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TOPOLOGY OPTIMIZATION OF ENGINEERING STRUCTURES: SUPPRESSION OF DISCRETIZATION ERRORS

Theses of the PhD Dissertation

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1. Background, aims and scope of the research

Topology optimization of engineering structures is currently a very popular and important part of the scientific research. The rapid expansion of this field is supported by a fast improvement of the computational capabilities and their decreasing cost. Thus in the not so distant future a more accurate **F**inite **E**lement (FE) analysis of even the largest ground structures will be both feasible and economical. But discretized FE-based optimization methods also require some control of computational errors causing incorrect solutions. These problems justify further research and modified algorithms.

One of the most severe computational difficulties in FE-based topology optimization is caused by solid (or "black") ground elements connected only through a corner node. This configuration may appear in checkerboard patterns, diagonal element chains or as isolated hinges. Corner contacts in nominally optimal topologies are caused by discretization errors associated with simple (e.g. four-node) elements, which grossly overestimate the stiffness of corner regions with stress concentrations. In fact, it was shown by Gáspár, that both checkerboard patterns and diagonal element chains may give an infinite compliance, if the latter is calculated by an exact analytical method. This makes them the worst possible solution, if an exact analysis is used in compliance minimization.

Corner contacts may be suppressed by:

(a) ***a more accurate FE analysis of the ground elements***, where the process may use several simple FE's per **G**round **E**lement (GE), or higher order elements. Disadvantages of this approach are

- greatly increased **D**egree **O**f **F**reedom (DOF) for a given number of ground elements and
- some diagonal chains remain in the solution.

(b) ***Modification of the original problem by using geometrical constraints or "diffused" sensitivities (filters)***, e.g. perimeter control or sensitivity filtering of the original topology optimization problem. These usually results in a lower resolution, which may – in some cases – be highly nonoptimal in terms of the original problem.

The aim of this investigation has been to determine an effective and computationally economical numerical method, which is able to (i) suppress the corner contact error and (ii) converge to the exact analytical solutions.

2. A short summary of the research program

This program has contained three main parts as follows.
(a) The weight-increasing effect of the topology simplification has been analyzed. The effect of simplifications is evaluated for both two and three dimensional elements. The effects of (i) applying a greater number of simple finite elements per ground elements and (ii) applying a higher order finite element per ground element are also presented. The analysis has been developed for structures for four ground elements and with arbitrarily formed and sized structures.
(b) The **Extended-SIMP** algorithm based on the well known **SIMP**[1] method with several four-node finite elements per ground elements is presented, which can suppress the checkerboard appearance of topology optimization of plane structures built up from quadratic elements. The efficiency of the program is demonstrated by numerical examples (eg. Michell cantilever and Michell bicycle wheel) with different tuning parameters. The effects of these parameters on the optimal layouts are also discussed. The Extended-SIMP algorithm was also applied to topology optimization of structures with some pre-existing members or elements (**SIMP-NDR**[2]).
(c) Some new **Corner Contact Functions** (CCFs) are presented, which are able to detect and suppress not only checkerboarding, but also diagonal chains and isolated hinges. Employing these functions a new mathematical programming process **Co-SIMP**[3] has been developed. The CCFs have been extended to three dimensional topologies as well.

3. An overview of the new scientific results

Weight increasing effect of topology simplification

Using known finite element models, an algorithm and program was developed for calculating arbitrarily supported and loaded plane structures built-up from quadrate elements:

a. by n^2 four-node finite elements per ground elements ($n = 1, 2, \ldots$), and
b. by a k^2-node finite element per ground elements ($k = 1, 2, \ldots$).

In addition an algorithm and program was developed for calculating arbitrarily supported an loaded cubic structures built-up from cubic elements:

c. by n^3 eight-node finite elements per cubic ground elements ($n = 1, 2, \ldots$).

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[1] **S**olid **I**sotropic **M**icrostructure with **P**enalization

[2] **N**on-**D**esign **R**egion

[3] **Co**rner **Co**ntact **Co**ntrol

It has been found that in case of given displacements increasing the n and k values the strain energy (and/or the compliance) decreases strongly.

Analyzing ***plane structures*** the following conclusions have been reached:

- the adaptation of n^2 four-node finite elements per ground elements for energy decrease is as good and efficient as the usage of several higher order (k^2-node) finite elements as ground elements, but
- the adaptation of higher order (k^2-node) finite element per ground elements is more effective in the energy reduction than the usage of several n^2 four-node finite elements per ground elements.

