Ph.D. Thesis

Investigation of Multicarrier Schemes for Cognitive Radio Applications

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Abstract

The frequency spectrum is an essential resource of wireless communication. Special sections of the spectrum are used for military purposes, some frequency bands are sold by governments to broadcasting and mobile communications companies for commercial use, others – such as ISM (Industrial, Science and Medical) bands – are available for the public free of charge. As the spectrum becomes overcrowded, there seem to be two possible solutions: pushing the frequency limits higher to frequencies of 60 GHz and above, or the reaggregating the densely used licensed frequency bands. Both ideas lead to difficulties: the use of higher frequencies requires the design and manufacturing of special analog devices, while the reallocation of the spectra requires complex, intelligent and adaptive systems.

As a result of the digital switchover of TV broadcasting, certain frequency bands become unused. According to recent plans, intelligent and opportunistically communicating radio systems aiming data transmission applications might be implemented on these frequencies. As the primary (incumbent) user of these frequencies will still be the broadcast sector, data communication systems operating in these frequencies will have to incorporate sophisticated intelligence and fast spectrum sensing capabilities to prevent interference. The physical layer of these communication systems must meet some special requirements implied by these special constraints. Cognitive radio based opportunistic exploitation for spatially and temporally unused frequencies is considered to be a feasible approach to improve the spectral efficiency and to introduce new services in the legacy bands.

The selection of the appropriate modulation technique is a major issue. The frequency-selective nature of the radio transmission channel and the need for multiuser applications call for multichannel modulation schemes, which are the subject of this thesis. Today the Orthogonal Frequency Division Multiplexing (OFDM) modulation scheme is the most widespread technique for high-speed wireless data transmission. It is used in many broadcasting and communication systems such as Digital Video Broadcasting (DVB), Digital Audio Broadcasting (DAB) and certain types of IEEE 802.11 Wireless Local Area Networks (WLAN). Using OFDM, low-complexity demodulation and modulation can be performed by Fast Fourier Transform (FFT) and Inverse FFT (IFFT) operations, respectively. Using Cyclic Prefix (CP) Inter Symbol Interference (ISI) can be eliminated which is the precondition of efficient channel equalization in the frequency domain. Nevertheless, this scheme has some drawbacks. Due to the large dynamic range of the transmitted signal, OFDM is highly sensitive to the nonlinear characteristics of the Power Amplifiers (PA). This nonlinearity induces in-band and out-of-band spurious products, which might degrade the system's performance. OFDM system performance can also be severely degraded by the frequency mismatch of the transmitter and receiver local oscillators, therefore, a very robust synchronization technique is required. Moreover, without filtering the transmitted signal, the out-of-band radiation of OFDM is considered as only moderate, which is a major drawback in the opportunistic context, where significant adjacent-channel leakage leads to interference.

There are many issues regarding cognitive radio scenarios which cannot be met by OFDM. This is why other multichannel schemes have become a point of interest. In this thesis, besides OFDM, the focus is on three possible alternatives: DFT-Spread OFDM (DFTS-OFDM), Constant Envelope OFDM (CE-OFDM) and Staggered MultiTone (SMT). Each of these schemes have some advantages over OFDM which makes them beneficial for use in cognitive radio applications.

In this thesis the above mentioned multichannel schemes are closely investigated and compared. First, their signal model and the required signal processing blocks are introduced. Then the advantages and
drawbacks derived from the transmitted signal properties are presented. Later, the effects of nonlinearities, synchronization impairments and multipath propagation will be investigated on the baseband transceiver model. Novel channel equalization schemes are also presented for SMT exhibiting better system performance than conventional techniques. Peak-to-Average Power Ratio (PAPR) reduction by clipping is discussed for OFDM signals. Iterative compensation techniques employing the *Turbo principle* are introduced to improve the performance of amplitude limited OFDM and SMT signals. OFDM based techniques are also presented for the SMT scheme in order to reduce the large dynamic range of the transmitted signal. Finally, applications of recursive discrete fourier transform in cognitive radio receivers for filter bank multicarrier modulation are investigated.
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1 Introduction

The purpose of communication is to transmit information from the information source to the sink. In digital communication systems the data to be transmitted is a binary information stream usually representing a digital multimedia stream or data file. The binary information is transformed to an analog signal which can be transmitted through the transmission medium. To achieve the best performance possible, a crucial task is to choose the most suitable modulation technique that matches the requirements of analog components, standards, regulations and also the properties of the transmission medium. The nature of the data transfer (bidirectional or omnidirectional) also strongly influences the design and complexity of the terminal equipment in these communication systems.

Depending on the transmission medium, three major scenarios can be distinguished: cable, terrestrial and satellite transmission. Each scenario calls for different criteria for the applied modulation scheme. Very high data transmission rates can be achieved using cable. A disadvantage of this transmission medium is that it can only be used with fixed communication nodes. In satellite communication the amplifier and the antenna are the most critical in system design and the overall system cost is high. The terrestrial scenario is the most widespread communication form in broadcasting, mobile communication and WLAN. Some of these technologies use single carrier-based modulation, others apply multicarrier schemes.

Networks have to face new challenges due to the growing need for fast and high quality data transfer induced by the users. Existing standards such as DVB [1] and DAB [2] are constantly improved and extended to meet the users’ needs. The second generation of terrestrial, cable and satellite versions of DVB have already appeared in the broadcasting domain. In mobile communications, current and upcoming systems such as High Speed Packet Access (HSPA) and Long Term Evolution (LTE) [3] have been introduced. These developments were enabled by more efficient analog devices, faster DSPs and FPGAs, which allow the use of more complex and advanced digital signal processing algorithms.

For wireless communication systems the radio spectrum is considered as the key resource. Throughout the world, the spectrum is divided into licensed and unlicensed bands. The growing need for bandwidth is pushing the communication systems to higher frequencies. The most recent target of the communication society is the 60 GHz band [4], where an unlicensed spectral section is opening up. The trend of using higher frequencies is currently limited by the scarce availability and the high cost of analog components supporting such frequencies. The performance of such devices can only be rated as moderate [5, 6].

There is another direction of development for wireless communication networks. Since Mitola introduced the concept of cognitive radio [7, 8] in 1999, the interest in this topic emerged rapidly. In general, the unlicensed bands are becoming overcrowded and in some cases the licensed bands are unutilized. This is especially the case of the TV broadcasting bands [9], where – due to the analog-to-digital switchover – a considerable amount of spectra is left unused in many countries around the world. This switchover was first completed in the US in 2009. This will be probably followed by the UK by the end of 2012. Communication Committees in these two countries are strongly encouraging the cognitive radio based utilization of this unexploited TV spectrum. In the US the Federal Communications Commission published documents concerning the intelligent use of the licensed spectra by opportunistic devices [10] in 2004 and [11] in 2010. The Office of Communications in the UK revealed similar interest in 2009 [12]. The changeover has been started in Hungary as well, it is assumed to be completed by 2015, when all the analog TV transmissions will be shut down.
The unused spectral section is called White Space (WS). Bands which are partially used are called Gray Spaces (GS) and the ones which are continuously occupied are designated as Black Spaces (BS). Opportunistic systems target WSs and GSs in the TV bands. In these newly introduced and flexible systems – besides bidirectional communication – spectral sensing also becomes a highly supported feature. The additional capability for spectral sensing allows these systems to adapt to the current radio environment, which makes them especially suitable for opportunistic wireless communications operating in the WSs and GSs of the licensed bands. Researchers predict that these systems will play a key role in future wireless communications [13].

Cognitive radio can be viewed as an intelligent wireless system built up of software defined radios which are able to constantly monitor the environment and adapt their communication in order to efficiently allocate the unused spectra [14]. Cognitive radio systems are secondary – opportunistic – systems. They make use of the free spectra in the licensed band which are currently unused by the primary – incumbent – systems. An opportunistic system has to operate totally hidden from the incumbent one and it should not disturb it by any means.

The need for such networks has also triggered the interest of standardization committees. The standardization of cognitive radio, which is currently at its draft stage [15, 16], has been started by the IEEE 802.22 Working Group on Wireless Regional Area Networks (WRAN). IEEE 802.22 WRAN mainly addresses the unused TV bands. The specified network can operate in WRAN environments, with path lengths up to 100 km, where a Base Station (BST) is managing the cognitively operating opportunistic users – the Consumer Premises Equipment (CPE) [17]. Each CPE features spectral sensing capabilities. The results of CPE spectral sensing are transmitted to the BST. The BST can be considered as the "brain" of the network, where special algorithms are used to optimally use the available spectrum, manage coexistence and coordinate communication between the CPEs. The standardization of cognitive radio equipment is also influenced by the industry. The first draft of the standard for the PHYsical (PHY) and Medium Access Control (MAC) layers was published by ECMA International and the Cognitive Networking Alliance standard in 2009 [18, 19].

The two most researched areas of cognitive radio equipment are the PHY layer and the MAC layer [20]. An open issue regarding the MAC layer is an intelligent way of detecting incumbent transmissions besides "blind" energy detection. Most approaches also include a geolocation combined with a database for more efficient spectrum sensing [21]. In addition to sensing, the spectrum allocation, management and coexistence with incumbent and other opportunistic networks are also open issues. In the PHY layer the choice of the applied modulation scheme is critical. In general, OFDM is considered as the main candidate for up- and downlink communication as well. OFDM is a well understood technique which has an extensive literature. Nevertheless, in some aspects such as out-of-band radiation, behavior in case of imperfect synchronization and performance in the presence of nonlinearity, OFDM shows some weaknesses. In cognitive radio applications the primary objective is the efficient spectral band allocation. To achieve a spectrum as compact as possible, other modulation schemes had to be investigated as well. Although these alternative schemes provide better spectral characteristics, their additional properties are less attractive. Besides spectral efficiency power efficiency is another important factor in CPEs, so having a limited dynamic range is preferred in case of the transmission signal.

There has already been some research for comparing alternative modulation schemes to OFDM for cognitive radios [22, 23, 24]. This thesis mainly deals with this topic, concerning various aspects – such as
nonlinear effects, influence of the channel, imperfect synchronization – of multicarrier modulation schemes which can be suitable for the PHY layer of the cognitive radio transceiver architecture.

1.1 Motivation

The research leading to the results presented in the thesis was derived from the European Community’s Seventh Framework Programme (FP7) under Grant Agreement number 248454 (Quality of Service and MObiity driven cognitive radio Systems (QoSMOS) [25, 26]). The research group of BME in the fourth work package (WP4) was supervised by Dr. Péter Horváth, where it had to collaborate with the Technische Universität Dresden (Germany), Commissariat à l’Energie Atomique (France), Fraunhofer-Gesellschaft zur Förderung der angewandten Forschung e.V. (Germany), Instituto de Telecomunicações (Portugal) and NEC Corporation (Japan).

The research group was given the task of studying schemes alternative to OFDM that enable a more power efficient operation and reduced interference to the adjacent bands in case of nonlinear distortions caused by power amplifiers. The ongoing research targets channel modeling, equalization techniques and synchronization issues. The construction of the signal structure and piloting scheme was also one of the major tasks. In the future an efficient synchronization scheme will be proposed for the physical layer waveforms chosen for the QoSMOS system, and interference resilience methods will be further investigated as well.

Together with the above mentioned partners, some of the results of the ongoing work at WP4 have already been published in joint papers such as [27, 28, 29].

1.2 Thesis organization

First, in Section 2, a brief history and overview of the multicarrier modulations schemes using filter banks is given. The evolution of these techniques is discussed, presenting the analog and digital signal models. Then, the most important and most researched variant of the Filter Bank MultiCarrier (FBMC) family, the OFDM is explained in details. Besides OFDM two other OFDM-based schemes are also introduced: the DFTS-OFDM and the CE-OFDM. These two techniques are closely related to OFDM and they differ only in minor additional signal processing modifications. An other variant of FBMC is described as well: the SMT technique. Following this overview, the results achieved by the author are presented in 4 sections.

Section 3 investigates the previously mentioned four multicarrier modulations (OFDM, DFTS-OFDM, CE-OFDM and SMT) from the point of transceiver complexity and the statistical characteristics of the transmitted signal. The digital baseband model of the transceiver chain is discussed as well, including the channel model. The model simulating the errors arising from synchronization imperfections caused by the non-ideal analog components is also described. Finally, a comparison of the four investigated modulation schemes is given regarding their performance in the presence of residual synchronization errors and nonlinear effects. Also channel equalization for these schemes is presented and compared. Novel methods for SMT modulation are introduced especially for radio channels with large delay spreads: first, a subcarrier-based equalization methods are shown and compared, then an iterative decision feedback technique is presented which can further improve the equalization performance.

In Section 4, a solution for reducing the high Peak-to-Average Power Ratio (PAPR) is shown for OFDM systems, where the amplitude range of the time domain signal is limited. Then, with the aid
of iterative error correction coding based on the *Turbo principle*, the nonlinear distortion is suppressed and the received signal is reconstructed in the receiver. This shifts large amount of the signal processing operations to the base station and enables a power efficient operation of the mobile terminals. This receiver scheme is investigated in terms of convergency based on EXtrinsic Information Transfer (EXIT) charts. The scheme shows divergence, therefore minor modifications are proposed to it which make it convergent.

In Section 5 PAPR reduction techniques suitable for SMT are described and compared. First clipping-based transmitter oriented iterative PAPR reduction methods are presented, where the receiver algorithms may remain unaltered without performance degradation. Then a modified version of the iterative receiver presented in the previous section is also given for clipped SMT signals.

In Section 6 the possible usage of Recursive Discrete Fourier Transform (R-DFT) in cognitive radio applications is investigated. Two possible implementations of the R-DFT are presented: the filter bank approach and the observer-based. The benefits of R-DFT over conventional FFT in cognitive radio receivers are described in terms of complexity gain, spectral sensing and channel equalization.

