MŰEGYETEM 1782

Budapest University of Technology and Economics

Faculty of Civil Engineering

Department of Geodesy and Surveying

Revision and refinement of functional models used in geodesy

Theses of PhD Dissertation

Csaba ÉGETŐ

M.Sc. in Surveying and Geoinformatics Engineering

Budapest, 2014

Table of Contents

I. Introduction

From the aspect of the geodetic and surveying measurements, the use of the correct functional and stochastic model is an essential assumption. The functional model should contain in all the cases the relevant parameters, otherwise the model errors would result in systematic errors of the results regardless the accuracy of the measurements. The functional models, applied in surveying and geodesy were derived according to the technical level of the era when certain methods were developed. However, by time measurement tools and methods are in constant development, which requires the revision and – in case of significant difference – the refinement of the functional model time after time.

II. Refinement of the fundamental equation of the torsion balance

II.1. Introduction

In the first section I have dealt with the functional model of the torsion balance measurements. According to the literature, in practice only linear functional models are used (when linearity is assumed for the spatial variation of the gravity acceleration). However, during processing torsion balance measurements we have experiences that in case of 'large' gravity gradients the linear assumption may be insufficient. The fundamental equation of the torsion balance has been extended with the inclusion of the higher terms of the Taylor series of the gravity acceleration. Due to the refinement of the functional model, the 4+1 unknowns have been extended by further 14 unknowns. These additional unknowns cannot be determined by inclusion of more measurements due to their azimuth dependence. Therefore these unknowns can only be determined by direct modelling based on a mass model of the region.

Another investigation refers to the definition of the azimuth of a measurement. The azimuth is set by turning the body of the torsion balance to the demanded azimuth with the balance being fixed. However, when the balance is released, it rotates to its equilibrium position governed by the local features of the gravity field. In essence, the equilibrium azimuth is not identical with the nominal azimuth. We define the difference of the two as *azimuth deflection*. The addition of the azimuth deflection is a further attempt to refine the the functional model of the torsion balance.

II.2. Objectives

- Revision and analysis of the functional model used to process torsion balance measurements.
- Numerical consequences of the refinement of the function model for practical use.
- Developing a method to include the extended, refined functional model.
- Refinement of the fundamental equation of the torsion balance with the inclusion of the azimuth deflection, and simulation of its effect in practice based on actual torsion balance observations.

II.3. Results

The results of the investigation are presented with an actual example. The test area is the Gravity and Geodynamic Observatory of the former MFGI in the cave of the Mátyáshegy. The investigation was performed on the points of the gravimetric micro-baseline of the observatory.

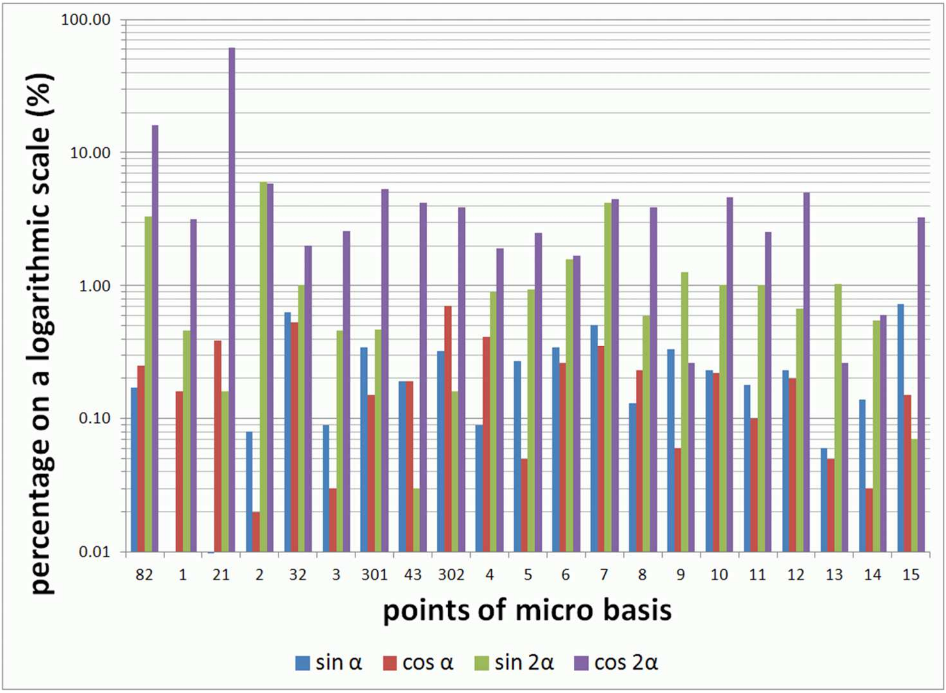

Fig. 1. *Amplitude of the sum of the third and fourth order momentum components with respect to the second order component for the E-54 torsion balance in percentage, i.e. 100*(third order term + fourth order term)/(second order term)*

The resulted high order momentum error (i.e. sum of the third and fourth order components) can reach even the 50% of the second order component (c.f. Figure 1) based on the investigation performed for the observatory of the Mátyáshegy. The results of this investigation can be applied as a supportive tool to define in certain cases whether the extension of the fundamental equation of the torsion balance makes relevant difference on the processing.