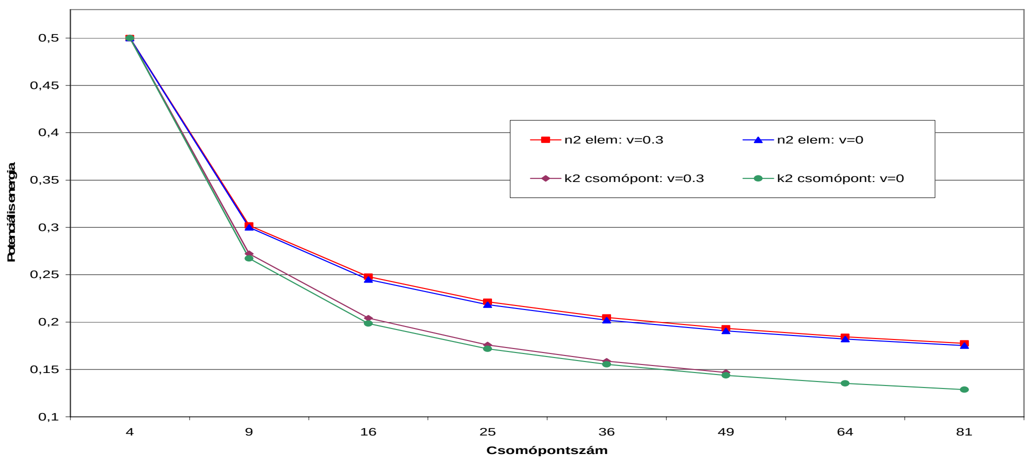


Figure 1: Strain energy decrease

Ithas been shown that the element chains are caused by neither a coarse FE discretization of ground elements nor a coarse GE mesh.

Analyzing ***three dimensional, cubic structures*** the following conclusion has been reached:

- the usage of several simple cubic finite elements per ground elements is good and effective for energy and/or compliance reduction.

Considering a ***square plate of four ground elements*** in plane stress it has been found that the compliance extremizations are presenting different and sometimes deceptive results. It has also been shown that the energy decrease generated by refining the finite element mesh per ground elements has an effect on the results.

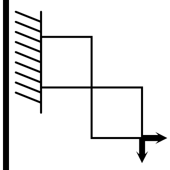
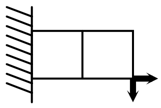
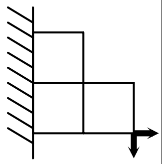
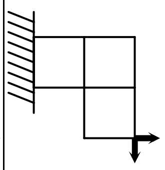
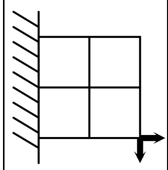
Number	**1**	**2**	**3**	**4**	**5**
Structural forms with supports and with displacement limits					
Pattern	1 0 0 1	0 0 1 1	1 0 1 1	1 1 0 1	1 1 1 1
Volume fraction (*f*)	0,5	0,5	0,75	0,75	1

Figure 2: Ground elements, supports and loading of the conceptual example

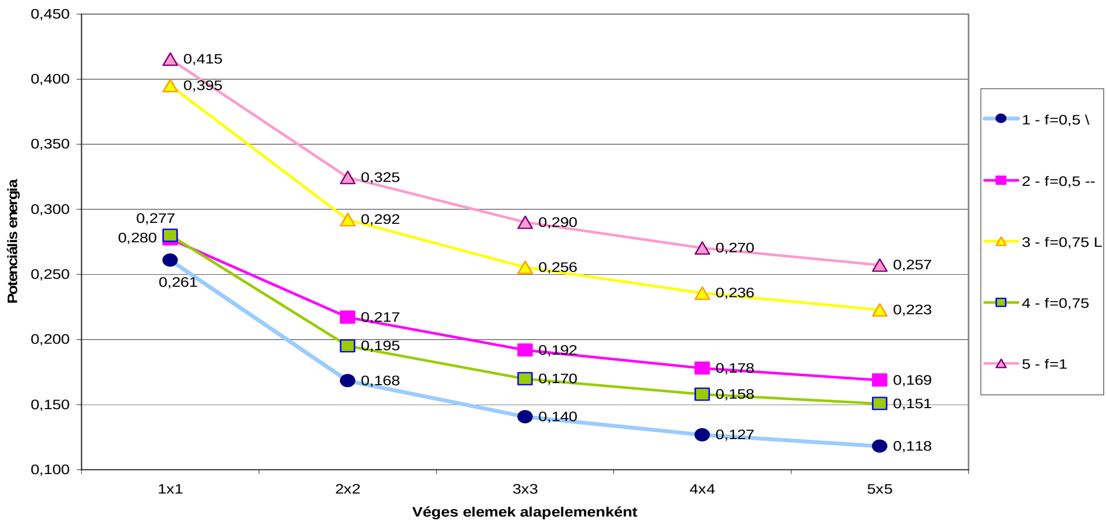


Figure 3: Strain energy decrease of the conceptual example

Extended optimality in topology design (Extended-SIMP)

