Department of Control Engineering and Information Technology
Budapest University of Technology and Economics

Formation Control of Autonomous Aerial Vehicles

PhD Thesis Summary

Autonóm légi járművek formációszabályozása

PhD értekezés tézisei

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Motivation

Recently, autonomous unmanned aerial vehicles (UAVs) have gained significant attention and their integration to everyday life is one of the most actively investigated problem in numerous countries. These vehicles can perform various challenging tasks efficiently, either alone or in cooperation with other similar vehicles. However, numerous open questions exist in this field of research due to the versatility of applications and the emerging problems. This work focuses on control related problems that include single vehicles and vehicle groups in indoor environment, while state measurement and estimation are also of importance.

Basic Notions, Methods and Tools

The results of the thesis are based upon the following mathematical and control theoretic methods and tools.

Backstepping control [6, 7]. The technique is applicable to the special class of nonlinear systems shown below:

$$\begin{aligned}
\dot{x} &= f_x(x) + g_x(x)z_1 \\
\dot{z}_1 &= f_1(x, z_1) + g_1(x, z_1)z_2 \\
&\vdots \\
\dot{z}_i &= f_i(x, z_1, \ldots, z_i) + g_i(x, z_1, \ldots, z_i)z_{i+1} \\
&\vdots \\
\dot{z}_k &= f_k(x, z_1, \ldots, z_k) + g_k(x, z_1, \ldots, z_k)u,
\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, z_1,. . . ,z_k are scalars and u is the control input.

It can be observed that the ith equation depends only on x and z_i from the preceding ones, which allows the designer to construct a stabilising control law including the dynamic equations step by step until reaching the last equation containing the control input. Stability is guaranteed by nonlinear control theoretic results [8]. Each step consists of constructing of a partial Lyapunov function and a virtual control law design to the virtual control input z_{i+1}, while during the last step the control law becomes the real one, hence the name of the design process. The choice of partial Lyapunov function candidates and the corresponding virtual control laws can be chosen in various ways, which makes the method powerful and flexible.

Robust ($\mathcal{H}_\infty$ and μ) control methods [11]. The objective is to find the controller K in the feedback interconnection depicted in Fig. 1 so that it minimises the worst case gain between the external input w and the error signal z in the system

$$\begin{aligned} \begin{bmatrix} z \\ y \end{bmatrix} &= \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \\ u &= K(s)y, \end{aligned} \tag{2}$$

while the closed loop system satisfies robust stability (in the case of μ-controllers also robust performance) conditions.

This is in tight relation with the $\mathcal{H}_\infty$ norm of $\mathcal{F}_l(P,K)$ appearing in $z = \mathcal{F}_l(P,K)w$. This $\mathcal{H}_\infty$ norm can be expressed in both frequency and time domain:

$$\begin{aligned} \|\mathcal{F}_l(P,K)\|_\infty &= \max_\omega \bar{\sigma}(\mathcal{F}_l(P,K)(j\omega)) \\ &= \max_{\|w(t)\|_2 \neq 0} \frac{\|z(t)\|_2}{\|w(t)\|_2}, \end{aligned} \tag{3}$$

where $\bar{\sigma}$ denotes the maximal singular value and $\|z(t)\|_2 = \sqrt{\int_0^\infty \sum_i z_i^2(t)dt}$.

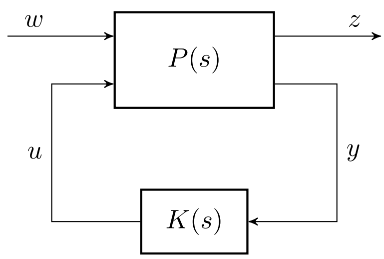

Figure 1. Robust control design setup.

Constrained nonlinear optimisation [1, 2]. During the research, several of the emerging problems have lead to the following one:

$$\begin{aligned} x_{opt} &= \arg\min f(x) \\ g(x) &= 0 \\ h(x) &\leq 0, \end{aligned} \tag{4}$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $g : \mathbb{R}^n \to \mathbb{R}^m$ and $h : \mathbb{R}^n \to \mathbb{R}^p$ are smooth functions. Occasionally, the optimisation problem does not involve $h(x) \leq 0$ type constraints (second type problems).

