tr>
<tr>
<td>Maximum carrier frequency</td>
<td>20 kHz</td>
<td>20 kHz</td>
</tr>
<tr>
<td>Dead time</td>
<td>3 µs (fixed)</td>
<td>≥ 2 µs (variable)</td>
</tr>
<tr>
<td>DC link voltage</td>
<td>540 V</td>
<td>310 V</td>
</tr>
<tr>
<td>CPU</td>
<td>TMS320F2808</td>
<td>dsPIC3FJ32MC204</td>
</tr>
<tr>
<td>PWM</td>
<td>NS-SVM</td>
<td>NS-SPWM, NS-THI-PWM, NS-SVM</td>
</tr>
</tbody>
</table>

R-L Load

Based on the assumptions described previously (in point 1.7.1 assuming \( R_s \neq 0 \)) to study the effect both of the DC and subharmonic components, the USIM can be modelled by a star-connected three-phase \( R-L \) circuit.

The \( R-L \) circuit has the following parameters: \( R = 1.7\Omega, L = 87\text{mH} \) (one single device had the ohmic resistance of the inductance as well). The measurements were carried out using Conv B (\( V_{DC} = 315 \text{V}, T_d = 2\mu s \)). The rated fundamental frequency was \( f_{in} = 250 \text{ Hz} \). For the better visualization simulation results are also plotted in some cases, where the dead time was also taken into consideration.

The operation with inductive load is investigated by applying different \( f_c \) carrier frequencies to obtain different \( m_f \) values (\( f_c = 2\text{kHz} \) for \( m_f = 8 \), \( f_c = 2.5\text{kHz} \) for \( m_f = 10 \), \( f_c = 3.5\text{kHz} \) for \( m_f = 14 \)).

**DC component:** Figure 1.36 and 1.37 show the simulated and the measured time functions of the phase currents when NS is assumed and the carrier frequency is \( f_c = 2 \text{kHz} \), thus \( m_f = 8 \), respectively. The measured and the simulated results are practically the same.

DC currents with considerable magnitude are generated in phase \( b \) and \( c \) by applying THI-PWM and SVM. There is no DC component in phase \( a \) as \( \varphi_{0,a} = 0 \) (see Fig.1.36). By applying SPWM, the value of the DC voltage and current is negligible.

THI-PWM generates DC voltage in phase \( b \) and \( c \) with opposite sign with value according to (1.36) (\( T_d = 0\mu s \)) \( V_{DC,THI-PWM} = 0.001V_{DC} = 0.315\text{V} \) resulting in \( I_{DC,THI-PWM,b} = -I_{DC,THI-PWM,c} = 0.1853\text{A} \). Similar value can be read from Fig.1.36(b) and 1.37(b), respectively.

When SVM is used the theoretical (\( T_d = 0\mu s \)) DC voltage component according to (1.36) is \( V_{DC,SVM} = 0.0064V_{DC} = 2\text{V} \) resulting in \( I_{DC,SVM,b} = -I_{DC,SVM,c} = 1.17\text{A} \). From Fig.1.36(c) and 1.37(c) the DC currents are only \( I_{DC,SVM,b} = -I_{DC,SVM,c} = 0.6\text{A} \). The main reason for the difference is the non-zero blanking time, as the DC components rapidly decreases by increasing the blanking time.

By increasing \( m_f \) the value of the DC component is diminishing rapidly. Figure 1.38(a) shows the measured phase currents, when THI-PWM is applied and \( m_f = 10 \). It can be seen, practically there are no DC currents. By applying SVM the DC currents are still considerable even for \( m_f = 10 \) (Fig.1.38(b)) and \( m_f = 14 \) (Fig.1.38(c)).

**Effect of dead-time:** The DC voltage component (\( V_{DC,SVM} = I_{DC,SVM}R \)) were measured in phase \( b \) assuming \( \varphi_{c,a} = 0 \) for different \( T_d \) dead-time and \( m_f \) values using SVM (Fig.1.39). As the effect of dead time depends on the carrier period as well, the DC voltage component is plotted as the function of the ratio of \( T_d/T_c \) in Fig.1.39. For the better visualization the theoretic DC components belonging to \( T_d = 0 \) calculated from (1.36) were also plotted. The measurement results verify the theoretic assumptions and the simulation results discussed
previously: The DC current rapidly decreases by increasing the dead-time, but its value can still be considerable.

Subharmonics: Figure 1.40 and 1.41 shows the simulated and measured trace of the space vector $I_s$ when $f_1 = 249.7$ Hz ($m_f = 8.01$) for the three different NS-PWM techniques, respectively. In all three cases $f_{sub} = 2.4$ Hz, resulting in $Z_{sub} = 1.7 + j1.312 \, \Omega$ ($|Z_{sub}| = 2.15 \, \Omega$). Again it can be concluded, the measured and the simulated results practically are the same.

The amplitude of the subharmonic voltage component near $m_f = 8$ is almost zero for NS-SPWM (see Table 1.1) resulting in negligible subharmonic currents (Fig.1.40(a) and 1.41(a)). Subharmonic voltage with amplitude $\hat{V}_{sub} = 0.34V$ and $\hat{V}_{sub} = 2.3V$ is generated by applying NS-THI-PWM and NS-SVM (for $T_d = 0 \mu s$ see Table 1.1), respectively. These theoretical subharmonic voltage across the small per-phase equivalent impedance at $f_{sub}$ generates subharmonic currents with amplitudes $\hat{I}_{sub} = 0.16 \, \text{A}$ for NS-THI-PWM, and $\hat{I}_{sub} = 1$
A for NS-SVM. The simulated and measured subharmonic current amplitudes are similar: \( \hat{I}_{\text{sub}} = 0.1 \) A for NS-THI-PWM (Fig.1.40(b) and 1.41(b)) and \( \hat{I}_{\text{sub}} = 0.75 \) A for NS-SVM (Fig.1.40(c) and 1.41(c)). The difference is caused by the non-zero value of the blanking time.

Figure 1.42 shows the simulated and measured trace of the space vector \( \mathbf{I}_s \) when \( m_f = 10.012 \) (Fig.1.42(a)) and \( m_f = 14.016 \) (Fig.1.42(b)). It can be concluded that by applying NS-SVM, even at higher \( m_f \), the amplitude of the subharmonic currents can be considerable.

**Motor Load**

Laboratory measurements were performed with Motor B with rated speed 18 krpm. Similarly to the simulation analysis the loading torque during the laboratory test was \( \tau_{\text{load}} = 0.9 \) Nm.
The measurements were carried out with Conv A ($V_{DC} = 540$ V, $T_d = 3\mu s$) and the carrier frequency was selected to be $f_c = 2$ kHz.

In Figure 1.43 the time function of the measured stator phase current in phase $b$ can be seen, when $f_1 = 250$ Hz ($m_f = 8$) and $m_a = 0.955$. The time function is practically the same as the simulated one (see Fig.1.33) validating the theoretic result.

In Figure 1.44 and 1.45 the time function of the measured stator phase current and the space vector of stator flux $\Psi_s$ measured by Hall sensors can be seen when $f_1 = 249.8$ Hz ($m_f \approx 8$) and $f_1 = 199.9$ Hz ($m_f \approx 10$), respectively. When $m_f \approx 8$, the frequency of the subharmonic component is $f_{sub} = 1.6$ Hz. The measured amplitude of the subharmonic current and flux are $I_{sub} = 3.2$ A and $\Psi_{s,sub} = 0.08$ pu, respectively. Practically the same results are obtained by measurement as by simulation (see Fig.1.34). When $m_f \approx 10$, the frequency of the subharmonic component is $f_{sub} = 1$ Hz. The measured amplitude of the subharmonic current and flux are $I_{sub} = 2.1$ A and $\Psi_{s,sub} = 0.05$ pu. Again the similar results are obtained as with simulation (see Fig.1.35).

Figure 1.43: Time function of phase current, NS-SVM, $f_1 = 250$ Hz, ($m_f = 8$)

Figure 1.44: Time function of phase current and the measured trajectory of $\Psi_s$, NS-SVM, $f_1 = 249.8$ Hz, ($m_f \approx 8$)
The exact value of the DC component generated by carrier-based PWM algorithms can be calculated by the summation of the value of the reference signal $v_{ref,x}$ at the switching instants $\alpha_i$

$$V_{DC,x} = \frac{1}{4m_f} \left( \sum_{i=1}^{2m_f} v_{ref,x}(\alpha_i) \right) V_{DC}$$

where $x$ denotes the particular PWM techniques.

In single phase system no DC component is generated for Natural Sampling carrier-based PWM, when the frequency ratio is odd. Furthermore DC component is generated using Oversampled carrier-based PWM techniques only when the number of samples during one carrier period $n \geq 4$. DC component is generated even both for Regular Sampled Third Harmonics Injection PWM and Regular Sampled Space Vector Modulation when the frequency ratio is $m_f = 3$ and $m_f = 3, 5, 7, 9, 15, 21,...$, respectively. In three-phase system, when the frequency ratio $m_f$ is multiple of 3 or Regular Sampled PWM techniques are applied the DC components are cancelled in each phase voltage and no adverse effect is done. The dead time reduces the DC component.

Even though the level of subharmonic voltage is low, the subharmonic stator flux and current can be surprisingly high due to the very low subharmonic frequency. It can cause serious detrimental effects as the subharmonic flux and current components results in considerable additional stator and rotor copper loss. The special parameters of ultra high speed machines further accentuate the detrimental effect of subharmonics.
A novel algorithm, which calculates the intersection points of the carrier and the reference signal in real-time, is developed in digital microcontroller realizing the carrier based PWM techniques applying Natural Sampling with high precision in open loop.

Related publications of the author: [A1, A2, A3, A4, A5, A6, A7, A8]

**Practical significance of the results**

It is occasionally stated that the naturally sampled carrier-based PWM techniques do not generate DC components in the output phase voltage for synchronous PWM, when the frequency ratio is integer. The same results can be obtained by calculating the DC component using the double Fourier expansion method. In the previous chapter just the opposite was proven: when $m_f$ is low and integer considerable DC components can be generated for natural sampled carrier-based PWM techniques when $m_f$ is even and not multiple of 3. As it was shown the DC components can be avoided for even $m_f$ if the number of samples during a carrier period is limited to be $n = 2$ (Doublesampled).

However, in most of the literature it is suggested to apply synchronous PWM and integer $m_f$ if $m_f \leq 12 - 15$ even when $f_1$ varies, in most of the commercially available three-phase inverters the switching frequencies can be varied only in discrete steps (e.g. 3-6-12-16 kHz) resulting in asynchronous PWM. It gives the practical significance of investigation of the effect of low and non-integer $m_f$. Furthermore, as it was shown, subharmonic voltage component can results significant additional loss when $m_f \leq 15$. The presented results have to be evaluated in the light of many publications stating that the effect of subharmonics can be neglected.

In the last decade increasing attention has been given to high speed and high pole count motor drives. In both cases the high fundamental frequency and the limited carrier frequency result in low frequency ratio $m_f$. Natural Sampled carrier-based PWM can be a favourable modulation for low $m_f$ as no distortion or delayed response to the reference signal are introduced. Recently many researchers deal with the digital realization of NS utilizing the parallel computation properties of FPGA by increasing the number of samplings during one carrier period. The presented method for digital implementation of NS can be used even for low-cost microprocessor, where the registers of the PWM peripheral can be updated only twice during a carrier period.
Chapter 2

Stability Analysis using Auxiliary State Vector

2.1 Motivation

As a student I participated in the development of a multimedia material for teaching and e-Learning in Nonlinear Dynamics. It led me into a very challenging wonderful new world. At first time I thought that Nonlinear Dynamics and specially Chaos Theory are challenges for only mathematicians and they have little practical relevance to mechanical, electrical and mechatronic engineering applications. However, I quickly encountered with engineering experiences, which had hidden and unforeseen surprises due to their nonlinear property. In some cases the source of the strange behavior was a kind of general nonlinearities while in some other cases the feedback controlled systems were inherently nonlinear as the switching times depend on one or more state variables. These led me to the decision to learn more about the new results and the recent development in the theory of Nonlinear Dynamics mainly focusing to its application in the field of electrical drive systems and power electronics converters. Nonlinear Dynamics gives a better understanding of the operation of nonlinear engineering applications, which is valuable in producing more reliable and more effective systems.

The target of my research work was twofold. Firstly, to use and illustrate the application of the so-called auxiliary state vector proposed by my supervisor István Nagy in [48] to analyse the stability of different piecewise linear nonlinear mechatronic systems. Secondly, to compensate the instabilities and expand the stable operation range of the studied systems. Two systems, a current controlled DC motor drive and a digitally controlled Power Factor Correction (PFC) with boost converter were analysed. Both systems are realized in the laboratory to verify the theoretic and simulation results.

2.2 Introduction

One kind of nonlinear dynamic systems are described by a set of first-order nonlinear differential equations and can be represented in state space. The system can be autonomous and nonautonomous. The autonomous model is described by coupled independent differential equations

$$\frac{dx}{dt} = \dot{x} = v = f(x, \mu)$$

(2.1)

where the time does not appear explicitly, $x$ is the state vector with dimension $n$, and its time derivative is the speed vector $v$. $\mu$ is the parameter vector of the system and $t$ is the time. Finally $f$ is a linear or nonlinear vector function.

The nonautonomous model includes time, for instance an external excitation (forcing) function $g(t)$. Now the coupled differential equations are

$$\frac{dx}{dt} = \dot{x} = v = f(x, \mu(t), g(t))$$

(2.2)
The nonautonomous system can be transformed into an autonomous model by introducing a new variable
\[
\frac{dx_{n+1}}{dt} = \dot{x}_{n+1} = \frac{dt}{dt} = 1
\] (2.3)

Nonlinear dependence of \( f \) on \( x \) makes the model nonlinear. However nonlinear function \( \mu(t) \) and/or \( g(t) \) does not make the model nonlinear.

Equation (2.1) and (2.2) can be solved analytically or numerically for a given initial condition \( x_0 \) and parameter vector \( \mu \). The solution describes the state of the system as a function of time. The solution can be visualized in a reference frame where the state variables are the coordinates. It is called the state space or phase space. At any instant a point in the state space represents the state of the system. As the time evolves the state point is moving along a path called trajectory or orbit starting from the initial condition [49].

2.2.1 Poincaré concept

The tools used to analyse the behaviour of nonlinear dynamical systems are inherently different from the ones applied in linear systems. In many cases it is easier to use the discrete time model (DTM) for analysis and design than the continuous time model (CTM). This can be achieved by a method called Poincaré Map Function (PMF) invented by Jules Henri Poincaré. Poincaré was one of France greatest mathematicians and theoretical physicists, and a philosopher of science. He is often described as a polymath, and in mathematics as "The Last Universalist", since he excelled in all fields of the discipline as they existed during his lifetime [50]. He became the first person to discover a chaotic deterministic system which laid the foundations of modern chaos theory.

In autonomous models the definition of the PMF is the following: consider a trajectory in the state space, then take an arbitrary plane (it is called Poincare plane, see Fig.2.1(a)). The only requirement is that the trajectory has to intersect this plane transversally. The consecutive intersection points are \( x_m, x_{m+1}, ... \). The discrete mathematical function, called PMF gives the relation between \( x_m \) and \( x_{m+1} \)
\[
x_{m+1} = P(x_m)
\] (2.4)

The point \( x^* \) that satisfies \( x^* = P(x^*) \) is the fixed point of the map function.

The PMF reduces the dimensionality of the system by one and describes it by difference equation rather than differential equation. On the other hand, PMF retains the essential information of the system dynamics [49]. PMF is a useful tool even when the system behaviour is described by a trajectory which closes on itself in steady-state.

In nonautonomous model we assume that periodic forcing function is applied (see Fig.2.1(b)). If the forcing function period is \( T_s \), discrete observations are made at time \( t_m = mT_s \), \( t_{m+1} = (m + 1)T_s \). This method called sampled-data modeling. The PMF is given again by (2.4), like in autonomous system.

If the continuous-time orbit is periodic, there are a finite number of points on the Poincaré plane. In nonautonomous systems, the periodicity is given by the repetitiveness of the orbit as integral multiples of the period of the external input, and not by the number of loops in the continuous-time orbit. Thus, if the continuous-time orbit shows one loop in the state-space, but there are two cycles of the forcing function within that period, the orbit will be said to be period-2 [51].

In the case of quasiperiodic orbits, where the ratio of the repetitiveness of the orbit and the period of the forcing function is irrational, the points on the Poincaré plane will not fall on each other and will be arranged in a closed curve. For a chaotic orbit the asymptotic behavior in discrete time shows an infinite number of points, contained within a finite surface [51, 49].

The stability can be investigated on the basis of PMF. First the fixed-point should be determined, then the discrete system can be linearized in the neighborhood of a fixed point by obtaining the Jacobian matrix by differentiating PMF at the fixed point [49]. Based on the eigenvalues of the Jacobian matrix the fixed points can be classified. The system is stable if all of the eigenvalues lay within the unit circle.

It is important to note that in linear systems theory the loss of stability implies that the state diverges without limit. However in nonlinear systems the outcome of a stability loss, or in other words bifurcation, does not lead to an unlimited explosion of the variables.
2.2.2 Variable structure piecewise-linear nonlinear system

Most of the power electronics circuits belong to the variable structure piecewise-linear nonlinear systems. They change their structure after each switching and the sequence of structures succeeds each other periodically in periodic steady-state. Generally these structures can be modelled by linear time-invariant (LTI) models therefore the systems are piecewise-linear. The overall systems are nonlinear due to the dependence of the switching instants on state variables, or in some other cases due to saturations or other nonlinearities. As the switching instants in most cases are obtained by Pulse Width Modulation (PWM) technique, where a fix frequency carrier signal $v_{\text{car}}$ is applied, these variable structure systems are nonautonomous systems.

Figure 2.2 presents the simplified block diagram of the type of systems discussed. The controlled object is the variable structure part with controlled and uncontrolled switches, incorporating periodically changing subcircuits or structures. After each switch, another linear circuit arrangement emerges and the sequence of linear circuits is repeated in the next $T_s$ period (see Fig.2.2(b)). The duration of the $i^{\text{th}}$ sequence denoted by $\tau_i$. Of course the sum of $\tau_i$ intervals equals to $T_s$. As the state of the two switches in the permanent magnet DC drive system are always complementary and the PFC converter has only one switch, the dissertation focuses on the case when the number of structures is two.

