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New Performance Evaluation Methods with Telecommunication Applications

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Abstract

The results of the dissertation consist of three parts. In the first part we define a new queueing model, called BMAP/D/1-timer multiplexer, based on the queueing behavior of the ATM AAL2 multiplexer. The most important performance measures of the ATM AAL2 multiplexer, the distribution of the waiting time and the multiplexing efficiency are analyzed with this queueing model. However the presented queueing model, the BMAP/D/1-timer multiplexer can be applied to model other engineering systems as well.

In the second part of the dissertation we provide methods to approximate the performance of multiclass queueing systems. In these queueing systems customers belonging to different classes can have different traffic parameters or service requirements. We present methods that give fast and numerically stable approximations for the mean and for the squared coefficient of variation of the waiting time.

The third part presents new results regarding to the extension of Markov reward models. We extend the modelling power of Markov reward models in two ways: we allow varying (so called second order) reward accumulation, and we make is possible to lose a portion of the reward accumulated during the last state sojourn at state transitions. We show that these extensions does not entail significantly larger computation complexity compared to ordinary Markov reward models.

1 Background

Performance evaluation plays an important role in the optimization process of the existing systems and in the dimensioning process of new systems as well. For the sake of economical operation and customer satisfaction it is crucial to understand the behavior of the system and its reaction to the changes of the operational environment.

One method of performance evaluation – besides simulation and measurement – is the mathematical, stochastic modelling based approach. In the last 2-3 decades the theory of phase type distributions, markovian arrival processes, efficient solution of structured Markov chains went through on a large improvement, which made it possible to apply markovian modelling in modern telecommunication systems.

Besides Markov chains, an other popular modelling technique is the application of Markov reward models. In Markov reward models a continuous quantity, the reward is accumulated with a speed depending on the state of the background Markov chain. For example, in a telecommunication application the accumulated reward can represent the amount of data transmitted on the network, while the state transitions of the background Markov chain correspond to the changes of the available bandwidth. One of the reasons of the increasing popularity of Markov reward models is that numerically efficient algorithms are present to compute the related performance measures. Due to the results of the recent research activity the class of problems that can be analyzed by Markov reward models is growing, and the investigation of further possible extensions is an important research task.

2 Research Aims

2.1 The Analysis of the ATM AAL2 Multiplexer

The ATM AAL2 protocol has been developed to transmit low bit rate multimedia traffic ([1]). Since the protocol is used by delay sensitive applications, the most important performance measure is the distribution of the waiting time. The AAL2 standard defined a timer based mechanism to improve the multiplexing gain. Due to this mechanism the system is non-work conservative, which makes the stochastic analysis difficult with the available techniques.

Our goal was to construct a stochastic model based on the queueing behavior of the multiplexer, and to compute the distribution of the waiting time. Emphasis has been laid on the numerical efficiency of the computation method.

2.2 Analysis of Multiclass Queueing Systems

In multiclass queueing systems jobs can be grouped to job classes. Jobs belonging to different classes can have different behavior, e.g. different arrival process, or different service requirement. In server stations the server can take the class of the jobs into considerations when deciding the service order. For example, non-preemptive priority scheduling (also referred to as head of line priority scheduling, HOL-PS) defines strict priorities: always the highest priority job is selected by the server, but there is no job preemption if a higher priority job arrives. An other popular scheduler algorithm is the weighted fair queueing (WFQ). In WFQ systems there are weights assigned to each class. The ratio of server capacity available for a class is given by the ratio of weights of the classes that are "active" (there are customers waiting in the queue belonging to that class). So the "importance" of the customers is regulated by the weight assigned to their class. Both scheduling algorithms are widely used in telecommunication systems.

The solutions in the literature all have restrictions, which can be too restrictive for practical application. Some of them allows only Poisson arrival process ([4, 6, 10, 3, 8]), while the others provide only the mean waiting time (e.g. [9]). The solution in [11] is general enough, but that method has some unsolved numerical issues.

Our goal was to develop methods to analyze the non preemptive priority and WFQ systems without the above mentioned restrictions.

2.3 Extensions of Markov Reward Models

In ordinary Markov reward models the reward accumulation follows a linear function (it is first order), and the amount of reward is maintained at state transitions. Some practical problems might need a more general model. For example, in telecommunication systems the bandwidth experienced by the customers can be varying even if the state of the system is not changing. In other systems it might be required to model the partial loss of the accumulated reward (loss of the "completed job" due to an error). The analysis of such extensions of Markov reward models were our goal in the third set of theses.