Interior point methods are usually suitable for the general problem, while in the latter case, Lagrange multipliers can be applied effectively. MATLAB's Optimization Toolbox and KNITRO offer powerful tools for solving such problems [2].

Graph theoretic notions and Frobenius's theorem [3, 5]. The following notions lie behind the ideas presented in the chapter dealing with safe formation change coordination.
Normalised graph Laplacian. $L = I - A$, where L is the graph Laplacian itself, A is the weighted adjacency matrix (all row sums equal 1), while I is a compatible identity matrix.
Maximum clique. A full subgraph of a graph is called clique. The largest clique of a graph is called maximum clique.
Perron – Frobenius theorem. Let A be a nonnegative irreducible matrix of order n. Then it has a positive eigenvalue r with maximum magnitude. The corresponding eigenvector is positive. In the case of imprimitive matrices, if h distinct eigenvalues exist with the same maximum magnitude, these eigenvalues are the solutions to the equation $\lambda^h - r^h = 0$.

For verification and implementation purposes, various software tools have been applied. The MATLAB/Simulink environment provides powerful tools for simulating and verifying control systems, developing algorithms and also generating code for embedded processors. Hardware-in-the-loop simulations may conveniently be carried out by the aid of hardware and software products of dSPACE. Advanced graphical interfaces can effectively be implemented using the NI LabWindows/CVI IDE.

Summary of the New Scientific Results

Control of a Quadrotor Helicopter

Due to its versatile applicability and simple mechanical structure, the quadrotor helicopter has become a popular research platform in the recent years. This motivated us to choose it as a testbed for nonlinear and cooperative control.

The quadrotor helicopter is an unstable and underactuated system (it has less independent control inputs than degrees of freedom), which is inherently difficult to control. Two schemes are proposed for stabilising the vehicle. One of them is based on a backstepping controller, which is an approach suitable for a certain class of nonlinear systems. An Extended Kalman Filter (EKF) based state estimator makes the control system complete, which provides the control algorithm with all the required signals including those that are not directly measured, based on measurements from various sensors. The other control algorithm is based on robust control theory and requires a linearised system model. A suitable model is obtained by linearising the system about a stationary point. The consequence of linearisation is that the system splits up into four independent subsystems, thus allowing us to design controllers with reduced complexity. The controller obtained after model order reduction (necessary because of the high computation demand) guarantees robust stability and the closed loop achieves the required robust performance. The helicopter thus can be stabilised in an arbitrary spatial point and yaw angle by both control methods. A complex manoeuvre is shown

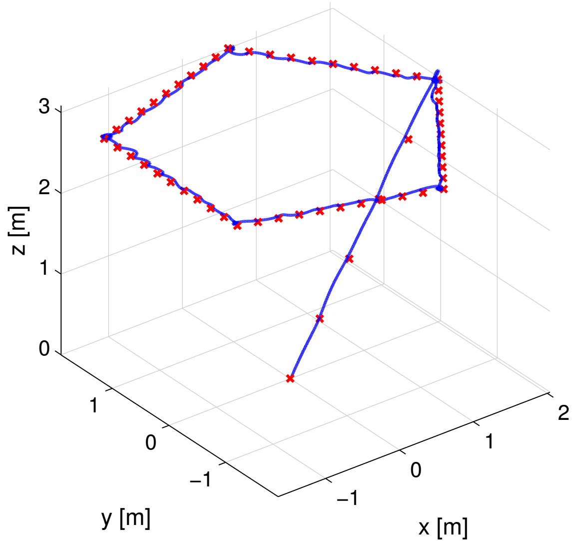

Figure 2. Spatial trajectory (nonlinear control).

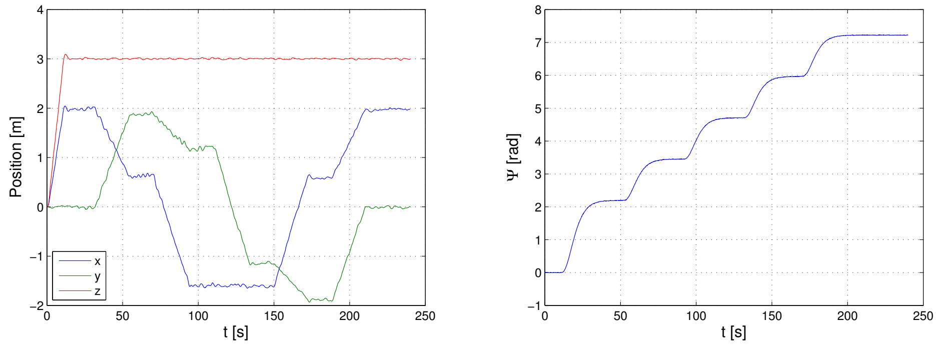

Figure 3. Position and yaw angle (nonlinear control).

in Figs. 2 and 3.

