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**BUDAPEST UNIVERSITY OF TECHNOLOGY AND ECONOMICS
DEPARTMENT OF STRUCTURAL MECHANICS**

**BUCKLING ANALYSIS OF
COLD-FORMED STEEL MEMBERS WITH THE
CONSTRAINED FINITE STRIP METHOD**

PhD dissertation

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Budapest, 2019

INTRODUCTION

During the last several decades thin-walled structural members made of cold-formed steel (CFS) have increasing popularity due to its structural efficiency i.e. high strength/weight ratio, low production and logistics costs and wide range of shapes available, especially in the fields of construction and vehicle engineering. Due to the high slenderness of these members, buckling is the governing mode of failure, which is a phenomenon with often neither no easily detectable indication prior to the occurrence of the failure, nor has post critical reserves which would allow a certain threshold of safety. Furthermore, the thin walls of the cross-sections usually ~1-3mm means that the traditional design methods aren't always applicable and special design methods are required. Due to these, the understanding and reliable prediction of such member's buckling resistance is very important.

Buckling in general is a combination of two effects, the idealised phenomenon known as elastic buckling and the effect of imperfections. In the case of cold-formed thin-walled steel members, it is usually classified in three major modes: local-plate-, distortional- and global buckling. In real life, the various buckling modes hardly ever appear separately, mode-coupling is practically always present. However, in the current design standards [1-3] and methods, different formulae or methodology apply to determine the resistance belonging to the various modes, furthermore, the post-critical reserve also differs among the different modes. Therefore, the identification of the buckling modes is important in the buckling analysis of cold-formed thin-walled members. A relatively simple and easily automated method for mode classification is by preparing the so-called signature curve of a cross-section. The signature curve is established by calculating the critical loads for different buckling lengths and then the results are plotted as illustrated in Figure 1.

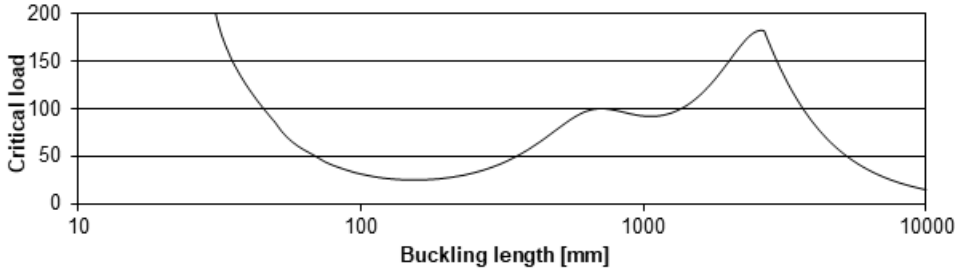


Figure 1: Signature curve of a C cross-section

The first minimum point of the curve belongs to the local-plate buckling mode, the second to the distortional buckling and the third, descending part of the signature curve at larger buckling lengths indicates global buckling.

The current design standards are generally based on analytical solutions for the different modes; these are, however, often very complicated and not always totally reliable in capacity prediction. A new and simple method proposed and already integrated in the North American Standard (NAS) is the Direct Strength Method (DSM) [4-5]. The DSM design is based on critical loads obtained from numerical analysis e.g. with the finite strip method (FSM) [6-8] for the three major modes, then the final member resistance is calculated in a few simple steps. However, in some cases since the signature-curve doesn't have both minima, or doesn't have one at all, the critical load of a mode cannot unambiguously be determined. To overcome this problem, an extension of the FSM called constrained FSM or cFSM [9-12] was proposed where the pure buckling modes can be analysed. The principle of cFSM is similar to that of the Generalized Beam Theory (GBT) [13-15], i.e. it is based on constraining the deformations according to mode-specific mechanical criteria describing the different modes. The necessary calculations in the research were performed by using the open source software CUFSM [16], directly or embedded in research specific routines.

COMPARISON OF COUPLED AND PURE BUCKLING MODES

As described above, cFSM solves the problem of always having a critical load for each buckling modes. However, *all-mode* or coupled (FSM based) and *pure-mode* or uncoupled (cFSM based) critical loads differ as presented in Figure 2.