3 Research Methodology

In the first part, during the analysis of the ATM AAL2 multiplexer, the applied methodology was the markovian modelling. The system – in some embedded instants – had markovian behavior. The regular (M/G/1-type) structure of the generator matrix made it possible to apply matrix geometric techniques in the solution. To compute the waiting time distribution in a numerically efficient way, we used randomization method to calculate the matrix exponential function and its integrals.

In the second part the Markov model of the multiclass queues lead to regular structured Markov chain generator again. We solved it using the theory of structured state spaces, and quasi birth-death processes.

In the third part in the analysis of second order Markov reward models we expressed the differential equations of the system by conditioning on the duration of the first transition. The moments are computed by Laplace transforming and taking the derivative of the differential equation. In the numerical method we applied randomization, the error bound has been derived using combinatoric considerations.

The results of the presented analytical methods have been compared to simulation results. The simulation tools have been developed using the OmNet++ simulation framework ([2]). In the first part we used the simulation results to check the correctness of the arisen quite complex expressions. In the second part, which presents an approximate solution, we checked whether our simplifying assumptions were reasonable, and we also checked the accuracy of the approximations by simulation.

4 New Results

1. Set of Theses: The Analysis of the BMAP/D/1-Timer multiplexer

The ATM AAL2 multiplexer is used by slow bit rate, delay sensitive (typically multimedia) traffic. The multiplexer forms and transmits an ATM cell in each deterministic time interval (whose duration is determined by the link capacity). To improve the multiplexing gain the following timer based mechanism has been introduced: if the data in the buffer is too few to form a full ATM cell, the server stops, and waits for more data to arrive. The timer ensures that this break (additional delay for the data) can not be arbitrarily long: as the timer elapses, the data is transmitted in a partially filled ATM cell ([1]).

To model the behavior of te ATM AAL2 multiplexer, I defined a queueing system, the BMAP/D/1-Timer multiplexer.

Thesis 1.1 *I provided the generator of the Markov chain that describes the behavior of the BMAP/D/1-Timer multiplexer at departure instants; and I determined the steady state probabilities of this Markov chain.*

With proper state partitioning the Markov chain becomes of type M/G/1, with the following block-structure:

$$\mathcal{X} = \begin{bmatrix} \boxed{\quad\quad\mathcal{B}\quad\quad} & \cdots \\ \boxed{\quad\quad\mathcal{A}\quad\quad} & \cdots \\ \quad\boxed{\quad\quad\mathcal{A}\quad\quad} & \cdots \\ \quad\quad\boxed{\quad\quad\mathcal{A}\quad\quad} & \cdots \\ \quad\quad\quad\boxed{\quad\quad\mathcal{A}\quad\quad} & \cdots \\ \quad\quad\quad\quad\quad\ddots & \vdots \end{bmatrix}$$

Matrix $\mathcal{B}$ corresponds to the states where the amount of data in the buffer is between 0 and $L-1$. Therefore the effect of the timer has to be taken into consideration during the computation of matrix $\mathcal{B}$. The timer plays no role in case of matrix $\mathcal{A}$, since in those states the buffer contains enough data to fill a complete packet. Reviewing the literature of M/G/1 type

Markov chains we found that the Ramaswami formula is the most appropriate to compute the steady state probabilities (see [7]).

Thesis 1.2 *I have computed the waiting time distribution of the data waiting in the BMAP/D/1-Timer multiplexer, and gave a numerical method to compute it efficiently.*

The waiting time of the data is the function of the buffer length and of the state of the timer at arrival. The conditional waiting time distribution is computed, by keeping the buffer length and timer state fixed. By using the steady state probabilities mentioned above, the waiting time distribution is computed by unconditioning. The resulting expressions contain integrals at many points. The numerical evaluation of these integrals can be slow and not accurate enough. With proper rearrangement, we could transform most of the integrals to a form of $\int_a^b e^{Qt} dt$. This form can already be evaluated efficiently using the randomization algorithm.