Exact optimal topologies of Michell-type structures have been replicated by the Extended-SIMP numerical computational program, based on the well known SIMP method and on ground elements divided into simple four-node finite elements. The following conclusions have been reached:

The Extended-SIMP method is suitable for suppressing the checkerboard areas of the optimal layouts:

- In case of a given structural form and loading the use of simple four-node finite elements as ground elements increases the size of the checkerboarded area,

- the adaptation of several four-node finite elements per ground elements decreases the size of the checkerboarded area. Moreover, applying a good combination of the ground element number and of the compliance limit the checkerboard pattern can be completely eliminated, but
- diagonal chains and/or isolated hinges remain.

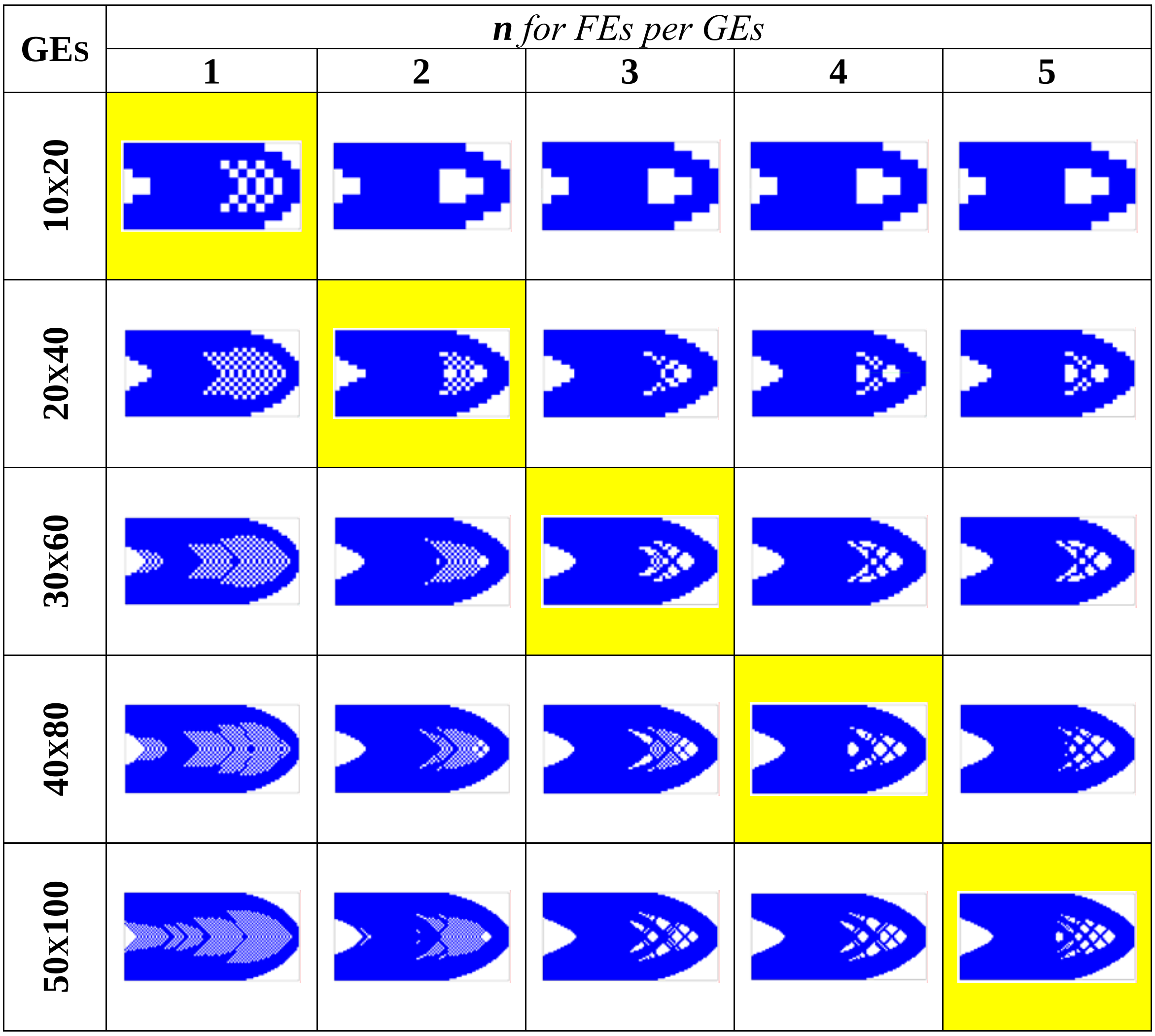


Figure 4: Michell cantilever with increasing number of GEs and FEs and with a very low compliance limit