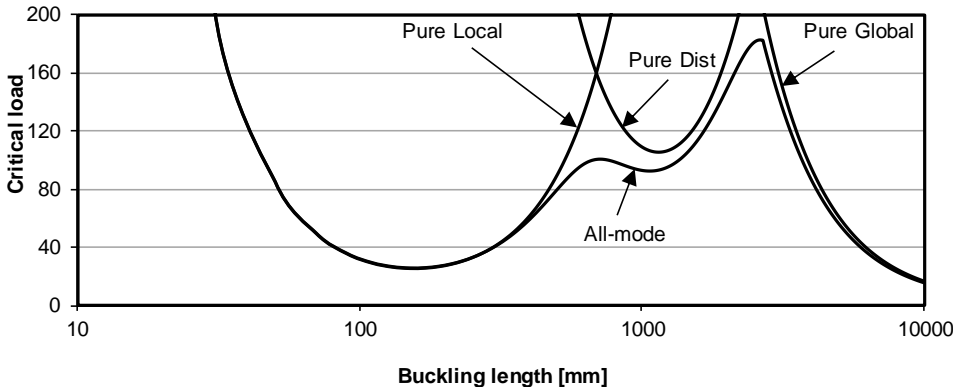


Figure 2: Comparison of all-mode critical loads by FSM and pure-mode critical loads by cFSM

Since the DSM formulae have been calibrated to the *all-mode* critical loads, the analysis of this difference deemed necessary to evaluate the possible impact on the final member resistance. This difference is natural due to the lack of the mode-coupling in the pure-mode results, however, since the values predicted by cFSM are always higher, hence on the unsafe side.

The first part of the research studies and evaluates this difference based on statistical analysis of parametric studies performed on a wide range of cross-sections and geometries. Three load types were applied, pure compression (column), pure bending (beam) and compression-bending (column-beam). Based on the first study the following conclusions were drawn: (i) although the difference appears in all of the modes, the distortional buckling results are those affected most due to a more important mode-coupling effect, (ii) the difference is larger in those cases where at least one minimum point of the signature-curve doesn't exist. It was also found that the above observations are basically independent from the applied load type.

The magnitude of the differences for C cross-sections under pure compression load separately for local and distortional buckling is presented in Figure 3 and Figure 4.

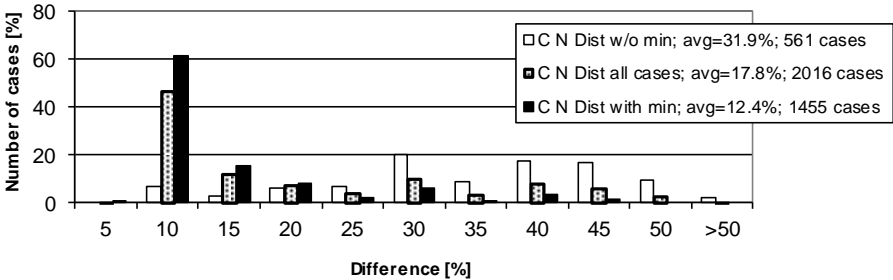


Figure 3: Distribution of differences in critical loads: C section, N load, D mode

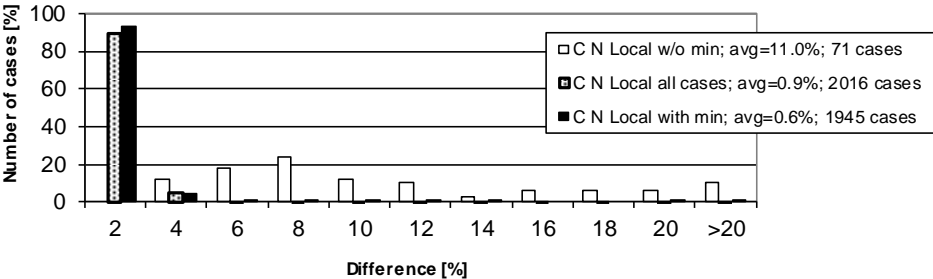


Figure 4: Distribution of differences in critical loads: C section, N load, L mode

The next step of the study was to evaluate the effect of the difference in critical loads on the final load bearing capacity. A second large scale parametric study was conducted on six different cross-section types. Since the member resistance was determined with the DSM, the geometry of the studied cross-sections were filtered to be compliant with the DSM

prescriptions. The difference between all- and pure-mode results in member resistance is less than in that of the critical loads', but a non-negligible difference even remains when comparing the final member resistance values as presented in Figure 5 for a C member under pure compression load. Table 1 presents the results for all of the cross-sections analyzed. It can thus be concluded, that cFSM indeed has the practical advantage of being able to differentiate pure buckling modes and the associated critical loads, however, the deviation of the results in the unsafe direction are non-negligible and appear even in the final member resistance.