The state variables of the object, e.g. rotational speed or output voltage, is controlled by PWM. Having only one switch in the controlled object the switching signals can be obtained if the output of the controller $v_{\text{con}}$ is compared to a saw-tooth carrier signal with fixed $T_s$ period (see Fig.2.2(c)). Switching occurs in the controlled object at each transition in $v_{\text{switch}}$ from 0 to 1 or vice-versa. Two kinds of switching take place: asynchronous A-switching and synchronous S-switching (Fig.2.2(c)).

It is common to analyse the stability and dynamical behaviour of variable structure piecewise-linear nonlinear systems by discarding the switching details and retain only the average dynamics of the system. While this average model can be analysed easier using several tools available from linear control theory, it fails to capture the instabilities that occur within the PWM period $T_s$. Furthermore in linear systems theory the loss of stability implies that the state diverges without limit. However, in nonlinear systems the outcome of a stability loss, or in other words bifurcation, does not lead to an unlimited explosion of the variables. The stability analysis method using the auxiliary state vector is able to detect the subharmonic and chaotic dynamics of the state variables that could occur in variable structure piecewise-linear nonlinear systems.

2.3 Theoretical Background

2.3.1 Auxiliary State Vector

The idea of the auxiliary state vector was introduced and discussed in more detail in [48].
Figure 2.2: Operation of the variable structure piecewise-linear feedback system controlled by PWM

The behaviour of the piecewise-linear nonlinear system between the switching instants can be modelled with the general state-space representation

$$\dot{x}(t) = \dot{z} = A_j x + B_j g$$  \hspace{1cm} (2.5)

where $j = 1, 2$ is the structure number, $x$ is the state vector, $z$ is the velocity vector, $g$ is the input or excitation vector, $A_j$ and $B_j$ are the parameter matrix.

At the switching instant the velocity vector is suddenly changed as both $A_j$ and $B_j g$ can be different

$$\dot{z}^{12} = \dot{z}_{s} - \dot{z}_{e}$$  \hspace{1cm} (2.6)

where suffixes $s$ and $e$ stand for start and end, respectively. $\dot{z}^{12}$ acts upon the system as a "force" and the direction of the system trajectory is abruptly varied in the state space.

Let time $\tau$ elapsed from the last switching instant. The behaviour of the system in time domain between two switching instants can be described by the solution of (2.5). Starting the system from $x_s = x(0)$ and assuming constant input vector the time function of the state-vector is

$$x(\tau) = e^{A_j(\tau)} x_s + \int_0^\tau e^{A_j(\vartheta)} d\vartheta B_j g = W_j(\tau) x_s + M_j(\tau) B_j g$$  \hspace{1cm} (2.7)

where $W_j(\tau) = e^{A_j(\tau)}$ is the weighting or base matrix. When $A_j$ is a regular matrix

$$M_j(\tau) = A_j^{-1} [W_j(\tau) - I]$$  \hspace{1cm} (2.8)

and where $I$ is the identity matrix with the same size of $A_j$. For singular $A_j$ matrices the Taylor series of the matrix exponential function can be used to calculate the integral:

$$e^{A_j(\vartheta)} = \sum_{n=0}^{\infty} \frac{A_j^n \vartheta^n}{n!}$$  \hspace{1cm} (2.9)
and therefore
\[
M_j(\tau) = \int_0^\tau e^A(\theta) d\theta = \sum_{n=0}^{\infty} \frac{A^n\tau^{n+1}}{(n+1)!}
\] (2.10)

The system trajectory keeps circulating along the same closed loop in state space in the periodic state. The values of the state vector at the start of \((k+1)^{th}\) period and at the end of \(k^{th}\) period are equal
\[
\mathcal{X}_{(k+1)1, s} = \mathcal{X}_{k2, e} \tag{2.11}
\]
\[
\mathcal{X}_{(k+1)1, e} = \mathcal{X}_{(k+1)2, s} \tag{2.12}
\]
Suffix 1 and 2 stand for structure 1 and 2, respectively.

The calculation of the Jacobian matrix is carried out by following the effects of a small initial perturbation. The deviation from the periodic trajectory during one period is calculated by taking into consideration the change of the A-switching instance and its effect as well.

As a result of a small change \(\Delta\mathcal{X}_{k1, s}\) from the state vector \(\mathcal{X}_{k1, s}\) at the start of the \(k^{th}\) period the state vector deviates from the steady-state space vector with \(\Delta\mathcal{X}_{k1, s}\) at the end of structure 1. Simultaneously the switching instant \(\tau_1\) belonging to the periodic solution will change by \(\Delta\tau_k\) as well. The actual state vector at the A-switching instant is \(\mathcal{X}_{k1, e}(\tau_1 + \Delta\tau_k)\). \(\Delta\tau_k\) is changing from period to period.

The calculation of Jacobian matrix is greatly facilitated by the introduction of the auxiliary state vector \(\mathcal{X}_{k2, s}(\tau_1)\) and its change \(\Delta\mathcal{X}_{k2, s}(\tau_1)\) instead of using \(\mathcal{X}_{k1, e}(\tau_1 + \Delta\tau_k)\) and \(\Delta\mathcal{X}_{k1, e}(\tau_1 + \Delta\tau_k)\) (see Fig.2.3). The auxiliary state vector points to a virtual initial state \(D_3(\tau_1)\) where starting the system at \(\tau_1\) from, the dynamics of structure 2 would drive the trajectory along the same orbit as the perturbed one after \(\tau_1 + \Delta\tau_k\).

The state vector change at \(\tau_1\) (between point \(D_1\) and \(D_2\))
\[
\Delta\mathcal{X}_{k1, e}(\tau_1) = W_{1}(\tau_1)\Delta\mathcal{X}_{k1, s} \tag{2.13}
\]

The commutation between the two structures takes place at \(\tau_1 + \Delta\tau_k\) at point \(D_2\). The distance between \(D_1\) and \(D_2\) is \(\mathcal{X}_{k1, e}\Delta\tau_k\), where \(\mathcal{X}_{k1, e}\) is the velocity of the state vector at \(\tau_1\) in structure 1. After the switching the velocity at the start of structure 2 is \(\mathcal{V}_{2, s}\). Due to the linearity of the structures the new trajectory can be projected backward in time from time \(\tau_1 + \Delta\tau_k\) to \(\tau_1\). In other words the trajectory will start from point \(D_3\) rather than point \(D_2\) by extending the new trajectory at the start of the structure 2 toward the negative time in the direction of velocity vector \(-\mathcal{V}_{2, s}\). Point \(D_3\) is reached this way at distance \(-\mathcal{V}_{2, s}\Delta\tau_k\) from \(D_2\). The auxiliary state vector change is obtained between \(D_3\) and \(D_2\). Now applying \(\Delta\mathcal{X}_{k2, s}(\tau_1)\) instead of \(\Delta\mathcal{X}_{k1, e}(\tau_1 + \Delta\tau_k)\) the trajectory will start in structure 2 at the same \(\tau_1\) instant as in the case of the periodic trajectory prior perturbation.

From Fig.2.3 the auxiliary state vector change reads
\[
\Delta\mathcal{X}_{k2, s}(\tau_1) = \Delta\mathcal{X}_{k1, e}(\tau_1) + (\mathcal{V}_{1, e} - \mathcal{V}_{2, s})\Delta\tau_k \tag{2.14}
\]

### 2.3.2 Calculation of velocity vectors

To calculate the velocity vectors \(\mathcal{V}_{1, e}\) and \(\mathcal{V}_{2, s}\) from (2.5) first the steady-state solution \(X_s\) of the state variable \(x\) at the start of structure 1 or in other word the fixed point should be determined. Assuming that \(\tau_1\) is known the value of the state vector at the end of structure 1 in the \(k^{th}\) period from (2.7)
\[
\mathcal{X}_{k1, e} = W_{1}(\tau_1)\mathcal{X}_{k1, s} + M_{1}(\tau_1)B_{1}g = \mathcal{X}_{k2, s} \tag{2.15}
\]

The value of the state vector at the end of structure 2
\[
\mathcal{X}_{k2, e} = W_{2}(\tau_2)\mathcal{X}_{k1, e} + M_{2}(\tau_2)B_{2}g = \mathcal{X}_{(k+1)1, s} \tag{2.16}
\]
Figure 2.3: Auxiliary state vector change $\Delta x_{k2,s}(\tau_1)$

Being $X_s = x_{k1,s} = x_{(k+1)s}$, the steady-state solution for known $\tau_1$ and $\tau_2 = T_s - \tau_1$ is

$$X_s = \left[ I - W_2(\tau_2) W_1(\tau_1) \right]^{-1} \left[ W_2(\tau_1) P_1 g + M_2(\tau_2) B_2 g \right]$$  (2.17)

By knowing $x_{(\tau_1)} = x_{k1,e} = x_{k2,s}$ from (2.15) the velocity vectors yield from (2.5)

$$v_{1,e} = \frac{dx}{dt} \bigg|_{\tau_1} = A_1 x_{(\tau_1)} + B_1 g(\tau_1)$$  (2.18)

$$v_{2,s} = \frac{dx}{dt} \bigg|_{\tau_1} = A_2 x_{(\tau_1)} + B_2 g(\tau_1)$$  (2.19)

Generally $\tau_1$ is not known and it should be determined by an iterative calculation using suitable mathematical software. Later on the calculation procedure will be presented on two practical examples.

### 2.3.3 Determination of the Jacobian matrix

The relation among the small deviations of the state vector around the fixed point $X_s$ at the start of consecutive periods

$$\Delta x_{(k+1)1,s} = J_k \Delta x_{k1,s} = J_k^h \Delta x_{11,s}$$  (2.20)

where $\Delta x_{11,s}$ is the initial deviation of the perturbed state vector from $X_s$ and $J_k$ is the Jacobian matrix. The stability criteria is that the absolute value of the largest eigenvalues of $J_k$ has to be less than one.

To determine $J_k$ by using the auxiliary state vector the $\Delta \tau_k$ should be calculated from (2.14). It can be done by using the switching condition for periodic and for the perturbed trajectory and utilizing that from the triangle $DD_1D_2$ (see Fig.2.3)

$$\Delta x_{k1,e}(\tau_1 + \Delta \tau_k) = \Delta x_{k1,e}(\tau_1) + v_{1,e} \Delta \tau_k$$  (2.21)

In this way, as it will be shown later, the auxiliary state vector change can be expressed as

$$\Delta x_{k2,s}(\tau_1) = M \Delta x_{k1,e}(\tau_1)$$  (2.22)

By applying the previous equations, (2.20) can be expressed as

$$\Delta x_{(k+1)1,s} = \Delta x_{(k)2,e} = W_2(\tau_2) \Delta x_{(k)2,s} = W_2(\tau_2) M \Delta x_{k1,e}(\tau_1) = W_2(\tau_2) M W_1(\tau_1) \Delta x_{k1,s} = J_k \Delta x_{k1,s}$$  (2.23)

The Jacobi matrix can be calculated as

$$J_k = W_2(\tau_2) M W_1(\tau_1)$$  (2.24)
2.3.4 The benefits of the method

The rewards of this method are as follows [48]:

1. The Jacobi matrix can be obtained without the calculation of the derivatives of the PMF.
2. The Jacobian matrix is obtained directly from the relations among the small differences of state vectors. The equations have graphical interpretation.
3. The auxiliary state vector preserves the constant switching instants depending on state variables even after the small excursion of the state variables from the steady state and the weighting matrices \( W_1(\tau_1) \) and \( W_2(\tau_2) \) without change can be applied.

2.4 Analysis of a current controlled DC drive system

2.4.1 Introduction

The investigation of nonlinear dynamics and chaos theory in power electronic circuits was launched in the late 1980s, focusing mainly on various kinds of switching DC/DC converters [52]. By extending the work to DC drive systems that involve a speed-dependent load voltage, chaotic behavior in the voltage-controlled DC drive system was first investigated in 1997 [53]. Chan et. al. showed in [54] that, the current controlled DC drive system using a proportional speed controller can exhibit chaotic behavior. In [55] the Filippovs-method is used to analyze the instabilities of a current controlled DC motor drive with full-bridge converter. Furthermore it was shown by adding a sinusoidal signal to the speed reference signal the stable period-1 range can be extended. Similar to [54] a proportional speed controller is used in [55]. The bifurcation behavior and the route to chaos of a voltage-mode controlled DC motor drive system applying a proportional-integral speed controller was presented in [56]. Unlike the conventional period-doubling bifurcation and its route to chaos which have been extensively reported in the literature, it was shown in [56] that Hopf bifurcation and chaos through torus breakdown can occur. The stability analysis of a DC shunt drive is carried out using the state transition matrix over one switching cycle (the monodromy matrix) including the state transition matrices during each switching (the saltation matrices) in [57]. Furthermore a new controller that can significantly extend the parameter range for stable period-1 operation has been proposed. In [58] an adaptive time-delayed feedback control is proposed and implemented to stabilize a steering system of an electric vehicle from chaos to stable operation. In the steering system also a permanent magnet DC motor is applied. The analysis of the border collision bifurcation in a DC drive applying a single phase thyristorised full converter is introduced in [59]. Paper [60] discusses the stability analysis of a two-cell Buck converter driven DC motor by deriving a nonlinear map and analyse its fixed points.

In the following the stability analysis of a current controlled DC drive system is presented using the auxiliary state vector. The Jacobian matrix determined from the state matrices without the derivation of the PMF. Furthermore, contrary to most of the papers in the literature here a Proportional-Integral (PI) speed controller is applied. It will be demonstrated by adding a ramp signal to the current loop the stable period-1 operation can be extended. The slope of the ramp signal is calculated also by using the auxiliary state vector. The calculation results are verified by simulation and experimental results as well.

2.4.2 Current controlled DC drive system

The schematic block diagram of the two-quadrant buck-chopper-fed permanent magnet DC motor drive is shown in Fig. 2.4(a). The speed and current control signals can be expressed as (the ramp signal with dotted line now is omitted)

\[
y(t) = g_\Omega (\Omega_{ref} - \Omega(t)) + \frac{g_\Omega}{T_i} \int (\Omega_{ref} - \Omega(t)) dt \tag{2.25}
\]

\[
u(t) = g_i i_a(t) \tag{2.26}
\]
where \( i_a \) is the armature current of the motor, \( \Omega \) is the speed of the rotor, \( \Omega_{\text{ref}} \) is the reference speed, \( g_i \) is the current gain, \( g_\Omega \) is the proportional gain and \( T_i \) is the integral time constant of the PI controller. Both \( u(t) \) and \( y(t) \) are fed into a comparator. Its output signal is connected to the reset and the clock pulse with period \( T_s \) to the set terminal of the RS latch. Both power switches are controlled by this RS latch. Once the latch is set by the clock pulse, \( S1 \) is turned on and \( S2 \) is turned off until the \( u(t) \) signal exceeds \( y(t) \) and then the RS latch is reset (Fig.2.4(b)). The switch \( S1 \) remains open until the arrival of the next clock pulse, while \( S2 \) is on.

![Figure 2.4: Current controlled DC drive](image)

The DC drive is a variable-structure piecewise-linear system. After each switch, another linear circuit arrangement emerges and the sequence of linear circuits is repeated in the next \( T_s \) period. The duration of structure 1 and 2 is \( \tau_1 \) and \( \tau_2 = T_s - \tau_1 \), respectively. The switching condition according to Fig.2.4(b) is

\[
y(\tau_1) = u(\tau_1)
\]

It will be assumed later on that both \( \Omega_{\text{ref}} \) and the supply voltage \( V_{\text{in}} \) are constant provided that it is not stated otherwise.

As the switching condition depends on the state vector the DC drive exhibits nonlinear dynamics.

### 2.4.3 Mathematical Background of DC drive system

The behaviour of the DC drive system can be described by the set of two first-order differential equations expressing the torque and the armature voltage balance, respectively:

\[
\frac{d\Omega}{dt} = \frac{K_T}{J} i_a - \frac{B}{J} \Omega - \frac{1}{J} T_{\text{load}}(t)
\]

\[
\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_E}{L_a} \Omega + \frac{1}{L_a} V_{\text{in}}
\]

Here \( V_{\text{in}} \) is the input voltage, \( T_{\text{load}} \) is the loading torque, \( B \) is the viscous damping, \( J \) is the inertia, \( K_E \) is the back-EMF constant, \( K_T \) is the torque constant, \( R_a \) and \( L_a \) is the armature resistance and inductance, respectively.
The behaviour of the DC drive system can be modelled with the general state-space representation
\[ \mathbf{x}(t) = \dot{\mathbf{x}} = A_x \mathbf{x} + B_x \mathbf{g} \]  
where \( j = 1, 2 \) is the structure number (\( j = 1 \), when \( S1 \) conducts (\( S2 \) is off) and \( j = 2 \) when \( S2 \) conducts (\( S1 \) off)), \( \mathbf{x} = [\Omega \; i_a] \) is the state vector, \( \dot{\mathbf{x}} \) is the velocity vector, \( \mathbf{g} = [T_{load} \; V_{in}] \) is the input vector, \( A_x \) and \( B_x \) are the parameter matrices:
\[
A_x = \begin{bmatrix} -\frac{B}{T_i} & \frac{K_E}{T_i} \\ \frac{K_E}{T_i} & -\frac{B_{es}}{T_i} \end{bmatrix}; \quad B_x = \begin{bmatrix} -\frac{1}{T_i} & 0 \\ 0 & \frac{1}{T_i} \end{bmatrix}; \quad \mathbf{g} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

When the \( S1 \) and \( S2 \) switches are turned on and off directly by open loop control independently on the state variables and therefore the values \( \tau_1 \) and \( \tau_2 = T_s - \tau_1 \) are preset, the periodic state of the DC motor can be calculated as described in 2.3.2.

