DISQUISITION ON PRICING OF
TELECOMMUNICATION SERVICES AND BILLING
SYSTEM FUNCTIONALITIES

Collection of Ph.D. Theses
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1 Introduction

The mobile telecommunication market has gone through significant changes since the first commercial cellular network was launched in 1979. As mobile phone penetration approached or passed 100% in most developed countries, the judgment of the service has been slightly lowering from premium category to generally available, thus mobile network operators are continuously introducing novel value added services, complex, attractive and customizable tariff packages to sustain the level of service and the temptation of the market for consumers and investors. These new services brought new players to the market and altered the legacy business models [4, 16]. However, the majority of the income is still generated by the subscribers and managed by the point of sale (POS) and billing system of the provider.

Due to the new requirements the legacy billing systems were exchanged with flexible, future proof solutions in the last few years, as currently the customer relationship modules are being replaced in most network operators’ infrastructure. These novel systems are capable to handle the aforementioned new services and models, but the general idea and architecture of rating (defining the price of a consumed service) and billing (creating and managing invoices or topups) did not change significantly since the very beginning.

Most network operators are serving pre-paid and post-paid customers. The main different is that while pre-paid customers are paying their service consumption in advance, post-paid subscribers are allowed to settle their bill at the end of the billing period. This fundamental contrast implies that the approach and architecture of the serving IT systems are significantly different in these two cases. Offline charging is mostly used to rate post-paid subscribers and post-paid services. The price of the post-paid services are calculated from their call detail record, which is sent to the billing system after the call was made. These records (also known as charging detail records or event detail records and often abbreviated as CDRs, EDRs, or more generally xDRs) are grouped together by the Mobile Switching Centers (MSCs) or other service enabler modules of the network and sent to the
billing system through an offline, file based protocol [8, 6]. Once the records arrive to the system, the appropriate module determines the price of the calls using the information stored in the records, the rating logic of the purchased tariff packages and discounts of the customers and the accumulated usage information of the subscribers in the given billing period.

With the advent of GPRS (General Packet Radio Service), 3G services, IMS (IP Multimedia Subsystem) and LTE (Long Term Evolution), the services are no longer necessary priced according to the length of the call. It is possible, that a rather long session costs a small amount of money, or on the contrary, a short session represents an expensive service. From business point of view, long and expensive service consumptions would have high risks, since the network operator would only be notified when the call has ended, and huge debits could be accumulated without the possibility of any intervention. To overcome this problem, standards define partial CDRs for long calls [5, 7]. Partial CDRs are generated while the call is made, and they are carrying information about the service consumption since the last partial CDR was issued.

When the early mobile telephony and GSM were introduced, pre-paid billing was managed by the serving network elements. As novel services were introduced and the rating (pricing) logic of these services got more and more complex, the need for a centralized pre-paid billing platform emerged. Currently in most operators’ system an intelligent node (often referred as IN) is responsible for managing and charging the pre-paid subscribers [1, 2, 9]. The rating approach is radically different for session and for event based services [5, 10]. Event based services (such as SMS, MMS, Mobile payment or e-Gambling) allow easier rating mechanism. Once the user would like to consume the service, the pre-paid platform rates the service in advance and if the subscriber’s balance is above this value, then the call is authorized and the value of the service is deducted from the balance. If the subscriber does not have enough money on her account, then the call is rejected during the call admission control process [9, 11, 14].
The main problem with the pre-paid session based services (such as voice call, GPRS/data session, video telephony, and so on) are that the price is unknown until the service has ended, since the price is highly depending on the length of the call (the length of the call is not necessary restricted or refers to the length of session in minutes, but to the length of session in the measured unit – e.g.: the amount of kilobytes transferred). The legacy approach was to deduct the balance only after the particular call has ended, but this method clearly carries the risk, that the account of the given subscriber does not cover fully the price of the service \[12, 15\]. Nowadays the reservation and rating is done in smaller chunks, allowing the operator to gain control over the long services and eliminating or lowering the before mentioned risk \[17, 5\]. The serving network elements are reserving a predefined amount of service from the account and assures that the services are enabled as long as the length of the service does not exceed the reserved amount. When the reserved amount is exhausted, the network elements are asking for another portion from the IN. When the service ends, the reserved amount is released and the total amount of service consumption is reported to the online billing system to re-rate the call and determine the final price in order to deduct it from the subscribers’ balance.