Finally, in the last section the conclusions are drawn and possible research directions are shown in which the investigations should proceed in the future in order find the most suitable solution for PHY layer of cognitive radio applications.
1.3 List of publications

This thesis is the results of the work which has been published in the following journals and selected conference contributions.

1.3.1 Journal papers

   Independently cited by [30].


1.3.2 Conference papers

   Independently cited by [35].

   Independently cited by [30].

   Independently cited by [38].


1.4 Scientific prizes, achievements

- Young Scientist Award for best young presenter at the Radioelektronika 2012 Conference

1.5 New scientific results – Theses

**Thesis I.** In case of Staggered MultiTone (SMT) systems highly advantageous spectral properties can be achieved by a specially chosen prototype filter. Since the individual symbols overlap in the time domain in SMT systems, the cyclic prefix used in Orthogonal Frequency Division Multiplexing (OFDM) systems cannot be applied, otherwise the orthogonality between the symbols would be violated. The absence of cyclic prefix may cause severe difficulties especially when the duration of the radio channel’s impulse response is comparable with the length of the symbol period. In such cases inter-symbol interference may occur in SMT systems, resulting a degraded bit error rate.

I investigated the efficiency of the channel equalization methods used in SMT systems. I have proven via simulations that the bit error rate of conventional channel equalization methods (i.e. which compensate individually per subcarrier) dramatically degrades in case of impulse responses whose length is comparable to the duration of the symbol period. Due to inter symbol interference the bit error rate could not be further improved beyond a certain limit by increasing the signal-to-noise ratio. I elaborated new methods to improve the efficiency of channel equalization in SMT systems. I compared these methods with each other by simulations for channel profiles B and C defined in the standard IEEE 802.22 [15].

I. a) I made a recommendation to consider inter symbol interference as noise in case of conventional systems, which enables the channel equalization procedure to yield better bit error rates.

I. b) I propose an averaging-based channel equalization method, which further improves the bit error rate. To reduce the computational requirements of this procedure, I recommended the application of an observer-theory based recursive DFT algorithm, which is also advantageous for cognitive radio applications in case of continuous spectrum monitoring.

I. c) To further increase the efficiency of channel equalization, I elaborated an iterative decision feedback procedure. Regarding the bit error rates, a further improvement can be achieved with this method.

(Please refer to Section 3.4.7 and Section 6.)

Related own publications: [31], [39], [40].

**Thesis II.** One of the disadvantageous features of OFDM systems is their high peak-to-average ratio. The nonlinearities of power amplifiers may severely distort the signal, leading to interference between the subcarriers and degrading the efficiency of the system. A possible method to overcome this problem is to limit (clip) the baseband signal samples. This clipping causes nonlinear distortion, which may reduce the bit error rate. To eliminate these negative effects, the Bussgang Noise Cancellation (BNC) receive procedure, which operates on an iterative basis, was elaborated for power-limited and coded OFDM signals [45, 46]. The convergence of iterative decoding procedures is characterized by the EXtrinsic Information Transfer (EXIT) functions of the decoders, which enables an efficient tracking of the iterative process [47].

II. a) I have proven that the EXIT function of the BNC detector is not ascending monotonously, which leads to divergence (degradation of the bit error rate) during the iterative decoding procedure in case of certain signal-to-noise ratios and code rates.
II. b) I further improved the receive procedure, obtaining a monotonously ascending EXIT function for it, making it convergent for any signal-to-noise ratio and code rate.

(Please refer to Section 4.)

Related own publications: [33], [34], [41].

Thesis III.: Just like OFDM modulation, SMT signals are also sensitive for nonlinear distortions and may have a large dynamic range, therefore their dynamic range must be reduced at a given amplifier gain. To this end, I investigated the opportunities of reducing the crest factor of modulated signals in SMT systems. I elaborated crest factor reduction methods suitable for SMT signals, on the basis of techniques applied in OFDM systems [48] and procedures used in measurement technology for the reduction of the crest factor of multi-sine signals [49]. I investigated the efficiency and applicability of power-limiting based procedures for SMT modulation. I elaborated three possible methods by adopting techniques used in OFDM:

- A technique applying iterative compensation in the receiver to counteract power limiting,
- A carrier-allocation based procedure,
- A procedure using active constellation extension.

III. a) I adopted the modified BNC receiver procedure (presented in Thesis II.) to SMT systems.

I have proven that it is convergent not just for channels with only additive white Gaussian noise but also for channels with Rayleigh signal paths. The bit error rate increases after each iteration step during iterative decoding.

(Please refer to Sections 5.2. - 5.3.)

III. b) I have shown that the lowest crest factor can be attained by power limiting and filtering.

In this case, however, compensation is necessary in the receiver. If the intention is to achieve the same bit error rate without modifying the receive algorithms, then the lowest crest factor can be ensured by the simultaneous application of carrier-allocation and active constellation extension. The results were verified through simulations. I defined a metric which enables the determination of the optimal iteration number of the iterative crest factor reduction procedures in the transmitter.

(Please refer to Section 5.1. and Section 5.3.)

Related own publications: [33], [41], [42].
2 Multicarrier modulation schemes

In this section the idea of multiplexing parallel data streams is described. First a short history and overview is given of the multicarrier modulation schemes. Then the digital modulation of a single carrier is explained on the basis on which the continuous and discrete time formulations of the spectrally efficient FBMC schemes are presented. This section concludes with two types of FBMC schemes: the OFDM and the SMT scheme, both allowing the transmission of complex modulation values.

2.1 Brief history and overview

Parallel transmission and multiplexing of data in digital wired and wireless communication is an important topic of communication theory. Parallel data transmission was first implemented by Frequency Division Multiplexing (FDM), where the available spectra is split up to transmission channels. These channels are separated by guard intervals in order to avoid mutual interference. Due to these guard bands FDM suffer from poor spectral efficiency. In his pioneering work, Chang [50] introduced (1966) the idea of orthogonal overlapping channels which can be used for parallel data transmission. Chang proposed the transmission of Pulse Amplitude Modulated (PAM) signals with Vestigial Sideband (VSB) modulation. Applying cosine-based filterbanks, this method will be referred to as Cosine-modulated MultiTone (CMT). This technique is closely related to the Discrete Wavelet MultiTone (DWMT) [51]. Saltzberg [52] extended the work of Chang with Quadrature Amplitude Modulated (QAM) signals using Double-SideBand (DSB) modulation and alternatingly in-phase or quadrature modulation on the subsequent carriers. The technique described by Saltzberg is called Staggered MultiTone (SMT) or Offset-QAM (OQAM) OFDM modulation. The variety of naming and terminology of these inventions is due to the fact that they were developed in parallel for wired and wireless communications. Such techniques, where a set of orthogonal subchannels with filterbank implementation are used, will be referred to as FBMC.

FBMC techniques are closely related to signal processing methods used in transform coding and subband coding of speech and images [53, 54]. These signal processing techniques use analysis filters for spectral analysis and data compression. Synthesis filterbanks are used for the reconstruction of the previously manipulated signal. In digital communications these filters are applied in reverse order. The synthesis filter bank is used to generate the time domain signal in the transmitter, and an analysis filter bank is used to separate the subchannels in the receiver.

2.2 Single carrier IQ modulation

In this subsection, the digital IQ (In-phase and Quadrature) modulation of a single carrier is explained. A simplified block diagram of an IQ modulator is depicted in Fig. 1. The input signal of a single carrier IQ modulator can be expressed as

\[ X(t) = \sum_{m=-\infty}^{\infty} X[m] \delta(t - mT), \quad (2.1) \]

where \( T \) represents the signalling time, which is the time interval between two consecutive modulation values \( X[m] \) and \( X[m+1] \). \( \delta(t) \) is the Dirac delta function. \( X[m] \) is selected from the modulation alphabet.
Figure 1: Model for single carrier IQ modulator using complex signalling.

$C$ for the $m^{th}$ symbol. Each complex modulation value $X[m]$ can be expressed as

$$X[m] = a[m] + jb[m], \quad (2-2)$$

where $a$ represents the real (in-phase, I) and $b$ the imaginary (quadrature-phase, Q) component of the modulation value $X$ and $j$ is the imaginary unit defined as $j = \sqrt{-1}$. The input signal $X(t)$ is filtered by the transmitter filter $p_T(t)$ resulting the output baseband signal $s(t)$ as

$$s(t) = X(t) \ast p_T(t) = \sum_{m=-\infty}^{\infty} X[m]p_T(t - mT). \quad (2-3)$$

The carrier frequency $F_c$ is modulated with the output of the filter to form the complex transmitted passband signal $v(t)$ as

$$v(t) = s(t)e^{j2\pi F_c t}. \quad (2-4)$$

Assuming an ideal transmission medium, the output signal after downconversion and filtering with the matched receiver filter $p_R(t) = p_T(-t)$ can be express as

$$\hat{X}(t) = (v(t)e^{-j2\pi F_c t}) \ast p_R(t) = s(t) \ast p_R(t). \quad (2-5)$$

To retrieve the transmitted modulation values, the output signal must be sampled with the rate of the signalling time $T$ as

$$\hat{X}[i] = \hat{X}(iT) = X[m]. \quad (2-6)$$

In order to fully recover the transmitted modulation values $\hat{X}[m] = X[i]$ the following equation must be fulfilled

$$p_T(t - mT) \ast p_R^*(t - iT) = \delta_{mi}, \quad (2-7)$$

where $(\cdot)^*$ represents the complex conjugation and $\delta_{mi}$ is the Kronecker delta defined as

$$\delta_{mi} = \begin{cases} 1, & m = i, \\ 0, & m \neq i. \end{cases} \quad (2-8)$$
Figure 2: Single carrier IQ modulator used in practice.

A practical realization of the complex signalling model shown in Fig. 1 is depicted in Fig. 2, where the I and Q branches are handled separately. Eq. (2-1) can be also expressed using Eq. (2-2) as

\[ X(t) = \sum_{m=-\infty}^{\infty} (a[m] + jb[m])\delta(t - mT) = a(t) + jb(t). \] (2-9)

The \( a(t) \) and \( b(t) \) components are treated as two separate signals. The real passband signal \( v_s(t) \) can be expressed as the sum of the filtered \( a(t) \) and \( b(t) \) branches, which are modulated using a sine and a cosine wave. The two branches are then summed and passed through the transmission medium:

\[ v_s(t) = \left( \sum_{m=-\infty}^{\infty} a[m]p_T(t - mT) \right) \cos(2\pi F_c t) - \left( \sum_{m=-\infty}^{\infty} b[m]p_T(t - mT) \right) \sin(2\pi F_c t). \] (2-10)

The real valued passband signal can also be calculated directly from the complex passband signal \( v(t) \) as

\[ v_s(t) = \Re\{v(t)\}. \] (2-11)

In the receiver, the I and Q components can be separated following a multiplication by the corresponding sine and a cosine waves respectively, and low pass filtering. The real and imaginary parts of the modulation values \( X[m] \) can be retrieved by sampling the signals \( \hat{a}(t) \) and \( \hat{b}(t) \) at the sampling times \( iT \) as given in Eq. (2-6). For the purposes of modeling and simulation, the complex signal representation from Eq. (2-4) will be used.

2.3 Filter bank multicarrier modulation

In this section the general structure of the FBMC technique will be introduced for analog and digital realization based on the previously introduced concepts. The computationally efficient polyphase structure will also be explained in detail for the digital implementation. Then FBMC modulation schemes, which allow the use of a complex valued modulation alphabet, are introduced. OFDM, a special FBMC scheme is presented along with the main signal processing steps. This section concludes with another FBMC scheme: the SMT.
2.3.1 Continuous time synthesis and analysis filter bank

The basic block diagram of the FBMC scheme is depicted in Fig.3. The input signal of the \( k^{th} \) synthesis filter can be expressed based on Eq. (2-1) as

\[
X_k(t) = \sum_{m=-\infty}^{\infty} X_k[m] \delta(t - mT),
\]

where \( X_k[m] \) corresponds to the modulation value selected from the modulation alphabet \( \mathcal{C} \) for the \( k^{th} \) subcarrier band in the \( m^{th} \) symbol. \( T \) represents the signalling time, which is the time spacing between two consecutive modulation values. The input signal of each subcarrier band is filtered by the transmitter filter \( g_{T,k}(t) \) corresponding to the \( k^{th} \) subcarrier band. Each filter is based on a specially designed prototype filter \( p_T(t) \) which is modulated to the \( k^{th} \) subcarrier band. Modulation is performed using the complex exponential of a frequency of \( F_k \) and a phase of \( \varphi_k \). The modulated transmitter filter \( g_{T,k} \) for the \( k^{th} \) subcarrier can be expressed as

\[
g_{T,k}(t) = p_T(t)e^{j(2\pi F_k t + \varphi_k)}.\]

The output of the synthesis filter bank can be expressed as the sum of \( N \) components:

\[
v(t) = \sum_{k=0}^{N-1} v_k(t), \tag{2-14}
\]

where \( v_k(t) \) can be expressed as the convolution of the modulated filter and input signal as

\[
v_k(t) = X_k(t) \ast g_{T,k}(t), \quad k = 0, \ldots, N - 1. \tag{2-15}
\]

Equation (2-14) can be reformulated using equations (2-12), (2-13) and (2-15):

\[
v(t) = \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} X_k[m] p_T(t - mT)e^{j(2\pi F_k (t - mT) + \varphi_k)}. \tag{2-16}
\]
The components of the $k^{th}$ subcarrier are separated using an analysis filter bank in the receiver. In this filter bank matched filters $g_{R,k}(t) = g_{T,k}(-t) = p_T(-t)e^{-j2\pi F_k t}$ are applied for each subcarrier:

$$\hat{X}_k(t) = v(t) * g_{R,k}(t).$$  \hspace{1cm} (2-17)

The modulation values $\hat{X}_k[i]$ can be retrieved by sampling the outputs of the analysis filter bank at sampling times $iT$. The precondition $\hat{X}_k[i] = X_k[i]$ must be followed. To this end, the subcarriers must constitute an orthogonal basis set, meaning that the following equation must apply for all subcarrier:

$$g_{T,k}(t - mT) * g_{R,i}(t - nT) dt = \delta_{k,i} \delta_{m,n}. \hspace{1cm} (2-18)$$

To maximize bandwidth efficiency and satisfy the condition of orthogonality the subcarrier band spacing must be $\Delta F = \frac{1}{2T}$. A larger spacing would result in a reduction of the data rate degrading the system to a special type of FDM called Filtered MultiTone (FMT) [55, 56].