The calculated optimal layouts depend upon the tuning parameters of the optimization process. The effect of a given compliance and of the applied number of ground elements and/or finite elements is as follows:

- In the case of low number of ground elements with low compliance limit a large number of finite elements is preferable, whereas for a large number of ground elements less finite elements per GEs are preferable, even $n = 2$ could be enough.

- Specifying a relatively low compliance limit to a given set of ground elements the optimal layout will take large solid areas internally.
- Specifying a relatively high compliance limit or applying an unrealistic finite element number to a given set of ground elements the optimal layout will be orderless inside. The generated diagonal chains are usually unbroken, moreover floating elements can appear.

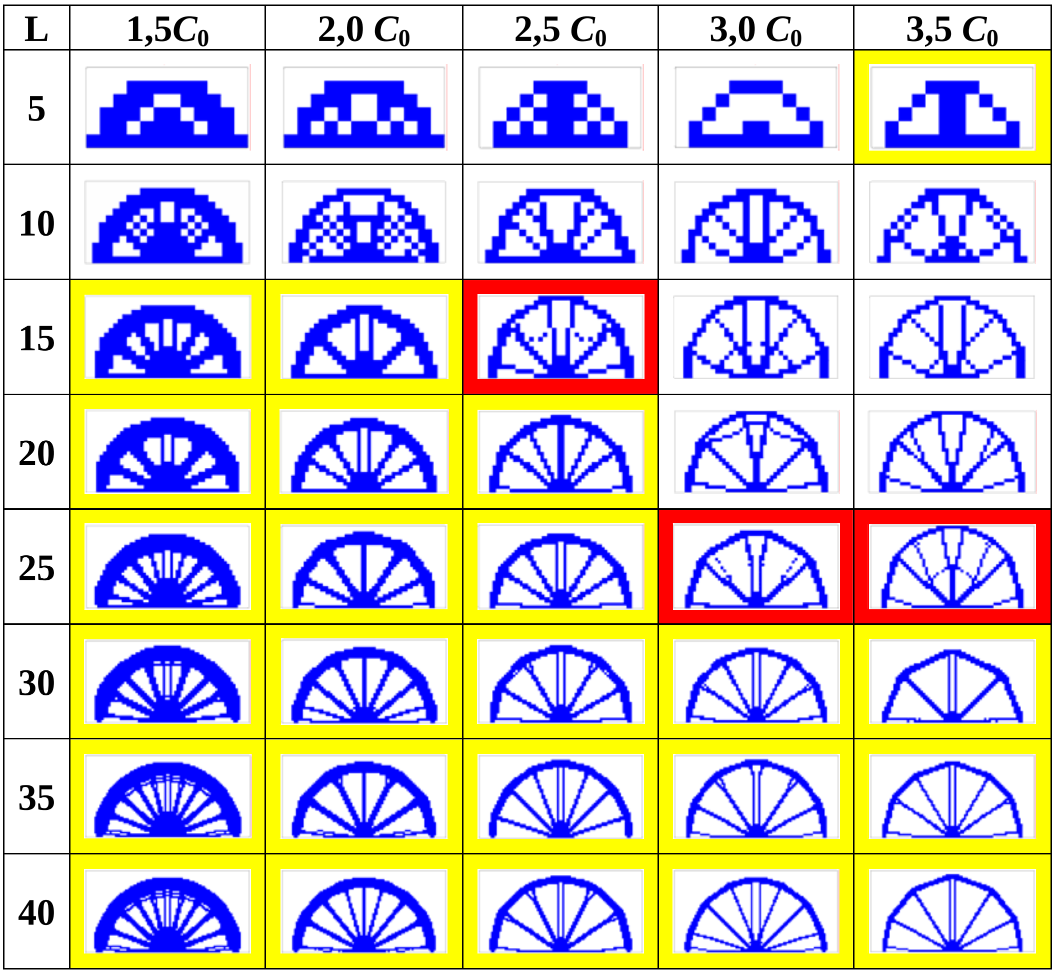


Figure 5: Michell's bicycle wheel with 2x2 FEs to an increasing number of GEs and to an increasing compliance limit

For proving the efficiency of the Extended-SIMP method the values of volume fraction and the compliance efficiency were determined.