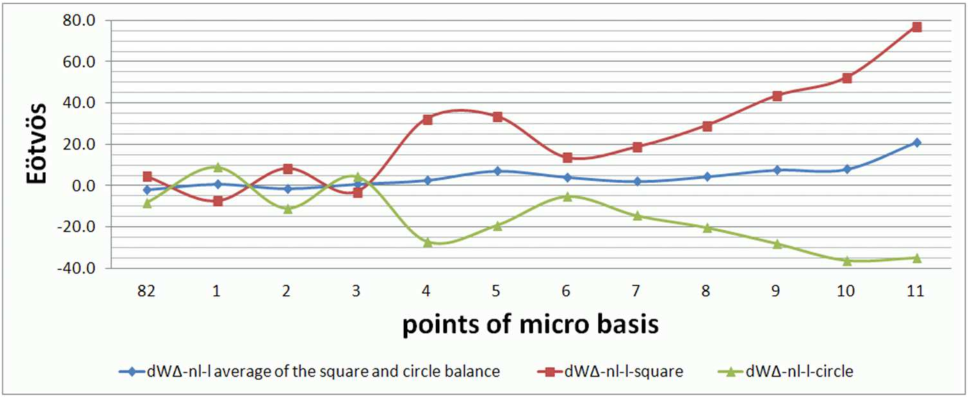

Fig. 2. *Difference of processed torsion balance measurements with linear and nonlinear equations:* dW_{Δ} *(E)*

The inclusion of azimuth defection term in the fundamental equation of the torsion balance has been found to result in different gradients reaching even 20 E (c.f. Figure 2). Since systematic errors of a processing of measurements are generally consequences of functional model errors due to exclusion of certain physical or geometric principles, the detailed analysis of systematic errors is essential.

III. Accuracy analysis of mass modell effects by Monte-Carlo-method

III.1. Introduction

Nowadays the forward (or direct) modelling of the gravity field becomes a more routinely applied tool of geodesy. Calculation of terrain correction of gravity (or gravity gradient) measurements, e.g. for geophysical prospecting, can efficiently be done by forward modelling of the gravity field. The forward modelling makes use of mass model of the region. Similarly, forward modelling using a mass model can be applied for determining the additional 14 unknowns of the extended functional model of the torsion balance (c.f. section II) For that analysis, the statistics of the accuracy of the forward modelling and of the used mass model should be determined.

III.2. Objectives

- Determination of how the errors of the coordinates of the terrain model (geometry of the mass model) affect the accuracy of the derived gravity field parameters, such as potential, gravity acceleration, gravity gradient.

III.3. Results

The basic concept of the analytical solution has been introduced, which is purely derived by the error propagation law for the present problem. Afterwards, the Monte-Carlo-method has been adopted for this task (c.f. Figure 3). Finally, the method has been applied for an actual task, that is the accuracy analysis of the mass model of the observatory in the Mátyáshegy owned by the MFGI (Table 1.).

There were five investigations performed on the random errors of the model. Among them, in four cases the coordinates of the model were contaminated with the same magnitude of the error regardless its position (i.e. same for the outer bulk of the mountain, for the internal cave), these mean errors were ±1cm, ±3cm, ±5cm and ±10cm. In the fifth case the heterogeneity of the observing method was taken into account. So the points of cave, which were measured by a total station, were considered to have an accuracy of ±5cm, while the rest of the points, points of the outer bulk of the mountain were assumed to have an accuracy of ±100cm.