The majority of filter design methods [57, 58, 59] propose symmetric prototype filter design where $p_T(t) = p_T(-t) = p_R(t) = p_0(t)$. This symmetry enables the substitution of indices $T,R$ with zeros resulting in a simple notation $p_0(t)$. Consequently the synthesis and analysis filters can be expressed as

$$g_{T,k}(t) = p_0(t)e^{j(2\pi F_k t + \phi_k)}, \hspace{1cm} (2-19)$$

$$g_{R,k}(t) = p_0(t)e^{-j(2\pi F_k t + \phi_k)}. \hspace{1cm} (2-20)$$

The exponential multiplication factors $e^{j(2\pi F_k t + \phi_k)}$ and $e^{-j(2\pi F_k t + \phi_k)}$ can be used for modulation and demodulation of the filtered modulation values in the $k^{th}$ subcarrier band.

### 2.3.2 Digital implementation of the synthesis filter bank

Implementing the synthesis filter bank with analog components implies many difficulties as each modulator uses a separate oscillator. These oscillators might have unmatched frequencies and non-synchronized phases causing loss of orthogonality between the subchannels.

The digital implementation of the synthesis filter bank is very simple in the baseband. After construction of the complex baseband signal $s(t)$ it is upconverted to the transmission band using a single carrier frequency $F_c$ as

$$v(t) = s(t)e^{j2\pi F_c t}, \hspace{1cm} (2-21)$$

meaning that the frequency of the $k^{th}$ subcarrier can be express as $F_k = F_c + f_k$. After transmission the signal is downconverted to the baseband, and the subchannels are separated via a baseband analysis filter bank. The filter banks in the digital baseband assume ideal analog-to-digital conversion, ideal transmission channel and perfect up- and downconversion. Therefore, the received complex baseband signal can be expressed as

$$r(t) = v(t)e^{-j2\pi F_c t} = s(t). \hspace{1cm} (2-22)$$

The baseband received signal is sampled with the sampling frequency $F_s$ resulting in a sampling time of $T_s = \frac{1}{F_s}$. In order to simplify the description, the sampling frequency will be normalized and a discrete time formulation will be used as $r[n] = r(nT_s) = r(n)$. The available complex baseband $[-F_s/2, F_s/2]$
is split into $N$ subcarriers, so the discrete frequency spacing between two adjacent subcarriers can be expressed with normalized sampling frequency as $\Delta f = F_s/N = 1/N$.

To obtain a causal discrete prototype filter $p_0[n]$, the continuous time prototype filter $p_0(t)$ has to be limited to a time interval $[-L_{p_0}/2, (L_{p_0}/2)]$ and it must be delayed by $((L_{p_0} - 1)/2)$ units:

$$p_0[n] = p_0\left(n - \frac{L_{p_0} - 1}{2}\right), \quad n=0,\ldots, L_{p_0} - 1.$$  \hfill (2-23)

The length $L_{p_0}$ of the discrete prototype filter must be an integer multiple of the number of samples $N$ during a symbol time $T$:

$$L_{p_0} = KN,$$ \hfill (2-24)

where $K$ represents the overlapping factor, which gives the number of consecutive overlapping symbols in the time domain.

The discrete representation of the continuous time filter bank (Fig. 3) can be seen in Fig. 4. The input of each subchannel is oversampled by a factor of $M = N$, then each subchannel is filtered by the digital prototype filter $g_{T,k}[n]$ aligned to the $k^{th}$ subcarrier. The modulation is performed with the aid of a complex exponential having a normalized discrete frequency of $f_k = \frac{k}{N}$ and a phase of $\varphi_k$:

$$g_{T,k}[n] = p_0[n]e^{j(2\pi f_k n + \varphi_k)}. \hfill (2-25)$$

The transmitted signal can be expressed as:

$$s[n] = \sum_{k=0}^{N-1} s_k[n]. \hfill (2-26)$$

The received signal $s[n]$ is multiplied by the complex exponentials corresponding to the subchannels. These subchannel signals are filtered with the matched form of the discrete prototype filter. The outputs of these filters are decimated by a factor of $M$ in order to retrieve the modulation values $X_k[m]$.  

---

**Figure 4:** Digital implementation of the FBMC scheme.
2.3.3 The polyphase implementation

Calculating the transmitted signal in the time domain is a computationally demanding task. Frequency
domain calculation is simpler using the Z-transform of the prototype filter \( p_0[n] \):

\[
P_0(z) = \sum_{l=0}^{L_{\nu_0}-1} p_0[l] z^{-l}.
\]  

(2-27)

Equation (2-27) can be reformulated using eq. (2-24) as

\[
P_0(z) = \sum_{p=0}^{N-1} P_{0,p}(z^N) z^{-p}.
\]  

(2-28)

\( P_{0,p}(z^N) \), the \( p \)th polyphase decomposition of the prototype filter \( P_0(z) \) can be expressed as

\[
P_{0,p}(z^N) = \sum_{q=0}^{K-1} p_0[qN+p] z^{-qN}.
\]  

(2-29)

Using the polyphase representation in eq. (2-27), the Z-transform of the \( k \)th digital synthesis filter \( g_T,k[n] \) can be expressed as

\[
G_{T,k}(z) = \sum_{l=0}^{L_{\nu_0}-1} p_0[l] e^{j\left(\frac{2\pi}{N} kl + \varphi_k\right)} z^{-l} = e^{j\varphi_k} \sum_{p=0}^{N-1} e^{j\frac{2\pi}{N} kp} z^{-p} \sum_{q=0}^{K-1} p_0[qN+p] e^{j\frac{2\pi}{N} qN} z^{-qN} =
\]

\[
e^{j\varphi_k} \sum_{p=0}^{N-1} e^{j\frac{2\pi}{N} kp} P_{0,p}(z^N) z^{-p}.
\]  

(2-30)

(2-31)

The filters \( G_{T,k}(z) \) can also be expressed – without the phase rotation factors \( e^{j\varphi_k} \) – in matrix form using

\( W = e^{j\frac{2\pi}{N}} \) as

\[
\begin{pmatrix}
G_{T,0}(z) \\
G_{T,1}(z) \\
\vdots \\
G_{T,N-1}(z)
\end{pmatrix} =
\begin{pmatrix}
1 & 1 & \ldots & 1 \\
1 & W^{-1} & \ldots & W^{-(N+1)} \\
\vdots & \ldots & \ddots & \vdots \\
1 & W^{-N+1} & \ldots & W^{-(N+1)^2}
\end{pmatrix}
\begin{pmatrix}
P_{0,0}(z^N) \\
P_{0,1}(z^N) z^{-1} \\
\vdots \\
P_{0,N-1}(z^N) z^{-(N-1)}
\end{pmatrix}.
\]  

(2-32)

The first vector gives Inverse Discrete Fourier Transform (IDFT) in matrix notation and the second one
is a polyphase decomposition with delays. The resulting structure of the polyphase implementation of
the digital synthesis filter bank is shown in Fig. 5.

The significantly faster IFFT can be used instead of IDFT if the filter banks have \( N = 2^p \) subchannels.
In the demodulator the analysis filter bank is implemented by a similar polyphase decomposition using
FFT instead of DFT.
2.4 OFDM

The OFDM scheme can be considered as an FBMC scheme where the prototype filter has a rectangular impulse response: \( p_0(t) = 1, t \in [0, T] \). The \( m \)th OFDM symbol can be expressed in the time domain as

\[
v^m(t) = \sum_{k=0}^{N-1} X_k[m] e^{j2\pi nk}, \quad t \in [0, T].
\]  

(2-33)

Each OFDM symbol is a sum of complex subcarrier frequencies having integer signal periods over a symbol time \( T \). The phases of the complex exponentials are irrelevant, they do not affect the orthogonality, so the phases can be chosen as \( \phi_k = 0 \). The time domain symbols in the OFDM signal do not overlap, due to the rectangular impulse response.

By implementing the OFDM scheme in discrete time, the corresponding polyphase structure becomes simpler. The discrete prototype filter will have a length of \( L_{p0} = N \) and an overlapping factor \( K = 1 \). So each polyphase decomposition will be equal to unity:

\[
P_{0,p}(z^N) = 1.
\]  

(2-34)

As a result, the time domain samples of the \( m \)th OFDM symbol can be calculated using only an IFFT operation as

\[
s^m[n] = \sum_{k=0}^{N-1} X_k[m] e^{j2\pi nk}, \quad n \in 0, \ldots, N - 1.
\]  

(2-35)

2.5 OFDM-related techniques

The OFDM scheme has some drawbacks, one of these is the high PAPR of the transmitted signal. Some techniques extend the OFDM scheme with additional signal processing blocks in order to counteract this effect. One of these techniques is the DFTS-OFDM where an extra DFT operation is introduced to reduce the PAPR (The term PAPR will be explained in details and it will be also investigated in
Section 3.2) of the transmission signal. Another technique which also has a low PAPR is the CE-OFDM, having a constant amplitude. These techniques will be presented and explained in details in Section 3.

2.6 Staggered multitone

FBMC schemes generating complex modulation values with prototype filters having a non-rectangular impulse response are called SMT. To maintain orthogonality, the real and imaginary parts of the modulation values \( X = a + jb \) are transmitted by time staggering with an offset of half the signalling time: \( T/2 \). To implement staggering, the filter bank is implemented using two separate banks, one for the real and another for the imaginary components. The basic block diagram of the SMT scheme can be seen in Fig. 6. Identical prototype filters are used for the real \( g^R_k(t) \) and the imaginary \( g^I_k(t) \) parts:

\[
g^R_k(t) = p_0(t) e^{jk(\frac{2\pi}{T} t + \frac{2}{T})}, \quad (2-36)
\]

\[
g^I_k(t) = p_0(t - T/2) e^{jk(\frac{2\pi}{T} t + \frac{2}{T})}. \quad (2-37)
\]

The resulting transmission signal can be formulated as

\[
v(t) = \sum_{k=0}^{N-1} v_k(t) = \sum_{k=0}^{N-1} a_k(t) * g^R_k(t) + j b_k(t) * g^I_k(t), \quad (2-38)
\]

Applying Eq. 2-2, Eq. 2-36 and Eq. 2-37 the signal \( v(t) \) can be fully expressed as

\[
v(t) = \sum_{m=\infty}^{N-1} \sum_{k=0}^{N-1} (a_k[m]p_0(t - mT) + j b_k[m]p(t - T/2 - mT)) e^{jk(\frac{2\pi}{T} t + \frac{2}{T})}. \quad (2-39)
\]
In the analysis filter bank, the real and imaginary parts of the modulation values of the $l^{th}$ subcarrier band in the $i^{th}$ symbol can be calculated from the signal of the corresponding filter:

$$\hat{a}_l(t) = \Re \left\{ p_0(t) * v(t) e^{-j(l \frac{2\pi}{2T} t + \frac{\pi}{2})} \right\}, \quad (2-40)$$

$$\hat{b}_l(t) = \Im \left\{ p_0(-(t + T/2)) * v(t) e^{-j(l \frac{2\pi}{2T} t + \frac{\pi}{2})} \right\}. \quad (2-41)$$

The convolution must be evaluated at discrete time intervals $t = iT$, therefore (2-40) and (2-41) can be reformulated as

$$\hat{a}_l(iT) = \Re \left\{ \int_{-\infty}^{\infty} p_0(-(iT - t))v(t)e^{-j(l \frac{2\pi}{2T} t + \frac{\pi}{2})} \, dt \right\}, \quad (2-42)$$

$$\hat{b}_l(iT) = \Im \left\{ \int_{-\infty}^{\infty} p_0(-(iT - T/2 - t))v(t)e^{-j(l \frac{2\pi}{2T} t + \frac{\pi}{2})} \, dt \right\}. \quad (2-43)$$

Substituting (2-39) into (2-42) and (2-43) the sampled outputs can be expressed as

$$\hat{a}_l[i] = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{N-1} \int_{-\infty}^{\infty} \Re \left\{ p_0(t - iT) \left( a_k[m] p_0(t - mT) e^{j(k-l)\left( \frac{2\pi}{2T} t + \frac{\pi}{2} \right)} \right) \right\} \, dt,$$

$$\hat{b}_l[i] = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{N-1} \int_{-\infty}^{\infty} \Im \left\{ p_0(t + T/2 - iT) \left( a_k[m] p_0(t - mT - T/2) e^{j(k-l)\left( \frac{2\pi}{2T} t + \frac{\pi}{2} \right)} \right) \right\} \, dt. \quad (2-44)$$

The orthogonality conditions of Eq. (2-44) and Eq. (2-45), which are necessary for the prototype filters to achieve perfect reconstruction ($\hat{a}_l[i] = a_{k=m}[i] = m$ and $\hat{b}_l[i] = b_{k=m}[i] = m$), are given in Appendix A. It can be shown that if a real valued, symmetric Nyquist filter is used as the prototype filter, then orthogonality is fulfilled.