The proposed control methods stabilise the nonlinear dynamics of the helicopter. Furthermore, the closed loop can be approximated by a linear model, which makes it suitable for higher level formation control design.

Thesis group 1. *I developed novel control and state estimation algorithms for quadrotor helicopters. The control algorithms make use of the backstepping and μ-synthesis techniques. The developed methods outperform the available similar techniques in terms of model accuracy, noise suppression (backstepping controller) and robustness (μ-controller).*

The corresponding results are published in conference papers [S02, S03] and journals [S04, S07, S08].

Thesis 1.1. *I developed a new nonlinear control algorithm that renders an unstable and underactuated quadrotor helicopter stable in a specified spatial position and yaw angle. The controller is based on a novel backstepping algorithm. The required but unmeasured signals are estimated by an extended Kalman filter based state estimator.*

Thesis 1.2. *I proposed a new robust μ-controller based stabilising controller for quadrotor helicopters. The closed loop achieves robust stability and robust performance under realistic assumptions about model uncertainty and noise characteristics.*

I verified the controllers against the nonlinear quadrotor model in simulation environment.

Application and implementation related results that are not tightly related to the thesis are published in further conference papers [S01, S11, S12].

The results are presented in detail in Chapter 2 of the thesis.

Novel Marker Detection Based External State Estimation

A sensory system that provides position and attitude information about the vehicles is necessary for navigation. The most frequently applied systems in indoor environments are based on video image processing techniques. These systems often require special light conditions and dedicated area (see the sensor system of BME IIT [S02,S03,S04]). The indoor positioning system developed at MTA SZTAKI aims at tackling these problems by its novel light sensing method. Each marker's light is modulated by a unique frequency. A marker can be located by displaying a small-sized transparent window on a TFT panel mounted in front of the photosensors, as shown in Fig. 4.

The calibration process and the spatial reconstruction methods are presented in

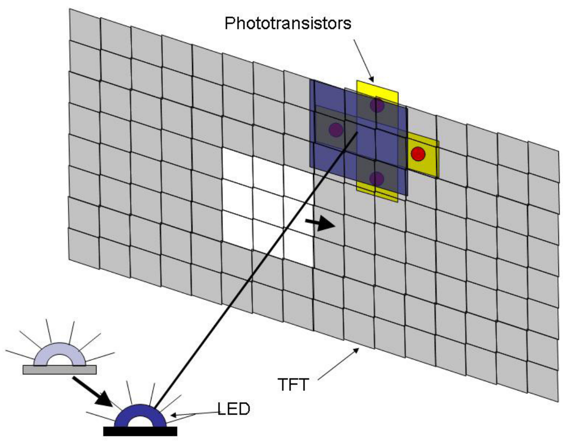

Figure 4. Operating principle of the sensor units.