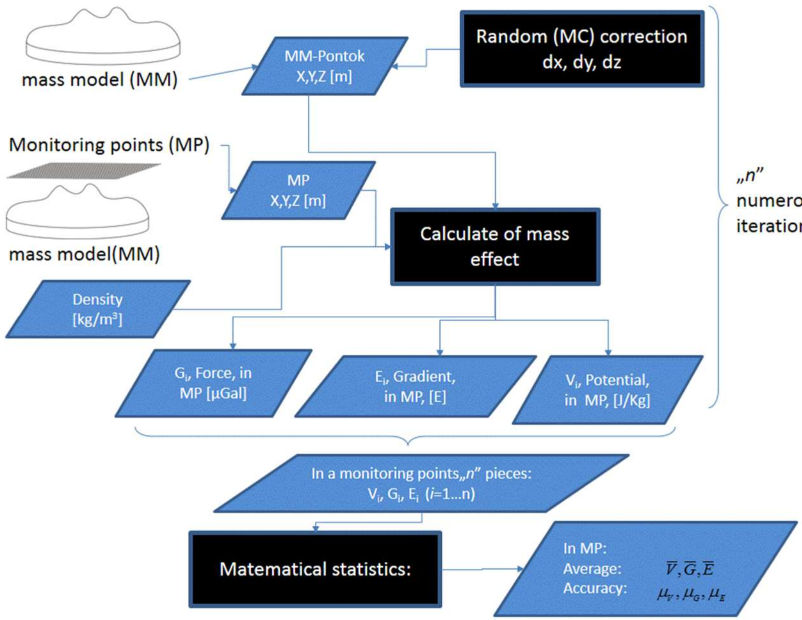

Fig. 3. *Flowchart: accuracy analysis of a mass model by Monte-Carlo-method*

Table 1. *Results of the accuracy analysis*

		assumed mean errors of the points				
		± 1 cm	± 3 cm	± 5 cm	± 10 cm	± 5/ ± 100 cm
reaction of the model mean errors at p=0.99 confidence level	V_{xz} [E]	± 0.7	± 2.0	± 3.4	± 7.0	± 6.9
	V_{yz} [E]	± 0.6	± 1.9	± 3.2	± 6.6	± 6.5
	V_{Δ} [E]	± 1.0	± 3.0	± 5.0	± 11.0	± 9.3
	$2V_{xy}$ [E]	± 1.1	± 3.4	± 5.7	± 12.1	± 9.8
	V_{zz} [E]	± 0.9	± 2.7	± 4.6	± 10.6	± 9.5
	G_x [μGal]	± 0.4	± 1.2	± 1.9	± 4.1	± 19.8
	G_y [μGal]	± 0.4	± 1.3	± 2.2	± 4.3	± 21.2
	G_z [μGal]	± 0.7	± 2.0	± 3.3	± 6.2	± 31.1
	V [μJ/kg]	± 1.1	± 3.3	± 5.1	± 10.3	± 57.1

IV. Effects of temporal variations of surface levels on repeated vertical control levelling measurements

IV.1. Introduction

In the third section the functional model of repeated levelling measurements for vertical control was considered. The general model is very simple: take a “back” and “forth” reading on two vertically held staffs at the points of interest, and determine the height difference as the difference of the two readings. In case of high precision requirements, this functional model refined by taking into account the local features of the gravity field with the inclusion of observed gravity acceleration values over the region. In both cases, when repeated measurements are performed, implicitly the shape of the local horizontal surface levels (and equivalently, that of the vertical plumb-lines) is assumed to be constant in time. However, the shape of the surface level is changing by time. Due to the change of the gravity surface levels, the reference frame of the observations is changing, influencing the results of the measurements (c.f. Figure 4).

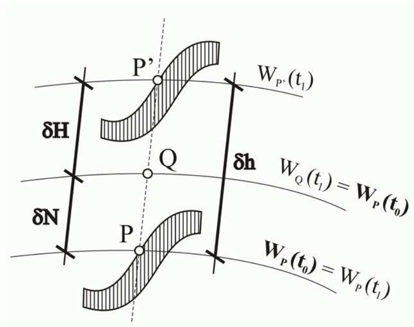

Fig. 4. *Vertical displacement (δH) and the displacement of a surface level of the gravity field (δN) consists of the actual vertical deformation of the surface (δh).*

Classically, vertical monitoring is done by levelling measurements in a vertical control network, which is repeated time after time and compared with the results of the initial state. Since due to the vertical displacements of the surface also the shape of the surface levels is changing, the repeated measurements reflects not only the actual deformation but also this latter effect.

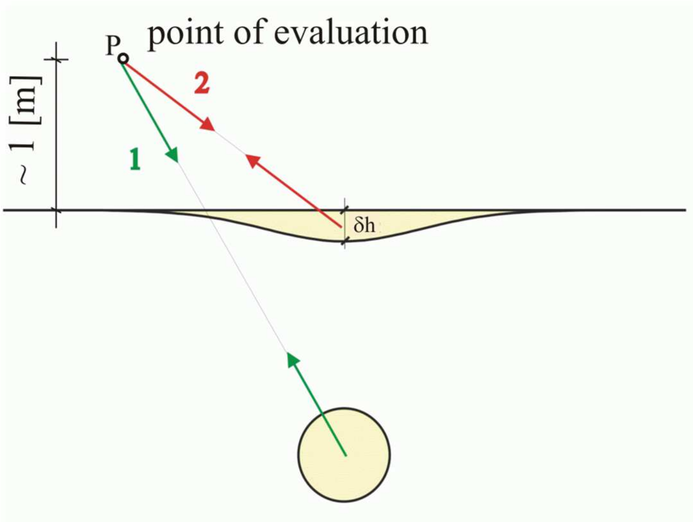

Fig. 5. *In a point P outside of the mass variation the temporal variation of the gravity field can be observed due to the (1) excavation, and due to the (2) accompanied δh deformation.*

According to Biró (1983) and Figure 4., the deformation of the physical surface (δh) is (the symbol δ refers to change by time between t_0 and t_1 epochs):

$$\delta h = \delta H + \delta N \tag{1}$$

where δH is the change in orthometric height (surface measureable deformation) and δN is vertical displacement of the level surface.

In case of excavation (e.g. mining, c.f. Figure 5.), this model consists of three components:

- change of the level surface due to the excavation: δN_1
- vertical displacement of the physical surface due to the excavation: δh
- change of the level surface due to the vertical displacement of the physical surface: δN_2.

IV.2. Objectives

- Refinement of the functional model of repeated levelling measurements for vertical control by considering the temporal variation of the surface levels due to excavation and accompanied vertical displacements, and illustration by a practical example.

IV.3. Results

The introduction of the temporal variations of the surface levels on repeated levelling measurements was done based on Biró (1983), which has been further developed by a new formula as described in [2] and [12].