- The compliance efficiency is maximum, it gives a constant value concerning to the given limit and it is independent from the number of ground elements.

- With a given ratio for the compliance limit the volume fraction decreases when the number of the ground elements increases.
- Increasing the number of ground elements the numerically calculated quantity of total compliance multiplied by the structural volume is not convergent to the quantity of the exact solution.[4]

Topology optimization with some pre-existing members or elements (SIMP-NDR)

Numerical topology optimization methods are often verified by comparing discretized optimal solutions for perforated plates in plane stress[5] with analytical solutions for least-weight trusses. The presented examples show that the solutions are not trivial. The exact optimal topology of Michell-type structure changes from a complicated one into a two-bar system, where there is a full use of the existing and therefore costless horizontal bar. Thus only the bottom stiffening involves extra cost.

It follows that pre-existing members cause a complete change of the optimal topologies of other examples.

Numerical confirmation of the proposed theory was reached by comparing result of the Extended-SIMP (where all elements are parts of the design region) and the SIMP-NDR methods (where there are pre-existing members, which are not parts of the design region). The results are very similar to the exact analytical solutions in both cases. The very thin diagonal bar in the middle of the SIMP-NDR solutions is due to the fact that the top horizontal bar is in beam action, which is not the case in the truss modeling.

With the above examples it has been proved that the application of pre-existing members do not modify the efficiency of the extended optimization method, and the appearance of diagonal chains.

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[4] It was shown later that the solution converges to the analytical one if we use a large and constant value of the FEs.