In closed-loop operation \( \tau_1 \) is not known, however in steady-state both (2.27) and (2.17) hold true. Furthermore the start and the end value of the output of the PI controller in a switching period must be the same \( y_s = y_e \). For determining \( \tau_1 \), first \( \mathbf{X}_s \) is calculated from (2.17) by an initially selected \( \tau_1 \). Then from (2.25)
\[
D(\tau_1) = y_e - y_s = -g_3 k_1^T (\mathbf{X}_e - \mathbf{X}_s) + \frac{g_0}{T_i} \left( \Omega_{ref} T_s - \frac{k_1^T}{k_1} \left( \int_0^{\tau_1} \mathbf{x}_1(\tau) d\tau + \int_{\tau_1}^{T_s} \mathbf{x}_2(\tau) d\tau \right) \right) = 0
\]
where \( k_1^T = [1 \; 0] \), \( \mathbf{x}_j(\tau) = W_j(\tau) \mathbf{X}_j + M_j(\tau) \mathbf{B}_j \mathbf{g} \) (see (2.7)), \( \mathbf{X}_1 = \mathbf{X}_s \) and \( \mathbf{X}_2 = \mathbf{X}(\tau_1) \). In steady-state \( \mathbf{X}_e - \mathbf{X}_s \) is zero and it can be omitted. Likely \( D(\tau_1) \) will not be zero at the first estimation of \( \tau_1 \). The value of \( \tau_1 \) can be determined by an iterative calculation using suitable mathematical software.

2.4.4 Stability Analysis

Extended System Matrices

The integration property of the controller involves a new state variable. The output of the speed controller \( y \) as a new state variable has to be included in an extended state vector \( \mathbf{x}^* = [\mathbf{x}^T \; y] \). Later on the marking * denotes vectors and matrices in the extended space. According to (2.25)
\[
\dot{\mathbf{y}} = -g_0 \Omega + \frac{g_0}{T_i} (\Omega_{ref} - \Omega) \tag{2.32}
\]

The extended system matrices are
\[
A^*_x = \begin{bmatrix} A_x & 0 \\ 0 & A_x \end{bmatrix}, \quad B^*_x = \begin{bmatrix} \frac{1}{T} & 0 \\ \frac{1}{T} & 0 \end{bmatrix}, \quad \mathbf{g}^* = \begin{bmatrix} \frac{g}{\Omega_{ref}} \end{bmatrix}
\]

Calculation of \( \Delta \tau_k \)

According to (2.27) the switching condition for periodic trajectory
\[
E_k^T \mathbf{x}^*(\tau_1) - g_3 k_3^* \mathbf{x}^*(\tau_1) = 0 \tag{2.33}
\]

and for perturbed trajectory
\[
E_k^T \left( \mathbf{x}^*(\tau_1) + \Delta \mathbf{x}^*_{k1,e}(\tau_1 + \Delta \tau_k) \right) - g_3 k_3^* \left( \mathbf{x}^*(\tau_1) + \Delta \mathbf{x}^*_{k1,e}(\tau_1 + \Delta \tau_k) \right) = 0 \tag{2.34}
\]
where $k_3^{*T} = [0 \ 1 \ 0]$ and $k_3^{*T} = [0 \ 0 \ 1]$.
Subtracting (2.33) from (2.34) and using (2.21), $\Delta \tau_k$ can be expressed as

$$\Delta \tau_k = \frac{k_3^{*T} - g_1 k_3^{*T}}{(g_1 k_2^{*T} - k_3^{*T}) v_1^{*e}} \Delta \tau_{k1,e}^{*}$$  \hspace{1cm} (2.35)

By substituting $\Delta \tau_k$ in (2.14) the $\mathbf{M}$ matrix in (2.22) can be obtained as

$$\mathbf{M} = \mathbf{I} + (v_1^{*e} - v_2^{*s}) \frac{k_3^{*T} - g_1 k_2^{*T}}{g_1 k_2^{*T} v_1^{*e} - k_3^{*T} v_1^{*e}}$$  \hspace{1cm} (2.36)

and the Jacobian matrix according to (2.24) is

$$\mathbf{J} = \mathbf{W}_2^{*} (\tau_2) \left( \mathbf{I} + (v_1^{*e} - v_2^{*s}) \frac{k_3^{*T} - g_1 k_2^{*T}}{g_1 k_2^{*T} v_1^{*e} - k_3^{*T} v_1^{*e}} \right) \mathbf{W}_1^{*} (\tau_1)$$  \hspace{1cm} (2.37)

### Stabilizing ramp signal

The DC drive has a very colorful response depending on its parameters (see simulation and laboratory results). Its behavior can be chaotic from stable period-1 operation through bifurcation cascade. By using the stability analysis method derived in previous section the controller parameters ($g_\Omega$, $T_i$, $g_i$) can be calculated to maintain a stable operation at the rated parameters. However, sudden changes in the input signals (like loading torque, reference signal and input voltage) can result that the largest eigenvalue of the Jacobian matrix leaves the unit circle resulting an unstable operation.

It is a common method in DC/DC converters to add a periodic ramp signal with period $T_s$ synchronized to the clock signal to the current loop (see Fig.2.4(a) with dotted line) [61]. By properly selecting the slope of the signal, the DC drive can be stable in the whole operating range.

The equation of the current loop during one $T_s$ period is changed to

$$u(\tau) = g_i a(\tau) + m_c \frac{\tau}{T_s}$$  \hspace{1cm} (2.38)

where $\tau$ denotes the time elapsed from the last S-switching instant. This change has to be taken into consideration in (2.33) and (2.34) to calculate $\Delta \tau_k$, which results that $\mathbf{M}$ will be

$$\mathbf{M} = \mathbf{I} + (v_1^{*e} - v_2^{*s}) \frac{k_3^{*T} - g_1 k_2^{*T}}{g_1 k_2^{*T} v_1^{*e} - k_3^{*T} v_1^{*e} + \frac{m_c}{T_s}}$$  \hspace{1cm} (2.39)

It is obvious that by increasing $m_c$ the eigenvalues of the Jacobian matrix decrease provided that $W_2^{*} (\tau_1)$ and $W_2^{*} (\tau_2)$ are the same matrices (see (2.37)). For a given operation range of the DC drive the required value of $m_c$ to keep the largest eigenvalue of the Jacobian matrix in the unit circle can be calculated.

### 2.4.5 Results

To verify the theoretical results obtained by the stability analysis just described both simulations and measurements were carried out. The parameters of the DC drive system can be found in Table 2.1. The selected switching frequency ($f_s = 1/T_s = 250$ Hz) is quite low compared to the values applied in practical dc drives. The reason of the selected low switching frequency is to emphasize the nonlinear behavior of the speed control loop.
Table 2.1: Rated Parameters of DC drive system, Motor type: BALDOR M2240-A

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{in}$</td>
<td>Input voltage</td>
<td>46 V</td>
</tr>
<tr>
<td>$K_E$</td>
<td>Back-EMF constant</td>
<td>0.1196 V/rad/s</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Torque constant</td>
<td>0.1196 Nm/A</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Armature inductance</td>
<td>7.7 mH</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Armature resistance</td>
<td>6 Ω</td>
</tr>
<tr>
<td>$J$</td>
<td>Inertia</td>
<td>$6.5 \times 10^{-5}$ kgm$^2$</td>
</tr>
<tr>
<td>$B$</td>
<td>Viscous damping</td>
<td>$2.1 \times 10^{-4}$ Nm/rad/s</td>
</tr>
<tr>
<td>$I_n$</td>
<td>Nominal current</td>
<td>2.1 A</td>
</tr>
<tr>
<td>$n_n$</td>
<td>Nominal speed</td>
<td>3500 rpm</td>
</tr>
<tr>
<td>$g_l$</td>
<td>Current gain</td>
<td>1</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Integral time constant</td>
<td>0.1 sec</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Clock period</td>
<td>4 msec</td>
</tr>
</tbody>
</table>

Simulation and Calculation Results

Computer simulation were carried out in MATLAB/Simulink environment. Figure 2.5 shows the speed and the current time functions of the drive system at different $g_\Omega$ proportional gain value. Figure 2.5(a) shows the stable period-1 orbit. The absolute value of the largest eigenvalues of the Jacobian matrix now is $\lambda = 0.95$. By increasing the $g_\Omega$ gain the largest eigenvalue of the Jacobian matrix leaves the unit circle. Figure 2.5(b) and 2.5(c) show a period-2 and period-4 orbit, when $\lambda = 1.07$ and $\lambda = 1.56$, respectively. For $g_\Omega = 3$ the system is in chaotic state (Fig.2.5(d), $\lambda = 1.78$) and the time functions never repeat themselves.

An overall picture of the behavior of a nonlinear system is offered by the bifurcation diagram showing the various states and the sudden changes or bifurcations of the system due to the variation of bifurcation parameter. For this case the bifurcation diagram is obtained by sampling the speed $\Omega$ and the current $i_a$ signal at the start of every switching period in steady state and plotting these sampled values $\Omega_k = \Omega(kT_s)$ or $i_k = i(kT_s)$ as a function of the bifurcation parameter. Figure 2.6 shows the bifurcation diagram of the drive when the bifurcation parameter is $g_\Omega$. The sampled $i_k$ and $\Omega_k$ of the time functions shown in Fig.2.5 are denoted by red points in Fig.2.6. As it can be seen in the simulation results - unlike linear systems - changes in parameters can result different responses.

To maintain stable period-1 state in the whole operation range a ramp signal is added to the current control loop as it was mentioned previously. The operating range of the drive system is selected as $T_{load} = 0 - 0.4$ Nm, $V_{in} = 40 - 70$ V, $\Omega_{ref} = 100 - 220$ rad/s and $g_\Omega = 0.4$. The other parameters of the drive are kept constant during the operation.

The required $m_c$ slope of the ramp signal was determined using (2.39) in the whole operation range of the drive (Fig. 2.7). During the calculation the value of $m_c$ is increased from 0 for a given $T_{load}$, $V_{in}$ and $\Omega_{ref}$ value until the largest eigenvalue of the Jacobian matrix does not enter the unit circle. Naturally, if the largest eigenvalue of the Jacobian matrix is already in the unit circle $m_c = 0$ is selected. From the figures it can be concluded by selecting $m_c = 10$, the drive system is stable for changes in the whole operation range.

To illustrate the effect of the ramp compensation (RC) Fig.2.8 shows the time functions of the speed and the current of the drive, when $\Omega_{ref} = 170$ rad/s, $T_{load} = 0.3$ Nm and $V_{in} = 45$ V. The motor without RC (red line) has a chaotic response with large ripples. The value of the largest eigenvalues of the Jacobian matrix is $\lambda = 4.43$. By increasing the switching frequency to $f_s = 1$ kHz, the ripple both in the speed and the current is reduced, however the response is still chaotic (red dashed line, $\lambda = 4.63$). By applying RC with slope $m_c = 10$ to keep $\lambda$ in the unit circle (black line, $\lambda = 0.96$) near the lower nominal switching frequency a stable response can be obtained with the same ripple in the speed and currents as at higher ($f_s = 1$ kHz) switching frequency.
Laboratory Results

The current controlled DC drive system based on Fig.2.11(a) was built and tested in the laboratory. Similar DC drive system with proportional gain was implemented in [54] using analog circuitry. Here a low-cost 16-bit Digital Signal Controller\textsuperscript{1} (DSC) using fixed-point arithmetic is applied to implement the control algorithm. The speed signal $\Omega$ by the in-built tachogenerator and the armature current $i_a$ are measured, and transformed with analog circuitry to keep their value within the operating range of the DSC ($0 - 3.3$ V). Then the signals are converted to digital signals by using the in-built 12-bit A/D converter of the DSC. The control algorithm calculating $y$ and $u$ and determining the state of the switches is called with a sampling rate $f_{samp} = 50$ kHz. It is 200 times higher than the switching frequency ($f_s = 1/T_s = 250$ Hz). Thus, it can be assumed that the digital implementation has practically

\footnote{dsPIC33EP512MU810 from Microchip company}
Figure 2.6: Bifurcation diagram for $g_\Omega$, $\Omega_{ref} = 150$ rad/s, $V_{in} = 46$ V, $T_{load} = 0.1$ Nm, Simulation

(a) $\Omega_k$ vs. $g_\Omega$

(b) $i_{ak}$ vs. $g_\Omega$

Figure 2.7: Required slope value of the ramp signal

(a) $m_c$ as a function of $\Omega_{ref}$ and $T_{load}$

(b) $m_c$ as a function of $V_{in}$ and $T_{load}$

(c) $m_c$ as a function of $\Omega_{ref}$ and $V_{in}$

the same performance as an analog one.

Figure 2.9 shows the response of the DC drive for the same $g_\Omega$ values as in Fig.2.5. The laboratory results are good agreement with the stability analysis and the simulation clearly demonstrating that the stability method is capable to determine the parameters, like $g_\Omega$ to obtain proper operation.

To demonstrate the effect of the stabilizing signal by measurements as well the required $m_c$ value is calculated for $g_\Omega = 1.3$ to keep the largest eigenvalue of the Jacobian matrix inside the unit circle. By applying $m_c = 15$ the same stable period-1 response can be obtained for $g_\Omega = 1.3$ as for $g_\Omega = 0.1$ without ramp signal (Fig.2.10).
2.5 Analysis of a digitally controlled PFC boost converter

2.5.1 Introduction

Power Factor Correction (PFC) AC-DC converters perform the rectification of grid voltage with near unity power factor and low current harmonic distortion that are required to meet
Figure 2.10: Effect of stabilizing signal, $m_c = 15$, Measurement

the EN61000-3-2 standard. By taking into account the current stress and the efficiency, the boost topology shown in Fig.2.11(a) is one of the most favourable and most popular solutions. Operating PFC in discontinuous conduction mode (DCM) has the advantage of simple control and small input inductor. However, DCM results in increased peak current stress of the switch and increased electromagnetic interference (EMI). Therefore, even in low power applications the continuous conduction mode (CCM) is preferred. Operating PFC in CCM mode requires a more complex control solution as the duty ratio should vary in a wide range due to the periodic change of the input voltage from zero to its peak value.

Generally, the conventional control system used for PFC consists of a slow outer voltage loop regulating the output voltage and a fast inner current loop forcing the inductor current to follow the shape of the rectified AC input voltage. Two kinds of current-mode controls, the peak and the average ones can be distinguished. In the paper PFC operating in CCM with average current mode control is discussed.

In the last decade the complex behaviour of PFC is intensively studied [62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72]. Many approaches have been proposed to analyse their stability. In the paper [62] the analysis is based on the charging and discharging energy or charge of the output capacitor, which is one of the main contributing parameter on the system stability. In [63] an analytical investigation is presented to predict the stability boundary of PFC under peak current mode control using RS latch. The same PFC arrangement is studied by state-transition matrix in [64]. Practical and accurate analytical expressions for the stability boundary are obtained for preregulator and two stage PFC in [65]. The bifurcation behaviour of a two-stage PFC using analog circuits was shown in [66]. Furthermore, a selective notch filter based controller is introduced to suppress instabilities. Paper [67] stabilizes the bifurcations developing in PFC with a "washout-filter-aided" method. Here again a commercially available analog control IC$^2$ is applied to achieve unity power factor. Increasing attention has been paid recently on digitally controlled PFCs with the development of faster and cheaper microcontrollers and digital signal processors as well [68, 69, 70, 71, 72]. Digital control provides many advantages over analog one, like ease of implementation of the sophisticated control algorithms and flexibility of design modifications [69]. The reference [70] experimentally compares three digital control methods: an adaptive nonlinear, the classical cascaded linear controller and linear controller with notch filter strategy. In [71] a positive current feed-forward compensator is proposed to improve the stability of the PFC, while in [69] an optimal Lyapunov-based control strategy is introduced. Asymmetrical oscillations caused by the time delay due to the sample-and-hold and the digital computation was investigated in [72].

Stability analysis and methods for improving delayed dynamical systems, not only feedback controlled systems like the PFC but machine tool vibrations as well, are in the center of interest of many researchers [73]. The main purpose of the stability analysis presented in the dissertation is to determine the stability border, where large oscillations starts in digitally controlled PFC by taking into account the effect of the time delay resulting from the digital implementation (see later) similarly to [72].

$^2$UC3854A
One of the main advantages of my method using the auxiliary state vector over the one presented in [72] is that, it inherently contains the feasibility to add a stabilizing input signal to extend the stability range. Furthermore, in the dissertation the time delay is approximated by the second-order Padé approximation as well. It determines the stability border more precisely as the first-order Padé approximation used in [72].

The analysis focuses on the investigation of the fast inner current loop and it is assumed that the outer voltage loop works properly.

### 2.5.2 Digitally Controlled PFC Boost Converter

The block diagram of the digitally controlled PFC can be seen in Fig. 2.11(a). The control algorithm is implemented in a low-cost 16-bit Digital Signal Controller (DSC) using fixed-point arithmetic. The signals $v_{in}$, $i_L$, and $v_o$ are measured, filtered, and transformed with analog circuitry (not shown in the figure) to keep their value within the operating range of the DSC (0 – 3.3 V). Then the signals are converted to digital signals by using the in-built 12-bit ADC module of the DSC shown in Fig. 2.11(a).

![Block diagram of Digitally Controlled PFC Boost Converter](image)

**Figure 2.11: Digitally controlled PFC boost converter**

The slow outer voltage loop operates with sampling rate $f_{s,v} = 2$ kHz. The measured output voltage $v_o$ is subtracted from the reference voltage $V_{ref}$ and the error is fed into a proportional-integral (PI) type controller with low-bandwidth [72, 74, 75]. It should be noted that often low-pass filter is applied as voltage controller [67, 70]. The $u_v$ output of the controller is multiplied with the normalized $v_{in}/V_{av}$ rectified input voltage, where $V_{av}$ is the calculated average of $v_{in}$, to obtain the reference current $i_{ref}$. The inner current loop has a higher sampling rate $f_{s,i} = 40$ kHz. It is the same as the switching frequency of MOSFET switching device $S$. The difference between the reference $i_{ref}$ and the actual instantaneous current $i_L$ is fed into a PI controller realizing average current mode control. The output of the PI current controller $u_i$ determines the duty ratio (between 0 and 1). It is sent to the digital PWM module of the DSC, where it is compared to a sawtooth signal with unit amplitude to generate the switching signal of $S$. The time function of $i_L$, $v_{in}$, and $u_i$ can be seen in Fig. 2.11(b).