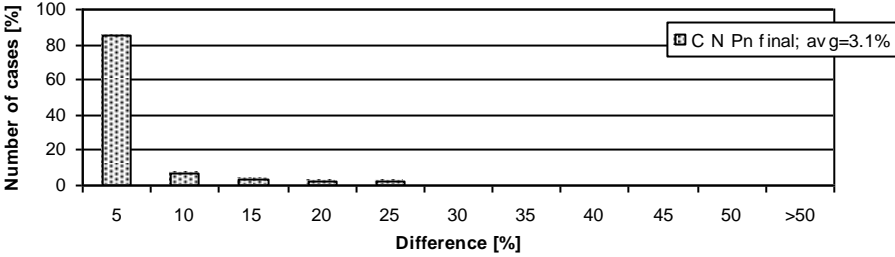


Figure 5: Distribution of differences in final load-bearing capacity: C section N load

Table 1: Differences of resistances for pure compression: FSM vs. cFSM

Cross section	No. of Cases	P_{nl}		P_{nd}		P_n	
		Average diff. [%]	Max. diff. [%]	Average diff. [%]	Max. diff. [%]	Average diff. [%]	Max. diff. [%]
C	1720	0.3	5.3	6.8	25.4	3.6	25.4
(w/ real D min)	(1233)			(4.1)	(11.4)	(2.0)	(10.6)
C w/ stiffener	781	0.6	2.9	2.9	5.8	2.8	5.6
Z	240	0.3	2.2	3.9	7.8	2.0	7.8
Hat	78	0.5	1.3	2.5	2.7	2.5	2.7
Rack	156	0.1	0.8	1.8	2.6	1.8	2.6
Rack w/ stiffener	147	0.1	0.8	1.6	2.3	1.6	2.3

Thesis 1

I compared the buckling analysis of cold-formed steel members with finite strip method and constrained finite strip method for local-plate and distortional buckling modes.

1a) I compared in a parametric study the elastic critical loads of cold-formed steel members on a wide geometrical range with finite strip method and constrained finite strip method for pure compression, pure bending and combined compression-bending loads both for local-plate and distortional buckling modes. I determined the characteristic differences for the various cross-sections and load cases. I concluded that in case of local-plate buckling the difference obtained

by the two methods is a few percent in average, while in case of distortional buckling the differences show an average of approximately 10-20 percent, with a higher scatter. I concluded that the differences are considerably higher when the signature curve has no definitive minimum point at the given buckling mode.

1b) I compared in a parametric study generally available cold-formed steel members' resistance under pure compression and pure bending loads; the resistance was calculated from elastic buckling critical loads determined by finite strip method and constrained finite strip method. I determined the typical differences for the various cross-sections and load cases. I concluded that the difference shows an average of approximately 2-3 percent, and the difference is mostly due to the difference in the critical loads of the distortional mode. I concluded that the differences are significantly higher in the case the signature curve does not have a definitive minimum point for distortional mode.

Related publications: [BZ1], [BZ2], [BZ3], [BZ4]

APPLICATION OF CFM WITH THE DIRECT STRENGTH METHOD

Thin-walled cold-formed steel members are due to the nature of the manufacturing process always produced with rounded corners. The different standards' approach to this is to simplify or neglect the rounded corners in the buckling design. Modelling rounded corners is done by dividing the corner into a series of close to parallel thin stripes. When the analysis is done with conventional FSM this discretization does not cause a problem, however, in case of cFSM analysis, although performing calculations with cFSM is technically possible, but due to the embedded mechanical criteria (limitation of displacements of nodes between non-parallel stripes) the results obtained are not compliant with the engineering expectations, therefore, they cannot be regarded as correct. This is demonstrated via two illustrations.

In Figure 6 pure-mode solutions are shown both with sharp- and rounded-corner models. While G solutions show the expected tendency, L and D solutions produce unexpected ones, e.g. pure L critical values of rounded-corner models are too high with regard to their sharp-corner counterparts. Another strangeness is that the pure D curve has two minima: one in the L region where it is unexpected, besides the one in the D region, furthermore, in this latter case the calculated minimal D critical stress with the rounded corners is unexpectedly lower compared to the similar D critical stress with the sharp corners. The notation 160-60-15-4-1.5 used in the figure indicates a cross-section with a web depth of 160mm, a flange width of 60mm, a lip length of 15mm, a corner radius of 4mm and a plate thickness of 1.5mm.