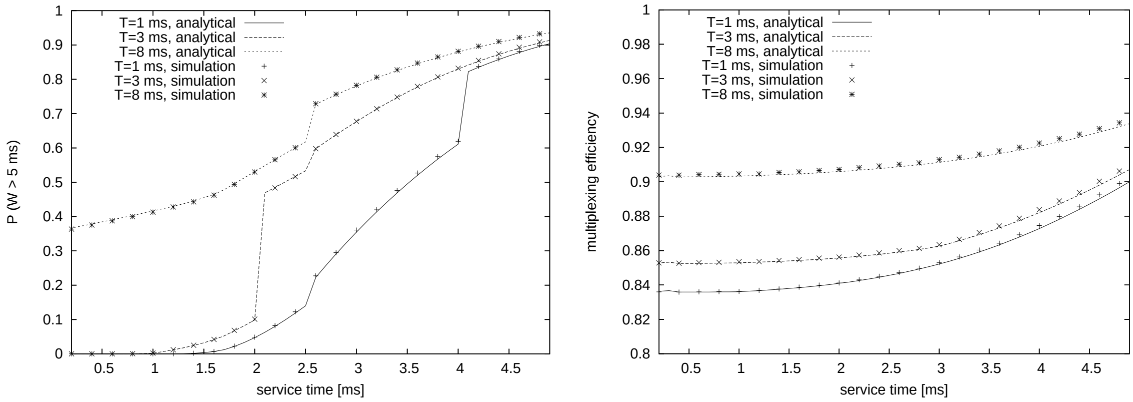
Thesis 1.3 *I have computed the efficiency of multiplexing of the BMAP/D/1-Timer multiplexer.*

I defined the multiplexing efficiency as $\eta = \lambda/(L \cdot \mu)$, where λ is the data arrival intensity, μ is the packet departure intensity, and L is the size of the packet payload measured in data units.

With this definition the multiplexing efficiency is the smallest ($\eta = 1/L$) when every data unit leaves the system in a separate packet. The multiplexing efficiency is the best (η is the largest: $\eta = 1$) when all the departing packets are fully filled. The mean packet departure rate μ can be computed from the steady state distribution of the Markov chain embedded at departures.

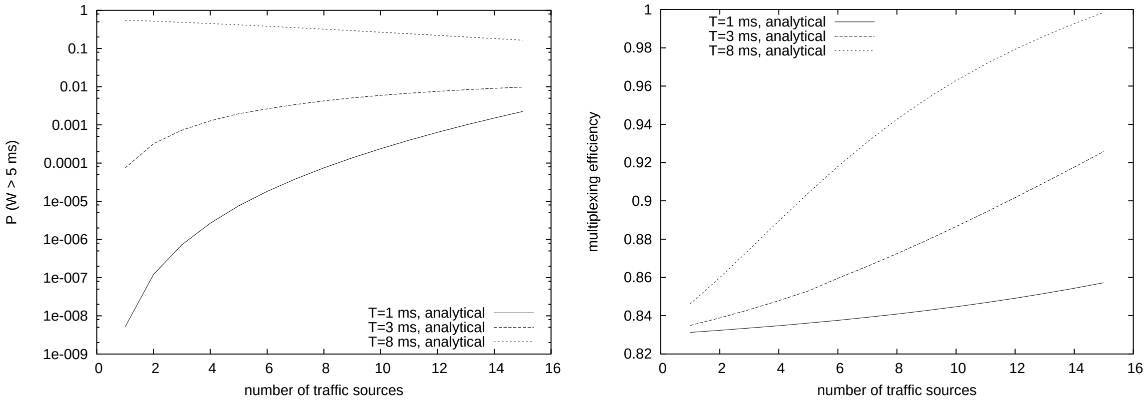
To investigate the behavior of the system, I constructed a numerical example, with real life like traffic and service parameters. Figure 1 depicts the effect of the service time (thus, the effect of the link capacity) on the probability of exceeding the delay limit. The larger the value of the timer is, the larger is the probability of exceeding the delay limit. An interesting feature of the system is that if the timer value is larger than the delay limit, the probability of exceeding the limit can not be decreased arbitrary low by increasing the link capacity.

An other practical problem is to determine the number of traffic sources allowed to enter the system while keeping low the probability of exceeding the delay limit. Figure 2 shows that if the timer value is smaller than the delay limit, then the probability of exceeding the delay limit increases by increasing number of traffic sources. But if the timer value is larger than the delay limit, we experienced the opposite.



(a) (b)

Figure 1: $P(W > 5\,\text{ms})$ vs. service time



(a) (b)

Figure 2: $P(W > 5\,\text{ms})$ vs. the number of sources

The results related to the 1st set of theses are published in [C10] and [J2].

2. Set of Theses: Approximate Analysis of Multiclass Queueing Systems

In this set of theses we consider multiclass queueing models given by a so-called two parameter description. This means that the inter arrival times of customers are given by two parameters (by the arrival intensity, and by the squared coefficient of variation of the inter arrival times), and the service time is given by two parameters (mean service time, squared coefficient of variation of the service time), too. The parameters of customers belonging to different classes can be different. The provided performance measures are the distribution of the number of customers in the system, and the mean and variance of the waiting time.