### 2.5.3 Mathematical Background of PFC converter

Figure 2.12 show the equivalent circuits of the PFC boost converter in states of switch $S$. The state variables of the converter is output voltage $v_o$ and inductor current $i_L$. The set of

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3. dsPIC33EP512MU810 from Microchip company
differential equations describing the operation of the PFC when the switch conducts (structure 1, Fig. 2.12(a))

\[
\frac{dv_o}{dt} = -\frac{1}{RC}v_o \quad (2.40)
\]

\[
\frac{di_L}{dt} = -\frac{R_{on}}{L}i_L - \frac{R_L}{L}i_L + \frac{1}{L}v_{in} \quad (2.41)
\]

and when the diode conducts (structure 2, Fig. 2.12(b))

\[
\frac{dv_o}{dt} = -\frac{1}{RC}v_o + \frac{1}{C}i_L \quad (2.42)
\]

\[
\frac{di_L}{dt} = -\frac{1}{L}v_o - \frac{R_D}{L}i_L - \frac{R_L}{L}i_L + \frac{1}{L}v_{in} - \frac{V_f}{L} \quad (2.43)
\]

Figure 2.12: Equivalent circuits of the PFC converter

Here \(R_{on}\) is the resistance of the MOSFET switch, when it conducts. \(C\) is the output capacitor, \(L\) is the input inductance and \(R_L\) is its resistance. \(V_f\) and \(R_D\) is the forward voltage drop and the resistance of the diode D, respectively. The behaviour of the boost converter can be modelled with the general state-space representation

\[
\dot{x}(t) = Ax + Bu \quad (2.44)
\]

where \(j = 1, 2\) is the structure number (\(j = 1\), when \(S\) is on and \(j = 2\) when \(S\) is off), \(x^T = [v_o \quad i_L]\) is the state vector, \(v\) is the velocity vector, \(g^T = [v_{in} \quad V_f]\) is the input vector, \(A_j\) and \(B_j\) are the parameter matrix:

\[
A_1 = \begin{bmatrix}
-\frac{1}{RC} & 0 \\
0 & -(\frac{1}{R_{on} L} + \frac{1}{R_L L})
\end{bmatrix} \quad A_2 = \begin{bmatrix}
-\frac{1}{RC} & \frac{1}{C} \\
0 & -(\frac{1}{L} + \frac{1}{R_D L})
\end{bmatrix}
\]

\[
B_1 = \begin{bmatrix}
0 & 0 \\
\frac{1}{L} & 0
\end{bmatrix} \quad B_2 = \begin{bmatrix}
0 & 0 \\
\frac{1}{L} & -\frac{1}{L}
\end{bmatrix}
\]

The rectified input voltage is

\[
v_{in} = V_m | \sin(2\pi f_1 t) | \quad (2.45)
\]

where \(V_m\) is the amplitude of the input phase voltage and \(f_1 = 50\) Hz is the line frequency. Since the line frequency \(f_1\) is a few orders of magnitude smaller than the switching frequency \(f_{s,i}\), the input voltage can be assumed constant over one switching period \(v_{in} = V_{in} = V_m | \sin \theta | = [64].\) In the \(n^{th}\) cycle from the last zero crossing of the input voltage

\[
v_{in} = V_m | \sin \theta | = V_{in} \sin(\omega_1(n - 1)T_{s,i}) \quad (2.46)
\]

Assuming analogue continuous current controller, control signal \(u_i\) determining the duty ratio of \(S\) can be expressed as

\[
u_i(t) = K_{pi}(i_{ref} - i_L) + \frac{1}{T_{ci}} \int (i_{ref} - i_L)dt \quad (2.47)
\]
and assuming ideal components the switching condition

\[ u_i(\tau_1) = d = 1 - \frac{V_m}{V_{ref}} = 1 - \frac{V_m}{V_{ref}} \sin \theta \] (2.48)

where \( \tau_1 = dT_{s,i} \). Equation (2.48) must be met if the outer voltage loop works as required. Note that \( d \) and \( \tau_1 \) is changing in wide range even in steady-state due to \( V_m \sin \theta \) unlike to the current controlled DC drive system where \( \tau_1 = \text{const.} \) in steady state.

Due to the nonideal circuit components the actual duty ratio should be higher than the theoretic one calculated from (2.48). During the stability analysis the required duty ratio is obtained by an iterative calculation using (2.17) and utilizing that \( [1 \ 0]X_s = v_o \approx V_{ref} \).

In the DSC the discrete version of (2.47) is implemented with a sampling rate of \( f_{s,i} = \frac{1}{T_{s,i}} \). As the discretization has influence on the stability it should be taken into consideration during the analysis.

The \( u_i \) duty ratio is calculated by using the sampled signal of \( i_L \) and \( i_{ref} \). The calculation takes less time than \( T_{s,i} \), however, it is advisory to update the duty ratio of the digital PWM module only in the next switching period. It results in a fixed delay, which can be expressed by \( G_{\text{delay}} = e^{-sT_{s,i}} \). In most cases it is approximated by using the first-order Padé approximation [70, 76]

\[ G_{\text{delay,1}} = e^{-sT_{s,i}} \approx \frac{1 - T_{s,i}s}{1 + \frac{T_{s,i}s}{2}} \] (2.49)

My analysis showed that this approximation is not accurate enough therefore the delay was approximated by using a second-order Padé approximation [76] as well

\[ G_{\text{delay,2}} = \frac{1 - T_{s,i}s + \frac{T_{s,i}^2 s^2}{12}}{1 + \frac{T_{s,i}s}{2} + \frac{T_{s,i}^2 s^2}{12}} \] (2.50)

During the analysis I approximated (2.50) as the serial connection of two one-energy storage elements

\[ G_{\text{delay,2}} \approx \frac{1 - T_{s,i}s}{1 + \frac{T_{s,i}s}{3}} \cdot \frac{1 - T_{s,i}s}{1 + \frac{T_{s,i}s}{4}} \] (2.51)

The effect of the Zero-Order Hold (ZOH) at the output of the digital PI controller can be given by the following transfer function [77, 34]

\[ G_{\text{ZOH}}(s) = \frac{1 - e^{-sT_{s,i}}}{T_{s,i}s} \] (2.52)

Equation (2.52) can be further simplified by using (2.49) the first-order Padé-approximation

\[ G_{\text{ZOH}}(s) \approx \frac{T_{s,i}s}{0.5T_{s,i}s^2 + T_{s,i}s} = \frac{1}{1 + 0.5T_{s,i}s} \] (2.53)

Taking into consideration the digital implementation, Fig.2.13 shows the block diagram of the current control loop.

In steady-state the reference current \( i_{ref} \) will be in phase with the rectified voltage

\[ i_{ref} = I_m \sin 2\pi f_1 t \] (2.54)

The current work focuses on the stability issues of the inner current loop. Let us assume that the outer voltage loop works properly and \( v_o \approx V_{ref} \). If \( i_L \) follows the shape of \( i_{ref} \)

\[ P_{in} = \frac{I_m V_m}{\sqrt{2} \sqrt{2}} = \frac{V_{ref}^2}{R} = P_{out} \] (2.55)

as the power factor is unity, where \( \eta \) is the overall efficiency. Similarly to the input voltage, the reference current can be assumed constant over a switching frequency

\[ I_{ref} = \frac{2V_{ref}^2}{RV_m \eta} \sin \theta \] (2.56)
Figure 2.13: Block diagram of the current control loop

Figure 2.14: Forward part of current control loop with state variables

2.5.4 Stability Analysis

Extended System Matrices

Figure 2.14 shows three different models of the forward path of the current control loop up to signal $u_i$. In Fig. 2.14(a), 2.14(b) and 2.14(c) the order of the blocks are rearranged to express the additional state variables by first-order differential equations but of course it is the same as the forward part of the original current control loop depicted in Fig. 2.13.

The integration property of the PI controller, the ZOH and the computation delay involve new state variables. They have to be included in an extended state vector $x^h$. Later on the marking $^h$ denotes vectors and matrices in the extended space, where $h$ refers to how the effect of digital implementation is approximated. When $h = 1$ and $h = 2$ the delay is approximated by using the first and second-order Padé approximation, respectively. In both cases the effect of ZOH is taken into consideration. When $h = 3$ the computation delay is approximated by the second-order Padé approximation, but the effect of ZOH is not taken into consideration. When $h$ is missing the statement holds true for all cases. Sign “/” denotes “OR”. Of course (2.44) holds true for extended state vector and matrices as well.

By using the first-order Padé approximation and taking into consideration the effect of the ZOH ($h = 1$) (see (2.49) and (2.52)) three additional state variables $x_3$, $x_4$ and $x_5$ have to be
included into the extended state vector: \( x^{*T} = [x^T \ x_3 \ x_4 \ x_5] \). Based on Fig.2.14(a)

\[
\begin{align*}
\dot{x}_3 &= \frac{2}{T_{s,i}}(I_{ref} - i_L - x_3) \\
\dot{x}_4 &= \frac{2}{T_{s,i}}(x_3 - x_4) \\
\dot{x}_5 &= \frac{x_4}{T_{ci}}
\end{align*}
\]

The extended system matrices are

\[
A_{1}^{*1} = \begin{bmatrix}
\frac{A_1}{A_2} - \frac{2}{T_{s,i}} & 0 & 0 & 0 \\
0 & -\frac{2}{T_{s,i}} & -\frac{2}{T_{s,i}} & 0 \\
0 & 0 & -\frac{2}{T_{s,i}} & 0 \\
0 & 0 & 0 & -\frac{1}{T_{ci}} & 0
\end{bmatrix}
\]

\[
B_{1}^{*1} = \begin{bmatrix}
\frac{B_1}{B_2} - \frac{2}{T_{s,i}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[g^* = \begin{bmatrix}
\frac{g_{I_{ref}}}{T_{ci}} \\
0 \\
0
\end{bmatrix}
\]

The \( u_1 \) control voltage is (see Fig.2.14(a))

\[ u_1 = x_5 + K_{pi}x_4 - \frac{T_{s,i}}{2}\dot{x}_5 - \frac{T_{s,i}K_{pi}}{2}\dot{x}_4 \]  \hfill (2.57)

By substituting the relations of \( \dot{x}_4 \) and \( \dot{x}_5 \) into (2.57), it can be expressed as \( u_1 = k_{1}^T x^{*1} + k_{g1}^T g^* \), where

\[ k_{1}^T = \begin{bmatrix}
0 & 0 & -K_{pi} & 2K_{pi} - \frac{T_{s,i}}{2T_{ci}} & 1
\end{bmatrix}; \quad k_{g1}^T = \begin{bmatrix}
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  \hfill (2.58)

When the digital delay is modelled by the second-order Padé approximation and the effect of ZOH is also taken into consideration (\( h = 2 \)) (see (2.50) and (2.52)) four additional state variables \( x_3, x_4, x_5 \) and \( x_6 \) should be included into the extended state vector: \( x^{*2T} = [x^T \ x_3 \ x_4 \ x_5 \ x_6] \). Based on Fig.2.14(b)

\[
\begin{align*}
\dot{x}_3 &= \frac{2}{T_{s,i}}(I_{ref} - i_L - x_3) \\
\dot{x}_4 &= \frac{3}{T_{s,i}}(x_3 - x_4) \\
\dot{x}_5 &= \frac{4}{T_{s,i}}(x_4 - x_5) \\
\dot{x}_6 &= \frac{x_5}{T_{ci}}
\end{align*}
\]
The extended system matrices are

$$
\frac{A^*}{A^2} = \begin{bmatrix}
\frac{A_1}{A_2} & 0 & 0 & 0 & 0 \\
0 & -\frac{2}{T_{s,i}} & -\frac{2}{T_{s,i}} & 0 & 0 \\
0 & 0 & -\frac{3}{T_{s,i}} & 0 & 0 \\
0 & 0 & 0 & T_{s,i} \frac{1}{T_{ci}} & 0 \\
0 & 0 & 0 & 0 & T_{s,i} \frac{1}{T_{ci}}
\end{bmatrix}
$$

and

$$
\frac{B^*}{B^2} = \begin{bmatrix}
\frac{B_1}{B_2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$g^* = \begin{bmatrix}
g \ref{Iref} \\
0 \\
0
\end{bmatrix}
$ (2.59)

The $u_{i2}$ control voltage is (see Fig.2.14(b))

$$u_{i2} = x_6 + K_p x_5 - \frac{T_{s,i}}{3} (\dot{x}_6 + K_p \dot{x}_5) - \frac{T_{s,i}}{4} (\dot{x}_6 + K_p \dot{x}_5) + \frac{T_{s,i}^2}{12} (\ddot{x}_6 + K_p \ddot{x}_5)
$$

By utilizing that

$$\ddot{x}_6 = \dot{x}_5 \quad \ddot{x}_5 = \frac{4}{T_{s,i}} (\dot{x}_4 - \dot{x}_5)
$$

and taking into account the relations of $\dot{x}_4$, $\dot{x}_5$ and $\dot{x}_6$, (2.59), it can be expressed as $u_{i2} = k_2 T x^* + k_2^T g^*$, where

$$k_2^T = \begin{bmatrix} 0 & 0 & K_p & -\frac{14 K_p}{3} + \frac{T_{s,i}}{3 T_{ci}} & \frac{14 K_p}{3} - \frac{11 T_{s,i}}{12 T_{ci}} & 1 \end{bmatrix} \quad k_2^T = 0
$$

If the effect of ZOH is not taken into consideration and the delay is approximated by the second-order Padé approximation ($h = 3$) (see (2.50)) three additional state variables $x_3$, $x_4$ and $x_5$ should be included in the extended state vector: $x^{*3T} = [x^T \ x_3 \ x_4 \ x_5]$. Based on Fig.2.14(c)

$$\dot{x}_3 = \frac{3}{T_{s,i}} (I_{ref} - i_L - x_3)
$$

$$\dot{x}_4 = \frac{4}{T_{s,i}} (x_3 - x_4)
$$

$$\dot{x}_5 = \frac{x_4}{T_{ci}}
$$

The extended system matrices are

$$
\frac{A^*}{A^3} = \begin{bmatrix}
\frac{A_1}{A_2} & 0 & 0 & 0 \\
0 & -\frac{3}{T_{s,i}} & -\frac{3}{T_{s,i}} & 0 \\
0 & 0 & T_{s,i} \frac{1}{T_{ci}} & 0 \\
0 & 0 & 0 & T_{s,i} \frac{1}{T_{ci}}
\end{bmatrix}
$$

and

$$
\frac{B^*}{B^3} = \begin{bmatrix}
\frac{B_1}{B_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

$g^* = \begin{bmatrix}
g \ref{Iref} \\
0 \\
0
\end{bmatrix}
$ (2.60)
The \( u_{i3} \) control voltage is (see Fig.2.14(c))

\[
u_{i3} = x_5 + K_{pi} x_4 - \frac{T_{s,i}}{3} (\dot{x}_5 + K_{pi} \dot{x}_4) - \frac{T_{s,i}}{4} (\ddot{x}_5 + K_{pi} \ddot{x}_4) + \frac{T_{s,i}^2}{12} (\dddot{x}_5 + K_{pi} \dddot{x}_4)
\] (2.61)

By substituting the relations of \( \dot{x}_4 \) and \( \dot{x}_5 \) into (2.61), it can be expressed as

\[
u_{i3} = \frac{k_{i3} T_x}{k_{g3} T_g} = \begin{bmatrix} 0 & -K_{pi} \frac{T_{s,i}}{3} + \frac{14K_{pi}}{3} T_{ci} - \frac{11T_{s,i}}{12T_{ci}} 1 \\ \end{bmatrix}
\]

(2.62)

Calculation of \( \Delta \tau_k \)

To determine \( \Delta \tau_k \) the switching rule has to be used. The commutation between the two structures takes place when \( u_{ih} \) (which actually determines the duty ratio and can be changed between 0 and 1) intersects the ramp signal with slope \( m = 1/T_{s,i} \) (Fig.2.15). Figure 2.15 shows the perturbed waveform of the control signal denoted by \( u'_{ih} \) as well. It intersects the ramp signal at \( \tau_1 + \Delta \tau_k \). Index \( h \) is left out in Fig.2.15 in \( u_{ih} \) and \( u'_{ih} \) for simplicity.

![Figure 2.15: Calculation of \( \Delta \tau_k \)](image)

The switching condition for periodic trajectory

\[
k_{h}^T z^*(\tau_1) + k_{gh} g^* - m \tau_1 = 0
\] (2.63)

and for perturbed trajectory

\[
k_{h}^T (z^*(\tau_1) + \Delta z_{k1,e}^*(\tau_1 + \Delta \tau_k)) + k_{gh} g - m (\tau_1 + \Delta \tau_k) = 0
\] (2.64)

Subtracting (2.63) from (2.64) and using (2.21), \( \Delta \tau_k \) can be expressed as

\[
\Delta \tau_k = \frac{k_{h}^T \Delta z_{k1,e}^*}{m - k_{h}^T z_{1,e}^*}
\] (2.65)

By substituting \( \Delta \tau_k \) into (2.14) matrix \( M \) can be obtained from (2.22)

\[
M = I + \left( z_{1,e}^* - z_{2,e}^* \right) \frac{k_{h}^T}{m - k_{h}^T z_{1,e}^*}
\] (2.66)

and the Jacobian matrix according to (2.24) is

\[
J_{dc} = W_{2}^*(\tau_2) \left( I + \left( z_{1,e}^* - z_{2,e}^* \right) \frac{k_{h}^T}{m - k_{h}^T z_{1,e}^*} \right) W_{1}^*(\tau_1)
\] (2.67)
Steps for the calculation of $J_{\text{stab}}$

To determine the Jacobian matrix for a given set of parameters the following steps should be done in each $T_{s,i}$ cycle as $\tau_1 = dT_{s,i}$ is changing in wide range with $x_2 = i_2$.

1. Determination of the duty ratio $d$ by an iterative calculation\(^4\) using the original system matrices $(A, B)$

2. Calculation of $W^*_1(\tau_1)$ and $W^*_2(\tau_2)$ using the extended system matrices and $\tau_1 = dT_{s,i}$ ($\tau_2 = T_{s,i} - \tau_1$)

3. To calculate first the matrix $M$ and next the Jacobian matrix $J_{\text{stab}}$ from (2.24) the velocity vectors $\dot{u}^*_1, \dot{u}^*_2, u^*_2, s$ should be calculated. It can be done in two steps

   (a) As the extended system matrices $A^*$ are not invertible, the last state variable ($x_5$ for $h = 1$ and $h = 3$, and $x_6$ for $h = 2$) in the extended state vector are omitted\(^5\), and the velocity vectors are calculated using (2.18) and (2.19).