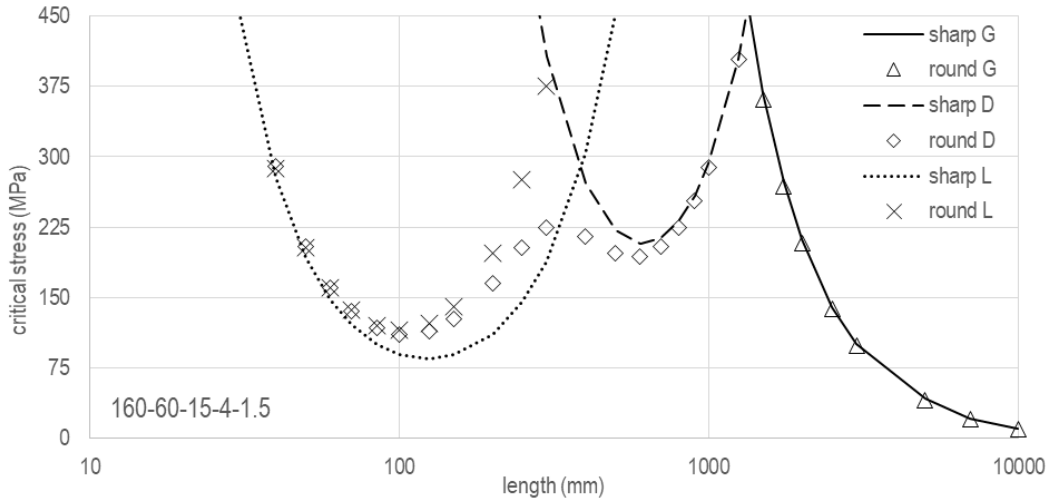


Figure 6: Pure mode solutions for sharp and rounded models, 160-60-15-4-1.5 member

The problem is also illustrated on a C member under pure compression, Figure 7 a) shows the cFSM analysis results of a sharp corner model, b) shows the results of the cFSM analysis of a rounded corner one, while c) represents the expected deformed shape. The deformation shown in b) indicates that the results aren't correct.

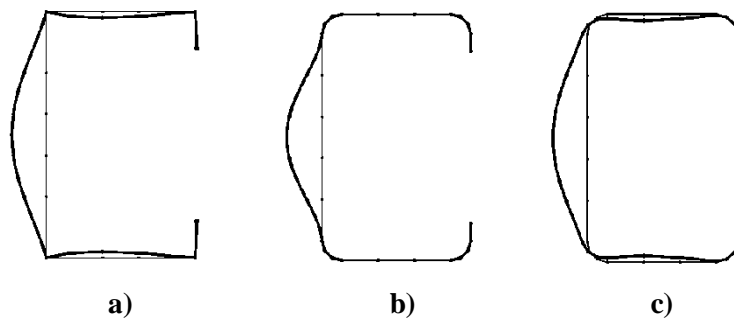


Figure 7: Cross-section deformations for various L buckling modes

Practically this means that obtaining pure-mode results for real cross-section models is not possible. To overcome this obstacle a so-called extrapolation method is proposed to estimate the numerical values corresponding to the pure modes of rounded corner members. The principle of the solution is to establish coefficients which take into account the effects of (i) the difference between rounded- and sharp corners in terms of the cross-sections' geometrical properties, (ii) the difference between rounded- and sharp corners on the critical stress and (iii) mode coupling. Based on these coefficients, the numerical analysis results of cross-sections in *all-mode with rounded corners* and in *pure-mode with sharp corners* can be extrapolated to obtain results in *pure-mode with rounded corners* as follows:

$$\text{rounded cFSM} \approx \text{sharp cFSM} * k_{c,W} * k_{c,L/D} * k_{m,L/D}$$

$k_{c,W}$: effect of rounded corners on the cross-section properties

$k_{c,L/D}$: effect of rounded corners on σ_{cr}

$k_{m,L/D}$: effect of mode-coupling on σ_{cr}

In order to verify the proposed extrapolation method, numerical analysis was performed on a set of cross-sections for which existing laboratory experimental data [17-18] was also available. The experiments were conducted in a way to receive pure buckling mode resistance values, hence served as a good basis for verification purposes. Naturally, since these were actual, commercially available steel members the corners were rounded. The pure-mode member resistance results of the rounded corner models obtained via the extrapolation method were compared to the experimental results and also verified against the DSM based predictions and then statistically evaluated. The extrapolation method was found to perform at least equally as good as the DSM, hence the approach may be regarded as an acceptable alternative solution for the problem. The results of the comparison is summarized in Table 2.