Thesis 2.1 *I have developed an approximate solution method to compute the distribution of the number of customers in the system, and the mean and variance of the waiting time in case of two class non preemptive priority scheduling.*

The concept is to approximate the two class system as the classes were separated, and construct a service process for both classes that approximately imitates the behavior of the original server.

From the point of view of the low priority customer class, the exact number of high priority customers does not play any role. When there are no high priority customers, the server is available, and when there are high priority customers, the server is not available for low priority customers. Therefore, during the analysis of the low priority queue, the two dimensionally infinity state space is eliminated such, that the number of high priority customers is modeled by only 2 states: zero, and more than zero. This approach is reflected by Figure 3, which depicts the structure of the approximate Markov chain model of the low priority queue.

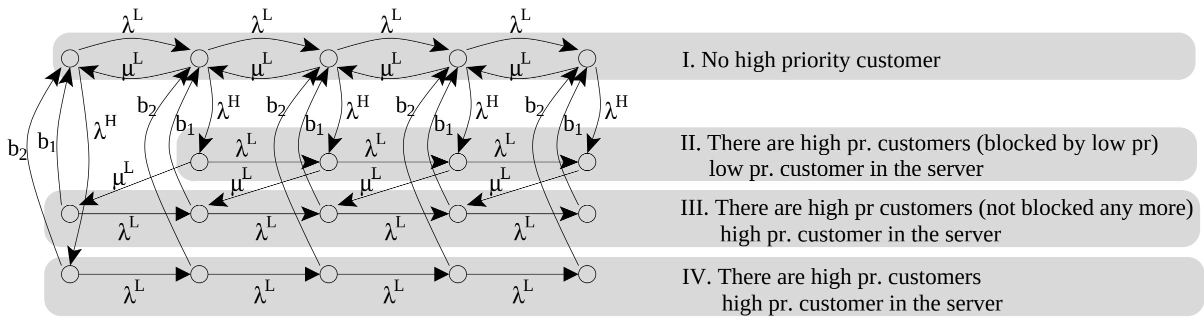


Figure 3: The approximate QBD model of the low priority queue

The high priority customers can be affected by the low priority customers only at one point: when the high priority queue is empty at the arrival of a high priority customer, and a low priority customer is in the server. In this case the arrived high priority customer has to await the remaining service time of the low priority customer, since the service is non-preemptive. The probability of this event (q) will be computed from the queue model of the low priority class. Figure 4 shows the structure of the corresponding Markov chain.

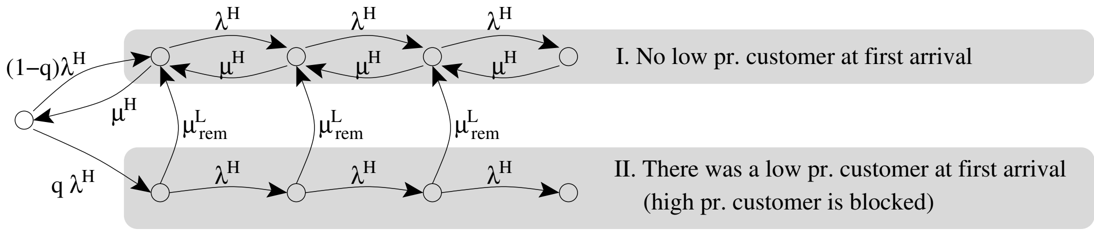


Figure 4: The approximate QBD model of the high priority queue

Thesis 2.2 *I have developed an approximate solution method to compute the distribution of the number of customers in the system, and the mean and variance of the waiting time in case of two class weighted fair queueing scheduling.*

As in the previous case, the idea of the approximation is to separate the classes. From the point of view of a customer class, the capacity of the server is varying depending on the presence of customers belonging to the other class (Figure 5) This behavior is reflected on Figure 6, which shows the structure of our approximating Markov chain.