[5] Truss-like structures

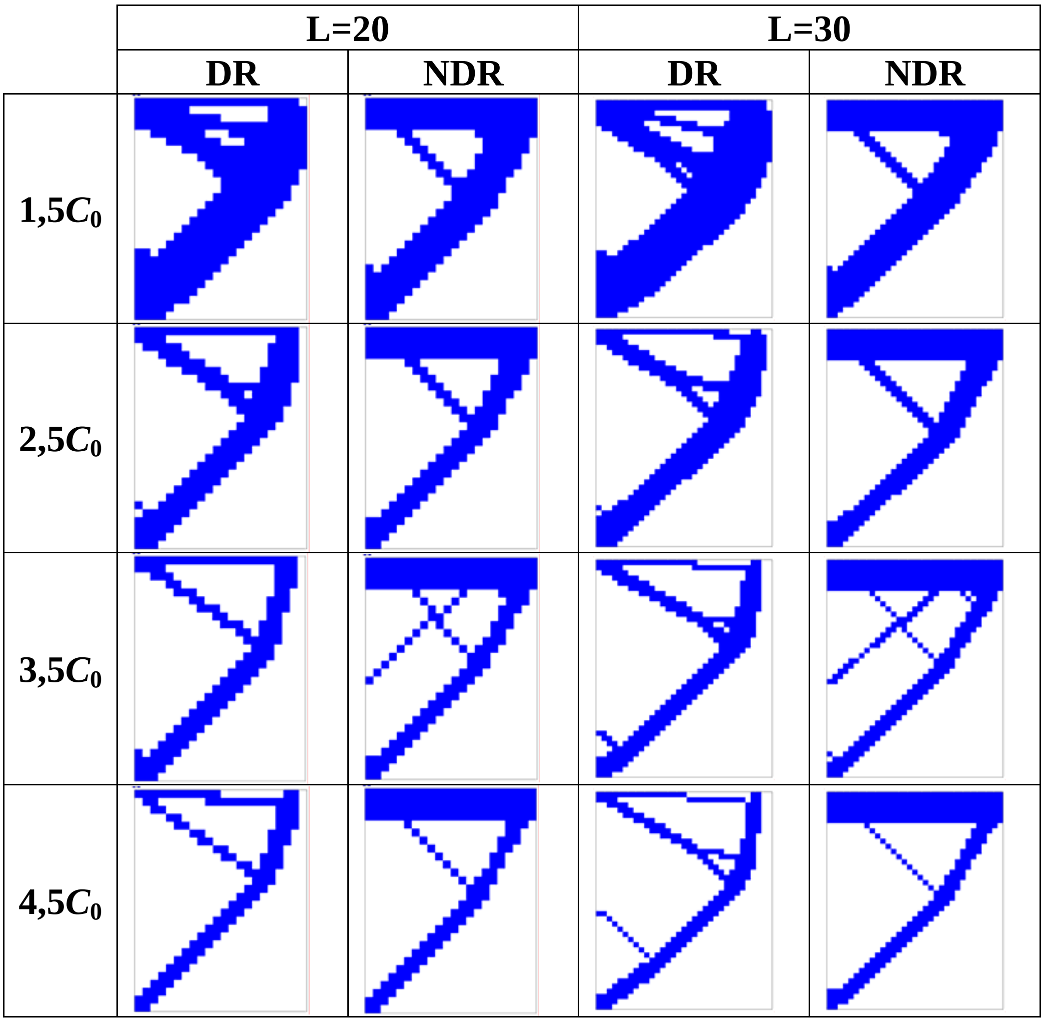

Figure 6: Results with the Extended-SIMP and SIMP-NDR of a unit loaded short cantilever

Topology optimization with direct Corner Contact Control (Co-SIMP)

As mentioned before, the checkerboard patterns caused by discretization errors can be suppressed by the Extended-SIMP and by the SIMP-NDR methods, but in the outputs diagonal chains or isolated hinges remain.

For this reason a new Co-SIMP[6] algorithm and program has been developed to control directly the corner-contact error. The problem description, the reasons, the formulations of the optimization and the iteration method are discussed in the dissertation. Moreover, review of corner contact functions both in plane and in space, which are able to control and assess the corner connections at each

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[6] **Co**rner **Co**ntact **Co**ntrol

internal node and are able to modify the optimal layouts of the example structures is given.

The treatment includes two **C**orner **C**ontact **F**unctions in plane and one (with three sub-formulas) in space. The two CCFs in plane have been included into the Co-SIMP and tested. The results of the numerical calculations are presented.

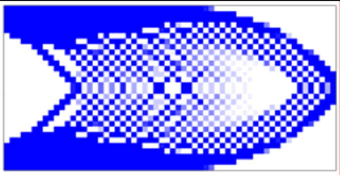
Value of *m* for increasing the effect of $H(\mathbf{t})$		
0	75	100
		

Figure 7: Co-SIMP results of the Michell's cantilever with the author's CCF

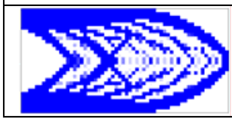
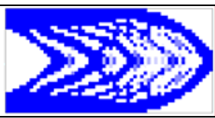
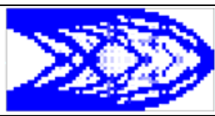
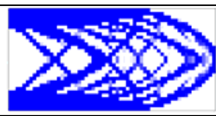
Value of *m* for increasing the effect of $H(\mathbf{t})$				
50	100	150	200	250
				

Figure 8: Co-SIMP results of the Michell's cantilever with the author's modified CCF

By the numerical results it has been proved that the Co-SIMP algorithm and a corner contact function is able to suppress the checkerboarding. The presented examples show that the type and the quality of the applied CCF has a big contribution to the optimization result. The outside form of the optimal topologies always compares well to the exact solutions, but the internal trussed part is changing according to the applied CCF and to the parameter which controls its efficiency. The presented examples and results confirmed that the proposed Co-SIMP method is able to present many optimal solutions for a given set. To determine that which one is the real optimum additional adequate conditions are necessary.

4. The theses of the PhD Dissertation

For the theses of my PhD research it is important to note that although the numerical methods of theses 1-3 are known from the scientific literature, all of the presented numerical calculations have been made by self developed and written algorithms and programs.

THESIS 1

The efficiency and the validity of employing several simple finite elements per ground element has been proved by numerical calculations. Calculation of compliance was made by algorithms with known square finite element models applied to arbitrarily supported and loaded structures with a given displacement and built up from quadratic elements in plane and on three dimensional structures built up from cubic elements.

1.a. By the adaptation of several of the simplest (four-node in plane and eight-node in space) finite elements per ground elements (n^2 in plane and n^3 in space, $n = 1, 2, \ldots$) the compliance of the structure with a given displacement limit strongly decreases.

1.b. It has been shown that the diagonal chains as remaining discretization error are caused by neither a coarse finite element discretization of the ground elements nor a coarse ground element mesh.

1.c. It has been proved that the energy decrease generated by the refining of the finite element mesh per ground element has an effect on the results, including the optimal structural forms.

The relevant publications: [1], [2], [3].