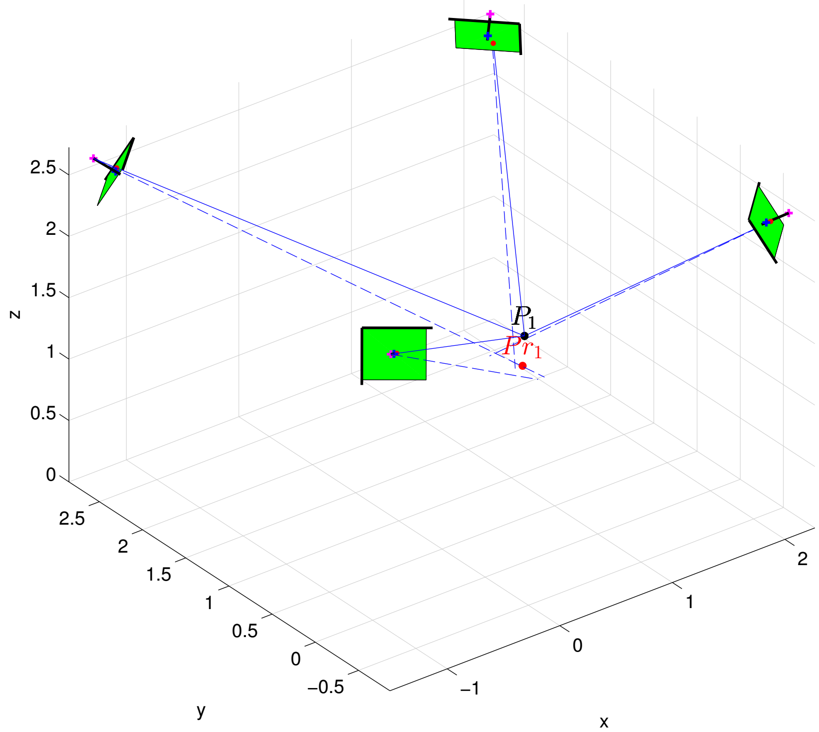

Figure 5. Estimating a single marker's location.

detail in the thesis. Following the steps of the proposed calibration method, the internal and external parameters of each sensor unit can simultaneously be estimated by a "virtual" calibration object. The full calibration can be performed by a single marker.

Localisation methods are application specific. A single marker is sufficient for the simplest case when only the position information is relevant (Fig. 5). Finding the location of a single marker leads to a least squares method. The sizes of the transparent windows on the TFT panels characterise the reliability of the measurements. If these pieces of information are also taken into account, iterative optimisation is required. The location of a single marker can thus be calculated with an accuracy of a centimetre.

In the case of aerial vehicles, however, three-dimensional position and attitude estimation is often required. Attitude estimation requires multiple markers. If information on the relative position of the markers is available, the position and attitude of the vehicle can accurately be calculated. The proposed reconstruction method can be considered as the improved version of the localisation of a single marker. In this case, the relative location of the markers mounted on the vehicle are included into the estimation problem. Depending on the application, the estimation problem leads to either a constrained or a fourth order unconstrained optimisation problem. The result of the optimisation problem is the spatial location of each marker, from which the attitude can be calculated.

The methods are available in test and real-time environments. Due to the careful coding, the speed of the reconstruction algorithm does not limit the operating frequency of the system.

Thesis group 2. *I developed calibration and reconstruction methods for marker based indoor positioning systems. Precise indoor navigation can be performed by the accurately calibrated indoor positioning systems, which can estimate object's position and attitude with an accuracy of $1-2\,\mathrm{cm}$ and $1-2°$. The estimation methods utilise LS and unconstrained optimisation methods in the case of one marker, while constrained and unconstrained optimisation techniques in the case of multiple markers.*

The corresponding results are published in conference papers [S05, S06].

Thesis 2.1. *I elaborated a two-step calibration method for accurately estimating the internal and external parameters of indoor positioning systems. The calibration process involves only one marker. The first step of the algorithm aims at reconstructing a "calibration object", while the parameters of the positioning system are simultaneously estimated for each sensor unit during the second.*

Thesis 2.2. *I proposed methods for multi-view marker based three-dimensional position and attitude estimation. The numerical optimisation based reconstruction algorithms take into account a priori information about the relative position of the markers that are mounted on the object to be tracked. The achieved accuracy and update frequency make the solution suitable for indoor navigation of small-scale ground and aerial vehicles.*

The results are presented in detail in Chapter 3 of the thesis.

Robust Formation Control and Safe Formation Change

Numerous UAV related applications require that the task be solved by a group of identical agents in tight cooperation with each other. The reason for this can either be that the agents are unable to solve the task alone due to physical limitations, or the task can be performed more efficiently in a cooperative fashion. An example to the latter case is the formation flight, where the total fuel consumption can drastically be reduced by synchronised motion.

Therefore, the research has been extended to problems related to the control of a group of vehicles. This is fundamentally different from single vehicle control since the stability of single vehicles does not guarantee the stability at the group level. Here we investigate how a large number of vehicles can perform missions together efficiently in a limited space. The main task will be to develop a control algorithm that is capable of coordinating a formation change manoeuvre, guaranteeing that vehicles avoid collisions with each other and the environment, while certain robustness requirements are satisfied.