Table 2: Comparison of test results with the extrapolation method and DSM based predictions

	Mode	average	st. deviation	min.	max.
DSM/experiment	All-L	0,976	0,084	0,837	1,246
Extrap./experiment		0,989	0,080	0,826	1,228
DSM/experiment	All-D	0,992	0,070	0,829	1,164
Extrap./experiment		1,018	0,099	0,863	1,218

Thesis 2

I proposed a simple, fully automated dimensioning method to determine the member resistance of cold-formed steel cross-sections with the constrained finite strip method and the direct strength method. In the proposed procedure, I introduced modification factors to take into account the differences between the finite strip method and the constrained finite strip method as well as the difference between rounded- and sharp corner cross-sections. Based on the results of a parametric study, I provided the necessary modification factors for the various profiles of generally available cold-formed steel members. I compared the results of the proposed design

procedure with experimental results and concluded that the proposed method is statistically equivalent to the widely applied direct strength method.

Related publications: [BZ3], [BZ4]

MECHANICAL MODELLING OF ROUNDED CORNERS

Rigid corner model

Although the extrapolation method’s performance seems fairly good, since it is based on certain assumptions which cannot be directly verified, a solution to directly model the rounded corners was sought. The aim of the corner modelling was to eliminate the nodes of the rounded corner itself to allow performing pure buckling mode numerical analysis. Two corner models were elaborated and evaluated.

The first model was a so-called rigid corner model where the displacement of the corner nodes are assigned to a virtual (reference) node at the intersection of the flat parts, creating a sharp corner cross-section equivalent to the original rounded corner one as shown in Figure 8.