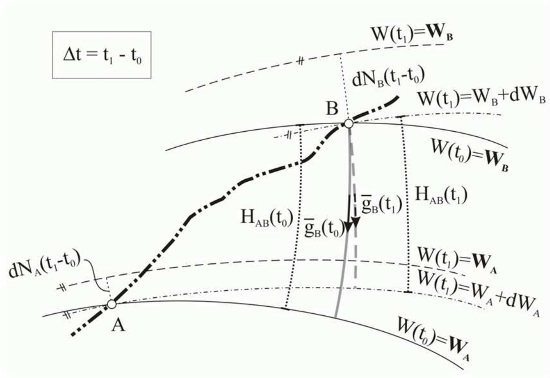

Fig. 6. *The scheme of the temporal variations of a surface level.*

In Figure 6., most important quantities are shown for a case of repeated levelling between two benchmarks, A and B. The temporal change of the height difference (δH_{AB}) and that of the level surfaces (δN_{AB}) in the Δt period, referring to the time difference between the initial t_0 and the subsequent t_1 epochs, are visualized in Figure 6.

Biró (1983) has derived the following formula:

$$\delta H_{AB} = -\delta N_{AB} - \frac{\tilde{g}_B(t_1) - \tilde{g}_B(t_0)}{\tilde{g}_B(t_1)} \cdot H_{AB}(t_0). \tag{2}$$

In the investigation instead of the height difference at the t_0 initial epoch, access to the t_1 epoch was available. For this epoch the following formula has been derived [2]:

$$\delta H_{AB} = -\delta N_{AB} - \frac{\tilde{g}_B(t_1) - \tilde{g}_B(t_0)}{\tilde{g}_B(t_0)} \cdot \left(H_{AB}(t_1) + \delta N_{AB}\right). \tag{3}$$

Based on (2) and (3), a relationship between the height difference before and after the change has been derived [2]:

$$H_{AB}(t_1) = \frac{\tilde{g}_B(t_0)}{\tilde{g}_B(t_1)} H_{AB}(t_0) - \delta N_{AB}. \tag{4}$$

According to (4), temporal variations of the height difference are generated not purely by changes of geometrical, but also by physical quantities as well.
The theoretical results were applied for the newly built M4 subways construction (including earthworks of the tunnelling and of the excavation of the stations as well). The results are as follows:

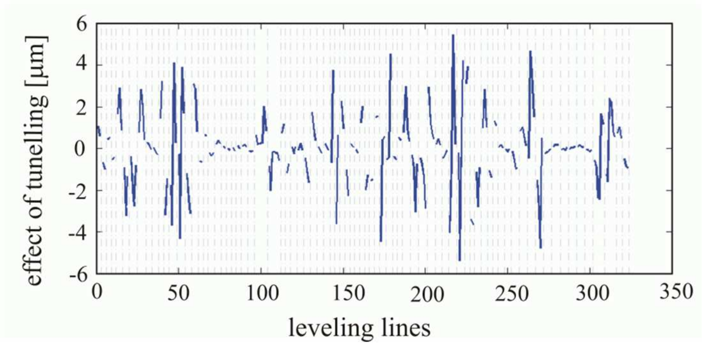

Fig. 7. *The effect of the tunnelling and of the excavation of stations on the repeated levelling measurements determined for the south-west part of the M4 subway line.*

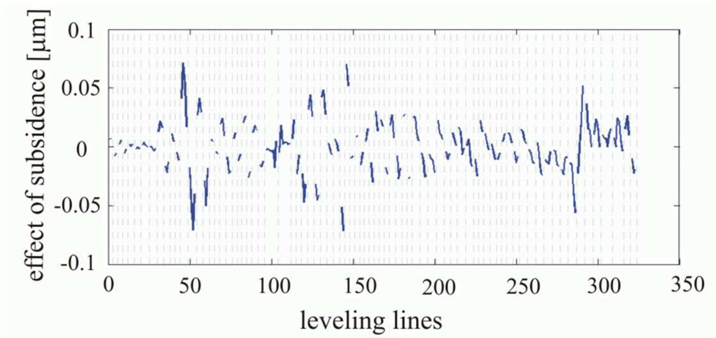

Fig. 7. *The effect of the vertical displacements of the physical surface on the repeated levelling measurements determined for the south-west part of the M4 subway line.*

According to Figures 7 and 8, the effect of the tunnelling and of the excavation of the stations is in the order of magnitude of some μm, while the effect of the vertical displacements of the physical surface does not exceed 0,1 μm.

The results were generalized by very simple simulation studies for the effect of the excavation and for that of the vertical displacements as well. Based on the simulation the effect of the excavation for very large volumes can be notable (0,2-0,3 mm), while the effect of the vertical displacement of the physical surface is always negligible.

V. Analysis of standard theodolite errors

V.1. Introduction

The fourth section deals with the functional model of processing angle measurements observed by theodolite. In practice, some of the known errors of the horizontal and vertical angle measurements can be neglected without any distortion of the accuracy. Others, however, can affect the measurements in a larger extent than the precision of the measurements. When measurements in one face are performed, these later errors essentially affect the measurements. Even though nowadays total stations enable the real time correction of the observations in one face with an occasionally determined summed value of all the errors, it does not mean that further understanding of the errors is not important, since no accurate correction can be obtained without the accurate functional model of the errors.