Several methods have been elaborated that solve certain problems related to multi-

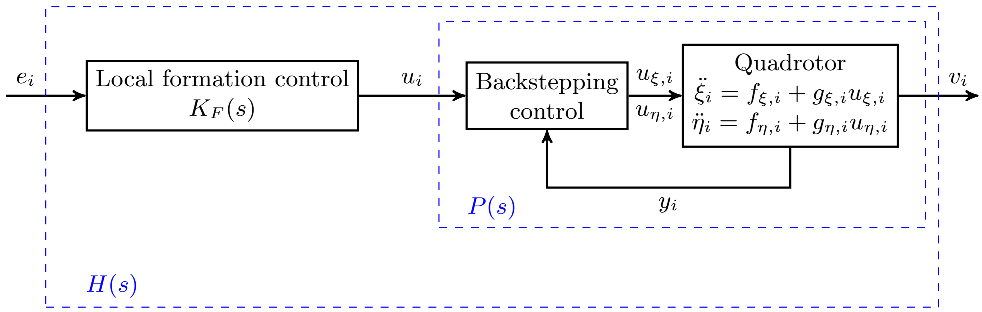

Figure 6. Single quadrotor with local controllers.

vehicle systems. Two of the most frequently applied methods are the model predictive control (MPC) and robust control techniques. Obstacle and collision avoidance is most often solved by applying MPC methods. MPC involves numerical optimisation (occasionally mixed integer programming) at every sample time instant and it is a flexible framework, various objectives can be included into the problem formulation. The cost is the increased computational complexity that may require more computational power than what currently exists in embedded systems.

The advantage of robust control based solutions over MPC based ones is that they can guarantee certain types of robustness and performance. However, they are not suitable for satisfying hard constraints, which typically occurs in collision avoidance problems. This is the motivation of the method we propose in the following. The starting point is a formation stabilising algorithm, which ensures that vehicles reach a desired formation, even if the communication topology can change arbitrarily rapidly, while certain connectivity properties are maintained. In the case of a group of quadrotor helicopters, the formation control method is integrated into the on-board control system as depicted in Fig. 6. The method is based on the work of Popov and Werner [9], which utilises the results of Fax and Murray [3]. However, the formation control method proposed by Popov and Werner does not guarantee that vehicles' trajectories are collision-free. Therefore, we extend this approach by a higher level method which effectively tackles the problem mentioned above, even for a relatively large group of vehicles.

The proposed two-level formation control system is depicted in Fig. 7. In the system interconnection, $L_{(p)}$ and $H(s)$ correspond to the communication topology and the vehicle dynamics together with the local controllers (see Fig. 6), respectively, while the rounded block and the connecting arrows represent the formation change logic. The formation change algorithm designs straight paths for the vehicles, along which they reach the target points in several steps. Vehicles are assigned target points dynamically during the design process. The method guarantees that the vehicles keep safe distance between each other throughout the whole formation change mission. Additionally, the proposed method takes into account static obstacles. If the distance ratio between the two closest starting points and the safety distance and the two closest target points

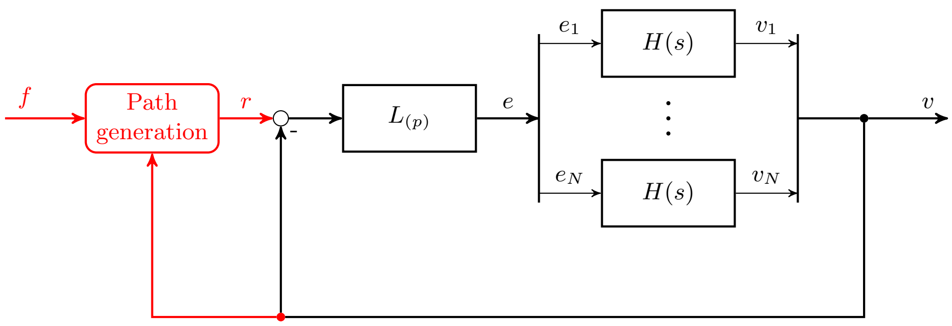

Figure 7. Formation change setup.

and the safety distance exceed a certain (relatively low) limit, it is guaranteed that the formation change manoeuvre can successfully be performed.

During the design of the manoeuvre, graphs are constructed and cliques in these graphs correspond to the vehicles that can move safely together at the same time. The vertices of the graphs correspond to the routes, while two edges are connected if vehicles can travel safely along the two routes.