The filter bank used for the real part can be expressed based on (2-36) as

$$g_{k}^{R}(t - T/2) = p_0(t - T/2) e^{j(k \frac{2\pi}{2T} t + \frac{\pi}{2})} \quad (2-46)$$

The filter bank for the imaginary modulation values has a similar structure as that of the real part. Substituting Eq. (2-37) into Eq. (2-46), the filter bank for the imaginary part can be expressed as

$$g_{k}^{I}(t) = g_{k}^{R}(t - T/2) e^{-j k \pi}. \quad (2-47)$$

This means that the filter bank for the imaginary part can be expressed as the real parts filter bank with time staggering and phase rotation applied. The components responsible for the phase rotation of the two polyphase filters can be expressed as

$$\phi_{k}^{R} = e^{j \frac{\pi}{2} k}, \quad (2-48)$$

$$\phi_{k}^{I} = e^{-j \frac{\pi}{2} k}. \quad (2-49)$$

Considering discrete time implementation, two polyphase structures operate in parallel. This polyphase
structure implementation for SMT can be seen in Fig. 7, the complexity may be further reduced with some additional signal processing considerations [60]. The same polyphase structure is used for the real and imaginary parts, the two implementations differ only in the extent of time delay and phase rotation.

2.7 Section summary

In this section a family of multicarrier modulation scheme, the FBMC was presented. The concept for the synthesis and analysis filters was presented and the orthogonality conditions were derived. Based on this orthogonality concept, a perfect reconstruction of the transmitted modulation values is possible. The continuous and the discrete time formulation of the filter bank was shown. The signal processing requirements of the digital implementation can be significantly reduced using the polyphase structure. Finally, two FBMC schemes were described in details: the OFDM, which applies prototype filters with rectangular impulse response, and the SMT, where Nyquist pulses are applied with time staggering between the real and imaginary components. For both methods the signal model and the corresponding polyphase implementation was described. In the further sections only the digital complex baseband equivalent representation of the transmitted signal (according to Eq. 2-26) will be considered, no up- and downconversion using the carrier frequency will be applied in the simulations.
3 Comparison of the investigated multicarrier techniques

In this section the detailed description of four multicarrier modulation schemes is given on a system level (OFDM, CE-OFDM, DFTS-OFDM and SMT). First, the transmitter model of each investigated modulation scheme is described, where the bit operation and the other signal processing steps are explained as well. Then the signal metrics and the spectral characteristics of the transmitted signals are investigated. The analog and digital models of the transceiver chain with possible impairments caused by the analog components are also presented. Channel equalization techniques for compensation of the linear distortions caused by the multipath channel are compared for the schemes are compared as well. At the end of the section, the investigated multicarrier modulation schemes are compared via simulations.

3.1 Transmitter models

![Diagram of transmitter models]

Figure 8: Transmitter models of the multicarrier schemes.

The block diagram of the four multicarrier modulations is presented in Fig. 8. For all modulations, the binary information stream $B$ generated by the source is first encoded using an error correction coding method. The encoded bitstream $C$ is then mapped to complex modulation values $X$ selected from the modulation alphabet. These binary signal processing steps are identical for all the presented modulations. The generation of the baseband transmitted signal $s$ for each technique is given in the next sections, separately for each. All modulation schemes include a block performing IFFT. The differences lie mainly in the surrounding signal processing blocks.

3.1.1 OFDM

The block diagram of the OFDM transmitter is depicted in Fig. 8(a). The modulation symbols are applied to the subcarriers and the time domain samples of the complex baseband OFDM symbol $x[n]$ are generated by IFFT as

$$x[n] = \sum_{k=0}^{N-1} X_k e^{j \frac{2\pi}{N} nk}.$$  \hspace{1cm} (3.1)
In practical systems some of the subcarriers remain unused (null carriers), while others carry a reference signal (pilot carriers). Before transmission, the CP is added to the symbol to form the discrete baseband time domain signal $s[n]$.

In OFDM systems the CP used to address the ISI caused by the multipath propagation in the radio channel. The CP is a repeated part of the last $P$ samples of an OFDM symbol which is copied before it as an extension. This ensures the continuity of the extended symbol. After passing through the channel filter, the CP is removed in the receiver. As long as the channel impulse response is shorter than the CP, the equalization can be performed in the frequency domain without any ISI and Inter Carrier Interference (ICI).

3.1.2 DFTS-OFDM

DFTS-OFDM is also known as Single Carrier Frequency Division Multiple Access (SC-FDMA) [61, 62]. This technique has been selected as the uplink scheme for LTE. In case of DFTS-OFDM systems, the complex modulation data set is preprocessed, the complex modulation values which will be transmitted are grouped and DFT operation is applied on them, as it can be seen in Fig. 8(b). Then the output of the DFT procedure is used to modulate the subcarriers. This technique can also be considered as a single carrier modulation scheme, where frequency spreading is applied through all modulated subcarriers. The result is a slightly lower PAPR value compared to OFDM transmissions.

3.1.3 CE-OFDM

CE-OFDM [63] aims to solve the high PAPR values of the OFDM signal. The basic idea is to generate a real valued OFDM signal, which can be used as an input for a phase modulator. The complex modulation symbols are aligned in a complex conjugated manner to achieve a real-valued IFFT output as depicted in Fig. 8(c). Subsequently, phase modulation is applied to the real-valued time domain signal and the CP is added to form the transmitted signal. The transmitted symbol $s'[n]$ before adding the CP can be expressed as:

$$x'[n] = e^{j2\pi hx[n]}, \quad n \in 0, \ldots, N-1,$$

where $m$ is the modulation index of the phase modulator and $x[n]$ is the output of the IFFT but with the restriction of $X_k = X_{N-k}^*$. A noticeable disadvantage is that the complex conjugated pairing reduces the data rate by a factor of two. Then the phase modulator is driven by this real valued signal that results in a constant envelope output signal. The power spectrum density (PSD) of the transmitted signal will be determined by the modulation index $h$ of the phase modulator.

3.1.4 SMT

SMT systems [64, 65] as described in Section 2 use a specially designed filter bank structure. First the complex modulation values are spread over several carriers and filtered by a prototype filter. This implies the necessity of a larger FFT to construct the transmission signal, which can be seen in Fig. 8(d). Due to the advantageous properties of the prototype filter bank, the spectral band efficiency of this scheme will be better than that of the OFDM signal. With the use of offset-QAM modulation, where the real and imaginary data values are offset by a half symbol duration, no data rate loss will occur. Prior to transmission, the symbols are overlapped in such a way that they can be separated at the receiver.
This is due to the fact that the filter bank is designed to fulfill the Nyquist criterion to minimize the inter-symbol interference. Although the symbols’ duration is longer compared to the OFDM symbols and they overlap, no data rate loss will occur. The other advantage of SMT is that no CP is used. However, more complex signal processing has to be applied, and the channel equalization in the receiver chain will be more complex than in case of other schemes. With the use of a so-called polyphase filter bank, the previously mentioned signal processing requirement can be reduced.

3.2 Transmission signal metrics

In order to design the analog circuits constituting the transceiver chain, a deep knowledge of the statistic properties of the transmitted signal has to be gathered. Such investigations are especially important in case of the design of power amplifiers which have to work as efficiently and as linearly as possible. A detailed investigation of such metrics for OFDM can be found in [66]. In this section the PAPR, Cubic Metric (CM) and kurtosis, as well as the spectral behavior of the transmitted signal will be investigated. The negative effect of nonlinear distortion in presence of a nonlinear power amplifier will also be shown.

For the simulations in this section a 64 point FFT with a cyclic prefix of 16 samples was applied with an oversampling ratio of 4. 16-QAM modulation was applied on 48 subcarriers from the maximally available 64, omitting the DC subcarrier and some carriers on the edge of the transmission band. In case of DFTS-OFDM, a 48 point DFT was used for preprocessing. For the CE-OFDM modulation a moderate modulation index of $h = 0.8$ was chosen.

3.2.1 PAPR

Two types of metrics are considered in the literature when describing the dynamic properties of the transmission signal $s(t)$: the PAPR and the CM. A simple technique to describe the dynamic behavior of the transmission signal $s(t)$ is to calculate the PAPR which is defined as

\[
\gamma_1 = \frac{\max\{|s(t)|^2\}}{E\{|s(t)|^2\}},
\] (3.3)
where $|s(t)|$ is the amplitude of the transmission signal and $E\{\cdot\}$ is the expectation value. The PAPR in dB is defined as:

$$\text{PAPR}(s[n])_{\text{dB}} = 10\log_{10}(\gamma_1).$$  \hspace{1cm} (3-4)

### 3.2.2 CM

The CM is described as

$$\text{CM}_{\text{dB}} = \frac{\text{RCM}_{\text{dB}} - \text{RCM}^\text{ref}_{\text{dB}}}{K_{CM}},$$  \hspace{1cm} (3-5)

where RCM$_{\text{dB}}$ is the raw CM of the analyzed signal and RCM$^\text{ref}_{\text{dB}}$ is the CM of the reference signal. The constant $K_{CM}$ and the reference signal are specified in [67]. For simplicity we will use the raw CM defined as:

$$\text{RCM}(s(t))_{\text{dB}} = 10\log_{10} \left( E \left\{ \left( \frac{|s(t)|^2}{E\{|s(t)|^2\}} \right)^3 \right\} \right).$$  \hspace{1cm} (3-6)

Using a Power Amplifier (PA), the third order distortions dominate, so the CM gives a better insight of the dynamic behavior of the signal in the presence of the odd order non-linearities. First the PAPR values of the entire transmission signal are analyzed in Fig. 9, where the complementary cumulative distribution functions (CCDF) of the PAPR values are depicted. It can be seen that CE-OFDM has the lowest PAPR value, being a constant 0 dB. SMT and OFDM perform worst, featuring an approximate PAPR value of 10 dB with the probability of $10^{-4}$. DFTS-OFDM, as mentioned earlier, has a lower PAPR figure, with a value of 8 dB at the same probability.

Another type of analysis can be also be performed by separating every transmission symbol, not looking at the entire transmission signal. The distribution function of PAPR and CM values for the four schemes regarding one symbol is depicted in Fig. 10. It can be seen that the metric values of OFDM and SMT are fairly similar. This is due to the fact that both systems use a sum of complex harmonics, resulting in Gaussian distributed amplitude values which can lead to high PAPR. The values for DFTS-OFDM are slightly lower due to the preprocessing of the complex modulation data. Again, CE-OFDM performs the best, due to its constant envelope resulting in a constant PAPR and a CM value of zero. Although for SMT and OFDM the CM curves lie below the PAPR curves, this does not apply to DFTS-OFDM signals, though they are still significantly lower when compared to SMT and OFDM. This can lead to better resistance to non-linear distortions as described in the next sections. It can be also concluded that all symbols of DFTS-OFDM, OFDM and SMT have a PAPR and CM value higher than 4 dB.

### 3.2.3 Kurtosis

In order to statistically describe the probability distribution of the transmitted signal, the kurtosis "excess", denoted as $\gamma_2$ of the random variable $\xi$ will be employed in the following discussion. Parameter $\gamma_2$ of $\xi$ is commonly defined as [68]

$$\gamma_2 = \frac{E\{\xi^4\}}{(E\{\xi^2\})^2} - 3.$$  \hspace{1cm} (3-7)

Note that for the Gaussian distributed signals, $\gamma_2 = 0$ [68].

Fig. 11 shows $\gamma_2$ of the real and imaginary part of the OFDM, DFTS-OFDM and SMT signals comprising different number of subcarriers. In this investigation CE-OFDM is not relevant, since it has a constant amplitude due to the phase modulation. It can be observed that $\gamma_2$ converges with increasing $N$ (fulfilling the requisition of the Central Limit Theorem) rapidly to zero for OFDM and SMT, i.e. it can
be assumed that the real and imaginary parts of the transmitted signal for SMT and OFDM are gaussian distributed. On the other hand, for DFTS-OFDM it remains constant, no change can be observed with the increasing number of subcarriers. This means that the DFTS-OFDM signal is non-Gaussian.

3.2.4 Spectral characteristics

Another important property of the transmitted signal is its spectral behavior, especially regarding its out-of-channel leakage. These characteristics are especially important when dealing with cognitive radio scenarios. In cognitive radio scenarios one of the most strict requirement are related to the out-of-band radiation. Different standardization communities, organizations define slightly different limits. For Ofcom (UK) mask a limit of -55 dB is set, while FCC (USA) inquires a level of -51...-60 dB depending on the frequency band. The Power Spectral Density (PSD) functions of the transmitted signal with a linear PA is presented in Fig. 12. The Adjacent Channel Leakage Ratio (ACLR) of SMT outperforms all the other modulation schemes, DFTS-OFDM and OFDM have fairly similar characteristics, but with larger amount of out-of-channel leakage. CE-OFDM has the largest out-of-band radiation, with a large DC component. This DC component is not preferred in wireless transmissions. The specially low side radiation and the efficient band allocation make SMT the most suitable solution for cognitive radio applications. A detailed comparison of the PSD of SMT and OFDM signals using filtering can be found in [69].