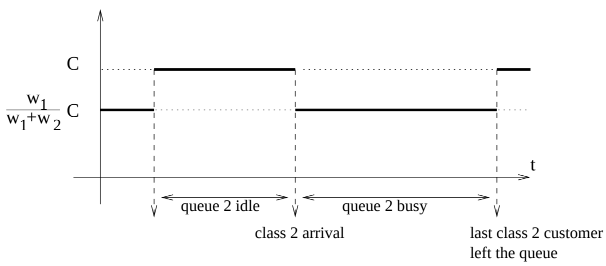


Figure 5: Server capacity seen by a class 1 customer

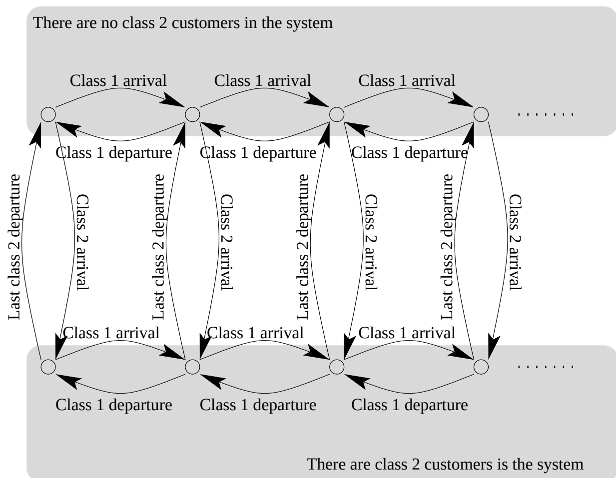


Figure 6: Structure of the Markov chain

During the analysis of both scheduling algorithms the inter arrival times and the service times are characterized by phase type distributions constructed based on the two parameter description (the figures above show only the macro structure of the Markov chains!). The duration of the busy periods are computed by matrix geometric methods, and approximated by a phase type distribution, too. The resulting Markov chain has a block tri-diagonal –

so-called quasi birth-death – structure, whose performance measures are studied extensively in the literature. (In the dissertation we use the algorithm descried in [5]).

We compared our analytical results to simulation results to check the accuracy of the approximations. To explore the limits of usability of the presented approximations, we checked the influence of all the system parameters on the accuracy. We found that the approximation of the non-preemptive priority scheduler is more accurate than the approximation of the WFQ scheduler. In both cases the mean waiting times are approximated reasonably accurate, the difference compared to simulation results is less than 5% in case of the priority system and less than 10% in case of the WFQ system. In the checked range of the parameters the error of the approximation of the squared coefficient of variation of the waiting time is mostly less than 10% in case of the priority scheduler, and less than 20% (except in some cases) in case of the WFQ scheduler. According to our experience at larger squared coefficient of variation of the inter arrival and service times the accuracy decreases, the approximation performs best in the exponential case. Figures 7 and 8 depict some of the results.

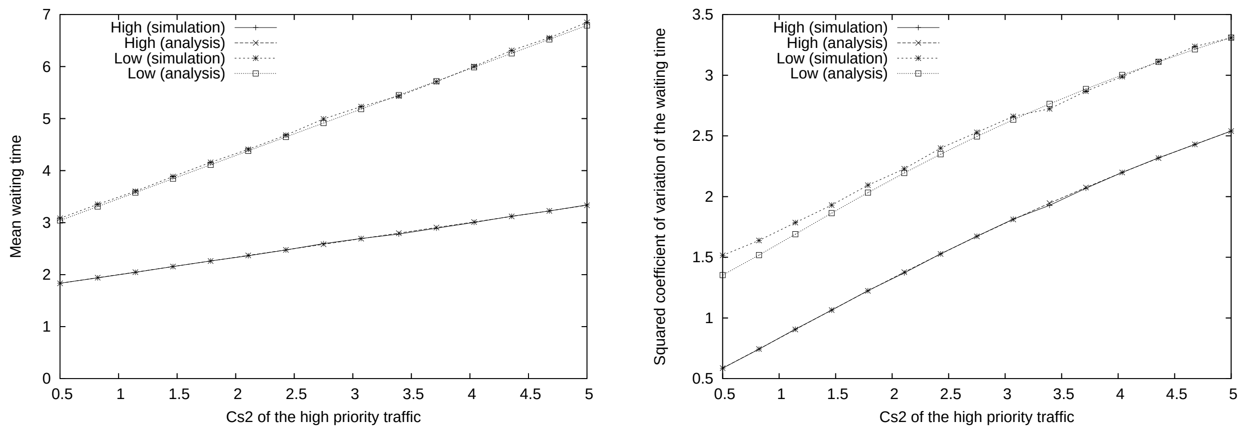


Figure 7: Non-preemptive priority queue; the effect of the squared coefficient of variation of the service time of the high priority customers

The results of this set of theses are published in [C2] (WFQ scheduling) and in [C12] (non preemptive priority scheduling).