   (b) The last element in the velocity vectors can be calculated using $\dot{x}_5(\tau_1) = x_4(\tau_1)/T_{ci}$ (for $h = 1$ and $h = 3$) or $\dot{x}_6(\tau_1) = x_5/T_{ci}$ (when $h = 2$)

4. $J_{\text{stab}}$ can be determined using (2.24) or (2.67)

One of the benefits of using the auxiliary state vector is that $\tau_1$ and the weighting matrices remain unchanged in the stability analysis in the same $T_{s,i}$ cycle.

Stabilizing signal

By using the stability analysis method derived previously the current controller parameters can be calculated to maintain a stable operation at a given set of parameters. However, sudden changes in the load resistance or in the $V_{\text{ref}}$ reference voltage signal can result that the largest eigenvalue of the Jacobian matrix leaves the unit circle resulting an unstable operation.

The stability analysis method inherently contains the feasibility to add a stabilizing signal to extend the stability range. It is a common method in DC/DC converters to add a periodic ramp signal synchronized to the carrier signal to the current loop [61]. However, in the case of a PFC converter, as the duty ratio continuously varies with time, the maximum value of the ramp signal should be also varied with time. Lets denote the stabilizing signal with $u_{\text{stab}}$ given by

$$ u_{\text{stab}} = -M_c \sin \theta \left( \frac{M_c}{T_{s,i}} \left( \frac{k_h T_{s,i}}{m + \frac{m_c}{T_{s,i}} - k_h T_{s,i} \Delta \tau_{s,i}} \right) \right) $$.  

which is subtracted from the output of the controller $u$, therefore $u_{i,\text{stab}} = u_i - u_{\text{stab}}$. $\tau$ is the time elapsed from the last S-switching instant. The actual value of $\sin \theta$ is obtained from the actual value of the input voltage $v_{in}$. For the better understanding the Fig.2.16 shows the operation of the stabilizing signal. It should be noted subtracting $u_{\text{stab}}$ from $u_i$ is the same if $u_{\text{stab}}$ is added to the ramp signal.

This change has to be taken into consideration in (2.63) and in (2.64) resulting in the following Jacobian matrix

$$ J_{\text{stab}} = W^*_2(\tau_2) \left( I + \frac{\left( \frac{v^*_1}{m} e^{x_{2,s}} \right)}{m + \frac{m_c}{T_{s,i}} - k_h T_{s,i} \Delta \tau_{s,i}} W^*_1(\tau_1) \right) $$

In the DSC the discrete version of the stabilizing signal should be implemented. Let us assume $u_i(n+1)$ is the output of the discrete PI controller calculated in the $n^{th}$ period, which

\(^4\)for initial value in iteration the duty ratio can be calculated from (2.48) for a given $V_m \sin \omega_t (n - 1) T_{s,i}$

\(^5\)The last columns and last rows are also omitted in the state matrices
is practically gives the duty ratio $u_i(n + 1) = d(n + 1) = \tau_1(n + 1)/T_{s,i}$. As the value of the stabilizing signal is required only at switching instant $\tau_1$, $u_i(n + 1)$ can be modified as

$$u_{i,\text{stab}}(n + 1) = u_i(n + 1) - u_{\text{stab}}(n + 1) = u_i(n + 1) + m_c(n + 1) - \frac{m_c(n + 1)}{T_{s,i}}\tau_1(n + 1)$$

which can be expressed as

$$u_{i,\text{stab}}(n + 1) = u_i(n + 1)(1 - m_c(n + 1)) + m_c(n + 1)$$

(2.70)

(2.71)

### 2.5.5 Physical roots of unstable operation

It is known that instability, subharmonic oscillation develops in the boost converter operated in continuous conduction mode and controlled by current controller if the duty ratio exceeds 50% or even in some cases of smaller $d$. My results are in line with this knowledge.

**Consideration on stability of boost converter using area $T_+$ and $T_-$**

Figure 2.17(a) represents the time function of the inductor current when a small $\Delta d$ deviation is applied around the steady-state duty ratio $d$. $m$ and $m_o$ are the slope of the current when the switch and when the diode conducts, respectively.

![Figure 2.17: Stability of boost converter using geometric consideration](image)

(a) Effect of small deviation $\Delta d$

(b) $T_+$ as the function of $d$

The area $T_+$ equals to the increment in the diode, and in this way the average load current:

$$T_+ = [1 - (d + \Delta d)](m + m_o)\Delta d \cong (1 - d)(m + m_o)\Delta d$$

(2.72)

As in steady-state $md = m_o(1 - d)$ the $T_+$ area is

$$T_+ = (1 - d)m(1 + \frac{d}{1-d})\Delta d = m\Delta d$$

(2.73)
The area $T_-$ equals to the reduction in the diode and the average load current

$$T_- \approx (md + I_{Lo})\Delta d$$  \hspace{1cm} (2.74)$$

where $I_{Lo}$ is the inductor current at the beginning of the carrier period in steady-state.

The ratio of the two area

$$\frac{T_+}{T_-} = \frac{1}{d(1 + \frac{I_{Lo}}{md})}$$  \hspace{1cm} (2.75)$$

Figure 2.17(b) represents the ratio of the two area as the function of the duty ratio $d$ when the parameter is $p = \frac{I_{Lo}}{md}$. The ratio implies instability if it is less than 1.

**Derivation of transfer function $\Delta v_o/d$ on the basis of average approach for small signals**

Figure 2.18(a) represents the equivalent circuit of the boost converter neglecting the resistance of the inductor, the switch and the diode and the forward voltage drop of the diode. In Fig.2.18(a) the series resistance of the capacitor $R_c$ are taken into consideration. The main effect of the capacitor series resistance in a transfer function is the appearance of a negative real zero at the frequency $\omega_z = 1/CR_c$.  

If switch $S$ is on

$$\frac{v_{in}}{v_o} = \frac{L}{R} + \frac{v_o}{R_c + X_c(s)} = 0 \hspace{1cm} \cdot d$$  \hspace{1cm} (2.76)$$

If switch $S$ is off

$$\frac{v_{in} - v_o}{v_o} = \frac{L}{R} i_L \hspace{1cm} \cdot (1 - d)$$  \hspace{1cm} (2.77)$$

![Diagram of Boost Converter](image)

(a) Boost converter  
(b) Equivalent circuit  
(c) Equivalent circuit referring to the output side

Figure 2.18: Small signal model of boost converter

After multiplying (2.76) by the weighting factor $d$ and (2.77) $(1 - d)$, respectively and adding the respective equations

$$v_{in} - (1 - d)v_o = v_{in} - d'v_o = L si_L$$  \hspace{1cm} (2.78)$$

$$v_o \left(\frac{1}{R} + \frac{1}{R_c + X_c(s)}\right) = (1 - d)i_L = d'i_L$$  \hspace{1cm} (2.79)$$
where \( d' = 1 - d \).

The equivalent circuit of the boost converter based on the equations (2.78) and (2.79) is shown in Fig.2.18(b), where the interpretation of the transformer in the DC circuit can be read from the two previous equation. Furthermore from (2.78) and (2.79)

\[
\frac{v_{in}}{d'} - v_o = \frac{L}{(d')^2} st_L
\]  

(2.80)

The equation is interpreted as follows: the input voltage \( v_{in} \) and inductance \( L \) are referred to the output side (Fig.2.18(c)).

Applying the principle of superposition as usual in an AC analysis all DC sources disappear therefore setting \( v_{in} = V_{in} = 0 \), the effect of a small change \( \Delta d \) in the duty ratio results in

\[
-(V_o + \Delta v_o)(d' - \Delta d) = L s(I_L + \Delta i_L)
\]  

(2.81)

or

\[
-\Delta v_o d' + V_o \Delta d = L s \Delta i_L
\]  

(2.82)

and in (2.79)

\[
G(V_o + \Delta v_o) = (d' - \Delta d)(I_L + \Delta i_L)
\]  

(2.83)

or

\[
G \Delta v_o = -\Delta d I_L + d' \Delta i_L
\]  

(2.84)

where letters \( V_o, I_L, d \) and \( d' = (1 - d) \) denote the steady-state values and \( \Delta \) refers to the small changes.

Substituting \( \Delta i_L \) from (2.82)

\[
G \Delta v_o = -\Delta d I_L + d' \frac{1}{L s} (V_o \Delta d - \Delta v_o d')
\]  

(2.85)

From here

\[
Y(s) = \frac{\Delta v_o(s)}{\Delta d(s)} = \frac{\frac{V_o d'}{L s} - I_L}{\frac{1}{R} + \frac{C_s}{1 + R_c C_s} + \frac{d'^2}{L s}}
\]  

(2.86)

In steady-state from (2.79)

\[
\frac{V_o}{R} = d' I_L \quad \rightarrow \quad I_L = \frac{V_o}{R d'}
\]  

(2.87)

Substituting the steady state value \( I_L \) into (2.86)

\[
Y(s) = \frac{\Delta v_o(s)}{\Delta d(s)} = \frac{V_o}{d'} \frac{(1 + R_c C_s)(1 - \frac{L/d'^2}{R} s)}{[(L/d'^2)C(1 + R_c/R)]s^2 + \left(\frac{L/d'^2}{R} + R_c C\right)s + 1}
\]  

(2.88)

It should be noted \( V_o/d' \) and \( L/d'^2 \) are included because the equivalent circuit was referred to the output side.

In voltage controlled closed-loop operation the transfer function

\[
\frac{\Delta v_o}{\Delta v_{ref}} = \frac{K(s) Y(s)}{1 + K(s) Y(s)}
\]  

(2.89)

where \( K(s) \) is the transfer function of the controller. The right hand side zero (RHSZ) \( (1 - \frac{L/d'^2}{R} s) \) implies instability in voltage controlled closed-loop operation due to the negative the sign in the denominator.

The stability picture is further aggravated because \( G_{delay} \) adds one or two additional RHSZ ((2.49), (2.50)).
2.5.6 Simulation, Calculation and Test Results

To verify the theoretical results obtained by the stability analysis just described both simulations, calculations and measurements were carried out. It should be noted that the stability analysis treated only the inner current loop and it was assumed that the outer slow voltage loop works properly. Naturally during the simulation and experiments the effect of the nonideal voltage loop are taken into consideration as well. The parameters of the PFC boost converter used in all three cases can be found in the Table 2.2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_m$</td>
<td>Amplitude of input voltage</td>
<td>34 V</td>
</tr>
<tr>
<td>$L$</td>
<td>Inductance</td>
<td>2.4 mH</td>
</tr>
<tr>
<td>$R_L$</td>
<td>Resistance of the inductance</td>
<td>0.2 Ω</td>
</tr>
<tr>
<td>$C$</td>
<td>Output capacitor</td>
<td>440 µF</td>
</tr>
<tr>
<td>$R$</td>
<td>Load resistance</td>
<td>200 Ω</td>
</tr>
<tr>
<td>$R_D$</td>
<td>On-state Resistance of the diode</td>
<td>0.025 Ω</td>
</tr>
<tr>
<td>$V_f$</td>
<td>Forward voltage drop of the diode</td>
<td>0.7 V</td>
</tr>
<tr>
<td>$R_{on}$</td>
<td>On-state Resistance of the MOSFET</td>
<td>0.55 Ω</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Efficiency</td>
<td>90 %</td>
</tr>
<tr>
<td>$T_{vi}$</td>
<td>Voltage controller integral time constant</td>
<td>20 ms</td>
</tr>
<tr>
<td>$K_{vi}$</td>
<td>Voltage controller gain</td>
<td>2</td>
</tr>
<tr>
<td>$f_{s,v}$</td>
<td>Sampling rate of voltage loop</td>
<td>2 kHz</td>
</tr>
<tr>
<td>$T_{ci}$</td>
<td>Current controller integral time constant</td>
<td>2.5 ms</td>
</tr>
<tr>
<td>$K_{pi}$</td>
<td>Current controller gain</td>
<td>1.6</td>
</tr>
<tr>
<td>$f_{s,i}$</td>
<td>Sampling rate of current loop</td>
<td>40 kHz</td>
</tr>
</tbody>
</table>

Simulation and Calculation Results

A complete PFC model was developed in MATLAB Simulink environment with the help of the SimPower Systems toolbox. To make the simulation more realistic the algorithm of the discrete outer voltage and inner current controllers with different sampling rate were implemented using the Embedded Matlab Function (EMF) block. The script in EMF is practically the same as the C code implemented in the DSC.

Figure 2.19(a) shows the inductor current when $K_{pi} = 1.6$. It can be seen that the PFC converter operates in its stable normal state. By increasing the gain $K_{pi}$ large oscillations begin to occur (Fig. 2.19(b)). The right side of Fig. 2.19(b) shows the close-up view of the inductor current around the critical angle $\theta_c \approx 20^\circ$ (when $V_{in} = V_m | \sin \theta_c |$), where the bifurcation occurs. The amplitude of the oscillation continuously increases until the system reaches a steady oscillating state similarly as in [72]. In order to see the loss of stability more clearly bifurcation diagrams are plotted when the bifurcation parameter is the $\theta$ angle of the input voltage. The inductor current $i_L$ and the $u_i$, which gives the duty ratio, are sampled at the beginning of each $T_{s,i}$ period both for $K_p = 1.6$ (blue dots) and $K_p = 1.9$ (red dots) as shown in Fig. 2.19(c) and 2.19(d), where the critical angle can clearly be identified. As it can be seen on Fig. 2.19(c) and 2.19(d) by applying $K_p = 1.9$ a bifurcation occurs as $\theta$ increases from zero to $\pi$. The response becomes unstable and the stable period-1 oscillations is changed to a period-3 and later to a period-6 oscillation.

The stability boundary is calculated using the auxiliary state vector discussed previously. The red dashed line ($h = 1$), the red continuous line ($h = 2$) and the red point-dash line in Fig. 2.20 show the critical angle $\theta_c$ as the function of $K_{pi}$. During the calculation the value of $K_{pi}$ was increased for a given input $V_m$ and $V_{ref}$ until the largest eigenvalue of the Jacobian matrix leaves the unit circle. The blue crosses denotes the stability border obtained by simulations. It can be concluded that the critical phase angles obtained by the stability analysis using second-order Padé approximation with ZOH and those found by simulations are practically the same. The stability borders using the first-order Padé approximation and ZOH or using second-order

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Padé approximation without taken into consideration of the ZOH deviate considerable from the simulated one.

Figure 2.21 shows the response for sudden change in the voltage reference signal. At \( t = 0.4 \) sec \( V_{\text{ref}} \) changes stepwise from its nominal value 55 V to 62 V. Without the stabilizing signal the largest eigenvalue leaves the unit circle at \( V_{\text{ref}} = 62 \) V and large oscillations begins to occur in the current signal (Fig.2.21(a)). By adding a stabilizing signal and using \( m_c = 0.35 \) (see (2.68)) the largest eigenvalues of the Jacobian matrix is kept in the unit circle and no oscillation occurs (Fig.2.21(b)). The value of \( m_c \) for \( V_{\text{ref}} = 62 \) V is calculated using (2.69) for \( h = 2 \) case. The value of \( m_c \) is increased from 0 until the largest eigenvalues of the Jacobian matrix enters into the unit circle. The inductor current \( i_L \) and the \( u_i \) are sampled after the stepwise change in steady state at the beginning of each \( T_{s,i} \) period both with (blue dots) and without (red dots) stabilizing signal as shown in Fig.2.21(c) and 2.21(d). Similarly to Fig.2.19(c) and 2.19(d) the bifurcation, where the stable period-1 oscillations is changed to a period-3 and later to period-6, can be clearly identified.
Laboratory Results

The PFC boost converter based on Fig. 2.11(a) was built and tested in the laboratory. Figure 2.22 shows the response of the PFC for the same $K_{pi}$ values as in Fig. 2.19. For simplicity in this case the input voltage $v_s$ and current $i_s$ were measured. Of course, practically they are the same before and after the rectification. The laboratory results are in good agreement with the stability analysis and the simulation clearly demonstrating that the stability method is capable to determine the maximum allowable $K_{pi}$ to obtain proper operation.

The effect of stabilizing signal was measured in the laboratory as well. Figure 2.23 shows the response of the PFC for sudden change in the reference voltage ($V_{ref}$ from 55 to 62 V) with ($M_c = 0.35$) and without stabilizing signal. Again for simplicity the input unrectified current $i_s$ is shown. It can be concluded that the measurement and the simulated results are in good agreement. The effect of the stabilizing signal is clearly demonstrated.
Figure 2.22: Measured input current $i_s$ and input voltage $v_s$ (Fig. 2.11(a))

(a) $K_{pi} = 1.6$

(b) $K_{pi} = 1.9$

Figure 2.23: Effect of stabilizing signal for sudden change in $V_{ref}$, $K_{pi} = 1.6$, Measurement

(a) without stabilizing signal

(b) with stabilizing signal $M_c = 0.35$
2.6 Thesis 2

2.a

The Jacobian matrix of the Poincaré Map Function (PMF) of the peak current mode controlled permanent magnet dc drive system can be straightforward determined without the derivation of the PMF by using the auxiliary state vector when proportional-integrator type speed controller is applied.

2.b

The critical phase angle, where oscillations start in the inductor current, of the digitally implemented average current mode controlled Power Factor Correction (PFC) converter can be calculated using the auxiliary state vector, when the digital computation delay was approximated by using the second-order Padé approximation and the effects caused by the Zero-Order Hold and the non ideal circuit elements were taken into consideration.

2.c

The stable period-1 range can be extended both in the DC drive and in the PFC converter system by adding a stabilizing ramp signal to the control loop. The slope of the ramp signal can be calculated by using the auxiliary state vector. The ramp signal for PFC converter should be modulated according to the sinusoidal input voltage.

Related publications of the author: [A9, A10, A11, A12, A13, A14]

Practical significance of the results

Switching-mode power converters are probably the most commonly used devices as they can be found from simple domestic applications (like PFC converter) to different high performance drive systems (like DC or AC motor drive). The ever increasing need for producing more reliable, more effective and less expensive systems make the analysis, understanding and design of such switch-mode power converters important, interesting and even imperative.