3.3 Transceiver model with impairments

A simplified model of the transceiver chain can be seen in Fig. 13. After the modulation, the In-phase (I) $s_I[n]$ and Quadrature (Q) $s_Q[n]$ components of the discrete complex baseband signal $s[n]$ are converted
to analog signals \( s_I(t) \) and \( s_Q(t) \) by D/A converters. The next step is the pass-band modulation which is performed by multiplying the cosine and sine signals generated by the transmitters’ local oscillator \( F_c^T \) and adding them together. The pass-band signal can be expressed as:

\[
v(t) = \Re \left\{ s(t)e^{j2\pi F_c^T t} \right\}, \tag{3-8}
\]

where \( \Re \{ \} \) means the real part of the argument. Next, the pass-band signal is amplified using a Power Amplifier (PA). The output signal \( u(t) \) is formed on the bases of the characteristics of the amplifier:

\[
u(t) = f_{PA}(v(t)). \tag{3-9}
\]

The radio channel is modeled by a tap-delay line having an impulse-response of \( h(t) \), which represents the multipath propagations and an Additive White Gaussian Noise (AWGN) term \( w(t) \). Having passed the channel, the signal \( u(t) \) forms the received signal \( y(t) \) as:

\[
y(t) = u(t) * h(t) + w(t). \tag{3-10}
\]

The received signal \( y(t) \) is converted down to the baseband using the receiver LO with the frequency \( F_c^R \). The signals I and Q are segmented from the downconverted signal using a Low Pass Filter (LPF):

\[
\begin{align*}
    r_I(t) &= \Re \left\{ y(t)e^{-j2\pi F_c^R t} \right\} \tag{3-11} \\
    r_Q(t) &= \Im \left\{ y(t)e^{-j2\pi F_c^R t} \right\} \tag{3-12}
\end{align*}
\]

where \( \Re \{ \} \) and \( \Im \{ \} \) mean the real and imaginary parts of the argument. The baseband I and Q signals are then sampled using an A/D converter forming the signals \( r_I[n] \) and \( r_Q[n] \), which are passed to the digital demodulator and processed according to the modulation scheme. The discrete complex baseband received signal can be written as

\[
r[n] = r_I[n] + j r_Q[n]. \tag{3-13}
\]

The digital baseband transmission path can be divided into three major parts as it is depicted in Fig. 14. Every component is processed in the complex digital baseband: the baseband equivalent of the channel is applied and the errors caused by the up- and downconversion are shifted to the receiver. In our model we only consider the error caused by the nonlinear PA at the transmitter side. The channel consists of the channel filter and an AWGN term. Finally, three types of errors are introduced at the receiver: frequency and phase errors of the local oscillators and IQ-mismatch. The model for each error is discussed in the following sections.
In this section the system parameters used for the simulation are given, then the effect of each imperfection on the system performance is analyzed separately and compared with the undistorted case. The system performances are compared via Bit Error Rate (BER) as a function of Signal-to-Noise Ratio (SNR). In our case the SNR values are defined as

\[
\text{SNR}_{\text{dB}} = 10 \log_{10} \left( \frac{E_s}{N_0} \right) \quad 10 \log_{10} \left( \frac{E_b}{OV (N + P)N_0} \right)
\]

(3-14)

with \( E_b \) being the bit energy, \( N_0 \) the noise variance, \( N \) is the number of the subcarriers available and \( N_D \) is the number of subcarriers used for data transmission. \( P \) is the length of the cyclic prefix in samples. \( OV \) is the oversampling ratio and \( M_b \) is the number of bits transmitted by one subcarrier. For the simulations a 64 point FFT with a cyclic prefix of 16 samples was applied with an oversampling ratio of 4. 16-QAM modulation was applied on 48 subcarriers from the maximally available 64, omitting the DC subcarrier and some carriers on the edge of the transmission band. In case of DFTS-OFDM, a 48 point DFT was used for preprocessing. For the CE-OFDM modulation a moderate modulation index of \( \hat{h} = 0.8 \) was chosen. The bit error rates are compared using the above mentioned simulation parameters, plotting the values in the function of the normalized \( \frac{E_b}{N_0} \).

3.3.1 Amplifier

The PA is a crucial element of the transmitter chain. These analog components are not ideal, they have a limited linear range, which may introduce nonlinear distortions. For the baseband equivalent of the PA a memoryless Saleh-model [70] was applied which has the following discrete amplitude transfer function \( A_{PA}(|v[n]|) \) and phase shift characteristics \( \phi_{PA}(|v[n]|) \):

\[
A_{PA}(|v[n]|) = \frac{\alpha_{PA}|v[n]|}{1 + \beta_{PA}|v[n]|},
\]

\[
\phi_{PA}(|v[n]|) = \frac{\rho_{PA}|v[n]|}{1 + \lambda_{PA}|v[n]|^2},
\]

(3-15) (3-16)

where \( |v[n]| \) is the amplitude of the input signal \( v_n \) and \( \alpha_{PA}, \beta_{PA}, \rho_{PA}, \lambda_{PA} \) are the parameters of the PA. The output signal \( u[n] \) can be expressed

\[
u[n] = A_{PA}(|v[n]|) e^{j(\phi_{PA}(|v[n]|))},
\]

(3-17)

where \( \phi(v[n]) \) is the phase of the complex baseband signal \( v[n] \).

First the negative effects of the nonlinear PA is investigated. In the simulation scenario the parameters similar to [71] were used but with slightly smoother nonlinearity: \( \alpha_{PA} = 1, \beta_{PA} = 0.1, \rho_{PA} = \frac{\pi}{2}, \lambda_{PA} = 0.125 \). The bit error rate results are depicted in Fig. 16. It can be observed that CE-OFDM is very robust against the effects of the PA, due to its constant envelope. On the other hand, the performance of SMT is severely degraded. A slightly better performance can be observed in case of the DFTS-OFDM system, which is also outperformed by OFDM beyond 17 dB. It can be concluded that with the exception of CE-OFDM, all other modulations are very sensitive to nonlinear distortions.

An equally important metric is the occupied bandwidth, which can be compared via power spectral density functions. The distorted spectra are depicted in Fig. 15. The non-linear amplifier adversely affects all modulations schemes, resulting in a similar spectral shape. Comparing it with the ideal case shown in
Fig. 15: Power spectrum density comparison of the transmitted signals with the effect of a nonlinear PA \( \alpha_{PA} = 1, \beta_{PA} = 0.1, \rho_{PA} = \frac{1}{10}, \lambda_{PA} = 0.125 \).

Fig. 16: Bit error rate of the four schemes over AWGN channel with the effect of a nonlinear PA \( \alpha_{PA} = 1, \beta_{PA} = 0.1, \rho_{PA} = \frac{1}{10}, \lambda_{PA} = 0.125 \).

Fig. 12, it can be seen that the power spectrum density function of SMT is highly distorted, although its out-of-band emission is still smaller than that of the other three schemes. The SMT is severely affected by the non-linear distortion, but the resulting out-of-channel leakage is still lower compared to OFDM and DFTS-OFDM. The only exception is CE-OFDM which is resilient to the effect of the amplifier due to the constant envelope of the transmission signal.

As it can be seen, due to the increased out-of-band radiation the strict requirements can not be fulfilled anymore. In cognitive radio applications it is very important to predict the extent of out-of-band radiation. Based on the amplifier characteristics and using Bessel functions similar to as in [72], it can be directly predicted as shown in [29].

### 3.3.2 Channel

For the channel, the digital baseband equivalent is used. It means that the output of the channel can be expressed as the discrete convolution of the input signal and the channel impulse response, and an added AWGN sample:

\[
y[n] = u[n] * h[n] + w[n].\tag{3-18}
\]

### 3.3.3 Timing

Timing error occurs if the estimated time \( T_0 \) of the received signal is not equal to the actual starting time of the transmitted signal. This estimation is usually performed based on a preamble known to the receiver. If a CP is used and the channel impulse response is shorter than the CP, then the result of the negative timing offset (earlier than the actual starting time) has only phase rotation effect which can be compensated by channel equalization, due to the fact that the channel estimation also includes this phase rotation. For this reason the effect of positive timing offset is investigated, in the presence of ISI with the phase rotation of the subcarriers compensated.

First a small timing offset of 0.5% of the OFDM symbol length is introduced. The effects of this impairment over AWGN channel can be seen in Fig. 17. SMT, due the longer symbol durations, suffers almost no distortions. Only a small degradation can be measured in case of OFDM and CE-OFDM
systems. On the other hand, the DFTS-OFDM scheme—with this relatively small timing error—converges to an error floor of $10^{-3}$.

### 3.3.4 Frequency and Phase Errors

Frequency and phase error occur during up and down conversion of the baseband signal to the passband. A detailed description is given in [73]. The frequency error $\Delta F_c$ is the result of local oscillator (LO) mismatches between the transmitter and receiver side:

$$\Delta F_c = F_T^c - F_R^c$$  \hspace{1cm} (3-19)

The frequency offset is measured in terms of carrier spacing. The frequency errors are usually estimated on the bases of known preambles, so during the simulation only small residual frequency errors are considered, without the effect of the phase shift caused by the frequency offset. The phase errors $\varphi_n$ are caused by the phase jitter of the local oscillators and the phase mismatch between them. The constant phase shift can be estimated via channel estimation and corrected in the channel equalizer. The focus is on the phase jitter which can be modeled as a variable $\varphi \sim \mathcal{N}(0, \sigma_\varphi^2)$ having a normal distribution. The resulting signal $z_n$ can be written as

$$z[n] = y[n]e^{j2\pi \Delta F_c \cdot n + \varphi[n]}.$$  \hspace{1cm} (3-20)

For the LO mismatch we choose a relatively small residual frequency difference of $\Delta \omega_c = 2\pi \Delta F_c = 0.03$ subcarrier spacing. The resulting bit error rate curves are depicted in Fig. 18. It can be seen that in this case CE-OFDM is the most sensitive, it hits a bit error floor of $2 \cdot 10^{-3}$ above 20 dB. Regarding the other schemes, the SMT still performs best, OFDM is about 1 dB and DFTS-OFDM is about 5 dB worse in performance.

The system performances in the presence of a normally distributed ($\varphi \sim \mathcal{N}(0, \sigma_\varphi = 8^\circ)$) phase noise is depicted in Fig. 19. The performance of CE-OFDM is affected most severely by the phase noise due to the phase modulation. The performance degradation of SMT, OFDM and DFTS-OFDM is similar as in the case of the frequency mismatch of the LOs.
3.3.5 IQ-mismatch

IQ-mismatch is also an adverse consequence of down conversion. The error is generated when an amplitude imbalance or a quadrature error (phase difference is not exactly 90°) occurs between the I and Q branches. In general, the IQ-mismatch causes interference between the I and Q branches. The IQ-mismatch can be simply modeled according to [74] by two parameters: the amplitude imbalance \(K_IQ\) and the quadrature error \(\phi_{IQ}\). The value \(K_{IQ}\) represents the power mismatch between the I and Q branches, which is represented by the constants \(K_I\) and \(K_Q\). So the model can be written as

\[
\begin{bmatrix}
    r_I[n] \\
    r_Q[n]
\end{bmatrix} = 
\begin{bmatrix}
    K_I & 0 \\
    -K_Q \sin \phi_{IQ} & K_Q \cos \phi_{IQ}
\end{bmatrix}
\begin{bmatrix}
    z_I[n] \\
    z_Q[n]
\end{bmatrix}
\]

(3-21)

The results for IQ-mismatch are plotted in Fig. 20. The applied parameters were: \(K_{IQ} = 0.87\) dB, \(K_I = 0.95\), \(K_Q = 1.05\), \(\phi_{IQ} = 10^\circ\). CE-OFDM has the largest immunity to IQ-mismatch. The performance of OFDM is about 3 dB and the performance of DFTS-OFDM is about 6 dB worse. SMT modulation reaches a bit error floor of \(5 \cdot 10^{-4}\), but its performance is better than that of DFTS-OFDM below 19 dB.

3.4 Channel equalization

Besides residual synchronization errors and the effects of nonlinearities, linear distortions caused by multipath propagation also have a strong impact on the overall system performance. In multicarrier systems frequency domain channel equalization is favored, due to its simple implementation and low complexity. In this section the most common channel equalization methods will be introduced for the previously discussed four schemes. The block diagrams of the receivers with channel equalization are shown in Fig. 21. For all modulations the received signal is first demodulated and equalized, then the estimated modulation symbols \(\hat{X}\) are demapped and the estimated coded bits \(\hat{C}\) are decoded to retrieve the estimated transmitted information bits \(\hat{B}\).
3.4.1 OFDM

The basic steps of an OFDM receiver are presented in Fig. 21(a). The received discrete baseband signal $r$ can be demodulated after removing the CP using an $N$-point FFT. As long as the CP is longer than the channel delay spread the following frequency domain equation is valid for one OFDM symbol after removing the CP:

$$Y_k = X_k H_k + W_k, \quad k \in \{0, \ldots, N-1\}. \quad (3.22)$$

Here $Y_k$, $X_k$, $H_k$ and $W_k$ are $N$-FFT's of the signals $y[n]$, $x[n]$, $h[n]$ and $w[n]$, respectively.

3.4.2 DFTS-OFDM

The receiver blocks of DFTS-OFDM are almost the same as for OFDM as it can be seen in Fig. 21(b). The difference is the extra IDFT operation which is used to "de-spread" the information after channel equalization. The remaining blocks of the receiver are identical to OFDM.

3.4.3 CE-OFDM

The receiver for CE-OFDM has a higher complexity compared to OFDM. First a frequency domain equalization is performed, similar to OFDM. Then the equalized signal is transformed back to time domain where phase demodulation can be performed. After phase demodulation, the $N$-FFT is applied to retrieve the complex modulation values.