2 Research Objectives

My preliminary research objective was to examine the effects of novel services and paradigm change on the existing billing architecture and billing systems. My objective was covered with the following three areas:

1. New services introduced to mass market might have a substantial effect on the performance of the billing systems. My aim was to find appropriate models and calculations to be able to compute the impact of these novel services on performance and on resource requirements.

2. A significant amount of network traffic and CPU power are consumed to
assure proper charging. Taking the existing standards and implementations into consideration, I have examined the possibilities to reduce the amount of administrative overhead, thus free some of the aforementioned resources.

3. Novel services and business models require a highly flexible and complex billing system. A major part of my research was to find a new rating model that fulfils these requirements and assures new functionalities and opportunities to service providers.

3 Research Methodology

During my research I have explored the issues in the mobile telecommunication era emerged by the changes of the market from scientific publications and industry forums and papers. However, the majority of the inspiration to my work was coming from the experiences I have gained as a billing system developer and later as a business and solution analyst in the industry. As a toolset I have used various techniques.

For dimensioning and overhead reduction I have created the appropriate mathematical model for the given problem. As the distributions of the call start and call length have a major impact on the results, I have assumed different distribution classes and calculated the results with the parameters of the distributions. For each model a simulation was created to prove the calculated results and to give a more comprehensive tool. With slight modifications, these simulations can be applied even if the actual distributions do not correspond to the assumed distributions used in the analytical approach.

For the novel rating model I have gathered and summarized the principles of rating and designed and built a new rating engine. Due to the graph-based approach, the rating engine is capable to predict the price of a service if the transition-probabilities are known. The mathematical model beneath was borrowed from the queuing theory.
Figure 1: General incoming CDR ($C(t)$) and processing power ($P(t)$) functions

4 New Results

As mentioned in Section 2, my research objectives were covered with three areas. My theses below are grouped accordingly.

**Thesis 1:** I have created a mathematical model to calculate the required processing power of the post-paid rater module and calculated the number of CDRs that has to be processed. The topic is detailed in Chapter 2 in the dissertation.

Pre-paid billing systems are real 24/7 systems, that are up and running continuously, since they play a major role in Call Admission Control. Post-paid rating systems are on the other hand running less than 24 hours a day, and the call detail records are not necessary processed immediately as they arrive to the system [3]. If $C(t)$ represents the distribution of the calls over time and $P(t)$ denotes the processing power (expressed with the amount of CDR it is capable to process), then their general relation is represented on
Figure 1. The maximum processing power generally does not exceed the number of incoming CDRs during peak hours, thus the two functions intersect four times (at time $m_1, m_2, m_3$ and $m_4$) as represented on the chart. The number of unprocessed CDRs in the processing queue increases as long as the processing power is less than the number of incoming CDRs and decreasing in every other case. I will assume in the later theorems that the CDR arrivals and the processing power of the post-paid billing system comply with these conditions. If $m_0 = 0$ and $m_5 = 24$, then let us introduce five different areas as follows:

$$A_i = \left| \int_{m_{(i-1)}}^{m_i} c(t) dt - \int_{m_{(i-1)}}^{m_i} p(t) dt \right|$$

where $i := \{1, 2, 3, 4, 5\}$. (1)

Let us denote the additional power of the system with $D$ and define as follows:

$$D = -A_1 + A_2 - A_3 + A_4 - A_5 = \int_0^{24} P(t) dt - \int_0^{24} C(t) dt. \quad (2)$$

It is straightforward, that the system is only capable to process the calls on a given date if $D \geq 0$.

I have proved, that if $\int_0^{24} P(t) dt > \int_0^{24} C(t) dt$, and the CDR processing queue was empty in the beginning, then the backlog will be zero at $m_2$ or $m_4$ every day. The maximal CDR queue size (backlog) in this case can be calculated as follows: $Q_{max} = \max(A_5 + A_1; A_3; A_5 + A_1 - A_2 + A_3; A_3 - A_4 + A_5 + A_1; 0)$.

The number of records and queue size are required parameters to properly size the serving IT systems. On the other hand, the business and security departments are interested in the record ages in the processing queue. A general business requirement is to not to have too old records in the backlog during the day, when the system is in normal operation (between $m_2$ and $m_4$).