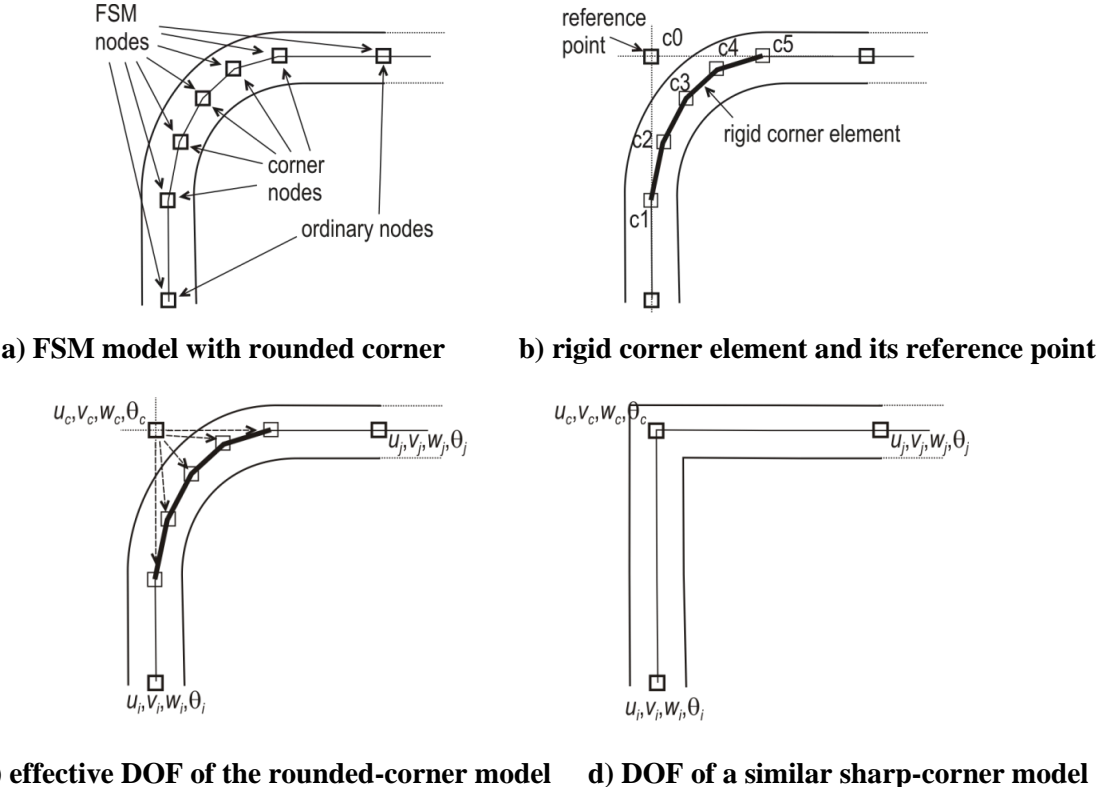


Figure 8: Illustration of the rigid-corner approach

The performance of the model is then compared to conventional and constrained FSM analysis results performed on both rounded and sharp corner members. Since, as mentioned above, the rounded corner cFSM analysis does not produce correct results, the extrapolation method served as the comparison basis in this case. Although this introduces some uncertainty in the interpretations of the results, it still renders drawing general conclusions possible.

To evaluate the performance of the rigid-corner approach, the differences between critical values delivered by the rigid-corner and rounded-corner models are determined. The difference $(\text{'rigid'} - \text{'rounded'}) / \text{'rounded'}$ is calculated for both the all-mode and pure-mode solutions, and presented in Figure 9. To help identifying the type of buckling modes, a signature curve for the given cross-section is also shown in the figure.

The results show that the rigid corner model leads to reasonable results; however, it tends to overestimate systematically the critical loads, especially if the corner radius is large. The reason of this lies in the simplicity of the model; the rigid corner brings an additional rigidity to the corner zones of the cross-section which modifies the overall behaviour.

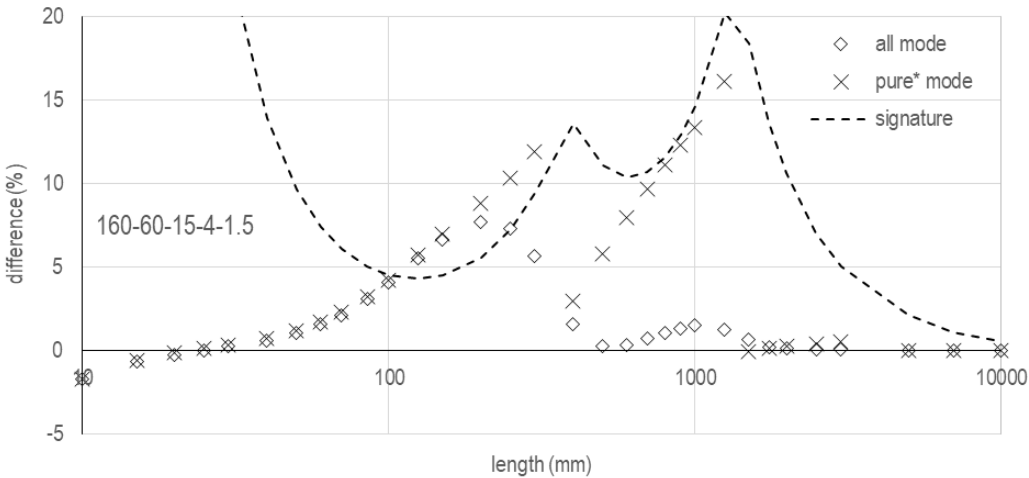


Figure 9: Differences between critical load results of rigid-corner and rounded-corner calculations, 160-60-15-4-1.5 member

Elastic corner model

The rigid-corner model proved that creating a mechanical model for the rounded corner element is a good approach. An improved modelling technique named elastic corner model was developed where the displacement of the corner nodes is derived from the displacement of the nodes of the flat plates, therefore, the corner nodes themselves were excluded from the numerical analysis. The principle of this approach is illustrated in Figure 10.

