MŰEGYETEM 1782

PhD Thesis Booklet

Correlation functions and non-equilibrium dynamics in one dimensional quantum systems

Dávid Horváth

Supervisor: Dr. Gábor Takács
Professor
BUTE Physics Insitiute
Department of Theoretical Physics

Budapest University of Technology and Economics
2019

Introduction

Nowadays low dimensional quantum systems are attracting considerable interest for several reasons. On the one hand, these systems are often strongly correlated as the role of quantum fluctuations is more dominant due to the reduced dimensionality. On the other hand many one dimensional systems are integrable, i.e. exhibit special, exotic features together with giving access to often their exact solution. Due to recent experimental techniques (e.g. realizing trapped cold atomic gases) such models and their unusual physics can be directly investigated in experiments in many cases.

A peculiar and actively studied phenomena is the lack of thermalization of integrable models in out-of-equilibrium situations. It is generally accepted, that in such cases the stationary state can be described by the generalized Gibbs ensemble (GGE). However, little is known about the sufficient class of quasi-local charges that are necessary for a proper GGE. Notable open problems are the eventual description of the time evolution in non-equilibrium situations and the role of integrability breaking, nevertheless these issues are of great importance from both a theoretical and an experimental point of view. My thesis focuses on mainly the question of the evolution, nevertheless my works also contributed to demonstrating the applicability of field theoretical models and the truncated conformal space approach, which can be applied to integrability breaking systems too, in out-of-equilibrium situations.

Goals

One of our first goal was the determination of the quench overlaps (i.e. scalar products of the initial state and the eigenstates of the post-quench Hamiltonian) in an interacting theory. Although the knowledge of the overlaps is often essential to describe the steady-state and they offer a natural way to characterize the time evolution after a quench too, there are only few examples of exact or even approximate expressions for the overlaps in interacting systems. Besides the determination of the overlaps, an important goal was their investigation and studying their properties too.

Our concrete aim was the determination of the overlaps in some quenches of the sinh- and sine-Gordon models by means of a hierarchy of integral equation in the former and by the truncated conformal space approach (TCSA) in the latter case.

It was an important objective to demonstrate that the pair overlaps possess a pole at zero rapidity (or momentum) if the expansion of the initial state contains a one-particle term in homogeneous quenches as the generic singularity was overlooked in the literature resulting in incorrect expressions for various quantities. Part of motivation was to correct the wrong results concerning the time evolution of one-point functions and to work out the correct procedure to handle the singularities.

Methods

In the works on which this thesis is based on I applied analytical and numerical methods too. During the study of the sinh-Gordon quenches for the derivation of the integral equations I used methods based on form factors such as regularization procedures based on finite volume form factors. The same method was used in the study of the singular overlaps and in the description of time evolution after quenches. To study quenches in the sine?Gordon-model, the truncated conformal space approach (TCSA) was used, which is a very effective treatment of many interacting integrable field theories.

New scientific results

1. I demonstrated that the pair overlap is singular in homogeneous quenches of massive integrable field theories if there is a one-particle term in the post-quench expansion of the initial state. The pole strength of the pair overlap can be related to the coupling of the one-particle term. Besides giving a general proof I used the phase quenches of the sine-Gordon model to demonstrate the validity of our arguments. The treatment of phase quenches also gives a perturbative proof of our statement. These results were published in [I].
2. I took part in the study of quenches of the sinh-Gordon model consisting of quenching from the free boson model by changing the physical mass and turning on the sinh-Gordon interaction. Based on integral equations containing form factors and the unknown overlaps I demonstrated that the integral equation can be solved by iteration if the initial state is assumed to be integrable. This result was published in [II].
3. By numerically determining of the overlaps and comparing them with an analytic Ansatz I showed that the Ansatz gives a good approximation of the overlaps in the studied regime of parameters. Using another version of the integral equations obtained by three-particle test-states I demonstrated that our assumption on the integrability of the quench is consistent; the Ansatz satisfied the the-particle equation to a good accuracy. I took part in a non-trivial check for the validity of the general, algebraic field theoretical recipe of deriving the integral equation via calculating the three-particle test-equation

using the finite volume regularization of form factors. These results were published in [II].

4. I showed that TCSA implemented for the sine-Gordon model can be used to study the mass quenches of the theory. By this numerical method I determined the pair overlaps of the first breathers in the model. These overlaps can be well approximated by an analytic continuation of the overlaps for the sinh-Gordon quench problem, discussed above. These results were published in [III].

5. I studied the time evolution of one-point functions in massive integrable field theories after homogeneous quantum quenches. If the one-particle term is present in the post-quench expansion of the initial state, then a linked-cluster expansion of the time dependent expectation value up the fifth order with respect to the particle number results in a $\sqrt{t}e^{-imt}$ term from the third order of the calculation for large times and in leading order in time. If the initial state is integrable then the linked cluster calculation in the fifths order (with respect to the total number of particles) yields a $t\ln t$ time dependent contribution besides the linear one for the one-particle oscillations e^{-imt} for large times in leading order. A non-oscillatory time dependent contribution is encountered in the fourth order of the calculation, which is of the type $1/t$ for large times in leading order. These results were published in [I].

Publications

[I] D.X. Horváth, M. Kormos and G. Takács, *JHEP* **08** (2018) 170, arXiv:1805.08132 [cond-mat.stat-mech].

[II] D. X. Horvath, S. Sotiriadis, and G. Takács, *Nucl. Phys.* **B 902** (2016) 508, arXiv:1510.01735 [cond-mat.stat-mech].

[III] D.X. Horváth and G. Takács, *Phys. Lett.* **B 771** (2017) 539–545, arXiv:1704.00594 [cond-mat.stat-mech].