I have shown, that if $\int_0^{24} P(t) dt > \int_0^{24} C(t) dt$, and the CDR processing queue is not empty at $m_2$ (but empty at $m_4$), then $P(t)$ shall fulfil the
following inequation in order to not to have older than $K$ records in the queue for every $m_2 \leq x \leq m_4$:

$$A_5 + \int_0^{\min(0,x-K)} c(t) dt \leq \int_0^x p(t) dt,$$

if the queue is empty at $m_2$, then the following inequation shall be fulfilled:

$$\int_{m_2}^{(x-K)} c(t) dt \leq \int_{m_2}^x p(t) dt,$$

for every $m_2 \leq x \leq m_4$.

The price of data services (for example) are not necessary depends on the length of the session. These sessions can be last for a couple of hours or days and for such long calls, partial CDRs are generated in post-paid environment. These partial CDRs are used to assure, that subscribers are not consuming too much service and allows the service provider to continuously control the spending of its subscribers.

The partial CDRs can be generated based on many parameters (such as amount of consumed service, location change, handover, etc.), but a general approach is to generate these interim CDRs in every $K$ time interval [5, 7]. Let $f(t)$ denote the call start, $g(t)$ denote the call length distribution, while $F(t)$ and $G(t)$ are their cumulative counterpart. $E_g(t)$ is the expected length of the call length. Let us use $P_i$ as the probability that a call length is between $iK$ and $(i + 1)K$. It is trivial, that

$$P_i = \int_{iK}^{(i+1)K} g(t) dt = G((i + 1)K) - G(iK).$$

**Thesis 1.1:** I have proved, that the average number of partial CDRs sent for a long call is between $(\frac{E_g(t)}{K} - 1)$ and $\frac{E_g(t)}{K}$ if partial CDRs are generated at every $K$ interval and $E_g(t)$ represents the expected value of the call length. Section 2.2.1 details the proof in the dissertation. [Ary2010ARR]

In order to size the CDR processing systems, the distribution of the CDR arrival (including the partial CDRs as well) shall be taken into consideration besides the total number of incoming CDRs.
Figure 2: Final and partial CDR arrival distribution as a function of time for different call length distributions

**Thesis 1.2:** I have proved, that the CDR arrival distribution (including the partial and final CDR) can be calculated as follows:

\[ h(\tau) = \frac{\int_{0}^{\infty} f(\tau - t)g(t)dt}{1 + N} + \frac{\sum_{i=1}^{\infty} f(\tau - iK) \sum_{j=i}^{\infty} P_j}{1 + N}, \]  

(6)

where \( f(t) \) represents the call start, \( g(t) \) the call length distribution, \( N \) is the average number of partial CDRs and \( P_i \) is defined in (5). Please see Section 2.2.3 in the dissertation for the proof. [Ary2010ARR]

For the simulated CDR arrivals (partial and final CDRs) please see Figure 2, where the call start distribution is normal and the call length distribution is lognormal with \( \sigma = 0.5 \).

The partial CDRs are stored in a database and correlated till the final CDR arrives. The correlation can be on-the-fly, meaning that the information stored in a particular partial CDR is correlated (added) to the information stored in the database, or it can be offline (all individual partial CDR is stored separately and summarized when the final CDR arrives to the system).
required database size is an important operational parameter of the system and it shall be computed prior a service is introduced. Since calls can be longer than a day (in a HSDPA session for instance), the required database size cannot be calculated based on one day, thus we have to take the previous days into consideration during dimensioning. Theoretically we have to take all the days into consideration since the service was introduced.

**Thesis 1.3:** I have created an algorithm to calculate (upper bound) the required database size, which takes a predefined number of days into consideration and omitting the rest of the days if partial CDRs are correlated on the fly and we have the required empirical data or we know the distribution class (and parameters) about the call start and call length distributions. The algorithm is detailed in Section 2.3.2 in the dissertation. [Ary2012TS]

The total and precise database size can be calculated if we add the effect of the rest of the days \( S_L \) to the results of our algorithm. This can be done, if the call length distribution is known. However, the normal distribution is a plausible choice, the corresponding publications are also suggesting the lognormal or the Erlang distributions [13].

If \( g_n(t) \) denotes the density and \( G_n(t) \) denotes the cumulative density function of the normal distribution, then let us introduce the **positive normal distribution** as follows:

\[
g_p(t) = \begin{cases} 
0 & \text{if } t < 0 \\
\frac{g_n(t)}{1-G_n(0)} & \text{if } t \geq 0.
\end{cases}
\] (7)

Let us further define \( \varsigma(r, L) \) as \( \sum_{i=L}^{\infty} \frac{1}{i^r} \), let \( T_d \) denote 24 hours and \( C \) the total number of calls on a day.