It is common to analyse the stability and dynamical behaviour of these variable structure piecewise-linear nonlinear systems by discarding the switching details and retain only the average dynamics of the system. While this average model can be analysed easier using several tools available from linear control theory, it fails to capture the instabilities as the effect of the switching action. Furthermore in linear systems theory the loss of stability implies that the state diverges without limit. However, in nonlinear systems the outcome of a stability loss does not lead to an unlimited explosion of the variables. These devices may exhibit undesirable irregular behaviours such as bifurcations and chaotic motion. The stability analysis method using the auxiliary state vector is able to detect the subharmonic and chaotic dynamics of the state variables that could occur in systems applying switch-mode power converters. The method inherently contains the feasibility to extend the stability range by adding a stabilizing signal in the control loop.
Chapter 3

Effect of Sampling of Space Vector Modulation in Field Oriented Control Drives

3.1 Motivation

As it was shown in Chapter 1 I investigated the effect of the sampling techniques on the harmonic content, mainly focusing on DC and subharmonic components generation of Space Vector Modulation (SVM) in open-loop. As Space Vector Modulation (SVM) facilitates the application of Field Oriented Control (FOC) [6] I decided to implement a field oriented controlled induction motor drive to investigate how the performance of the control loop can be improved only by the sampling technique of the SVM.

As there is a ”strong trend to avoid mechanical motion (speed/position) sensors because it reduces cost and improves reliability and functionality of the drive system” [6], my investigation is focused on a speed sensor-less FOC drive.

Furthermore, the analysis is concentrated on the case when shunt resistors are used to measure the stator currents as they are a popular solution due to the low system cost and exact current measurement. However, as in most cases shunt resistors are placed on the bottom of the inverter legs, the application of shunt resistor inherently limits the current sampling frequency as it is discussed later. It deteriorates the robustness of the control loop when the sampling over fundamental frequency ratio $F$ is low. It will be shown by simulation and laboratory experiment that by using Double Sampling (DS) SVM instead of the generally applied Regular Sampling (RS) technique the stability range of the drive system can be extended.

3.2 Introduction

In a modern closed loop controlled high speed drive systems, all the signal processes including the speed and current regulation loop and also the PWM block are implemented in the digital domain. Even with the up-to-date digital devices with clock frequency in the range of tens of MHz, the sampling frequency ($f_s$) is limited [12, 79, 80]. Its outcome is that the ratio of the sampling frequency and the actual fundamental frequency $F = f_s/f_1$ around the maximum speed of a high speed or high-pole count motor is also low, resulting in stability problems and sampling error in the regulation loop.

In [12] a complex proportional and integral controller with predictive active damping term is developed to stabilize the current loop, while a model-based error estimator is offered to compensate the sampling error. A novel $V/f$ control method with stabilising loops suitable for ultrahigh speed sensor-less drives with limited sampling frequency and computation time is derived in [79]. In [81] an enhanced stationary Proportional-Integrator (PI) technique is introduced to compensate the transport delay caused by the PWM process. The paper [82] analyzes the behavior of discrete-time current regulators for high-speed automotive drives and large-traction drives applying low sampling frequency. It investigates different design
methods and current regulator topologies. In [80] the speed sensor-less drive performance is investigated at high speeds, with very low sampling to fundamental frequency ratio \( F \) focusing on the problems of the rotor flux estimators. Field-angle correction method for speed sensor-less induction machine drives applying Field Oriented Control is discussed in [83]. Various compensation methods and a modified structure for a PI current controller is proposed in [84] to reduce the carrier frequency for the same fundamental frequency for large AC drive systems.

It can be concluded from the paper cited that the limited ratio of the sampling frequency and the actual fundamental frequency \( F = f_s/f_1 \) poses stability issues not only in the current regulation of FOC drives, but also in current control of active filters. Paper [85] discussed the multisampling approach using FPGA to improve the performance of the current control loop. The paper [86] presents an improved current regulation strategy for active filter using state feedback concept compensating the PWM delay resulting that the controller gain can be significantly increased.

The main purpose of this investigation presented in this chapter is to compare two sampling techniques of the SVM process on the performance of a digitally implemented field oriented controlled induction machine drive when \( F \) is low.

### 3.3 Speed Sensor-less Field Oriented Control

#### 3.3.1 Overview

The principal aim of the Field Oriented Control (FOC), proposed by Hasse [87] and Blaschke [88], is to control independently the flux and torque in the induction machine, in a similar way to the control of separately excited DC machine in order to change fast its electric torque [26, 6, 89]. As it was shown in Chapter 1 the three phase stator currents can be expressed as a current space vector \( i_s \) in the \( \alpha - \beta \) stationary reference frame (SRF) (Fig.3.1) where \( i_s \) revolves with synchronous angular speed \( \omega_1 = 2\pi f_1 \) in steady-state. Let us introduce a rotating reference frame (RRF) rotating at synchronous angular speed \( \omega_1 \) comprising the direct and quadrature axis (denoted by \( d \) and \( q \)). In RRF the current space vector \( i_{sd} = e^{-j\sigma}i_s \) can be decomposed into two dc quantities (Fig.3.1)\(^1\). If the real axis \( d \) of the RRF is coinciding with the direction of the rotor flux \( \Psi_r \), the \( d \) and \( q \) axis current components of \( i_s \) are the rotor flux and torque producing component, respectively. As the two current components \( i_{sd} \) and \( i_{sq} \) are orthogonal to each other, the rotor flux \( \Psi_r \) and the electric torque \( T_e \) can be controlled independently. They can be expressed as [26]

\[
\Psi_r = \frac{L_m}{1 + sT_r} i_{sd} \tag{3.1}
\]

\[
T_e = \frac{3}{2} p \frac{1}{1 + \sigma} \Psi_r i_{sq} \tag{3.2}
\]

where \( L_m \) is the mutual inductance, \( T_r = L_r/R_r \) is the rotor time constant for open stator circuits, \( L_r = L_m + L_{lr} \) is the rotor inductance, \( L_{lr} \) is the rotor leakage reactance, \( R_r \) is the resistance of one rotor phase, \( p \) is the number of pole pairs and \( \sigma = \frac{L_{lr}}{L_m} - 1 \). 3 stands for 3 phases, 2 stands for absolute values of space vector \( \Psi_r \) and \( i_s \).

By keeping the amplitude of the rotor flux constant, \( i_{sq} \) represents the motor electric torque \( T_e \) so the output of the speed controller can be directly used as a set point for the quadrature current component denoted by \( i_{sq}^\ast \). The rotor flux can be influenced by the direct component \( i_{sd} \) with the time constant \( T_r \). Thus, the reference point \( i_{sd}^\ast \) can be provided by the output of the flux controller. When the rotor flux is kept constant, in some cases the flux controller can be omitted and the reference point of \( i_{sd}^\ast \) is set to a constant value.

The schematic block diagram of the speed sensor-less Field Oriented Control investigated in the dissertation can be seen in Fig.3.2. The control loop contains two controllers in the outer loop, one for the flux and one for the speed. For simplicity the flux controller set the reference value of \( i_{sd}^\ast \) from \( \Psi_r^\ast \) and \( \Omega^\ast \) using a look-up-table. The value of the reference speed \( \Omega^\ast \) is required to operate the drive in field weakening region as well. The inner loop is

\(^1\)For better understanding the rotor of synchronous machine is drawn in Fig.3.1.
Figure 3.1: Stator current space vector $i_s$ in stationary reference frame (SRF) $\alpha - \beta$ and the rotating reference frame (RRF) $d - q$ fixed to the rotor flux space vector $\Psi_r$

formed by two separate PI current controllers. The outputs of the current controllers are the reference voltage $v^*_{sd}$ and $v^*_{sq}$ in RRF. First, the SVM applies inverse Park transformation to generate reference voltage $v^*_\alpha$ and $v^*_\beta$ in SRF. Second, it calculates the reference signal $v_{ref,i}$ ($i = a, b, c$) which are compared to the triangular carrier signal having frequency $f_c$ to produce the switching signals driving the inverter. The mechanical speed $\Omega_{est}$ and rotor flux angle $\rho_{est}$ are estimated using by measuring the stator currents in phase $a$ and $b$.

### 3.3.2 Estimator

As the speed sensor is missing (Fig.3.2), an estimator block is applied to determine the mechanical speed $\Omega$ and the flux angle $\rho$ (see Fig.3.1). The estimator can provide only their estimated values, $\Omega_{est}$ and $\rho_{est}$ (Fig.3.2). In order to estimate them the rotation angular speed $\omega_{1,est}$ of the rotor flux space vector has to be estimated as well.

The inputs of the Estimator block is the phase currents of the motor in the $\alpha - \beta$ stationary reference frame and the reference values $v^*_{\alpha}, v^*_{\beta}$ of the Space Vector Modulation block produced by the $d$ and $q$ axis PI current control blocks, respectively.

The squirrel cage induction machine in the stationary reference frame $\alpha - \beta$ can be described by the following two differential equations expressing the stator and rotor voltage balance

$$ v_s = R_s i_s + \frac{d\Psi_s}{dt} \tag{3.3} $$

$$ v_r = R_r i_r + \frac{d\Psi_r}{dt} - j\omega \Psi_r = 0 \tag{3.4} $$

and by the stator $\Psi_s$ and rotor $\Psi_r$ leakage flux relations

$$ \Psi_s = L_s i_s + L_m i_r \tag{3.5} $$

$$ \Psi_r = L_m i_s + L_r i_r \tag{3.6} $$

where $\omega$ is the mechanical angular speed, $R_s$ is the resistance in one stator phase, $L_s = L_m + L_{ls}$ and $L_{ls}$ is the stator total and leakage inductance, respectively. Once again it is emphasized that all space vectors, even $v_r$, $i_r$ and $\Psi_r$ are written in SRF.
As the estimator block does not require the stator flux $\Psi_s$ and the almost unmeasurable rotor current $i_r$, they have to be eliminated from the equations (3.3)- (3.6). After some manipulation the set of differential equations can be expressed as

$$v_s = R_s i_s + \sigma L_s \frac{di_s}{dt} + \frac{L_m}{L_r} \frac{d\Psi_r}{dt}$$

$$0 = - \frac{L_m}{T_{ss}} i_s + \left( \frac{1}{T_r} - j\omega \right) \Psi_r + \frac{d\Psi_r}{dt}$$

where $\sigma = 1 - \frac{L_s^2}{L_r T_s}$ is the total leakage factor and $T_{ss} = L_s/R_s$ is the stator time constant when the rotor circuits are open. The last term in (3.7) is the Back Electro Motive Force (BEMF) later denoted by vector $e_{\alpha\beta}$.

The two components of the $e_{\alpha\beta}$ in the $\alpha - \beta$ frame are

$$e_\alpha = v_\alpha - R_s i_{s\alpha} - \sigma L_s \frac{di_{s\alpha}}{dt}$$

$$e_\beta = v_\beta - R_s i_{s\beta} - \sigma L_s \frac{di_{s\beta}}{dt}$$

Space vector $e_{\alpha\beta}$ can be transformed from $\alpha-\beta$ SRF in the $d-q$ RRF by using the estimated flux angle $\rho_{est}$ by equation (see Fig.3.3)

$$e_{dq,est} = e^{-j\rho_{est}} e_{\alpha\beta}$$

or by

$$e_{d,est} = e_\alpha \cos(\rho_{est}) + e_\beta \sin(\rho_{est})$$

$$e_{q,est} = -e_\alpha \sin(\rho_{est}) + e_\beta \cos(\rho_{est})$$

As $e$ is leading by 90° the rotor flux $\Psi_r$ (see (3.7)) in steady-state its $d$ axis component should be zero. If the estimated $e_{dq,est}$ is not equal to the accurate $e_{dq}$, the angle difference $\Delta\rho$ between the accurate flux angle $\rho$ and the estimated flux angle $\rho_{est}$, that is, $\Delta\rho = \rho - \rho_{est}$ is not zero resulting in $e_{d,est} \neq 0$ [89]. Figure 3.3(a) and 3.3(b) shows when the $d$ component of the estimated $e_{dq,est}$ is negative ($\Delta\rho > 0$) and positive ($\Delta\rho < 0$), respectively.
To determine the $d$ and $q$ component of BEMF by the rotor flux let us transform $e_{\alpha\beta}$ = $L_m \frac{d\Psi_r}{dt}$ from SRF into RRF (see (3.7)). For simplicity suffix "est" will be omitted. Substituting $e_{\alpha\beta} = e^{j\rho}e_d$ and $\Psi_r = e^{j\rho}\Psi_{r,dq}$ in to the equation of $e_{\alpha\beta}$ (see (3.7))

$$L_r e^{j\rho}e_d = j e^{j\rho} \frac{d\rho}{dt} \Psi_{r,dq} + e^{j\rho} \frac{d\Psi_{r,dq}}{dt}$$

(3.14)

Cancelling the term $e^{j\rho}$ on both side and taking into account $\omega_1 = d\rho/dt$, the components of $e_d$

$$e_{d,est} = \frac{1}{1 + \sigma_r} \frac{d\Psi_r}{dt} \rightarrow 0$$

(3.15)

$$e_{q,est} = \frac{1}{1 + \sigma_r} \omega_1 \Psi_r$$

(3.16)

where we use suffix "est" again and we need $\rho_{est}$ to determine $e_{d,est}$ and $e_{q,est}$ (see (3.9) and (3.10)). Furthermore we have taken into account that space vector $\Psi_{r,dq}$ has only $d$ component $\Psi_{r,dq} = \Psi_{r,d}$. Remember: $\sigma_r = \frac{L_m}{L_r} - 1$ (see (3.2)).

The estimated rotor flux rotational speed from (3.16) is

$$\omega_{1,est} = \frac{1 + \sigma_r}{\Psi_r} e_{q,est}$$

(3.17)

Error in the estimation generates a non-zero $d$ axis component of the BEMF. The larger the value $e_{d,est}$, the larger error. It can be corrected by the following expression [89]

$$\omega_{1,est} = \frac{1 + \sigma_r}{\Psi_r} \left( e_{q,est} - sqn(e_{q,est}e_{d,est}) \right)$$

(3.18)

Depending on the direction of rotation Table 3.1 summarizes the effect of correction term.

The flux angle can be calculated

$$\rho_{est} = \int \omega_{1,est} dt$$

(3.19)

The mechanical angular speed $\Omega$ can be obtained from the difference of the synchronous angular speed $\omega_1$ and the slip angular speed $\omega_2$. The rotor voltage balance in reference frame rotating with $\omega_1$ is

$$0 = R_r i_{r,dq} + j(\omega_1 - \omega) \Psi_r$$

(3.20)
Table 3.1: Correction of $\omega_{1,\text{est}}$

<table>
<thead>
<tr>
<th>Direction of rotation</th>
<th>Sign of error</th>
<th>Correction term</th>
<th>Action on $\omega_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive speed ($e_{q,\text{est}} &gt; 0$)</td>
<td>$e_{d,\text{est}} &gt; 0$</td>
<td>$-e_{d,\text{est}}$</td>
<td>decrease ↓</td>
</tr>
<tr>
<td>Positive speed ($e_{q,\text{est}} &gt; 0$)</td>
<td>$e_{d,\text{est}} &lt; 0$</td>
<td>$-e_{d,\text{est}}$</td>
<td>increase ↑</td>
</tr>
<tr>
<td>Negative speed ($e_{q,\text{est}} &lt; 0$)</td>
<td>$e_{d,\text{est}} &gt; 0$</td>
<td>$e_{d,\text{est}}$</td>
<td>increase ↑</td>
</tr>
<tr>
<td>Negative speed ($e_{q,\text{est}} &lt; 0$)</td>
<td>$e_{d,\text{est}} &lt; 0$</td>
<td>$e_{d,\text{est}}$</td>
<td>decrease ↓</td>
</tr>
</tbody>
</table>

Figure 3.4: Estimator block diagram

where $\omega_1 - \omega = \omega_2$ is the slip angular speed. The rotor flux linkage

$$\Psi_r = L_m i_{s,dq} + L_r i_{r,dq} \quad (3.21)$$

Substituting $i_{r,dq}$ into th rotor voltage balance equation and taking into account only the imaginary part of the equation, the slip speed in the RRF can be expressed as [26]

$$\omega_2,\text{est} = \frac{R_{r,\text{est}}}{\sigma_r + 1} \frac{1}{\Psi_{r,\text{ref}}} i_{sq,\text{est}} \quad (3.22)$$

where suffix "est" was used again. Here constant rotor flux $\Psi_r = \Psi_{r,\text{ref}}$ was assumed. We have to be aware that all three quantities in (3.22) are estimated quantities.

The estimated mechanical angular speed is

$$\Omega_{\text{est}} = \frac{\omega_{1,\text{est}} - \omega_{2,\text{est}}}{p} \quad (3.23)$$

where $p$ is the number of pole pairs.

Figure 3.4 shows the block diagram of the estimator block based on the equations described in this subchapter and written in the relevant parts. The estimator is used in the speed sensorless FOC drive (see Fig.3.2) [89]. The estimated BEMF is low-pass filtered to reduce the noise generated by the currents derivation. The time constant of the filter should be selected to significantly reduce the noise but not to introduce dynamic changes in the control loop.
3.3.3 Current Sampling

In modern high-performance closed loop drive systems, all signal processes including the processes in speed and the current regulation loop and also the PWM block are implemented in the digital domain. Both the signal flow of the estimator presented in 3.3.2 in the continuous time domain and that of the continuous PI controllers have to be discretized.

Generally the stator phase currents are synchronously sampled once or twice in one carrier period to eliminate the current ripple without requiring low-pass filtering or complex ripple-elimination filters [34, 81]. To demonstrate this Fig.3.5 shows the measured and the sampled $i_{sa}$ and $i_{sb}$ stator currents in no-load obtained by me from the memory of the microcontroller when the sampling frequency was equal to the carrier frequency $f_s = f_c$ and double of the carrier frequency $f_s = 2f_c$ (in the figure $f_c = 1.4$ kHz, $f_1 = 70$ Hz). It is clearly visible that in both case the sampled currents are practically perfect sinusoidal signal while the stator currents have a high harmonic distortion due to the low $m_f$ ratio ($m_f = 20$). The sampling frequency is also limited as it is advisory to update the duty ratio of the digital PWM peripheral maximum twice during a carrier period to avoid intermediate PWM transitions and glitches in the switching signals.

The drawback of the solution is that the sampling frequency is limited and the delay caused by the PWM peripheral (see later) introduces a phase lag limiting the achievable control bandwidth and deteriorating the performance of the control loop. In general-purpose ac drive applications this effect can be neglected, but it can be crucial when both the sampling over fundamental frequency ratio $F$ and the carrier over fundamental frequency $m_f$ is low. In [85] a multisampling strategy is introduced for active filters by using FPGA. The disadvantage of the method is the required complex filter to eliminate the ripples in the current signal and the unwanted intermediate PWM transitions, that can occur.