3.4.4 SMT

The SMT receiver is more complex compared to the other three CP based techniques. Each SMT symbol is turned to the frequency domain using an extended FFT of $L_{\rho_0} = NK$ samples. The equalization is performed in the frequency domain. After equalizing all subcarriers in the subband separately, each subband is filtered with the corresponding analysis filterbank to get an estimate of the transmitted constellation value $X_k$ which can be fed to the demapper. The computationally efficient implementation based on the polyphase structure applying a separate phase and amplitude compensation stage is the Adaptive Sine/Cosine modulated filter bank Equalizer for Transmultiplexers (ASCET) presented in [75, 76].
3.4.5 Equalization techniques

The basic principle of channel equalization is to retrieve the transmitted modulation symbols $\hat{X}_k$ from the received values $Y_k$. If only linear distortions are present, the equalization can be simply performed using a complex multiplication coefficient $H_k^{eq}$ as

$$\hat{X}_k = Y_k H_k^{eq}.$$  \hspace{1cm} (3-23)

Zero forcing (ZF) is known to be the simplest method of channel equalization in the frequency domain. It is assumed that the received noise is zero in equation (3-22), so the transmitted complex constellation value on the $k^{th}$ subcarrier can be simply calculated by substituting

$$H_k^{eq} = H_k^{ZF} = 1/H_k$$  \hspace{1cm} (3-24)

into equation (3-23) as

$$\hat{X}_k^{ZF} = \frac{Y_k}{H_k}.$$  \hspace{1cm} (3-25)

The Minimum Mean Square Error (MMSE) technique gives a better result if the information about the AWGN term is also taken into account. The problem of ZF occurs when $H_k$ is small, in this case the noise components will also be amplified. The equalization coefficient $H_k^{MMSE}$ for the $k^{th}$ subcarrier is calculated through the minimization of the following expression:

$$\min_{H_k^{eq}} E \left\{ \sum_{k=0}^{N-1} |X_k H_k - H_k^{eq} Y_k|^2 \right\}, \hspace{1cm} (3-26)$$

where $E\{\cdot\}$ denotes the expected value of the argument. Using equation (3-22), the resulting channel compensation value for the $k^{th}$ subcarrier is calculated according to [77] as

$$H_k^{eq} = H_k^{MMSE} = \frac{H_k}{|H_k|^2 + \frac{N_0}{P_s}},$$  \hspace{1cm} (3-27)

where $N_0$ is the noise power and $E_s$ is the signal power. It can be seen that if $\frac{N_0}{P_s}$ is small compared to $|H_k|^2$, the MMSE solution is identical to ZF.

3.4.6 Simulation results

In this subsection simulation results of the effect of multipath propagation channel with equalization at the receiver are presented. During the simulations perfect synchronization and linear PA were assumed. The same simulation parameters were used as in the case of system impairments. The channel models B and C of the IEEE 802.22 standard [78] were applied with a 1MHz channel bandwidth. Channel model B has a shorter delay than the CP, enabling linear equalization. On the other hand, for channel C the channel delay is slightly longer, leading to considerable amount of ISI. The uncoded BER simulation results obtained by ZF as channel equalizer are shown in Fig. 22 and Fig. 23. It can be observed that for channel B the best performance is achieved by DFTS-OFDM, because it benefits from spreading the information over the frequency selective channel, leading to smaller BER results. OFDM and SMT have similar performance and they outperform DFTS-OFDM below 16 dB. The worst results are obtained by
CE-OFDM due to the halved data rate. For channel C, as ISI significantly affects the CP based methods, a performance degradation can be observed. The BER curves of OFDM, DFTS-OFDM and CE-OFDM are similar, only minor performance degradation can be seen. This is not the case for SMT: an error floor occurs, due to the missing CP and large amount of ISI caused by the longer channel delay spread. In the following section this error floor is investigated and novel equalization techniques are proposed for improving the BER characteristics.

3.4.7 Improved channel equalization techniques for SMT

In this section novel ideas are presented which can improve the system performance of channel equalization in SMT systems. First a technique base on the estimation of the power of the ISI is presented, which can improve the efficiency of the MMSE channel equalization, then a computationally more complex structure, a decision feedback equalization is presented, which outperforms all the previous techniques. The signal structure of the SMT modulation is presented in Fig. 24 where it is compared to the signal structure of the OFDM signal. In OFDM systems only one symbol is present in the transmitted signal in a given time slot. As a comparison, an SMT signal is given for an overlapping/oversampling factor of $K = 4$. It can be observed that the resulting transmitted signal is the sum of the overlapping SMT symbols generated by the filter banks.

Averaged MMSE

In case of SMT, the subcarrier-based MMSE equalization (3-27) has to be modified in order to take into account the ISI between the adjacent symbols as

$$
\frac{1}{H_{k}^{MMSE}} = \frac{H_{k}^{*}}{|H_{k}|^{2} + \frac{N_{0} I}{E_{s} - I}}.
$$

(3-28)

where $I$ is the power of the ISI, for which we present the following equation

$$
I = E_s \sum_{n=0}^{L_{ch}-1} \frac{\eta}{N} |h[n]|^2,
$$

(3-29)

where $L_{ch}$ is the length of the channel impulse response. The resulting estimation of the MMSE equal-
Figure 24: Signal structure of OFDM (a) and SMT (b) schemes.

Equalization using equation (3-28) is given by

$$\hat{X}_k^{\text{MMSE}} = \frac{Y_k}{H_k^{\text{MMSE}}}$$

(3-30)

By observing Eq. (3-29), it can be concluded that the ISI can be minimized by moving the observation window along all the possible positions of the channel impulse response and average the resulting estimates $\hat{X}_k^{\text{MMSE}}$. The ISI can be calculated in function of the observation window as

$$I(\Delta n) = E_s \sum_{n=0}^{L_{ch} - 1} \left| n - \Delta n \right| \frac{|h[n]|^2}{N}, \Delta n = 0, \ldots, L_{ch} - 1.$$  

(3-31)

The averaged MMSE is driven by the idea that the ISI can also be considered as noise, which can be eliminated by averaging. Therefore based on the idea of moving the observation window, the demodulation and MMSE equalization are performed for each $\Delta n$ positions of the possible $L_{ch}$ observation windows, and then all complex modulation values belonging to the same subband are averaged, in line with equations
Figure 25: SMT receiver with decision feedback.

(3-28) and (3-31) as

$$\hat{X}_k^{\text{AVG}} = \frac{1}{L_{ch}} \sum_{\Delta n=0}^{L_{ch}-1} \hat{X}_k^{\text{MMSE}}(\Delta n) = \frac{1}{L_{ch}} \sum_{\Delta n=0}^{L_{ch}-1} \frac{Y_k(\Delta n)}{H_k^{\text{MMSE}}(\Delta n)}. \quad (3-32)$$

With this calculation the effect of the ISI can be decreased.

**Iterative decision feedback scheme**

A second approach to improve the equalization of SMT systems with large delay spreads is to use a decision feedback technique, where the most reliable decision values are fed back after the decision to minimize the ISI in the received signal. This feedback scheme is presented in Fig. 25. The concept is to regenerate the transmitted signal, but using only those subbands which are reliable, and filter this signal with the known channel filter. The idea is visualized in Fig. 24: if a decision is to be made for the shaded $m^{th}$ SMT symbol of the cosine filter bank, then as much as possible are reconstructed from the surrounding symbols $m-3, m-2, \ldots, m+3$ which overlap with it (both sine and cosine) based on the selection criteria. Then, during the decision on the $m^{th}$ SMT symbol, the ISI of the known adjacent symbols can be subtracted, reducing the noise originating from ISI, leading to a better performance. The selection criterion is defined on the bases of the constellation diagram, by setting up a confidence interval around each constellation point based on the noise variance and the estimated interference. The complex modulation symbols which fall inside this interval are considered as reliable. During the iteration process the interval can be enlarged as the ISI is reduced. The selection of the optimal criteria and iteration number is still an open issue which has to be further investigated.

**Simulation results**

The BER simulations of this section are performed with the same parameters as presented in Subsection 3.4.6 over IEEE 802.22 channel profile C. The results of the investigated channel equalization schemes are presented in Fig. 26. It can be observed that the ZF technique achieves the worst BER result. The MMSE can improve the performance, but as the SNR values grow, its performance becomes similar to that of the ZF method. A small gain can be achieved if the ISI is also taken into account during the MMSE equalization. Applying the Averaging MMSE technique, the BER performance can be further improved, but only a minor gain can be reached. However, applying the iterative decision feedback scheme, after
the 5th iteration a considerable amount of BER performance gain can be achieved, but the error floor can not be fully eliminated.

### 3.5 Section summary

In this section four possible choices for the modulation scheme for cognitive radios in WS were investigated and a model was given for the complex discrete baseband transceiver chain including also the transmission channel.

First these schemes where compared from various aspects such as complexity, dynamic signal properties, spectra and bit error rate. A summary of the comparison for the various schemes is shown in Table 1, the best performance in each category is shown in bold letters. The lowest signal processing efforts are required in case of OFDM, DFTS-OFDM and CE-OFDM require some extra computation and SMT requires extreme amount of signal processing. CE-OFDM performs best in the presence of non-linear distortions, in other words, it has the best PAPR and CM figures. Considering spectral efficiency, SMT has the best properties and the lowest ACLR. Although DFTS-OFDM does not outperform the other systems in any categories it can be a good compromise as it has fairly acceptable performance in all categories.

Then, the schemes were compared by means of the effects of various imperfections in the transfer chain, namely: nonlinear PA, timing error, frequency offset, phase noise and IQ-mismatch. The influence of these errors on the bit error rate was examined and it has been shown that each modulation type has its own strength and weakness depending on the type of the introduced imperfection. In case of timing errors, only the performance degradation of DFTS-OFDM is severe. CE-OFDM is affected mostly in the

---

**Table 1: General comparison of the four modulation schemes in terms of system complexity, data rate, PAPR, CM and Spectral behavior.**

<table>
<thead>
<tr>
<th>Modulation scheme</th>
<th>System complexity</th>
<th>Data rate</th>
<th>PAPR and CM</th>
<th>Spectral behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM</td>
<td>Low</td>
<td>$1 - \frac{CP}{N}$</td>
<td>High</td>
<td>Compact</td>
</tr>
<tr>
<td>DFTS-OFDM</td>
<td>Medium</td>
<td>$1 - \frac{CP}{N}$</td>
<td>Moderate</td>
<td>Compact</td>
</tr>
<tr>
<td>CE-OFDM</td>
<td>Medium</td>
<td>$\frac{1}{2} (1 - \frac{CP}{N})$</td>
<td>Low</td>
<td>DC &amp; Sidebands</td>
</tr>
<tr>
<td>SMT</td>
<td>High</td>
<td>1</td>
<td>High</td>
<td>Very compact</td>
</tr>
</tbody>
</table>
Table 2: Comparison of the four modulation schemes in presence of impairments. Best performance depicted in white, the worst is given with black color, intermediate performance is with gray color.

<table>
<thead>
<tr>
<th>Modulation scheme</th>
<th>Nonlinear HPA</th>
<th>$T_0$</th>
<th>$\Delta \omega$</th>
<th>$\varphi$</th>
<th>$IQ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFTS-OFDM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE-OFDM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Presence of frequency offset and phase noise. SMT has its weakness when IQ-imbalance is present at the receiver. An overview of the results is given in Table 2. Black color indicates the worst performance in the presence of a given error, while white color represents the best behavior. Gray shading means worst performance than the best, however approximating it by a few dBs.

Finally, channel equalization performance of the four modulation schemes using conventional subcarrier based equalization techniques such as ZF and MMSE were compared. It has been shown that SMT performance is limited by the ISI caused by multipath channels having long delay spreads. Novel channel equalization methods for SMT were presented which can further improve the BER results.

Overall, these aspects have to be considered in case of cognitive radio applications and the solution best fulfilling the requirements should be chosen.

The results presented in this section for comparison of the four presented modulation schemes were published in the following papers: [24, 36, 37, 32]. The results of channel equalization were presented in [39, 31].
4 Iterative compensation of clipped OFDM signal

At it has been shown in the previous section, the large PAPR of the transmitted signal has a negative
drawback in the presence of nonlinearities in the transceiver chain. A wide range of PAPR reduction
techniques are presented in the literature, an overview of these is given in [48, 79, 80]. Each method
has its own advantages and disadvantages. One method is baseband clipping, which in its self introduces
distortions, while some other methods need an increased transmit power or cause data rate degradation,
and in some cases additional information is needed for the receiver to process the signals. More to the
point, the computation complexity can vary for each method, therefore the most suitable procedure that
fits to the system parameters and requirements should be chosen.

Clipping is one of the simplest method used to decrease the PAPR of the transmitted signal and
it has the lowest signal processing cost in the transmitter. In this section deliberate amplitude clipping
for OFDM signals and its compensation possibilities will be discussed. First the mathematical model
of clipping will be introduced, along with its effects on the system performance and spectrum. Then a
receiver oriented turbo principle based technique intended for the compensation of the clipped OFDM
signal is presented, and a convergence analysis is performed. It will be shown that the technique is
divergent, and modifications are proposed which lead to a convergent algorithm. Finally the algorithm is
verified through simulations.

4.1 Clipping

When clipping is applied, the amplitude values of the time domain OFDM signal are simply limited to
a threshold $A_{\text{max}}$, as shown in Fig. 27, but the phase values remain unchanged. As a result, the clipping

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig27}
\caption{Time domain of the original signal and the clipped (Cl 1 dB) signal. The dashed line represents the
clipping level ($A_{\text{max}}$) and the solid line is the square root of the average power ($\bar{s}_s$).}
\end{figure}

model for the soft envelope limiter $u(t) = f_{\text{dip}}(s(t))$ can be written as

$$f_{\text{dip}}(s(t)) = \begin{cases} s(t) & |s(t)| \leq A_{\text{max}} \\ A_{\text{max}}e^{j\phi(s(t))} & |s(t)| > A_{\text{max}} \end{cases},$$

(4.1)
with \( \varphi(s(t)) \) being the phase of the complex signal \( s(t) \). For a global characterization of such a limitation the Clipping Ratio (CR) is defined as

\[
\text{CR} = 20 \log_{10}(\gamma),
\]

where \( \gamma \) is defined as

\[
\gamma = \frac{A_{\text{max}}}{\sigma_s},
\]

with \( A_{\text{max}} \) is the clipping level and \( \sigma_s \) is the square root of the average power \( P_s \) of the transmitted signal prior to clipping.