Peter Májay has derived a method for the determination of the standard errors of theodolites (Májay, 1984). In this present section the refinement of the functional model of Májay's method is provided, considering aspects of the most recent total stations, and the effects are examined in two practical examples.

V.2. Objectives

- Analysis and interpretation of the standard errors of the theodolites, derivation of the corresponding mean errors, and the effect of temperature on the standard errors based on practical tasks. Numerical results with the refined functional model are compared with that of the classical model.

V.3. Results

The intersection of the horizontal and vertical axes of the theodolite (O) is displayed in the origin of a unit sphere (Figure 10). The collimation error of the telescope is $+\delta_k$. The „(δ_k)" effect of the collimation error can be deduced from the PQZ spherical triangle as derived in Sárdy (1964):

$$\frac{\sin(\delta_k)}{\sin 90°} = \frac{\sin \delta_k}{\sin(90° - \alpha)}. \qquad (5)$$

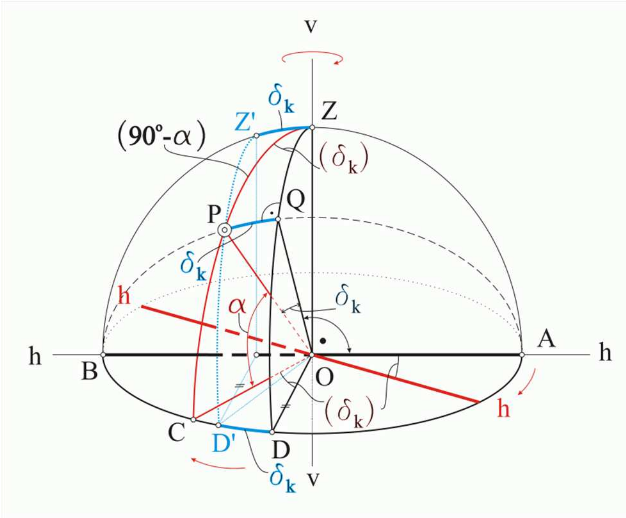

Fig. 10. *The interpretation of the collimation error in a unit sphere*

After simplifications and rearrangement, the effect of the collimation error is as follows (Sárdy, 1964):

$$(\delta_k) = \arcsin\left[\frac{\sin\delta_k}{\cos\alpha}\right] \tag{6}$$

The maximal value of the elevation angle, α has been redefined ($\alpha_{max} = 90° - \delta_k$), thus the maximal value of the collimation error has been also derived; from equation 7 it becomes: $(\delta_k)_{max} = 90°$.

$$(\delta_k) = \arcsin\left[\frac{\sin\delta_k}{\cos(90° - \delta_k)}\right] = 90° \tag{7}$$

Based on equation (7) it is obvious, that even if vertical axis of theodolites with collimation error are set exactly to the vertical, the telescope cannot be sighted precisely to the vertical ($\alpha = 90°$, $Z = 0°$). No literature on this issue was found. Even though the aforementioned error was found to be of theoretical importance only, the sketch on Figure 10 provides a proper addition for the better understanding of the collimation error.

The subsidiary errors of the Májay-method have also been investigated, which occur when the execution of the method is influenced by errors. Here we refer to such effects that line of collimation is not set exactly to horizontal, or the distance of the P_1 point is not exactly ∞, or that the distance of P_2 point is not adequately as it is ordained.

From the angle and distance measurement accuracy of a total station, the mean errors of the standard errors were derived considering the error propagation law. As an example, equation (8) shows the perpendicularity error of the horizontal axis. These mean errors were confirmed by Monte-Carlo-method. Subsequently, the accuracy estimate of the standard errors was extended with the subsidiary errors – again with the use of the Monte-Carlo method.

$$m_{\delta_h} = \left\langle \begin{aligned} & m_{sz}^2 (E_1)^2 + \frac{m_t^2 \sin^2 z_2^I \sin(fv_1) \tan^2 z_3^I}{t_{3f}^2 \sin^2 z_3^I (fv_3)} + \\ & + \frac{m_t^2 t_{2f}^2 \sin^2 z_2^I \sin^2(fv_1) \tan^2 z_3^I}{t_{3f}^4 \sin^2 z_3^I (fv_3)} + \\ & + \frac{m_{sz}^2 t_{2f}^2 \sin^2 z_2^I \cos^2(fv_1) \tan^2 z_3^I}{2 t_{3f}^2 \sin^2 z_3^I (fv_3)} + \\ & + m_{sz}^2 \tan^2 z_3^I \left[(E_2)^2 + (E_3)^2 + (E_4)^2 + \frac{(E_5)^2}{(fv_3)} + \frac{1}{2} \right] \end{aligned} \right\rangle^{\frac{1}{2}} \qquad (8)$$

where m_{sz} is the mean error of the angle measurement, m_t is the mean error of the distance measurement, fv_i and E_i are internal functions, which are dependents on the horizontal and vertical readings and on the observed distance.