Another factor influencing the sampling frequency is the applied current sensing circuit. Generally two main methods are applied. In the first one current transducer is used. By placing the transducers in the output phases the current signals can be continuously measured. Here there is no limitations regarding the sampling frequency.
The other method is the application of shunt resistor to measure the phase currents. It is a popular solution due to the low system cost and exact current measurement. The voltage across the resistor is proportional to the current. It is filtered, shifted and finally amplified and fed to the ADC module of the microcontrollers. In up-to-date IGBT power modules the current shunt resistors are placed directly on the printed circuit board. Several topologies can be used, but most often the resistor is placed on the bottom of the inverter legs\(^2\) (Fig.3.6) The drawback of the solution is that current flows across the resistor only when the lower switch conducts resulting a discontinuous \(v_{sh}\) voltage signal as shown in Fig.3.6. By applying SVM and linear modulation \((m_a \leq \sqrt{3}/2)\) all the three lower switches conduct at the positive peak of the triangular signal (see Fig.1.5(b) and 1.10). It results that the phase currents have to be sampled at the positive peaks of the carrier signal resulting in limited sampling frequency that is the same as the carrier frequency \(f_s = f_c\). It can be seen in Fig.3.6 that the width of the voltage pulses in \(v_{sh}\) are greater when the current is negative. Let us assume that the power factor is almost unity, \(\cos \varphi \approx 1\). Now the fundamental component of the reference signal of SVM has practically the same shape as the current signal in Fig.3.6. As the lower switch conducts when the triangular carrier signal is greater than the reference signal, the width of the voltage pulses are greater when the current signal and therefore the reference voltage is negative.

In another topology, similarly to the current transducers, the resistors are placed in the output phases. The drawback of this solution is that the resistors are working at floating potential requiring a more complex isolated sensing circuitry.

Due to the losses in the current shunt resistors the application of shunt resistors are confined to the low power applications. However by placing the resistors as close as possible to the heat sink and using special precision shunt resistor, the method are applied in medium and high power range, where mostly current transducers are utilised [90].

Later on the dissertation is focused only on the case when shunt resistor is applied to measure the stator currents. Furthermore the shunt resistors are placed in the bottom of the inverter legs.

Figure 3.7(a) and 3.7(b) show the digitally implemented signal flow by using two different SVM sampling techniques. In both techniques the current sampling is synchronized to the carrier signal. The current sampling takes place at the positive peak of the carrier signal \((T_s = T_c)\)(see bold sentence a few lines earlier). After measuring and sampling the stator currents \(i_{sa}\) and \(i_{sb}\) in two phases by using a 12-bit Analog-Digital Converter (ADC) the mechanical speed \(\Omega_{est}\) and the angle \(\rho_{est}\) are estimated using the values \(v^*_a(k-1)\) and \(v^*_b(k-1)\) obtained in the previous sampling period (see Fig.3.4). Proportional integral (PI) controllers regulate the stator voltages in the \(d - q\) frame by setting \(v^*_{sa}\) and \(v^*_{sq}\) to achieve the calculated reference stator currents \(i_{sa}^*\) and \(i_{sb}^*\) (Fig.3.2). The output signals of the controllers and the estimated flux angle \(\rho_{est}\) are the input signals for the Space Vector Modulation (SVM) block. It

\(^2\)The commercially available FSBB30CH60C IGBT module from the company Fairchild used during the measurements applied this topology.
calculates the reference voltage component \( v_{\alpha}^* \) and \( v_{\beta}^* \) and the reference signals \( v_{ref,i} \) \( i = a, b, c \) for each phase. They are latched into the Compare Registers \( CR_i, (i = a, b, c) \) of the PWM peripheral of the microcontroller to generate the switching signals.

It should be noted that the sampling of the current signals, the calculation of the estimator, the control and the SVM algorithm take less time than \( T_c/2 \). In spite of this the microcontroller vendor suggests to update the registers of the digital PWM peripheral only in the next half carrier period (see Fig.1.8 in Chapter 1). It means that the reference signals \( v_{ref,i} \) \( i = a, b, c \) calculated before the negative peak are latched into the \( CR_i, (i = a, b, c) \) registers only at the negative peak and vice versa. It results in a constant \( \frac{T_c}{2} \) time delay in the control algorithm between the current sampling and the update of the duty ratios independently of the applied SVM sampling technique (Fig. 3.7(a) and 3.7(b)).

### 3.3.4 Sampling Techniques of Space Vector Modulation

As it was discussed in Chapter 1 in Point 1.4 based on the sampling of the reference signals \( v_{ref,i} \) \( i = a, b, c \) that three different sampling techniques of carrier based PWM can be classified: Regular Sampling (RS), Natural Sampling (NS) and Oversampling (OS). In this chapter only RS and OS are studied. Furthermore, within the OS technique only the number of samples \( n = 2 \) (Doublesampling, DS) is discussed. As mostly Space Vector Modulation (SVM) is used as PWM technique in FOC drives the other two widely applied carrier based PWM technique SPWM and THI-PWM are not included in this chapter.

**Regular Sampling (RS) (Fig.3.7(a))

If the current sampling takes place only once during a carrier period the most common solution is the RS [89, 26]. After the current sampling at the positive peak of the carrier signal, the estimator, the control algorithm and finally the SVM block calculate the reference voltage \( v_{\alpha,est}^* \) and \( v_{\beta,est}^* \) and the reference signals \( v_{ref,i} \) \( i = a, b, c \) for each phase, which are latched into the Compare Registers \( CR_i, (i = a, b, c) \) of the PWM peripheral half period later, at the negative peak of the carrier signal. The value of the reference signals \( v_{ref,i} \) \( i = a, b, c \) are kept constant in one \( T_c = T_s \) period.

**Double Sampling (DS) (Fig.3.7(b))

Based on my investigations by using DS, the accuracy of the SVM algorithm and the stable range of the drive can be extended by recalculating the reference voltages \( v_{\alpha,est}^* \) and \( v_{\beta,est}^* \) and the reference signals \( v_{ref,i} \) \( i = a, b, c \) after the negative peak of the carrier signal as well. However, at this point there is no current sampling and in this way neither the estimator nor the control algorithm is called, only the change in the rotor flux angle over half carrier period has to be approximated. By assuming that \( \omega_1 \) is constant over a carrier period the rotor flux angle \( \rho_{est} \) at the negative peak of the carrier signal can be approximated in the \( k^{th} \) period as \( \rho_{est}(k) + \delta \rho(k) \) where \( \delta \rho(k) = \frac{\rho_{est}(k) - \rho_{est}(k-1)}{2} \). Here \( \rho_{est}(k) \) and \( \rho_{est}(k-1) \) is the calculated rotor flux angle by the estimator algorithm in the \( k^{th} \) and \( (k-1)^{th} \) sampling period, respectively. The division by two is required as the estimator algorithm calculates the rotor flux angle at the positive peaks of the carrier signal. By using \( \rho_{est}(k) + \delta \rho(k) \) the reference voltages \( v_{\alpha,est}^* \) and \( v_{\beta,est}^* \) and the reference signals \( v_{ref,i} \) \( i = a, b, c \) can be updated using \( v_{\alpha,i}^* \) and \( v_{\beta,i}^* \) calculated by the control algorithm after the positive peak of the carrier signal. The \( v_{ref,i} \) calculated after the negative peak of the carrier signal are latched into the \( CR_i, (i = a, b, c) \) of the PWM peripheral half period later at the positive peak of the carrier signal. It should be noted that the estimator algorithm called at the positive peak of the carrier signal, uses \( v_{\alpha,est}^* \) and \( v_{\beta,est}^* \) calculated after the previous negative peak to estimate the mechanical speed and the rotor flux angle (see Fig.3.4).

\(^3\)Using dsPIC microcontroller and fixed point arithmetic, it takes circa 20\( \mu \)s
3.3.5 Small Signal Laplace-Domain Analysis

Transfer function of digital PWM process

As it was mentioned all signal processes including the speed and current regulation loop and also the PWM block are implemented in the digital domain. In this subsection based on [34, 91, 85] the dynamic response considering the delay generated by digital PWM technique will be treated assuming triangular wave carrier signal.

The general block diagram of a digital PWM process is presented in Fig.3.8 [85]. The reference signal assumed to be sinusoidal, but the derived transfer function holds for other reference signals as well, e.g. for SVM. The continuous reference signal $v_{ref}$ is processed by a Zero-Order-Hold (ZOH). The sampling time is $T_c/N$, where $N$ is the number of samples during a carrier period. The output PWM waveform is generated by an ideal comparator. It compares the output signal of the ZOH and the triangular carrier waveform $v_{car}$.

Due to the sample and hold effect, the response of the modulator to any disturbance taking place between two consecutive sampling actions, like a stepwise change in the reference signal, appears only from and after the next sampling instant. This delay could result in a considerable difference to the analog modulator when $m_f$ is low, where the response takes place almost with negligible delay [34].
Assuming that the number of samples \( N = 1 \), the small-signal transfer function \( G_{PWM,N=1} \) of the digital PWM process can be derived based in Fig.3.8 by applying a small perturbation \( \Delta v_{ref} \) superimposed to the steady-state reference signal \( v_{ref} \) and following the effect of the corresponding perturbation \( \Delta d \) in the duty cycle of the output signal. As shown in [34, 91, 85] the calculation yields

\[
G_{PWM,N=1} = \Delta d(s) / \Delta v_{ref}(s) = \left( e^{-s(1-d)T_c/2} + e^{-s(1+d)T_c/2} \right)
\]  

(3.24)

where \( d \) is the steady state duty ratio belonging to \( v_{ref} \). Based on the transfer function it can be concluded that the time delay introduced by the digital PWM process is the time span between the last sampling instant taken from the reference signal \( v_{ref} \) and the instant when the output pulse is completely determined, when \( v_{ref} \) intersects \( v_{car} \). Equation (3.24) can be approximated as [34, 91, 85]

\[
G_{PWM,N=1} \approx e^{-sT_c/2}
\]

(3.25)

From (3.25) the equivalent delay time is the half the carrier period. The time delay can change from zero to \( T_c \). The value of \( T_c/2 \) is an average one.

By doubling the number of samples \((N = 2)\) (Fig.3.8) the equivalent delay time decreases. The transfer function of the digital PWM process when \( N = 2 \) is [34, 91, 85]

\[
G_{PWM,N=2} = \Delta d(s) / \Delta v_{ref}(s) = \left( e^{-sdT_c/2} + e^{-s(1-d)T_c/2} \right) \approx e^{-sT_c/4}
\]

(3.26)

Similarly by increasing the number of samples \( N \) the equivalent delay time is reduced [34, 91, 85]

\[
G_{PWM,N} \approx e^{-sT_c/2N}
\]

(3.27)

If the number of sampling \( N \) tends to infinity, the digital PWM process approaches the performance of the analog PWM process and the equivalent delay time tends to zero.

Thus, if the number of samplings of the reference signal is doubled as it was presented in Point 3.3.4 the delay caused by the PWM process can be reduced.

**Transfer function of SVM sampling techniques with the VSC**

In the next section the \( q \) axis speed control loop of the FOC will be analyzed using linear system theory. The aim of the analysis is to investigate the effect of the sampling technique of the SVM on the stability of the speed control loop. The dynamics of the SVM process together with the VSC (see Fig.3.2) by assuming ideal switches can be modelled by a pure time delay as was just shown. As it was discussed in the previous section the time delay depends on the...
number of samples $N$ of the SVM reference signal over a carrier period. In this chapter two SVM sampling techniques are studied: RS ($N = 1$) and DS ($N = 2$).

The input signals of the SVM Block process are the reference voltages $v_{sd}^*$ and $v_{sq}^*$ in the $d-q$ RRF (Fig.3.2). The SVM calculates the duty ratio values and generates the switching signal of the switches of the VSC producing the output voltage $v_{sd}$ and $v_{sq}$ acting on the induction machine in the RRF. Depending on the number of sampling in the reference signal over a carrier period, the dynamics of the SVM and the VSC can be approximated by a pure delay according to (3.27). By using the first order Padé approximation (see (2.49) in Chapter 2) the SVM process using different sampling techniques ($N = 1$ for RS, $N = 2$ for DS) can be given by the following transfer functions

$$G_{SVM, RS} = \frac{v_{sd}}{v_{sd}^*} = \frac{v_{sq}}{v_{sq}^*} \approx e^{-s \frac{T_c}{2}} \approx \frac{1 - \frac{T_c}{2} s}{1 + \frac{T_c}{2} s}$$  \hspace{1cm} (3.28)

$$G_{SVM, DS} = \frac{v_d}{v_d^*} = \frac{v_q}{v_q^*} \approx e^{-s \frac{T_c}{4}} \approx \frac{1 - \frac{T_c}{4} s}{1 + \frac{T_c}{4} s}$$  \hspace{1cm} (3.29)

**Small signal analysis of the speed control loop**

The approximate transfer functions $G_{SVM}$ can be used to investigate the effect of the sampling process on the stability of the FOC. The dissertation is focused on the $q$ axis speed control loop by assuming that the $d$ axis flux control loop works properly. Figure 3.9(a) and 3.9(b) show the block diagram and the time sequence of the digitally implemented speed control loop, respectively. The continuous stator current is sampled with a Sample&Hold element with frequency $f_s = 1/T_s$. As the mechanical speed is calculated by the estimator from the sampled stator current and stator voltage signals, the mechanical speed is also a discrete sampled signal. Furthermore the reference speed $\Omega^*$ is also a sampled signal as its changes are taken into consideration only when the discrete speed controller is calculated. As it was mentioned previously the register of the digital PWM peripheral can be updated only in the next half carrier period resulting in a $T_c/2$ fixed timed delay in the stator voltage signal.

Figure 3.9(c) shows the small-signal model of the $q$-axis speed control loop in the continuous domain by taking into account the effect of the digital implementation by approximate continuous transfer functions. During the small signal analysis we assume that the estimator discussed previously works properly and the estimated speed $\Omega_{est}$ is equal to the actual ones.

At the beginning of each sampling period the speed controller calculates the $i_{sq}^*$ reference current signal from the difference of the estimated and reference speed. The PI controller of the inner current loop calculates the reference voltage $v_{sq}^*$ by using the sampled and transformed signal of the stator phase current. $v_{sq}^*$ is the input reference signal of the SVM modulated VSC. The effect of sampling process, which is modelled by a Zero-Order Hold (ZOH), in the stator current $i_{sq}$, in the mechanical speed $\Omega$ and in the reference speed $\Omega^*$ should be taken into consideration. To simplify the analysis, it is desired to have a unity feedback, thus the effect of the ZOH is taken into consideration once at the output of the PI current controller (see Fig.3.9(c)). It is the same when ZOH elements are taken into consideration both in the feedback loop of $i_{sq}$ and $\Omega$ and the feedforward loop of $\Omega^*$. The ZOH can be given by the following transfer function [77, 34]

$$G_{ZOH}(s) = \frac{1 - e^{-T_s s}}{T_s s}$$  \hspace{1cm} (3.30)

which can be approximated by using the first order Padé-approximation (see (2.49) in Chapter 2)

$$G_{ZOH}(s) \approx \frac{1}{1 + 0.5T_s s}$$  \hspace{1cm} (3.31)

As it was mentioned that after the sampling process, the calculation of the estimator and the control algorithm takes less time than $T_c/2$, but it is advisory to update the registers of the digital PWM peripheral of the microcontroller only in the next half carrier period. It
(a) Block diagram of the digitally implemented q-axis speed control loop

(b) Time sequence of the q-axis speed control loop

(c) Continuous small-signal model of the q-axis speed control

Figure 3.9: Block diagram, time sequence and the continuous small-signal model of the q-axis speed control loop
results in a fixed delay expressed by $G_{delay} = e^{-\frac{s}{T_{c}}}$ (Fig.3.9(c)). By using the first order Padé approximation

$$G_{delay} = e^{-\frac{s}{T_{c}}} \approx \frac{1 - \frac{T_{c}}{4}s}{1 + \frac{T_{c}}{4}s} \quad (3.32)$$

As it was discussed previously the SVM sampling process introduces an additional delay which can be given by (3.28) for RS and (3.29) for DS.

By transforming the differential equation (3.7) in the d-q RRF, the $q$ axis component can be expressed as

$$v_{sq} = R_s i_{sq} + \sigma L_s \frac{di_{sq}}{dt} + e_q \quad (3.33)$$

The Laplace transform of (3.33) is

$$v_{sq} - e_q = (R_s + \sigma L_s) i_{sq} \quad (3.34)$$

Thus, as for small changes $e_q$ can be assumed constant, the $q$ axis stator current $i_{sq}$ follows the changes in $v_{sq}$ by the transient time constant $T'_c = \frac{\sigma L_s}{R_s}$, where $\sigma L_s$ is the so-called transient inductance. Assuming constant rotor flux the electrical torque is proportional to the $q$ axis current $i_{sq}$ (see (3.2)). The dynamic torque $T_d = T_e - T_{load}$ accelerates the rotor. Neglecting the damping, the transfer function $\frac{\Omega}{T_d} = \frac{1}{J s}$ gives the relation between the mechanical speed and the dynamic torque, where $J$ is the inertia of the rotor.

Figure 3.10 shows the open loop Bode diagram of the $\frac{\Omega}{T_d}$ speed control loop shown in Fig.3.9(c) using RS and DS for the same set of controller parameters (the parameters of the motor and the controllers can be found in the next section in Table 3.2 and 3.3). Here shunt resistor is applied to measure the stator current ($T_s = T_c$, and $f_c = 1.4$ kHz, Fig.3.10). During the analysis the $e_q$ BEMF term is assumed to be constant for small changes and it is considered that the load torque $T_{load}$ is zero.

Based on the continuous small-signal model of the $q$ axis speed control loop shown in Fig.3.9 and the Bode-plot (Fig.3.10) two main conclusions can be derived:

1. The small-signal model in Fig.3.9(c) takes into account only the effect of $T_c = 1/f_c$ and $T_s = 1/f_s$ and it is not capable to model the effect of the fundamental frequency $f_1$ and in this way the effect of the carrier to fundamental frequency ratio $m_f = f_c/f_1$ and the sampling to fundamental frequency ratio $F = f_s/f_1$. As it will be shown later by simulation and experimental results the stability highly depends on the ratio $m_f$ and $F$.