The mathematical model for clipping can be derived from the Bussgang theorem for memoryless nonlinearities with Gaussian inputs. The cross-correlation between the input and the output signal has a similar shape as the auto-correlation of the input signal for every time delay \( \tau \) [81]:

\[
\langle s(t + \tau)u(t) \rangle = \alpha \langle s(t + \tau)s(t) \rangle, \tag{4-4}
\]

where \( \alpha \) is a real constant representing the signal attenuation caused by clipping. Based on Eq. (4-4) the output of the nonlinearity can be defined as

\[
u(t) = \alpha s(t) + d(t), \tag{4-5}\]

where \( d(t) \) is the clipping noise, also called as Bussgang noise, which is assumed to be uncorrelated with \( s(t) \), i.e.

\[
\langle s(t + \tau)d(t) \rangle = 0. \tag{4-6}
\]

From (4-4), the attenuation factor \( \alpha \) can be calculated as

\[
\alpha = \left. \frac{\langle s(t + \tau)u(t) \rangle}{\langle s(t + \tau)s(t) \rangle} \right|_{\tau=0} = \frac{\langle s(t)u(t) \rangle}{\langle s(t)s(t) \rangle} = \frac{\langle s(t)u(t) \rangle}{P_s}. \tag{4-7}
\]

As shown in [81],[82] the attenuation factor \( \alpha \) can also be calculated using the envelope characteristics \( A(q(t)) \) of the nonlinearity:

\[
\alpha = \frac{\langle q(t)A(q(t)) \rangle}{P_s}, \tag{4-8}
\]

where \( q(t) \) is the amplitude value of \( s(t) \). The nonlinearity \( f_{\text{clipp}}(s(t)) \) and \( A(q(t)) \) are related through the first order Chebyshev transform [83]:

\[
A(q(t)) = \frac{2}{\pi} \int_{0}^{\pi} f(q(t) \cos \beta) \cos \beta d\beta. \tag{4-9}
\]

Based on (4-9), \( A(q(t)) \) can be expressed as

\[
A(q(t)) = \begin{cases} 
q(t), & 0 \leq q(t) < A_{\text{max}} \\
A_{\text{max}}, & A_{\text{max}} \leq q(t) \leq \infty
\end{cases}. \tag{4-10}
\]
With the aid of (4-8) and (4-10) the attenuation factor $\alpha$ can be calculated for OFDM signals as

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}}{2} \gamma \text{erfc}(\gamma).$$  

(4-11)

Detailed calculations for Eq. (4-11) are presented in Appendix B.

The output power is given by

$$P_{\text{out}} = (1 - e^{-\gamma^2}) P_s,$$

(4-12)

Detailed calculations for Eq. (4-12) are presented in Appendix C.

With the aid of (4-5) and (4-12), the power of the clipping noise can be calculated as

$$P_d = P_{\text{out}} - P_{\text{clipped}} = P_{\text{out}} - \alpha^2 P_s = (1 - e^{-\gamma^2} - \alpha^2) P_s.$$

(4-13)

The Signal-to-Distortion Ratio (SDR) can be calculated as

$$\text{SDR} = \frac{P_{\text{clipped}}}{P_d} = \frac{\alpha^2}{1 - e^{-\gamma^2} - \alpha^2}.$$  

(4-14)

The PAPR of the clipped signal will be limited by the upper boundary of

$$\text{PAPR}_{\text{clipped}} \leq \frac{A_{\text{max}}^2}{P_{\text{out}}} = \frac{\gamma^2}{1 - e^{-\gamma^2}}.$$  

(4-15)

The SDR of the clipped signal and the upper boundary of the PAPR in function of the CR can be observed in Fig. 28 and Fig. 29.

![Figure 28: SDR of the clipped signal in function of the clipping ratio.](image1)

![Figure 29: Upper bound for the PAPR (right) of the clipped signal in function of the clipping ratio.](image2)

The clipping ratio of 1 dB is marked in both figures because this value will be used in the further sections for clipping, as it represents a severe nonlinear distortion and as in this case the clipping noise can be considered to be exactly normal distributed (which will be explained in section 4.2.3). If clipping is performed in the baseband the distortion products only affect the inband signal. Clipping causes not only signal power attenuation ($\alpha$) but also clipping noise ($d(t)$) as previously shown. The attenuation of the ideal 16-QAM constellation points and the additional clipping noise can be seen for a clipping ratio of 1 dB in Fig. 30. Clipping also strongly affects the overall system performance, if it is not compensated.
in the receiver. The BER for an AWGN channel in the function of the SNR for OFDM symbols - with 64 samples where the subcarriers are modulated in 16-QAM, and the bits are uncoded - is shown in Fig. 31 for various clipping ratios. It can be seen that the system performance is strongly reduced even in the case of small clipping ratios (CR = 3 dB).

The baseband model equations modified in line with the clipping model will be presented in this section for OFDM symbols. The time discrete baseband model of the clipped OFDM symbols, distorted by the mobile radio channel is shown in Fig. 32. After applying IFFT to the modulation symbols $X_k$,

the time domain OFDM symbol $x[n]$ is clipped. Then the signal $x^c[n]$ is extended with the cyclic prefix. Assuming again that the synchronization perfect and the channel coefficient $h[n]$ are known – where the length of the channel impulse response is considered to be shorter than the CP –, the discrete incoming signal $y[n]$ after removing the prefix can be expressed as

$$y[n] = x^c[n] * h[n] + w[n], \quad n \in 0, \ldots, N - 1,$$

where $x^c[n]$, based on (4-5), can be written as

$$x^c[n] = \alpha x[n] + d[n], \quad n \in 0, \ldots, N - 1,$$

where $w[n]$ is the AWGN term with a variance $\sigma^2_w$, $\alpha$ is the attenuation factor caused by clipping and $d[n]$ is the Bussgang noise. Instead of (4-16), the frequency domain representation of the time discrete
received signal $y[n]$ for the $k^{th}$ subcarrier is given by

$$Y_k = X_k^* H_k + W_k = (\alpha X_k + D_k) H_k + W_k = \alpha X_k H_k + D_k H_k + W_k, \quad k \in \{0, \ldots, N - 1\},$$

(4-17)

where $X_k, X_k^*, D_k, H_k$ and $W_k$ are the discrete Fourier transforms of the sampled signals $x[n], x^*[n], d[n], h[n]$ and $w[n]$ respectively.

If the clipping of the transmitted signal is performed after the analog conversion and power amplification, out-of-band radiation will also appear, reducing the spectral efficiency of the system as shown in Fig. 33, which is not tolerable in cognitive radio applications. This can be removed by filtering, but this can cause peak regrowth, so clipping and filtering have to be reapplied several times to reduce the PAPR as shown in [84].

Clipping is a very simple technique to limit the PAPR of OFDM signals, but it introduces distortions to the signal, which has to be compensated in the receiver. It has been shown in [85] that with the use only of a Low-Density-Parity-Check (LDPC) code as forward error correction coding the improvement of the system performance is not significant. In the following sections discrete baseband clipping will be applied so no out-of-band radiation will appear.

### 4.2 Turbo receivers for clipped coded OFDM signals

Clipping is used to force the amplitude of the signal into the linear range of the PA. Although the PAPR of the signal can be well controlled by this, it causes power attenuation and errors (which may be considered as noise), therefore clipping needs to be compensated.

The receiver oriented turbo principle is a good candidate for compensation of the clipping effects. Two different methods are described in the literature:

- **Decision Aided Reconstruction (DAR),** where the receiver tries to rebuild the peaks of the time domain signal [86].
- **Bussgang Noise Cancelation (BNC),** where the objective is to remove the clipping noise in the frequency domain [87].

![Figure 33: Out-of-band radiation of clipped OFDM signal with various clipping ratios.](image-url)
Both methods were originally presented with a decoding procedure using hard decisions. The modified receivers applying soft decisions were presented in [45, 46]. With the use of soft information, the receiver takes full advantage of the turbo principle yielding better BER results than the methods based on hard decisions. In this thesis the focus is on the BNC algorithm using soft decisions, which outperforms the DAR method [45, 46].

In the next section the system model which is used for clipped, coded OFDM signals will be introduced. Then, the soft BNC receiver algorithm [45] is explained in detail and its convergence behavior based on the Extrinsic Information Transfer (EXIT) chart is discussed. At the end of this section some modifications are proposed to the described algorithms to further improve the system performance. Finally, the simulation results for the original and the improved BNC are presented and compared.

### 4.2.1 The Turbo-principle

The turbo principle is used for the decoding of serial or parallel concatenated codes. The system model of serial concatenated codes for turbo decoding is shown in Fig. 34. It can be also seen that the decoding method of such concatenated codes is an iterative process. The first encoder (Encoder I.) in the transmitter encodes the binary data, then the coded bitstream is interleaved using an interleaver and encoded again (Encoder II.). Interleaving is used to ensure statistic independency of the bits. The key to the turbo principle is in the receiver. The two decoders are Soft-In/Soft-Out (SISO) devices, which operate with probabilities or Log-Likelihood Ratios (LLRs) (soft values). The LLR for a binary variable \( u_k \in \{-1, +1\} \) is defined as

\[
L(u_k) = \ln \frac{P(u_k = +1)}{P(u_k = -1)},
\]

where the sign of the LLR value is the hard decision and its absolute value gives the reliability of the decision. From an LLR value it is possible to calculate the probability of the binary variables. With the help of

\[
P(u_k = +1) = 1 - P(u_k = -1),
\]

for the probability value we get

\[
P(u_k = +1) = \frac{e^{L(u_k)}}{1 + e^{L(u_k)}},
\]

The binary variable \( u_k \) can be easily assigned to digital bits \( B_k \in \{0, 1\} \); the binary variables \( u_k = -1 \) and and \( u_k = 1 \) represent digital bit \( B_k = 0 \) and \( B_k = 1 \).

The output of a SISO device, the a posteriori LLR \( (L_{AP}) \), can be written as the sum of a priori LLRs \( (L_A) \) and an independent information which can be considered as the gain of the decoding procedure,
the so-called extrinsic LLR \((L_E)\),
\[
L_{AP}(u_k) = L_A(u_k) + L_E(u_k).
\] (4-21)

The extrinsic output of the SISO device can be calculated by subtracting the input a priori information from the output a posteriori information. The extrinsic information of the second decoder (Decoder I.) – after interleaving again the fully decoded information – becomes the a priori input for the first decoder (Decoder II.). The a priori information is an independent information of the data, which has to be decoded. The independency is ensured by a sufficiently long interleaving. The a priori knowledge can not be just an extrinsic information provided by an other decoder, but, for example, the statistics of the source.

4.2.2 OFDM transmitter with clipping

The detailed structure model of the transmitter, presented in Fig. 34, for clipped, coded OFDM signals is shown in Fig. 35. The binary data are coded with a convolutional encoder (Encoder I.), then interleaving is applied. The coded, interleaved bits are mapped to complex symbols given by the signal constellation, which are used to modulate the subcarriers. After the set of modulation symbols \((X)\) belonging to one OFDM symbol is transformed to time domain using IFFT, clipping is applied and a CP is added to the signal. In this simple case the multipath channel - with modulation and clipping – is considered as the second encoder (Encoder II.).

4.2.3 Bussgang noise cancelation receiver for OFDM

The Bussgang noise cancelation (BNC) receiver performs iterative equalization and detection \([88]\). The basic block diagram of this iterative method is shown in Fig. 36. These blocks can be grouped into two main subblocks (Fig. 36.): the BNC detector and the channel decoder. The BNC detector consists of a forward and feedback signal processing path.

Forward-path

The extrinsic Log-Likelihood Ratio (LLR) value for each channel observation \(\hat{Y}_k\) are calculated according to \([89]\),
\[
L(b_{k,m}|\hat{Y}_k) = \ln \frac{\sum_{c_i \in C_{k,m}^1} p(\hat{Y}_k|c_k = c_i)}{\sum_{c_i \in C_{k,m}^0} p(\hat{Y}_k|c_k = c_i)},
\] (4-22)
where $C_{k,m}^1$ and $C_{k,m}^0$, $1 < k \leq M$ are the subsets of $C_k$, where the $m^{th}$ bit in $c_k$ takes the value 1 and 0, respectively. The conditional probability density function $p(\hat{Y} = c_i)$ is given by [86]

$$p(\hat{Y}_k|c) = \exp \left( \frac{(\hat{Y}_k - \alpha H_k c)^2}{N_0 + |H_k|^2 P_D} \right),$$

(4-23)

where $P_D$ is the power of the remaining clipping noise. Due to a large number of samples and the central limit theorem, the clipping noise term $d[n]$ can be modeled as a Gaussian distributed random variable, which is independent of the channel noise $w[n]$. The kurtosis of the real and imaginary parts of the clipping noise as a function of the CR can be seen in Fig. 37. It can be observed that at $\text{CR}=1\text{dB}$ the

Figure 37: Kurtosis of the real and imaginary parts of the clipping noise in function of the CR.

kurtosis of the real and imaginary parts of the clipping noise is almost equal to 3, i.e. it can be considered as gaussian distributed. Based on this assumption, passing through the linear channel filter, the power of the Bussgang noise $P_D$ is multiplied by the power of channel coefficient $H_k$. For the $0^{th}$ iteration $P_D$ is calculated according to (4-13). On the other hand, for each higher iteration and large number of samples, $P_D$ can be approximated as

$$P_D = E(|D_k - \hat{D}_k|^2).$$

(4-24)
Of course the receiver does not have knowledge of $D_k$, so for further implementation the power of the remaining clipping noise has to be estimated in another way. This will be discussed in subsection 4.3 in detail.