Path generation is split up into three phases. Phase 1 involves vehicles that can directly reach unoccupied target points simultaneously along straight paths safely (Fig. 8). During phase 2, correction routes are generated, when further targets are impossible to be reached directly. In this case, vehicles regroup along the intermediate points of the correction routes creating empty target points that the new vehicles can reach safely (Fig. 9). All vehicles remaining in their initial place reach a target in one step in the final phase. The steps of Phase 1 and Phase 2 are repeated as long as new vehicles can find their way to target points.

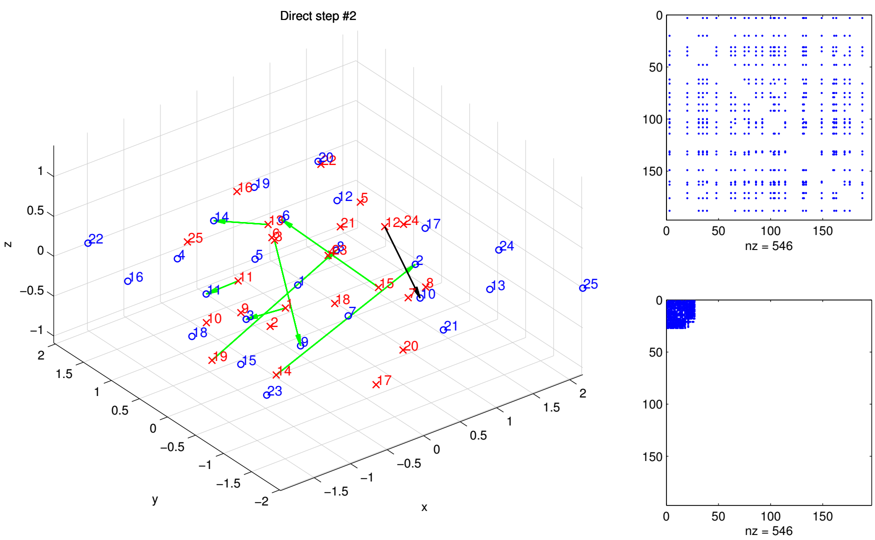

Figure 8. Example step (Phase 1).

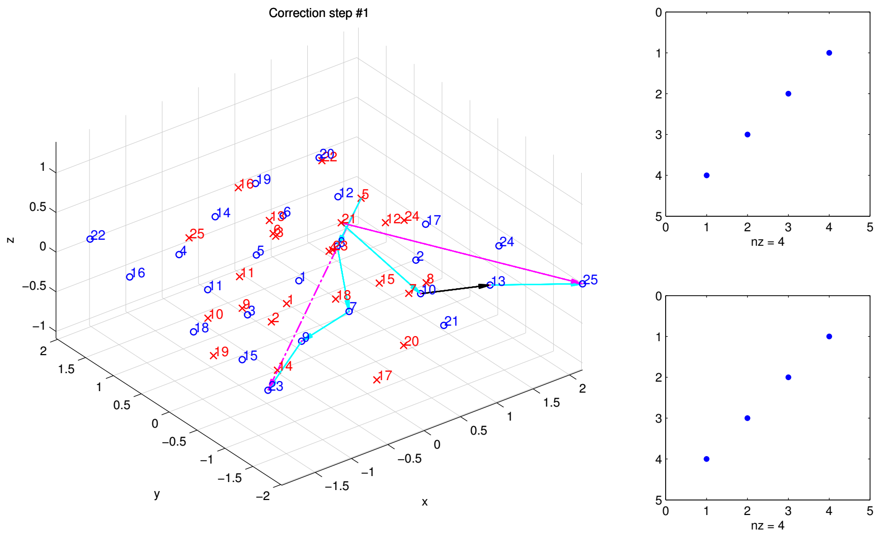

Figure 9. Example step (Phase 2).

Thesis group 3. *I proposed a novel two-level formation control strategy that can coordinate formation change manoeuvres of large groups of identical vehicles. The strategy allows for arbitrarily quick changes in communication topology and ensures that the distance between vehicles is never less than a predefined safety distance. The system consists of a formation-level controller and a path generation method. The controller is designed by $\mathcal{H}_\infty$ techniques to ensure robust formation stabilisation, while paths are generated that guarantee the vehicles safe motion even in the case when motion is restricted to a limited space and static obstacles are present.*

The corresponding results are published in the conference paper [S10] and journal [S09].