Moreover, corrections for the readings performed in two faces both on horizontal and vertical circles were extended with a temperature dependent term. The literature discusses some temperature dependent errors, but not the horizontal and vertical eccentricity of the telescope. This has been derived; in equations (9) and (10) correction for face left readings is presented:

$$\begin{aligned} l^{korrigált,\,I} &= l^I + \frac{\left(A_{\delta_k} \cdot T + B_{\delta_k}\right)}{\sin z} + \left[\arcsin \frac{\left(A_{\delta_{I,V}} \cdot T + B_{\delta_{I,V}}\right)}{t_f \sin z}\right] + \\ &+ \left(A_{\delta_h} \cdot T + B_{\delta_h}\right) \cot z + \left(\delta_V\right) \end{aligned} \qquad (9)$$

$$z^{korrigált,\,I} = z^I + \left(A_{\delta_z} \cdot \Delta T + B_{\delta_z}\right) + \left[\arcsin \frac{\left(A_{\delta_{I,H}} \cdot \Delta T + B_{\delta_{I,H}}\right)}{t_f}\right] \qquad (10)$$

where A_i are linear coefficients of the temperature, while B_i contain summed values of the bias due to the temperature and due to the (δ_v) error of the vertical axis.

In the practical tests the acclimatization of the total station for sudden temperature change was investigated, and it turned out to take 1.5 hour for a change of the temperature ΔT~13-15°C. When the acclimatization has been completed, the total station follows the outer temperature variations with a bias of 3-4 °C. The bias is interpreted as the consequence of the heat transport of the internal electronics (Figure 11.).

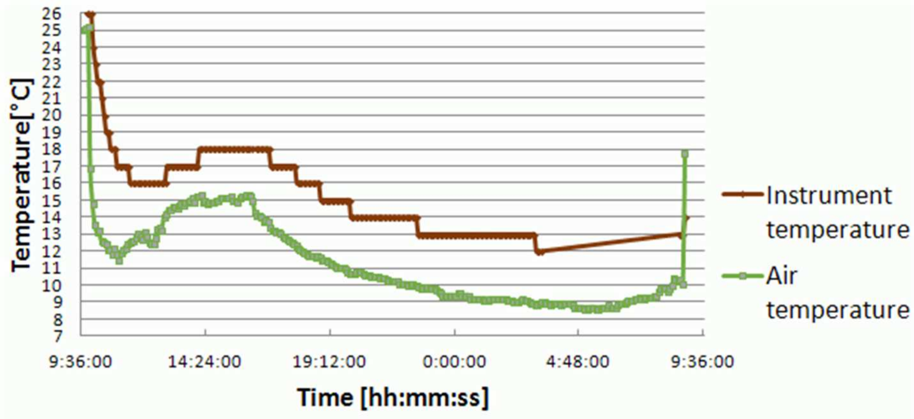

Fig. 11. *Internal temperature of a TS15i total station and the outer temperature ~24 hour long registration. Approximately +3 °C higher internal temperature is interpreted to be the consequence of the internal electronics.*

The automatic observation of the standard errors by the Májay-method has been exemplified with two practical tasks. The results show that by considering the dependence on the temperature (Figure 15.) and taking into account where its effect is obvious based on an accuracy estimate of the dependency (Figure 12.), a more accurate result can be achieved than when such a correction is not applied (Figure 13.). Further essential conclusion is that the mean of the difference of the two methods is approximately zero (Figure 14.).

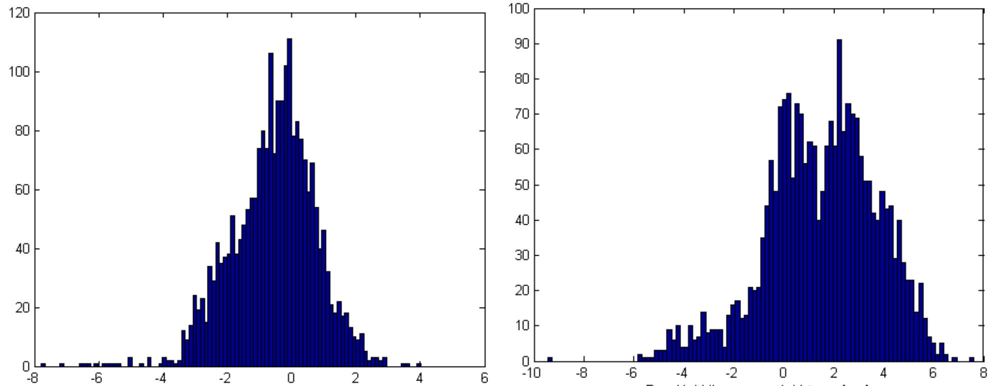

Fig. 12. *Histograms of the accuracy estimate of the temperature correction method a.) for the horizontal readings:* $d_l = l^{exact,I} - l^{temperature\_correction,I}$*, mean value: -0,6 [second], mean error: ±1,3 [second] b.) for the vertical readings:* $d_z = z^{exact,I} - z^{temperature\_correction.,I}$*, mean value: +1,5 [second], mean error: ±2,2 [second]*