**Thesis 1.4:** I have proved, that if the call length either follows the positive normal, lognormal or Erlang distributions, then the effect of long calls started more than \( L \) days ago on the database size can be upper bounded if \( L \) is large enough. The \( L_{\text{min}} \) and the additional database requirement \( (S_L) \) are displayed in Table 1 for the different distribution classes. The proof can be found in Section 2.3.3 in the dissertation. [Ary2012TS]
Table 1: Results for given distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$L_{\text{min}}$</th>
<th>$S_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive normal($\mu, \sigma$)</td>
<td>$\left\lceil \frac{\mu + 2\sqrt{2}\sigma}{T_d} \right\rceil$</td>
<td>$C \frac{0.25 e^{\sqrt{2}\mu/\sigma}}{\sqrt{\pi}} \frac{e^{-\sqrt{2}LT_d/\sigma}}{1 - e^{-\sqrt{2}T_d/\sigma}}$</td>
</tr>
<tr>
<td>lognormal($\mu, \sigma$) $\sigma &lt; \sqrt{2}$</td>
<td>$\left\lceil \frac{e^{\mu + 2\sqrt{2}\sigma}}{T_d} \right\rceil$</td>
<td>$C \frac{0.25 e^{\sqrt{2}(\mu - \log(T_d))/\sigma}}{\sqrt{\pi}} \zeta(\frac{\sqrt{2}}{\sigma}, L)$</td>
</tr>
<tr>
<td>lognormal($\mu, \sigma$) $\sigma \geq 1$</td>
<td>$\left\lceil \frac{e^{\mu + 2\sqrt{2}\sigma^2}}{T_d} \right\rceil$</td>
<td>$C \frac{0.25 e^{\sqrt{2}(\mu - \log(T_d))}}{\sqrt{\pi}} \zeta(\sqrt{2}, L)$</td>
</tr>
<tr>
<td>Erlang($\lambda, k$)</td>
<td>$\left\lceil \frac{1}{e^{\lambda T_d/k} - 1} \right\rceil$</td>
<td>$C \sum_{n=0}^{k-1} \frac{e^{-\lambda LT_d} (\lambda LT_d)^n / n!}{1 - e^{-\lambda T_d/k}}$</td>
</tr>
</tbody>
</table>

In many cases the distribution and distribution parameters of the calls allow us to simplify the calculations and give a simple rule-of-thumb, when calculating the database size. For rather short calls (when the majority of the calls are shorter than the partial CDR generation interval) the required partial database size is relatively small.

**Thesis 1.5:** I have calculated, that if the call length follows the positive normal distribution and $T_d > K > \mu + 2\sqrt{2}\sigma$, then the database size required to store partial CDRs is less than 0.00732$C$. If the call length follows the lognormal distribution and $T_d > K > e^{\mu + 2\sqrt{2}\sigma}$, then the database size
required to store partial CDRs is less than \(0.01502C\). Details can be found in Section 2.3.4 in the dissertation. \cite{Ary2012TS}

The proposed algorithm and analytical results can be used together to upper bound the required database size. However, if the call length distribution follows a more complex distribution, then the addition database size cannot be calculated with the equations above, moreover, we do not know the proper limit, which shall be used in the proposed algorithm. For such cases, the use of a simulation shall be considered.

**Thesis 1.6:** I have created a simulation to calculate the required database size for a given call length distribution and to prove the analytic calculations. The simulation is detailed in Section 2.3.5 in the dissertation. \cite{Ary2012TS}

The simulated and calculated database sizes are represented on Figure 3 for three different scenarios.

During pre-paid session based services (voice call, data session, etc.) the serving network elements are reserving credits (money) from the subscribers’
accounts for a predefined amount of units (minutes or kilobytes) [11, 10].

Each reservation requires a reservation message and at least a rating.

**Thesis 1.7:** I have proved, that if the reservation is done with $K$ intervals, then the average amount of reservation messages for a call is less than $E_a(t) + 2$. This result was proved with a simulation. Details can be found in Section 2.4 in the dissertation. [Ary2011PP]

Figure 4 displays the calculated and simulated number of unit reservation messages. The gray area represents the calculated interval for the given parameters, while the dots are representing the simulated values.