THESIS 2

The efficiency of the topology optimization process and the effects of so called tuning parameters of the applied algorithm have been analyzed by numerical calculations. The optimal topologies of Michell-type structures were determined by the Extended-SIMP numerical computational program, based on the well known SIMP method and on ground elements divided into simple four-node finite elements.

2.a. It has been shown numerically that the Extended-SIMP method is effective in suppressing the extensive checkerboard areas of the optimal layouts

2.b. It has been investigated how the applied number of ground elements and finite elements with the applied compliance limit are influencing the optimal layouts.

2.c. The adaptation of simple four-node finite elements per ground elements makes the optimization process effective in least-weight designs. It was also shown that the compliance constrain is active for the optimization design.

The relevant publications: [1], [3], [6], [11].

THESIS 3

The Extended-SIMP method was applied for calculating the optimal topologies of plate structures in plane stress with pre-existing elements of zero cost (SIMP-NDR).

3.a. It has been shown numerically that pre-existing members do not change the quality, the efficiency of the program and the dependence on tuning parameters. The relation between the compliance and the element number also exists in this case (as in Thesis 2.b).

3.b. The validity of the exact analytical solutions of GIN Rozvany for short cantilevers both in general and in NDR cases have been confirmed numerically. The optimal forms are very similar to the exact analytical solutions in both cases

3.c. The optimal topologies of the calculated short cantilever demonstrate the difference from the truss structures used in the exact solutions. The generated bars in plate structure are in beam actions, and there is a prop to reduce its bending moment.

The relevant publications: [7], [8], [9].

THESIS 4

Corner **C**ontact **F**unctions (CCFs) have been developed to detect and prevent checkerboard patterns, diagonal chains and isolated hinges. The CCFs can evaluate the connections by the densities of the adjacent elements and can measure the size of the checkerboard pattern and count the number of corner contacts.

4.a. A new CCF has been proposed for checking on patterns of plane problems. The analyses of the functional properties yielded some modification for the improvement of efficiency.

4.b. Alternative CCFs for plane problems proposed by other researchers have been tested. Their advantages, disadvantages have been discussed.

4.c. The CCF of plane proposed by the author was also used for the evaluation of three dimensional patterns:

- Planes have been linked to the eight cubic elements around the actual node. With special combinations of these plane evaluations new CCF in space have been made for detecting the direct corner contact and the edge contacts.
- Similar to the construction of the plane CCFs a space function has been obtained which can detect and evaluate three dimensional checkerboard patterns from densities of the adjacent elements.

The relevant publications: [4], [6], [8], [9], [10].

THESIS 5

Making use of the above CCFs a new goal, a new modified optimization problem has been proposed.

5.a. Based on the SIMP method and on the CCFs the Co-SIMP algorithm and program for topology optimization of plate structures in plane stress has been developed, in which the CCF is part of the objective function.

5.b. By numerical examples of the Co-SIMP algorithm with the CCFs of the author the efficiency of these functions has been demonstrated in checkerboard and corner contact suppression.

5.c. By numerical analysis it has been showed that how the optimal layout and the final structural form is influenced by a computational parameter for increasing the effect of the applied corner contact function and for managing the optimization efficiency.

The relevant publications: [4], [5], [6], [9], [12].

Proposal for the further research

The results summarized in the theses are important parts of the SIMP methodology. For further development of the completed programs it is important to find a method to assess the efficiency and the accuracy of them, to be able to measure and compare the results of the double meshed Extended-SIMP checkerboard suppressor processes to the results of the Co-SIMP corner contact method.

In the dissertation and in the previous research studies the corner contact functions have been mathematically analyzed from the a point of view of corner contact detection and assessment. The program efficiencies like run time, iteration number, etc. have not been evaluated. The comparison of the topology optimization results was performed only on a Michell-type cantilever and only with the author's two corner contact functions in plane. Thus it is a further aim to amplify the computational and comparability research for all of known CCFs and for other structural forms.

Based on the presented results and on the results of other researchers it is clear now that the number of the generated holes in the optimal layouts has to be regulated. There are known methods for it like perimeter control or other size controls like length scale. Furthermore it is known that these methods are able to suppress the checkerboard patterns on there own. My further aim is to create a method, which can controll both the number of the holes and the corner contact error.

In the dissertation and in the previous research studies structures in plane stress were analyzed mainly. The methods were extended to the three dimensional structures, but the optimization and result assessment were not. That happened because of the graphical difficulties of result presentation and because of the omission of measuring the efficiency and comparability. It is also planed to develop the necessary three dimensional calculations and test further ideas.