3. Set of Theses: Extended Markov Reward Models

In the third part of the dissertation we extend the ordinary Markov reward models in two directions. We allow varying reward accumulation (second order Markov reward model, see

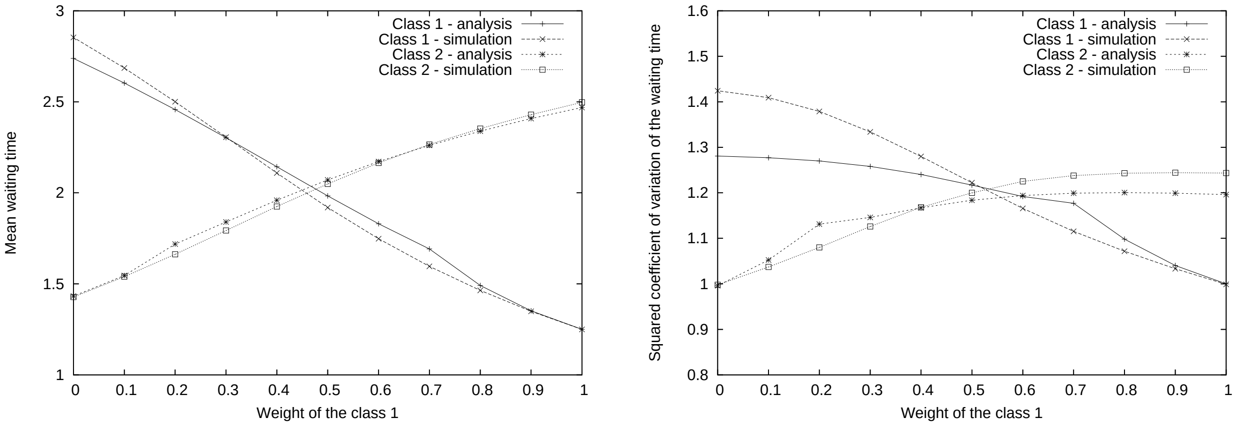


Figure 8: WFQ example; the effect of the weight

Figure 9), and we analyze partial loss reward models (Figure 10), where a portion of the reward accumulated during the last state sojourn is lost at state transitions.

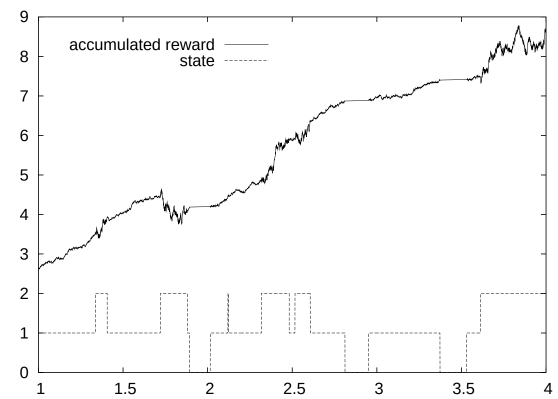


Figure 9: Second order Markov reward model

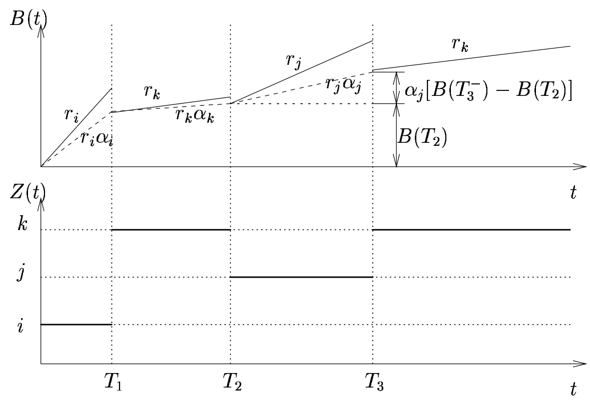


Figure 10: Markov reward model with partial increment loss

In second order Markov reward models the reward accumulation follows a Brownian motion with state dependent drift and variance parameters.