Thesis 3.1. *I developed a new path generation algorithm that is suitable for formation change manoeuvres involving a large number of vehicles with identical dynamics. The path generation method together with a robust formation-level controller guarantee collision-free motion throughout the whole formation change mission. The whole manoeuvre is performed in distinct steps and vehicles are assigned target positions during the path generation.*

Thesis 3.2. *I extended the capabilities of the algorithm of the previous thesis, making it capable of safe path generation even when static obstacles are present. The obstacle avoidance problem is solved by auxiliary virtual target points. The steps of path generation requires no change, only a slight modification to the correction route generation is made.*

The results are presented in detail in Chapter 4 of the thesis.

Applications and Future Development Directions

The results of the thesis may be applied to various practical problems. The primary motivation of the research was the quadrotor helicopter project initiated by BME IIT and MTA SZTAKI. Control and on-board state estimation algorithms developed during the research were tested on the quadrotor test bed. The research was supported by the project "Advanced Control Theory and Artificial Intelligence Techniques of Autonomous Ground, Aerial and Marine Robots" under grant No. OTKA K71762 at BME IIT.

The developed position and attitude estimation algorithms can be utilised in a variety of applications, including ground and aerial vehicles or vehicle groups. Currently, two working indoor positioning systems are assembled that make use of the algorithms, one at MTA SZTAKI, the other at BME KAUT. A waypoint tracking demonstration involving a miniature autonomous coaxial helicopter has been developed in collaboration with István Gőzse at MTA SZTAKI.

As of 2012, the research was also supported by the MTA – BME Control Engineering Research Group. The group focuses on optimal modelling and control of dynamic systems. Cooperative control algorithms developed during the research fit in the research objectives of the group and can be demonstrated in real applications.

The results of stereo image processing [10] as well as the control algorithms based on LQ, nonlinear input/output linearisation and $\mathcal{H}_\infty$ techniques [4], developed for a single quadrotor helicopter have been implemented in real time within the confines of the project OTKA K71762. The implementation of the backstepping and μ-synthesis based control algorithms as well as the safe formation change algorithm presented in the thesis are in progress. Testing may be started after completing the image processing and mechanics related developments.

I hereby wish to thank the OTKA K71762 project (BME IIT), the Systems and Control Lab of MTA SZTAKI and the MTA – BME Control Engineering Research Group for supporting the research.

Own Publications

[S01] P. Bauer, G. Regula, A. Soumelidis, and J. Bokor, "Applying CAN network in realizing low-cost small autonomous aerial vehicles," in *4th International Conference on Recent Advances in Aerospace Actuation Systems and Components (R3ASC)*, Toulouse, May 2010, pp. 78–82.

[S02] L. Kis, Z. Prohászka, and G. Regula, "Calibration and testing issues of the vision, inertial measurement and control system of an autonomous indoor quadrotor helicopter," in *17th International Workshop on Robotics in Alpe-Adria-Danube Region (RAAD'08)*, 2008, pp. 1–10.

[S03] L. Kis, G. Regula, and B. Lantos, "Design and hardware-in-the-loop test of the embedded control system of an indoor quadrotor helicopter," *6th Workshop on Intelligent Solutions in Embedded Systems (WISES'08)*, pp. 35–44, 2008.

[S04] L. Kis, Z. Prohászka, and G. Regula, "Calibration and testing issues of the vision, inertial measurement and control system of an autonomous indoor quadrotor helicopter," *International Journal of Mechanics and Control*, vol. 10, no. 1, pp. 29–38, 2009.

[S05] G. Regula, I. Gőzse, and A. Soumelidis, "3D position and attitude estimation using novel marker detection method," in *20th Mediterranean Conference on Control Automation (MED'12)*, 2012, pp. 1025–1030.

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[S07] G. Regula and B. Lantos, "Backstepping based control design with state estimation and path tracking to an indoor quadrotor helicopter," *Periodica Polytechnica Electrical Engineering*, vol. 53, no. 3–4, pp. 151–161, 2009.

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[S12] A. Soumelidis, P. Gáspár, G. Regula, and B. Lantos, "Control of an experimental mini quad-rotor UAV," *16th Mediterranean Conference on Control and Automation (MED'08)*, pp. 1252–1257, 2008.

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