In online charging, if the service reserves a large amount of credit from the user’s account, access to additional, parallel resources could be denied, because there is no credit left on the account for another resource usage request; even if some service terminates afterwards, and the unused credits are returned to the users [17, 15, 12]. In light of this, a more frequent unit reservation, with a smaller amount of credit should be applied. Because
CDRs indicate the used services/data, this problem does not occur during offline charging. This implies, that pre-paid billing systems and the regular online rating approach requires more network overhead, than the offline charging.

**Thesis 2:** I have proposed some protocol and architecture changes to reduce the amount of charging overhead in pre-paid billing systems. The topic is detailed in Chapter 3 in the dissertation.

**Thesis 2.1:** I have proposed a mode-switching model, where the system dynamically switches between offline and online charging. The model is detailed in Section 3.1 in the dissertation. [Ary2005HTS, Ary2005EUN, Ary2005CON]

The main idea of this model is to define the $C_{r,s}$ unit consumption speed for each $s$ service, which represents the maximum units (money), that can be consumed by that specific service and $r$ rating logic during one second. If we have some information on the propagation delay, then we can define the mode switching limit. If a subscriber’s balance is over this limit, then offline charging can be applied, while if the subscriber’s balance drops below this limit, then online charging shall be applied. In order to implement such solution, the same rating logic shall be applied in the pre-paid, and post-paid billing system and the serving network elements shall be modified to notify the post-paid billing system to initiate mode-switching.

**Thesis 2.2:** I have created a simulation to show the advantage of the mode-switching model. The simulation method and the results can be found in Section 3.1.2 in the dissertation. [Ary2005HTS, Ary2005EUN, Ary2005CON]

The number of rating is represented on Figure 5 for online, offline and mode-switching as the mode-switching limit changes. For the latter one, both the online, offline and total number of ratings are represented. Figure 6 displays the remaining account on the users’ balance.

The mode-switching model has its advantages, the implementation and
Figure 5: Number of ratings as function of mode-switching threshold for the different rating methods

the required modifications might be too difficult. A slightly easier modification is to define the unit reservation messages dynamically. The service can be further enhanced if we apply preemptive reservations, where services with higher priorities may initiate a redistribution of units.

**Thesis 2.3:** I have created an algorithm, that assures proper charging with dynamic unit reservation, and the average amount of unit reservation messages per calls is less or equal to \( \frac{E_g(t)}{K} + 2 + \frac{LP}{C} \), where \( L \) denotes the levels of dynamic unit reservations, \( P \) denotes the maximum number of service types and \( C \) denotes the average amount of charged calls or service requests in a topup period. If preemptive unit reservation is used, then the average amount of unit reservation messages per calls is less or equal to \( \frac{E_g(t)}{K} + 2 + \frac{DLP + LP}{C} \), where \( D \) represents the average number of preemptive calls. More information can be found in Section 3.2 in the dissertation. [Ary2011PP]

As new services were introduced around the millennium, the need for a future proof rating and billing system emerged. The telecommunication
network providers either developed or enhanced their legacy billing system or (as in most cases) replaced them with a third party application. In either case, the key factor is flexibility and the ability to support wide range of marketing ideas and services.

**Thesis 3:** *I have created a new flexible, yet powerful post-paid rating model, which is detailed in Chapter 4 in the dissertation.*

In most cases, post-paid rating systems are developed with the write-the-code concept, and the rating logic is coded with a high-level language. This assures the flexibility of the system, but sacrificing the ability to analyze the tariff packages with different mathematical tools.

**Thesis 3.1:** I have identified the main functionalities of a rating module and created a new flexible, yet powerful rating model based on the state-graph approach. Details are in Section 4.1 in the dissertation. [Ary2007MS]

The main steps performed when rating a call detail record is as follows:
1. Collect the CDR/EDR from the network elements (mediation or acquisition).

2. Find the proper customer and her purchased tariff package and discounts.

3. Determine the price of the call from the information stored in the xDR, the tariff packages, discounts and the accumulated service consumption in the given billing period.