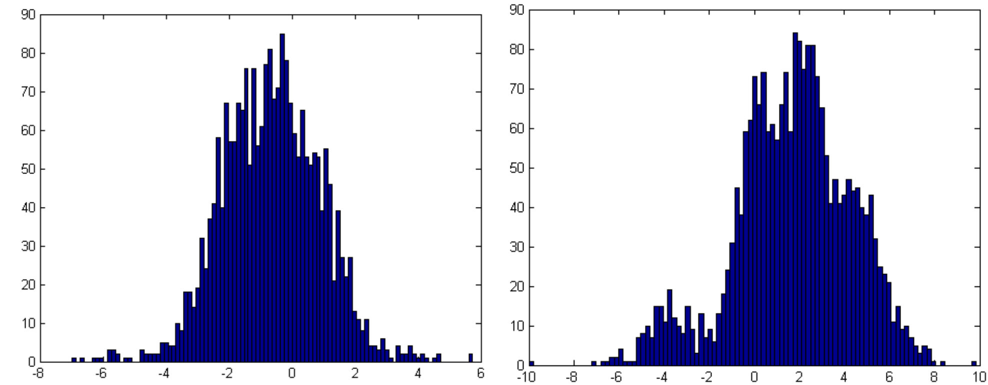

Fig. 13. *Histograms of the accuracy estimate of the classical method. a.) for the horizontal readings:* $d_l = l^{exact,I} - l^{classica\_correction,I}$*, mean value: -0,6 [second], mean error: ±1,6 [second] b.) for the vertical readings:* $d_z = z^{exact,I} - z^{classical\_correction,I}$*, mean value:+1,7 [second], mean error: ±2,6 [second]*

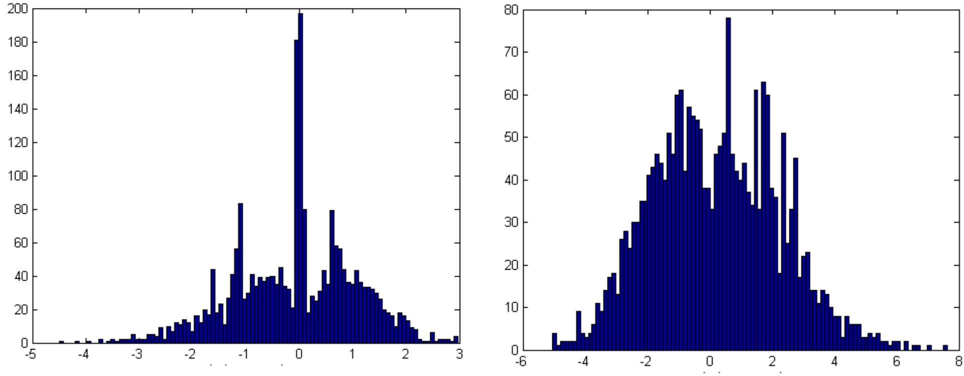

Fig. 14. *Histograms of the accuracy estimate of the full method, i.e. difference of the classical and of the temperature correction methods. a.) for the horizontal readings:* $d_{l-correction} = l^{classical\_correction,I} - l^{temperature\_correction.I}$*, mean value: -0,1 [second], b.) for the vertical readings:* $d_{z-correction} = z^{classical\_correction,I} - z^{temperature\_correction.I}$*, mean value: +0,2 [second]*

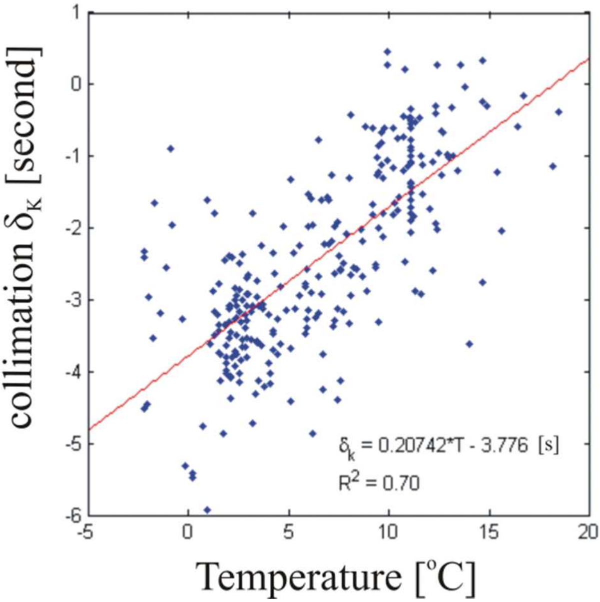

Fig. 15. *The relationship between the collimation error and the temperature in March, 2014, and the regression line, its coefficients and the* R^2 *value.*

VI. Summary

For the presented cases the functional model of the processing sequence has been revised, and extended with new terms. Based on examples, the practical effect of the refinement of the functional model was analysed. Some of the additionally introduced terms turn out to have only theoretical relevance, others were found to have notable effect. From practical point of view this later has obvious relevance. On the other hand, the theoretical results are also considered to have practical impact, as they can be considered later on as supplementary aspect for developing a new instrument construction, or can provide additional considerations for planning arrangement of measurements.

VII. Theses

My novel results in the field of refinement of functional models for geodesy and surveying are as follows:

1st Thesis The refinement of the linear functional model of the torsion balance with higher order terms has been investigated. A processing method has been developed, which applies the extended functional model, and has been demonstrated via a practical example. Further refinement of the fundamental equation of the torsion balance has been achieved by inclusion of the azimuth deflection. Finally, comparison of classical linear and the newly developed non-linear methods were presented on actual observation data.

Related publication: [6]

2nd Thesis Accuracy analysis of forward modelling of gravity field based on a mass model has been determined using the Monte-Carlo-method. An application of the method has been shown by a practical example.