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Publications on subjects of the theses

[1] Rozvany, G.I.N.; Querin, O.M.; Gáspár Zs.; Pomezanski V.: „Extended optimality in topology design”, in *Structural and Multidisciplinary Optimization,* Springer, Volume 24, Number 3, 2002. pp. 257-261. doi: 10.1007/s00158-002-0235-x

[2] Rozvany, G.I.N.; Querin, O.M.; Pomezanski V.: „Checkerboards, Element Chains and Hinges in Topology Optimization: Causes and Prevention”, in Gosling, P. (Editor) *Extended Abstracts of the 4th ASMO UK / ISSMO conference, Engineering Design Optimization, Product and Process Improvement,* Newcastle upon Tyne, UK, University of Newcastle upon, 2002. pp. 149-159.

[3] Rozvany, G.I.N.; Querin, O.M.; Gáspár Zs.; Pomezanski V.: „Weight-increasing effect of topology simplification”, in *Structural and Multidisciplinary Optimization*, Springer, Volume 25, Number 5-6, 2003. pp. 459-465. doi: 10.1007/s00158-003-0334-3

[4] Rozvany, G.I.N.; Pomezanski V.; Querin, O.M., Gáspár Zs.; Lógó J.: „Corner contact suppression in topology optimization”, in Querin, O.M.; Sienz, J.; Toropov, V.V.; Gosling, P. (Editors), *Extended Abstracts of the 5th ASMO UK / ISSMO conference, Engineering Design Optimization, Product and Process Improvement,* Stratford upon Avon, UK, 2004. pp. 33-40.

[5] Pomezanski V.; Querin, O.M.; Rozvany, G.I.N.: „CO-SIMP: extended SIMP algorithm with direct COrner COntact COntrol”, in *Structural and Multidisciplinary Optimization*, Springer, Volume 30, Number 2, 2005. pp. 164-168. doi: 10.1007/s00158-005-0514-4

[6] Pomezanski V.: "Topológiaoptimálási feladatokban megjelenő sakktáblamintázat kiküszöbölésének egy módja", in Barna Zs.; Józsa Zs. (Editors), *Doktori kutatások a BME Építőmérnöki Karán*, Budapest, Hungary, 2005. pp. 152-159. (in Hungarian)

[7] Rozvany, G.I.N.; Querin, O.M.; Lógó J.; Pomezanski V.: „Exact analytical theory of topology optimization with some pre-existing members or elements”, in *Structural and Multidisciplinary Optimization*, Springer, Volume 31, Number 5, 2006. pp. 373–377. doi: 10.1007/s00158-005-0594-1

[8] Rozvany, G.I.N.; Querin, O.M.; Pomezanski V.; Gáspár Zs.; Lógó J.: „Some basic issues of topology optimization” in Bendsoe, M.P.; Olhoff, N. and Sigmund, O. (Editors), *IUTAM Symposium on Topological Design Optimization of Structures, Machines and Materials, Series of Solid Me-*

chanics and Its Applications, vol. 137. Part. 2., Springer, 2006. pp. 77-86. doi: 10.1007/1-4020-4752-5_8

[9] Pomezanski V.: "Numerical Methods to Avoid Topological Singularities", in Topping, B.H.V.; Montero, G.; Montenegro, R. (Editors), *Proceedings of the Eighth International Conference on Computational Structures Technology*, Civil-Comp Press, Stirlingshire, UK, Paper 211, 2006. doi:10.4203/ccp.83.211

[10] Pomezanski V.: "Corner Contact Penalty Functions in Plane and in Space", in *Pollack Periodika, An International Journal for Engineering and Information Sciences*, Akadémiai kiadó, Hungary, Volume 2, 2007. pp. 39-50. doi: 10.1556/Pollack.2.2007.2.4

[11] Pomezanski V.: "Heuristic Features of the Extended SIMP Algorithm in Topology Optimization", in Topping, B.H.V.; Costa Neves, L.F.; Barros, R.C. (Editors), *Proceedings of the Twelfth International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press, Stirlingshire, United Kingdom, paper 62, 2009. doi:10.4203/ccp.91.62

[12] V. Pomezanski, "Heuristic Features of the Co-SIMP Algorithm in Topology Optimization: Numerical Examples", in B.H.V. Topping, J.M. Adam, F.J. Pallarés, R. Bru, M.L. Romero, (Editors), *Proceedings of the Tenth International Conference on Computational Structures Technology*, Civil-Comp Press, Stirlingshire, UK, Paper 320, 2010. doi:10.4203/ccp.93.320