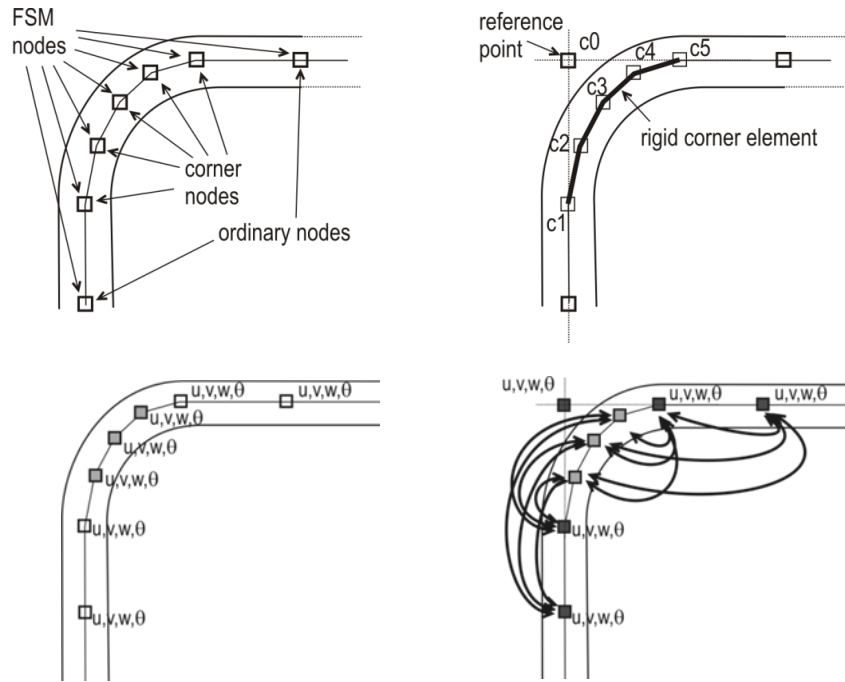


Figure 10: Principle of the elastic corner model

The performance of the elastic model was evaluated in the same way as the rigid corner model, illustrated in Figure 11.

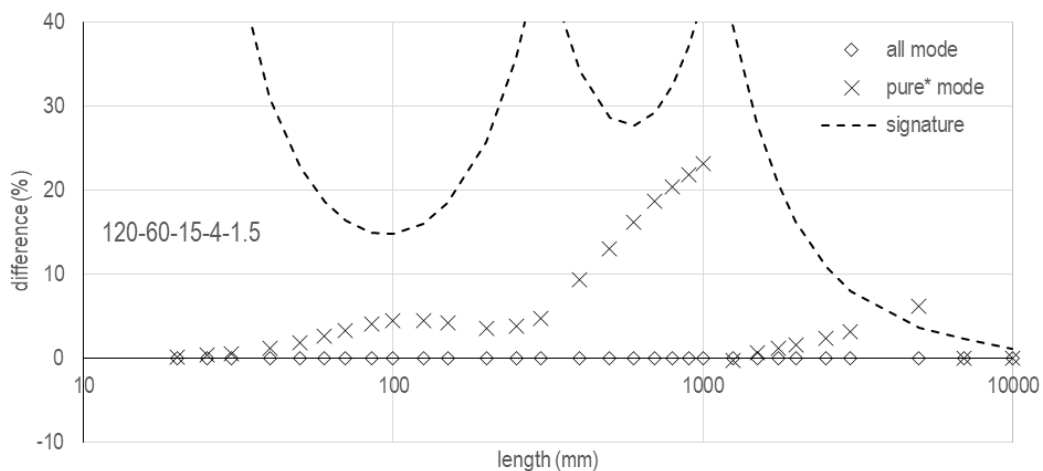


Figure 11: Differences between critical load results from elastic-corner and rounded-corner calculations, 120-60-15-4-1.5 member

The differences for all-mode options are negligibly small for any length and any cross-section shape; however, non-negligible differences exist between the results of pure-mode options. Since no theoretical solution is known for the exact pure-mode L or D critical load for cross-sections with rounded corners, if the calculated difference is non-zero, this is not necessarily due to the error of one or the other calculation. However, the pure G critical values from a

rounded corner model can be regarded as exact values; therefore, deviation from these values is indeed an error. By analysing the illustrative example, it was found that the warping distribution of the corner zone wasn't correct and this resulted in the higher than expected critical loads.

A potential solution for this problem was to apply the in-plane shear modes when calculating the pure critical values. Since the so-called warping shear modes do not involve cross-section distortion, only warping, thus, when added to the pure modes, they practically do not modify the cross-section's shape.

Consideration of in-plane shear seems to eliminate the error of the pure G and L modes, since the critical load values from the rounded-corner and the elastic-corner options are mostly identical. With in-plane shear modes added to pure modes, the difference between pure D results from elastic-corner and rounded-corner options is decreasing. The elastic-corner approach predicts somewhat larger pure D critical load values, however, as no theoretical solution is known for the exact pure D critical load in case of a cross-section with rounded corners, it is hard to judge whether one or the other prediction is better or more exact. The differences between critical values predicted by the elastic-corner and rounded-corner options are also shown in Figure 12.