Related publication: [6]

3rd Thesis The theory of temporal variations of the gravity field has been further refined, and has been applied for engineering applications requiring excavation of soil mass. The applicability of the refined functional model has been demonstrated in a practical example. The effect of excavation in some extreme cases can reach the precision of precise levelling, while the effect of the vertical deflection of the physical surface was found to be negligible.

Related publication: [1], [2], [5], [12]

4th Thesis Mean errors of the standard theodolite errors has been derived by the error propagation law and the Monte-Carlo-method. The dependence of the standard theodolite errors on the linear temperature has been determined in an empirical manner.

Related publication: [9]

VIII. Own publications

Reviewed papers in foreign language, pressed abroad

[1] Égető, Cs, Földváry, L, Huszák, T.: The effect of tunnelling on repeated precise levelling measurements for vertical deformation control of the Metro4 project, JOURNAL OF GEODETIC SCIENCE, 3(2): 95-102., (2013)

[2] Égető, Cs, Földváry, L.: Numerical accuracy analysis of modeling excavation induced gravity field variations. Proceedings in Global Virtual Conference: The 1st International Global Virtual Conference, Zilina, Szlovákia, 2013.04.08-2013.04.12. pp. 549-554. (2013), ISBN 978-80-554-0649-7

[3] Szabó, G, Égető, Cs: Kreiselmessungen mit dem MOM Gi-B3 im Gotthard-Basistunnel. Ingenieurvermessung 07, Graz, pp. 207-212. Paper 430., ISBN 978-3-87907-448-8 (2007)

Reviewed papers in foreign language, pressed in Hungary

[4] Csapó, G, Laky, S, Égető, Cs, Ultmann, Z, Tóth, Gy, Völgyesi, L: Test Measurements by Eötvös Torsion Balance and Gravimeters. PERIODICA POLYTECHNICA-CIVIL ENGINEERING 53:(2) pp. 75-80., IF: 0.222, ISSN 0553-6626 (2009)

Reviewed papers in Hungarian

[5] Égető, Cs, Földváry, L: A 4-es metróvonal alagútrendszerének hatása a Föld nehézségi erőterére. GEOMATIKAI KÖZLEMÉNYEK XIV:(1) pp. 17-25. (2011)

[6] Tóth, Gy, Égető, Cs: A Mátyáshegyi Gravitációs és Geodinamikai Obszervatórium átfogó gravitációs modellezése. GEOMATIKAI KÖZLEMÉNYEK XIII:(2) pp. 113-122. (2010)

[7] Völgyesi, L, Égető, Cs, Laky, S, Tóth, Gy, Ultmann, Z: Eötvös-inga felújítása és tesztmérések a budapesti Mátyás-hegyi barlangban. GEOMATIKAI KÖZLEMÉNYEK XII: pp. 71-82. (2009)

[8] Csapó, G, Égető, Cs, Kloska, K, Laky, S, Tóth, Gy, Völgyesi, L: Kísérleti mérések Eötvös-ingával és graviméterekkel – az Eötvös-inga mérések eredményei geodéziai célú hasznosításának vizsgálata céljából. GEOMATIKAI KÖZLEMÉNYEK XII: pp. 91-100. (2009)

[9] Szabó, G, Égető, Cs: Irányátvitel MOM Gi-B3 Giroteodolittal a svájci Gotthard-bázisalagút építésén. GEOMATIKAI KÖZLEMÉNYEK X: pp. 273-279. (2007)

Conference papers in Hungarian

[10] Égető, Cs: Mikrogeodéziai hálózat létesítése és meghatározása a BME Általános- és Felsőgeodézia Tanszékén. In: PhD Researches at the Faculty of Civil Engineering, BME. Budapest, Magyarország, 2006.02.28., pp. 5-13. (2006)

[11] Dede, K, Égető, Cs: Földalatti mérések oktatása a Budapesti Műszaki és Gazdaságtudományi Egyetemen, LI. Jubilee Conference of Minesurveying, Székesfehérvár 2012.06.13-15. pp. 28-35. (2012)

Accepted papers, in press

[12] Égető, Cs, Rehány, N, Földváry, L: Variations of the gravity field due to excavations of the Budapest Metro4 subway line. PERIODICA POLYTECHNICA-CIVIL ENGINEERING, accepted (2014)

Lecture notes, in electronic form

[13] Csemniczky, L, Czakó, J, Égető, Cs: Közműnyilvántartás (HEFOP lecture note) (2008)

TDK - Scientific student research study

[14] Égető, Cs: Mozgásvizsgálati mérések a Gellért-hegyen. TDK conference on BME, Faculty of Civil Engineering (2002)
[15] Szabó, G, Égető, Cs: Szabadálláspont koordinátáinak változása a meghatározására felhasznált irányok és távolságok számának függvényében. TDK conference on BME, Faculty of Civil Engineering (2001)

IX. References

Biró P (1983): Time variation of height and gravity. Wichmann Verlag, Karlsruhe, Akadémiai Kiadó, Budapest.160.

Májay P (1984): A teodolit vizsgálata. Oktatási segédanyag, Budapest, Kézirat, 6.

Sárdy A (1964): Geodéziai alapismeretek II., Kézirat, Tankönyvkiadó, Budapest, 392.