2. Based on the phase plots in Fig.3.10, it can be concluded that at the same set of controller parameter the DS has larger phase margin and in this way its performance is more robust. Thus, it is worth to resample the SVM signal as theoretically a more robust performance can be obtained, even when no current sampling, estimation and control loop calculation take place. As it will be shown in the next section the simulation and experimental results prove this statement.
3.4 Simulation and Experimental Results

To investigate the effect of the sampling technique of SVM on the operation of the speed sensor-less FOC drive both simulation and experimental analysis were carried out. The rated data and parameters of the motor used both during simulation and laboratory tests can be found in Table 3.2. The parameters of the PI controllers can be found in Table 3.3. The same controller parameters were used when RS and DS sampling technique were applied. The controller parameters are tuned based on the small-signal model of the q-axis speed control loop (Fig.3.9(a)) and the d-axis flux loop (not discussed in the dissertation) to obtain a fast and robust response.

The complete simulation model of the drive was developed in MATLAB Simulink environment with the help of the SimPower Systems toolbox. To make the simulation more realistic the algorithm of the estimator and the controllers were implemented using the Embedded Matlab Function (EMF) block. The script in EMF is practically the same as the C code, which was implemented in the microcontroller. The effect of the time delay caused by the PWM update was also taken into consideration by a “Unit delay” block available in the toolbox of the Simulink with a sample time $T_c/2$.

The drive using shunt resistor to measure the phase currents was built and tested in the laboratory as well. The discrete version of the estimator and the control algorithm was implemented in a dsPIC Digital Signal Controller\(^4\). In the experimental setup two identical induction machines were mechanically coupled. One of them was run as the test machine while the other acted as a generating load. By changing the synchronous $f_1$ frequency of the generator the load torque acting on the test machine could be varied. It should be noted as the two machines were directly coupled together, this two-mass mechanical system inevitably exhibits mechanical vibration, also called torsional oscillations. The effect of this oscillation on the dynamics of the system was ignored during simulation, but it could be observed as superimposed component on the measured time functions. However the ripples are considerable lower compared to the fundamental signals. As no speed sensor was applied later on only the time function of the measured electric torque $T_e$ and the trace of the measured stator current $i_s$ and stator flux $\Psi_s$ vector in the $\alpha - \beta$ coordinate system will be presented. The time function of the mechanical speed signal $\Omega$ was simulated only. The measurements were carried out by using a SILEX TMI-02 torque and fluxmeter and Agilent N2782A current probe.

As the analysis focused on the case when the ratio of sampling frequency over fundamental frequency $F$ is low, Tustin integration rule [34] was applied for the PI controllers and the estimator instead of the backward Euler method to improve the accuracy of the calculation.

Due to the maximum allowable DC link voltage of the three phase inverter available in the laboratory ($V_{DC} < 400V$) the maximum fundamental frequency was limited to $f_{\text{max}} \approx 200 - 210$ Hz therefore the motor could not be operated in field weakening region. Thus the maximum speed was around 12 krpm. To investigate the drive when both the current sampling over fundamental frequency ratio $F$ and the carrier over fundamental frequency $m_f$ are low the carrier frequency was reduced. The selected carrier and sampling frequency ($f_c = f_s = 1.4kHz$) was quite low compared to the values applied in practical ac drives at the same power level, but in this way the effect of sampling technique of SVM could be emphasized.

The performance of the drive system was investigated both by simulation and laboratory measurements for ramp like change in the load torque $T_{\text{load}}$. The load torque is changed from 0 Nm to 0.6 Nm in a ramp shape within 0.5 sec.

Figure 3.11(a) and 3.12(a) show the simulated time function of mechanical speed and simulated and measured time function of the electric torque during transient when $\Omega^* = 440$ rad/s (denoted by dotted line in 3.11(a) and 3.12(a)) for RS and DS SVM, respectively. As it was expected at higher frequency ratios (now $F = m_f \approx 20$) the difference between RS and DS is minimal. Both time functions of the mechanical speed and the electric torque are practically the same. For the better visualization the digitally filtered torque signal measured by the torquemeter is plotted. As it can be seen the time function of the measured torque signal almost equals to the average value of the simulated one. As there is no speed-sensor, the mechanical speed is estimated from the measured stator currents. The simulated mechanical

\(^4\)dsPIC33FJ256MC710
Table 3.2: Rated data and parameters of the induction machine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_n$</td>
<td>Nominal power 3 kW</td>
</tr>
<tr>
<td>$n_n$</td>
<td>Nominal rated speed 18/24 krpm</td>
</tr>
<tr>
<td>$f_{1n}$</td>
<td>Rated frequency 300 Hz</td>
</tr>
<tr>
<td>$V_{LL,rms}$</td>
<td>Nominal voltage 380 V</td>
</tr>
<tr>
<td>$I_{ph,rms}$</td>
<td>Nominal current 7.7 A</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>Rated torque 1.5 Nm</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Stator resist. 1.125 Ω</td>
</tr>
<tr>
<td>$R_r$</td>
<td>Rotor resist. 0.85 Ω</td>
</tr>
<tr>
<td>$L_{ls}$</td>
<td>Stator leak. induct. 25 mH</td>
</tr>
<tr>
<td>$L_{lr}$</td>
<td>Rotor leak. induc. 14 mH</td>
</tr>
<tr>
<td>$L_m$</td>
<td>Magnetizing ind. 84.82 Ω</td>
</tr>
<tr>
<td>$J$</td>
<td>Inertia 5 kgcm²</td>
</tr>
</tbody>
</table>

Table 3.3: Controller parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{p\Omega}$</td>
<td>Speed cont. prop. gain 1.5</td>
</tr>
<tr>
<td>$T_{i\Omega}$</td>
<td>Speed cont. time constant 0.085 s</td>
</tr>
<tr>
<td>$K_{p\eta}$</td>
<td>q-axis cont. prop. gain 2</td>
</tr>
<tr>
<td>$T_{i\eta}$</td>
<td>q-axis cont. time constant 8 ms</td>
</tr>
<tr>
<td>$K_{pd}$</td>
<td>d-axis cont. prop. gain 2</td>
</tr>
<tr>
<td>$T_{id}$</td>
<td>d-axis cont. time constant 9 ms</td>
</tr>
</tbody>
</table>

speed is maintained close to the constant reference speed with a good accuracy (the relative error is less than 0.5%).

Figure 3.11(b) and 3.12(b) show the trace of the simulated and measured stator current vector $i_s$. Figure 3.11(c) and 3.12(c) show the trace of the simulated and measured stator flux vector $\Psi_s$ in steady state when $T_{load} = 0.6$ Nm. The simulation and experimental result are in good agreement, but it can be seen the ripples in the current signal is higher in the measured signal. By comparing the two sampling techniques it can be concluded that the difference between RS and DS is negligible.

By increasing the reference speed $\Omega^*$ and keeping $f_c$ constant both the ratio of sampling to fundamental frequency $m_f$ and carrier to fundamental $F$ decreases. When $m_f$ or $F$ is low the difference between RS and DS becomes crucial. Figure 3.13 shows the simulated time function of mechanical speed and simulated and measured time functions of the electric torque during transient when $\Omega^* = 1005$ rad/s ($m_f = F \approx 8.75$) and RS technique is applied. As it can be seen in the figure that the operation of the drive applying RS becomes unstable when the load torque was increased. During the laboratory analysis the over-current protection was activated and it shut down the system to protect the machines from damages.

By applying DS instead of RS the stability range of the drive could be extended: by recalculating the reference signals $v_{ref,i} (i = a, b, c)$ of the SVM at the negative peak of the carrier signal (Fig.3.7(b)) where no current sampling took place, the performance of the drive was improved. Figure 3.14(a) shows the simulated mechanical speed $\Omega$ and the simulated and measured electric torque $T_e$ during transient when $\Omega^* = 1005$ rad/s ($m_f = F \approx 8.75$) and RS technique was applied. It can be seen, contrary to RS, the response was stable. Figure 3.14(b) and 3.14(c) show the simulated and measured stator current $i_s$ and stator flux $\Psi_s$. As it was expected at lower frequency ratio the ripples both in the currents and fluxes increased resulting in high total harmonic distortion (THD). As the frequency ratio is a low non-integer value, subharmonic components are generated both in the current and flux values, however, their amplitudes are negligible compared to the fundamental ones (see Fig.1.21(a) in Chapter I). Again the simulated and measured results were in good agreement.

Figure 3.15(a) and 3.16(a) show the simulated time function of mechanical speed $\Omega$ and simulated and measured time function of the electric torque $T_e$ during transient when $\Omega^* = 1100$ rad/s ($m_f = F \approx 8$) and $\Omega^* = 1162$ rad/s ($m_f = F \approx 7.6$), respectively. Based on
Figure 3.11: Simulated and measured responses for ramp like torque change applying RS SVM, \( \Omega^* = 440 \text{ rad/s, } F = m_f \approx 20, \Delta T_{\text{load}} = 0.6 \text{ Nm} \)

Figure 3.12: Simulated and measured responses for ramp like torque change applying DS SVM, \( \Omega^* = 440 \text{ rad/s, } F = m_f \approx 20, \Delta T_{\text{load}} = 0.6 \text{ Nm} \)

the figures the conclusion is that the control loop was stable even for such a low \( m_f \) and \( F \) values. Figure 3.15(b) and 3.16(b) show the trace of the stator current vector \( i_s \) and 3.15(c) and 3.16(c) presents the trace of the stator flux vector \( \Psi_s \). Due to the low \( m_f \) value the current and flux signals have high harmonic content.

By further increasing the reference speed the response of the drive applying DS sampling technique becomes also unstable. Figure 3.17 shows the time function of the simulated mechanical speed \( \Omega \) and the measured and simulated electric torque \( T_e \) when \( \Omega^* = 1193 \text{ rad/s } (F = m_f \approx 7.3) \). In the simulation model the torque starts to oscillating after the load-
ing torque increases. However, the oscillations settles down, such a large torque pulsation is unacceptable in practical drives. The laboratory result is again in good agreement with the simulated one: the response becomes unstable and the over-current protection was activated which shut-down the machines.

Based on the simulation and experimental results it can be concluded it is worth to resample the SVM at the negative peak of the carrier signal as more robust control performance can be obtained. The stability range is extended. The stability border of the control loop for the given set of controller parameters given in Table 3.3 is expanded from $\Omega^* = 160$ rad/s to $\Omega^* = 190$ rad/s. It should be emphasized that the sampling frequency is the same both for RS and DS.
Figure 3.15: Simulated and measured responses for ramp like torque change applying DS SVM, \( \Omega^* = 1100 \text{ rad/s}, F = m_f \approx 8, \Delta T_{\text{load}} = 0.6 \text{ Nm} \)

Figure 3.16: Simulated and measured responses for ramp like torque change applying DS SVM, \( \Omega^* = 1162 \text{ rad/s}, F = m_f \approx 7.6, \Delta T_{\text{load}} = 0.6 \text{ Nm} \)

Speed sensor-less FOC control methods must be robust to plant parameter variations. Figure 3.18 and 3.19(a) show the simulated mechanical speed \( \Omega \) and the simulated and measured electric torque \( T_e \) during transient for RS and DS, respectively. Now 20 % smaller mutual inductance value \( L_{\text{m,est}} \) was used during the estimation than the actual one and the stator resistance \( R_{\text{s,est}} \) was set to be only 40 % of the real one (the parameters of the motor can be found in Table 3.2). The reference speed was \( \Omega^* = 942 \text{ rad/s} \) (\( F = m_f \approx 9.3 \)) in 3.18 and 3.19. By using RS sampling technique for the SVM the response was oscillatory even at no-load. By increasing the load torque from 0 Nm to 0.6 Nm in a ramp shape, the amplitude...
Figure 3.17: Simulated and measured responses for ramp like torque change applying DS SVM, $\Omega^* = 1193 \text{ rad/s}$, $F = m_f \approx 7.3$, $\Delta T_{load} = 0.6 \text{ Nm}$

of the oscillations was increased and the drive practically was becoming unstable. During the laboratory test similar phenomenon was obtained: oscillations in the electric torque were developed and the over-current protection shut-down the drive. By applying DS instead of RS the response was stable (Fig.3.19(a)). By comparing the time functions of $\Omega$ in Fig.3.19(a) and in Fig.3.14(a) (the $m_f$ and $F$ value were practically the same in both cases) the conclusion is that the magnitude of the ripples in the mechanical speed and the relative error of the speed controller were increased when inaccurate machine parameters were applied. However, the response remained stable and was still acceptable.

Figure 3.19(b) and 3.19(c) show the trace of the stator current vector $i_s$ and the stator flux vector $\Psi_s$, respectively. The current and flux signals has a high harmonic distortion due to the low $m_f$. Again the measured and simulated trajectories are in good agreement.

The simulation and experimental results clearly demonstrated that the DS is more robust at low frequency ratios even for variations in the motor parameters.
Figure 3.18: Simulated and measured response for torque change applying RS SVM, $L_{m,est} = 0.8L_m$, $R_{s,est} = 0.4R_s$, $\Omega^* = 942$ rad/s, $F = m_f \approx 9.3$, $\Delta T_{load} = 0.6$ Nm

Figure 3.19: Simulated and measured responses for ramp like torque change applying DS SVM, $L_{m,est} = 0.8L_m$, $R_{s,est} = 0.4R_s$, $\Omega^* = 1162$ rad/s, $F = m_f \approx 7.6$, $\Delta T_{load} = 0.6$ Nm
3.5 Thesis 3

The stability range of a speed sensor-less Field Oriented Controlled induction machine can be extended by recalculating the reference signal of the Space Vector Modulation algorithm at the negative peak of the carrier signal by approximating the rotor flux angle change when shunt resistors, placed on the bottom of the three phase inverter legs, are applied to measure the stator currents and the current sampling, the estimation and the calculation of the controllers take place at and after the positive peak of the triangular carrier signal.

Related publications of the author: [A15, A16]

Practical significance of the results

Field Oriented Control technique is one of the mostly applied algorithm to control the speed and the flux of a three-phase electric drive thanks to its high performance and the advances in the semiconductors technology in both power and signal electronics. Due to its many advantage FOC driven electric machines are applied for drive systems of low-cost home appliances, like washing machine, to high performance and expensive systems, like transportation or manufacturing automation.

The application of high precision shunt resistors to measure the stator currents is a popular solution in Field Oriented Controlled electric drives due to the low system cost and exact measurements. Mostly the resistors are placed in the bottom of the inverter leg limiting the sampling frequency to be the carrier frequency ($f_s = f_c$).

In the last decade increasing attention has been given to high speed and high pole count motor drives. Even with the up-to-date digital devices with clock frequency in the range of tens of MHz, the sampling frequency is limited due to the shunt resistor measurement technique. It results that not only the ratio of the carrier to the actual fundamental frequency $m_f = f_c/f_1$ but the ratio of the sampling to the actual fundamental frequency $F = f_s/f_1$ is low around the maximum speed of a high speed or high-pole count motor. It deteriorates the performance of the closed loop drive.

This chapter demonstrated that the performance of the drive can be improved by applying Double Sampled Space Vector Modulation (DS SVM) technique instead of Regular Sampled one. The main advantages of using DS SVM is that the robustness of the drive can be improved and its stability can be extended without any additional hardware. Furthermore the extra calculation time required by the DS SVM is also very low so it requires very small time window in the precious processor time of the microcontroller or the DSP.
Plans of Future Research Work

In Chapter 1 the effect of the sampling techniques of three widely applied carrier based Pulse Width Modulation (PWM) techniques were investigated. In recent years, the development of the chaotic Pulse Width Modulation (PWM) techniques have received considerable attention and a number of ideas to generate random-like chaotic PWM signals have been tested in different systems [53]. In most cases the logistic map is applied to chaoize a frequency-modulated signal. Then it modulates the carrier frequency of the standrad PWM techniques, like Sinusoidal PWM or Space Vector Modulation [92]. One of the main advantages of the chaotic PWM techniques is that it can improve the Electromagnetic Compatibility of the converters applying high switching frequencies. Furthermore, it can also suppress the acoustic noise in PWM drive systems [53]. In the future I would like to implement chaotic carrier based PWM techniques applying other map functions than the Logistic map and investigate their effect on the operation of high speed drives.

In Chapter 2 a novel stability analysis method using the auxiliary state vector was successfully applied for DC servo motor and for a single phase Power Factor Correction (PFC) power electronic converter. Furthermore, the chaotic and oscillating response are stabilized by adding a stabilizing signal in the control loop. The parameters of the stabilizing signal were calculated by the auxiliary state vector as well. In the future I would like to apply the stability analysis for three phase motor drives, like Brushless DC motor or hysteresis controlled induction machine, and for three phase converters, like three phase PFC converters, as well. By applying different stabilizing signal I would like to extend the stability range of the investigated systems.

Single phase quadratic converter topology, where the voltage ratio is given as a quadratic function of the duty ratio, has many advantages [93], like greater voltage ratio in comparison with the basic configurations and better efficiency. Later I would like to investigate a single phase quadratic converter and design its controller parameters applying again the auxiliary state vector.

In Chapter 3 a speed sensor-less Field Oriented Controlled induction machine drive was investigated by comparing two SVM sampling techniques: Regular Sampling (RS) and Doublesampling (DS). Natural Sampling is often referred to as the best form of sampling technique in closed loop applications [34] when the sampling to fundamental frequency $F$ is low, as it does not introduce delay. As it was presented in Chapter 1 I developed and successfully implemented a method in a digital microcontroller for realizing SVM applying Natural Sampling with high precision in open-loop. In the future I would like to extend the method to closed loop operation as well and compare its performance with the RS and DS sampling techniques for low $m_f$ that is for high speed drives. Due to its parallel computation properties, FPGA has many advantage over microcontrollers. Later I would like to implement the closed loop algorithm applying NS SVM in an FPGA as well.

In the dissertation the carrier frequency $f_c$ is set to be constant resulting in variable non integer frequency ratio $m_f = f_c/f_1$ as $f_1$ is varied by the controller resulting in asynchronous PWM. In the future I would like to implement synchronous PWM technique and keeping $m_f$ constant by varying $f_c$ as well.
Author’s publications


References