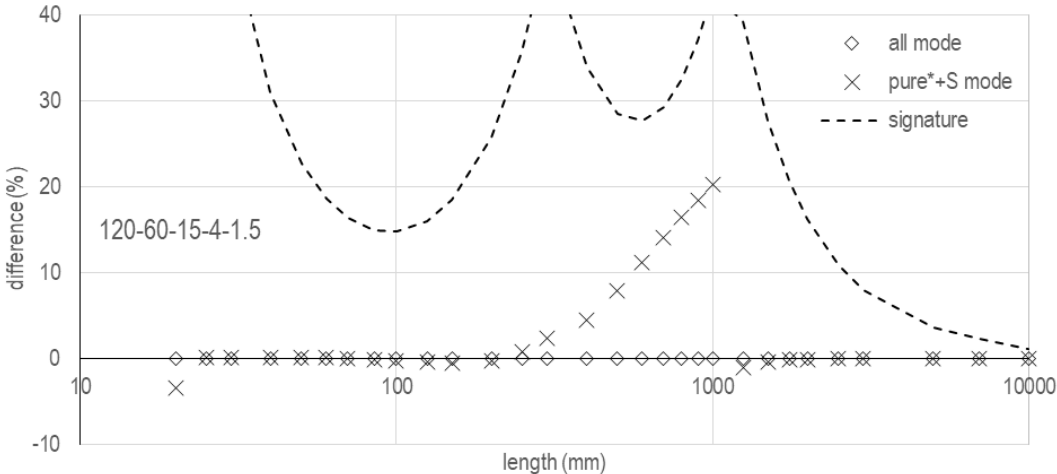


Figure 12: Differences between critical load results from elastic-corner and rounded-corner calculations, with shear deformations, 120-60-15-4-1.5 member

After carefully studying a typical cross-section, the overall performances of both the rigid and elastic models are verified by the statistical analysis of parametric studies on a large number of cross-sections. It was concluded that the primary aim, i.e. creating a rounded corner model on

which the constrained FSM analysis may be performed was achieved, although, in some cases certain overestimation of the critical loads was observed.

Thesis 3

I proposed rounded corner mechanical models for cold-formed steel members to be able to perform constrained finite strip method analysis on such members.

3a) I proposed a rigid corner element for modelling thin-walled members with rounded corners using the finite strip method. I have demonstrated that the rigid corner element makes the use of the constrained finite strip method possible, even when rounded corners are modelled directly. I have analysed the effect of the rigid corner element on the critical loads of pure buckling modes via illustrative examples and a parametric study with the constrained finite strip method. I concluded that the use of the rigid corner element provides buckling results according to engineering expectations, but in some cases, it overestimates the critical load. The magnitude of the overestimation depends on the cross-section geometry, primarily on the radius of the rounded corners and in several cases it isn't negligible. I concluded that the overestimation is partly caused by the rigidity of the corner element, thus when the rigid corner element is applied, the error cannot be eliminated.

3b) I proposed an elastic corner element for modelling thin-walled members with rounded corners using the finite strip method. I have demonstrated that the elastic corner element makes the use of the constrained finite strip method possible, even when rounded corners are modelled directly. I have analysed the effect of the elastic corner element on the critical loads of pure buckling modes via illustrative examples and a parametric study with the constrained finite strip method. I concluded that the use of the elastic corner element provides buckling results according to engineering expectations, but in some cases, it overestimates the critical load. I concluded that the overestimation was caused by the error of the longitudinal displacements, and I demonstrated that by applying the shear modes too, this error disappears. The results of the application of elastic corner elements show that the rounded corner somewhat increases the difference between the finite strip method and constrained finite strip method distortional buckling critical loads.

Related publications: [BZ5], [BZ6], [BZ7], [BZ8]

PUBLICATIONS RELATED TO THE THESES

- [BZ1] Ádány, S. and Z. Beregszászi, ‘Local and Distortional Buckling of Thin-Walled Members: Numerical Study to Compare Conventional and Constrained Finite Strip Method’, *Fifth International Conference on Thin-Walled Structures (ICTWS 2008)*, Brisbane, 2008, vol. 2, pp. 1121-1128.
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<https://doi.org/10.1016/j.compstruc.2011.07.003>.
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