4. Update the accumulated service consumption with the new call (total price, total length of calls, etc.).

5. Write and commit this information in the corresponding database.

The main idea of the state graph approach is that a specific tariff package can be represented with a graph. Each state of the graph represents the price of the service with similar conditions. The transitions between the states are triggered by the accumulated and by the current parameters of the call (let us denote them with transition-conditions). With this approach, the price determination of the call can be done by finding the actual state of the graph. An example state graph is represented on Figure 7 for the following theoretical price plan for voice calls:

**State A:** The first 20 minutes in the month is free.

**State B:** The price of the call is 20 units in peak periods.

**State C:** The price of the call is 10 units in off-peak periods.

**State D:** The price of the call is 8 units on weekends.

**State E:** After the 100th calls in the month, the price is 15 units in peak periods.

**State F:** After the 100th calls in the month, the price is 6 units in off-peak periods and on weekends.
Let $G_s$ denote the state-graph for a given service and for a given subscriber, $\bar{\alpha}$ stands for the state indicator vector (the actual state is represented with 1, while the other values are 0), and $\bar{u}$ denotes the price-vector of the service and indicates the price in each state ($\bar{u} = [20, 10, 8, 15, 6]$ in the example). We can also assign a probability to a transition and create a $\Pi$ state transition matrix, where the $i,j$ element represents the probability of transiting from state $i$ to $j$.

**Thesis 3.2:** I have shown, that if $\Pi_r$ denotes the transition probability matrix for a given $r$ rating logic, then the price of a service can be calculated as follows:

$$v(t) = \sum_{k=0}^{t-1} \bar{\alpha}\Pi_r^k\bar{u},$$

where $\bar{\alpha}$ is the state indicator vector, $\bar{u}$ is the price-vector, and $t$ denotes the length of the call. This is detailed in Section 4.3 in the dissertation. [Ary2007MS]

With this approach, the price of the service can be calculated before the actual consumption if the call length is assumed or known. However, the
main problem with this model is that the exact transition conditions are mapped into simple probabilities, and this simplification may ruin the result of the AoC functionality and may raise the deviation significantly.

**Thesis 3.3:** I have introduced three different models to enhance the quality of price prediction by coding the transition conditions to different states. The models are detailed in Section 4.3 in the dissertation. [Ary2007MS]

Advice of Charge with Exploded State Transition Matrix: The idea of this model is to code the transition conditions into the states of the state-graph. To be more exact, we create a new graph, where one state is exploded into as many states, as the pricing logic requires it, to hold some memory for the transition condition. For example, if the transition condition is to move from state \( i \) to state \( j \) after 5 units (seconds), then we substitute state \( i \) with a chain of five states (from state \( i_1 \) to state \( i_5 \)). The probabilities of these transitions are 1. The problem of this solution is that the state-graph may be enormous, if the required memory is huge and thus, it may have a significant impact on the computation speed and on the required memory from the IT system.

Advice of Charge with Time Layered Model: This model assumes that the transition-conditions can be either mapped into probabilities without causing significant deviation from the theoretical optimum, or they are depending only on the length of the call. Nowadays, most of the available tariff packages fulfil this requirement. We divide the state-graph into different layers. The transitions inside a layer will be mapped into probabilities, the transitions between the layers are deterministic, and they are depending only on the length of the call. Mainly we are using layer 1 for the first part of the call, layer 2 for the second part, and so on.

Stratified Advice of Charge: The main idea of this model is that some transitions may be left out from the transition graph, when we calculate the AoC. This simplification / condition should be published to the customers. Moreover, if the transition probability is quite small, but the value of the new state differs significantly from the previous one, then the precise advice
Figure 8: Number of simulation runs resulted the given price and the calculated expected price for the different models.

of charge may even cause problems.

All the above mentioned enhancements improve the quality of the estimation, however, the preferred option shall be chosen by the experts and needs to be aligned with the actual case.

**Thesis 3.4:** I have created a simulation to show the results of our price prediction and to show the differences of the different enhancements. The simulation and the results can be found in Section 4.4 in the dissertation. [Ary2007MS]

The results of the simulation as well as the calculated values are represented on Figure 8.

## 5 Applicability of the New Results

During my research I was endeavor to give fairly simple, yet effective mathematical formulas and algorithms. I strongly believe that the simpler a
solution is, the easier it can be understood and applied in real implementations.

The sizing results can be used to forecast the impacts of a new service on the billing system or it can be used during a green field implementation, when a new service provider tries to penetrate the market. The results from the overhead reduction can be used to fine-tune or enhance the system from charging overhead point of view, while the novel rating model can be a solid and powerful foundation for a very flexible and novel billing system.

References


Publications

Book chapters


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