Thesis 3.1 *In a second order Markov reward model the probability density function of the accumulated reward at time t (denoted by $\underline{b}(t,w)$) satisfies the following set of differential equations:*

$$\frac{\partial}{\partial t}\ \underline{b}(t,w) + \boldsymbol{R}\ \frac{\partial}{\partial w}\ \underline{b}(t,w) - \frac{1}{2}\ \boldsymbol{S}\ \frac{\partial^2}{\partial w^2}\ \underline{b}(t,w)\ =\ \boldsymbol{Q}\ \underline{b}(t,w),$$

where $\boldsymbol{Q}$ *is the generator of the background Markov chain;* $\boldsymbol{R}$ *matrix defines the state dependent drift,* $\boldsymbol{S}$ *defines the state dependent variance parameters (*$\boldsymbol{R}$ *and* $\boldsymbol{S}$ *are diagonal matrices). The ith item of vector* $\underline{b}(t, w)$ *corresponds the the ith initial state.*

Since this differential equation is second order and it has two variables, its numerical solution is problematic (it can be solved only with a few, about 100 states). However, instead of the distribution, the computation of the moments is much more efficient: its computational complexity is not significantly larger compared to ordinary Markov reward models. Using the following numerical method it is possible to solve models having more than 100000 states:

Thesis 3.2 *In a second order Markov reward model the nth moment of the accumulated reward at time t (denoted by* $\underline{M}^{(n)}(t)$*) can be computed by the following way:*

$$\underline{M}^{(n)}(t) \;=\; n!\; d^n \sum_{k=0}^{G} e^{-qt} \frac{(qt)^k}{k!} \underline{D}^{(n)}(k) + \underline{\xi}(G),$$

The $\underline{\xi}(G)$ *error term can be arbitrary small by setting G the following way:*

$$G \;=\; \min_{g} \left(2\; d^n\; n!\; (qt)^n \sum_{k=g+n+1}^{\infty} e^{-qt} \frac{(qt)^k}{k!} < \epsilon \right). \tag{1}$$

The $\underline{D}^{(n)}(k)$ *coefficients are computed using the following recursive formula:*

$$\underline{D}^{(n)}(k+1) \;=\; \boldsymbol{R'}\; \underline{D}^{(n-1)}(k) \;+\; \frac{1}{2}\; \boldsymbol{S'}\; \underline{D}^{(n-2)}(k) \;+\; \boldsymbol{Q'}\; \underline{D}^{(n)}(k), \tag{2}$$

We analyze the Markov reward models with partial increment loss by time reversal. The reason is that these models work exactly as ordinary Markov reward models with reward rates reduced by the loss until the last state transition. From the last state transition to time t the reward is accumulated with the lossless rate. (See Figure 10). With time reversal these models can be viewed as they were ordinary Markov reward models. The state space has to be duplicated. In the first part the reward is accumulated according to the lossless rates, in the second part it is accumulated according to the reduced rates. The accumulation is started in the first part, and at the first transition (which is the last one in "normal" time) the Markov chain enters (and stays in) the second part. This is the idea behind the next two theses:

Thesis 3.3 *In a Markov reward model with partial increment loss the distribution of the accumulated reward at time T (denoted by* $B(T, w)$*) is:*

$$B(T,w) = \sum_{i \in S} \left(\overleftarrow{\underline{X1}}_i(T,w) + \overleftarrow{\underline{X2}}_i(T,w) \right) \underline{\gamma}_i(0)\ ,$$

where vectors $\overleftarrow{\underline{X1}}(\tau, w)$ *and* $\overleftarrow{\underline{X2}}(\tau, w)$ *are the solutions of the following partial differential equations:*

$$\frac{\partial}{\partial \tau}\overleftarrow{\underline{X1}}(\tau, w) + \frac{\partial}{\partial w}\overleftarrow{\underline{X1}}(\tau, w)\mathbf{R} = \overleftarrow{\underline{X1}}(\tau, w)\mathbf{Q}_D \ , \tag{3}$$

and

$$\frac{\partial}{\partial \tau}\overleftarrow{\underline{X2}}(\tau, w) + \frac{\partial}{\partial w}\overleftarrow{\underline{X2}}(\tau, w)\mathbf{R}_\alpha = \overleftarrow{\underline{X1}}(\tau, w)(\mathbf{Q} - \mathbf{Q}_D)^T \ + \overleftarrow{\underline{X2}}(\tau, w)\mathbf{Q}^T \ . \tag{4}$$

These partial differential equations have two variables, using them we are capable to solve models having only a small number of states (few hundred). For the computation of the moments of the accumulated reward we could develop an efficient numerical method, which makes it possible to analyze models having more than 100000 states:

Thesis 3.4 *In a Markov reward model with partial increment loss the nth moment of the accumulated reward at time t (denoted by $M^{(n)}(t)$) can be calculated by:*

$$M^{(n)}(t) = \sum_{i \in S} \left(\overleftarrow{\underline{M1}}_i^{(n)}(T) + \overleftarrow{\underline{M2}}_i^{(n)}(T)) \right) \underline{\gamma}_i(0) \ ,$$

where $\overleftarrow{\underline{M1}}^{(n)}(T)$ *has the following closed form:*

$$\overleftarrow{\underline{M1}}^{(n)}(\tau) = \tau^n \ \underline{e} \ \mathbf{R}^n \mathbf{E}_D(\tau) \ , \tag{5}$$

($\mathbf{E}_D(\tau)$ is a diagonal matrix: $\mathbf{E}_D(\tau) = diag\langle e^{q_{ii}\tau} \rangle$), $\overleftarrow{\underline{M2}}^{(n)}(T)$ *is the result of the following sum with error term* $\underline{\xi}(G)$*:*

$$\overleftarrow{\underline{M2}}^{(n)}(\tau) = n! d^n \sum_{k=0}^{G} e^{-\lambda\tau} \ \frac{(\lambda\tau)^k}{k!} \ \underline{D}^{(n)}(k) + \underline{\xi}(G) \tag{6}$$

With adequately large G the error term can be made arbitrary small:

$$G = \min_{g > n} \left((\lambda\tau)^{n+1} d^n \sum_{k=g-n-1}^{\infty} e^{-\lambda\tau} \ \frac{(\lambda\tau)^k}{k!} < \varepsilon \right) .$$

The $\underline{D}^{(n)}(k)$ *coefficients are computed by the following recursive formula:*

$$\underline{D}^{(n)}(k) = \begin{cases} \underline{e} \ (\mathbf{I} - \tilde{\mathbf{Q}}_D^k) & n = 0 \\ 0 & k \le n, n \ge 1 \\ \underline{D}^{(n-1)}(k-1)\tilde{\mathbf{R}}_\alpha + \underline{D}^{(n)}(k-1)\tilde{\mathbf{Q}} + & \\ \qquad \binom{k-1}{n} \ \underline{e} \ \tilde{\mathbf{R}}^n \tilde{\mathbf{Q}}_D^{k-1-n}(\tilde{\mathbf{Q}} - \tilde{\mathbf{Q}}_D) & k > n, n \ge 1 \end{cases} \tag{7}$$

With the presented numerical methods it is possible to solve very large Markov reward models having the introduced extensions. With our implementation we were able to solve models with a background process having 200000 states in an hour.

We published the analysis of second order reward models in [C8]. The presented algorithms became the part of the reward model tool called MRMSolve 2.0, demonstrated in [C9].

5 Application of the Results

The results of the first set of theses can be used for the performance analysis of the ATM AAL2 multiplexer, and for the solution of the corresponding network dimensioning problems. The introduced stochastic model is general enough to model and evaluate other timer based practical systems as well.

The results of the second set of theses related to multiclass queueing systems can be used to analyze telecommunication networks that provide QoS (quality of service), since the discussed scheduling algorithms are commonly used in these systems.

The results of the third set of theses extend the set of modelling tools. They make it possible to efficiently analyze systems having varying reward accumulation with or without loss, whose analysis was not or not efficiently possible before.

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List of Publications

International Journal Publications

[J1] A. Horváth, G. Horváth, and M. Telek. **Analysis of Inhomogeneous Markov Reward Models**. *Linear Algebra and its Applications*, 386: 383-405, 2004.

[J2] G. Horváth, M. Telek. **Analysis of a BMAP/D/1-Timer multiplexer**. *Electronic Notes in Theoretical Computer Science*, 128(4): 25-44, 2005.

International Conference Publications

[C1] G. Horváth. **Approximate Waiting Time Analysis of Priority Queues**. In *Proc. of the Fifth International Workshop on Performability Modelling of Computer and Communication Systems*, Erlangen, Germany, Sep. 2001. Extended abstract.

[C2] G. Horváth, and M. Telek. **Approximate Analysis of Two Class WFQ Systems**. In *Proc. of the Sixth International Workshop on Performability Modelling of Computer and Communication Systems*, pages 43–46, Arlington, IL, USA, Sept 2003. Extended abstract.

[C3] G. Horváth, and Cs. Vulkán. **Analytical 3G RAN Transport Network Modeling with CALIPRAN**. In *Proc. of the 11th Microcoll*, Budapest, Hungary, Sep. 2003.

[C4] G. Horváth, M. Telek, and Cs. Vulkán. **AAL2 Multiplexing Delay Calculation in UTRAN**. In *Proc. of the 11th Microcoll*, Budapest, Hungary, Sep. 2003.

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