**Feedback-path**

After interleaving the extrinsic LLR values, the soft symbols are computed as [86]

$$ \hat{X}_k = \sum_{k=0}^{2^m-1} c_k \prod_{l=0}^{M-1} P(b_{k,l}), \quad c_k \in \mathcal{C}, $$

(4-25)

i.e. each constellation symbol is weighted by the probability of the mapped bits, then they are summed up. Using these soft symbols, the time domain estimate of the OFDM signal is formed by IFFT. Then, knowing the clipping level $A_{\text{max}}$, clipping is applied, and the signal is converted back to the frequency domain. Subtracting from these symbols the attenuated ones, the estimated clipping noise can be expressed as

$$ \hat{D}_k = \hat{X}_k^c - \alpha \hat{X}_k, \quad k = 0, \ldots, N - 1. $$

(4-26)

The estimated noise term $\hat{D}_k$, multiplied by the channel coefficient $H_k$, is then subtracted from the received symbols (Eq. (4-17)) to suppress the clipping noise

$$ \hat{Y}_k = \alpha H_k X_k + H_k(D_k - \hat{D}_k) + W_k, \quad k = 0, \ldots, N - 1. $$

(4-27)

The 0th iteration is considered as the case when no feedback loop is used, i.e. $\hat{Y}_k = Y_k$.

The BCJR channel decoder (named after its inventors: Bahl, Cock, Jelinek and Raviv) [90] has the task to compute the extrinsic information of the deinterleaved LLRs, which are provided by the BNC detector. These extrinsic LLRs will be used to suppress the clipping noise in the feedback path of the BNC detector.

**4.2.4 Convergence analysis**

Convergence analysis can be efficiently performed by a highly effective tool: the EXtrinsic Information Transfer (EXIT) chart, which was developed by Stephan ten Brink [91]. It is used to investigate the iteration behavior of the turbo loop based on mutual information exchange. With this powerful tool the mutual information exchange between the BNC detector and the channel decoder can be traced over the iterations. An detailed overview of the EXIT chart and its usage is given in [47].

**Application of the EXIT chart**

The first step of the convergence analysis is to split up the loop and investigate the two SISO decoders of the receiver presented in Fig. 34 separately as shown in Fig 38. The mutual information $I_E$ at the output of the SISO device is measured versus the input mutual information $I_A$. The output mutual information of Decoder II is also a function of the SNR of the AWGN channel which has an influence on the channel observation $Y$. The following relationship is used to match Fig. 36 and Fig. 38: the BNC detector is considered as Decoder II, and the channel decoder is considered as Decoder I.

The LLRs defined by (4-22) are modeled with an equivalent Gaussian channel [91]. The mutual information between these LLRs and the sent symbols $U$ which are the realizations of $u \in \{-1, +1\}$ can
be written with the [91] conditional probability density function as

\[ I_A(U;LLR) = \frac{1}{2} \sum_{u=-1,1} \int_{-\infty}^{\infty} p_A(\xi|U = u) \log_2 \left( \frac{2p_A(\xi|U = u)}{p_A(\xi|U = -1) + p_A(\xi|U = 1)} \right) d\xi, \] (4-28)

where 0 ≤ I_A ≤ 1 and the binary variable u_k can be easily matched to digital bits b_k ∈ {0, 1}; the binary variable u_k = -1 and u_k = 1 represents the digital bits b_k = 0 and b_k = 1, respectively. To measure the mutual information content of the output extrinsic LLR values, the following expression is applied

\[ I_E(U;LLR) = 1 - E \left\{ \log_2 \left( 1 + e^{-LLR} \right) \right\} \approx \]
\[ \approx 1 - \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + e^{-u_nLLR_n} \right). \] (4-29)

The EXIT function (f_EXIT) represents the relation between the input (I_A) and the output mutual information (I_E) of a soft input/soft output decoder. The EXIT function of the BNC detector is not just a function of the a priori mutual information I_A provided by the channel decoder, but is also dependent on E_b/N_0: I_{E1} = f_EXIT(I_{A1}, E_b/N_0). On the other hand, in case of the channel decoder the EXIT function is only dependent on the a priori LLRs provided by the BNC detector: I_{E2} = f_EXIT(I_{A2}). With the help of the two EXIT functions, the iteration steps of the turbo loop can be visualized. The output of the channel decoder becomes the input of the BNC detector, and the output of the detector will be the input of the decoder in the next iteration:

\[ I_{E1} = f_EXIT(I_{A1} = I_{E2}, E_b/N_0) \] (4-30)
\[ I_{E2} = f_EXIT(I_{A2} = I_{E1}). \] (4-31)

To observe the mutual information transfer of the turbo loop, the EXIT chart is constructed from the two EXIT functions. The EXIT function of the channel decoder is plotted with swapped x-y axes on top of the BNC detectors to visualize the iteration trajectory. An iteration trajectory can be seen for E_b/N_0 = 4 dB and E_b/N_0 = 12 dB with a channel decoder rate of \( \frac{1}{2} \) in Fig. 39. The divergence for the SNR value of 4 dB is clearly visible, the correction loop can not provide any improvement due to the "minimum" in the EXIT function of the BNC detector. After the first iteration, the mutual information
will converge to a lower value than the starting value of the 0th iteration. Despite the "minimum" for 12 dB, the convergence can be clearly observed, the starting mutual information is already high enough to overcome the "minimum" in the EXIT function of the BNC detector and lead to convergence.

In iterative receivers, to achieve convergence, the EXIT functions of both decoders have to be monotonically growing to achieve the mutual information of 1 or the intersection point of the two EXIT functions. Well designed decoders should produce a higher input information compared to the input information [91]. It can be observed in Fig. 39 that the monotony of the BNC detector is not satisfactory. As the input mutual information $I_{A1}$ is getting larger, one would expect a growing output mutual information $I_{E1}$, but this can only be observed if the input mutual information values exceed 0.4.

### 4.3 The modified BNC

To answer the question why the "minimum" is in the EXIT function of the BNC detector, the BNC feedback path has to be investigated more carefully. If the mutual information content of the input LLR values of the soft mapper is low, the output power $P_{\tilde{X}}$ will be small. If all constellation symbols have the same probability, the output power of the soft mapper will be zero. With small output power the clipping does not change the time domain signal significantly, since almost all peaks are under the clipping level $A_{\text{max}}$. Therefore based on these assumptions the clipping noise from Eq. (4-26) will be approximated as

$$\tilde{D}_k \approx \tilde{X}_k - \alpha \tilde{X}_k = (1 - \alpha)\tilde{X}_k,$$

which can be interpreted as an additional noise, for which the remaining clipping noise will be larger than for the 0th iteration Eq. (4-26)-(4-27): $P_{D-D} > P_D$. This effect causes the "minimum" in the EXIT function of the BNC detector. Therefore, a performance gain can be expected by setting dynamically the attenuation factor $\alpha$ according to the output of the soft mapper. The clipping ratio for the $i$th iteration is calculated as

$$\gamma_i = \frac{A_{\text{max}}}{\sqrt{P_{\tilde{X}_i}}}. \quad (4-33)$$
The new attenuation factor can then be expressed according to (4-11) as

$$\alpha_i = 1 - e^{-\gamma_i^2} + \frac{\sqrt{\pi}}{2} \gamma_i \text{erfc} (\gamma_i).$$  \hspace{1cm} (4-34)

During the iterations, the new attenuation factor will decrease from the value 1 to the value \(\alpha\) as the estimation becomes more and more accurate.

The simplest way to estimate the clipping noise is to create a lookup table for the remaining clipping noise power according to \(\alpha_i\) as \(P_{D,i} = f(\alpha_i)\). Therefore based on these assumptions, Eq. (4-23) is modified as

$$p(\hat{Y}_k|c) = \exp \left( \frac{(\hat{Y}_k - \alpha H_k c)^2}{N_0 + |H_k|^2 P_{D,i}} \right).$$  \hspace{1cm} (4-35)

The effect of these changes on the EXIT function are visualized in Fig. 40 for \(E_b/N_0 = 4\) dB and \(E_b/N_0 = 12\) dB. The "minimum" is fully eliminated and the EXIT function is monotonically increasing with the input mutual information, so the BNC receiver will converge.

4.4 Simulation results

For a reasonable comparison and to verify the results we used the same parameters for the simulations as in [45], where the soft decision based BNC was originally presented: the binary data are encoded with a code rate of 1/2, using a 4-state recursive systematic convolutional encoder with polynomials \((1, 5/7)_8\) in octal notation. The interleaved bits are mapped according to a 16-QAM constellation with Gray mapping, then OFDM modulated on 64 subcarriers and clipped with \(CR = 1\) dB. No cyclic prefix is used in the simulation for AWGN channel. Due to arithmetic overflow problems, a Log-map decoder [92] is used instead of the BCJR decoder [90]. It can be seen in Fig. 41 that both the original and the modified receivers can suppress the clipping noise with these code parameters, and the difference is not significant. Only a minor divergence can be seen at 4 dB.

In comparison, if a punctured rate code of 3/4 is used with the polynomials \((5, 7)_8\), the performance difference is noticeable. It is illustrated in Fig. 42 that the original BNC receiver does not converge any more under 14 dB, and over 14 dB only a small gain is visible over the subsequent iteration steps. On
the other hand, the proposed modified BNC algorithm can suppress the clipping noise, and the gain is clearly visible over 6 dB. In this section the comparison of the original and modified BNC receiver was performed over AWGN channel. Results over Rayleigh channel will be presented in Section 5.2.3.

4.5 Section summary

In this section PAPR reduction techniques for OFDM were discussed. Amplitude clipping as the simplest method was studied in details. A receiver oriented clipping compensation technique, the Bussgang noise cancellation was also investigated. The convergence behavior of the BNC receiver was described using the EXIT chart. The EXIT function of the BNC receiver revealed that the original algorithm is divergent thereafter the reason of this divergence was researched. To overcome this phenomenon modifications to the structure were proposed on the bases of the results of the EXIT functions. By these modifications the clipping noise can be suppressed also for higher code rates and beyond a certain $E_b/N_0$ value it can be fully eliminated.

The results of this section were published in [34].
5 PAPR reduction in SMT systems

As shown in the beginning of section 4, there are already numerous techniques presented in the literature for decreasing the high PAPR of the transmitted signal in OFDM systems. Each method has its own advantages and disadvantages. Clipping introduces distortion, which is in general not acceptable for cognitive radio driven TVWS communications where strict ACLR requirements are set. Some methods need increased power, others cause data rate loss and in some cases additional information is needed by the receiver.

In section 3 it has been shown that SMT suffers from the same PAPR problem and it is equally vulnerable to nonlinearities as OFDM. As a result, SMT requires also methods to reduce the large dynamic range of the transmitted signal. Most of the techniques used in OFDM are not applicable directly to SMT due to the fact that the time domain symbols overlap (unlike in OFDM, where each symbol can be treated separately). Some methods have already been proposed in the literature:

- reducing the number of subcarriers [93],
- SeLective Mapping (SLM) [94, 95],
- and DFTS-FBMC [96].

Each method has its own disadvantage: reducing the number of subcarriers leads to data rate loss, the SLM technique requires side information for demodulation and the DFTS-FBMC has only a moderate PAPR reduction performance.

In this section clipping based transmitter and receiver oriented techniques will be presented which may be used without any ACLR regrowth or severe performance degradation.

First two transmitter oriented techniques are discussed where most of the additional signal processing operations are preformed in the transmitter: in case of the Tone Reservation (TR) technique some subcarriers are allocated for PAPR reduction purposes leading to a loss of data rate, while in case of the Active Constellation Extension (ACE) technique the outer points of the constellation are allowed to be enlarged dynamically. Also a possible combination of the two schemes is presented. The negative side effect of both techniques is that they require an increased average power. It is important to mention that both techniques are part of the DVB-T2 standard [97] as well. After these a receiver oriented compensation scheme is presented: a modified IBNC presented in section 4 is adapted to SMT.

5.1 Transmitter oriented iterative PAPR reduction for SMT

In this section we focus on methods which can be used in the transmitter to reduce the PAPR of the transmitted signal. The targeted methods operate with unmodified receiver structure, i.e. enabling PAPR reduction without significant ACLR and BER performance degradation.

In system identification multisines are often used as measurement signal where a similar PAPR reduction is required. The main difference is that while in measurement technology the amplitude and phase of the sines are used for evaluation purposes, in wireless communications the amplitude and phase carry the data:

- In system identification a clipping aided iterative scheme has been developed to reduce the PAPR of multisines, presented in [49].
In wireless communications the use of clipping introduces strongly nonlinear distortions which significantly affect the overall system performance [98]. An iterative repeated Clipping and Filtering (CF) for OFDM was developed in [99], which can significantly reduce the PAPR but introduces considerable amount of nonlinear distortion.

Using these two ideas originating from different fields, incorporating their advantages, a similar algorithm suitable for SMT is presented in this section.

Clipping based PAPR reduction schemes presented in this section have common execution steps. In fact, these techniques can be represented using a common block diagram, shown in Fig. 44. The only difference of the methods is the implementation of frequency domain processing. According to the block diagram the $X_k$ data symbols are used to synthesize an SMT symbol, where $k$ represents the subcarrier index. Each symbol consists of $N$ subcarriers in two subsets: $N_D$ represents the subcarrier indices used for data transmission and $N_Z$ denotes the indices of the unused (zero-valued) band-edge and DC subcarriers. The index notation of the various subcarriers is depicted in Fig. 43. The subcarriers marked by $N_R$ are special reserved carriers, their role will be detailed later.

The general block diagram of the transmitter oriented PAPR-reduction scheme is presented in Fig. 44. The additional signal processing blocks are inserted into the signal path prior to D/A conversion. Following a conventional SMT modulation of the symbols $X$, the PAPR of the generated SMT signal $s$ is measured. If it is below the predefined limit, it can be transmitted. If the amplitude is larger, clipping is applied. After clipping, the signal $s^c$ is demodulated. Clipping decreases the signal power as shown in Eq. 4-5, this has to be compensated with a multiplication of $\frac{1}{\alpha}$, then it can be demodulated. The demodulated symbols $X^c$ are then processed using a special selection and processing algorithm. Then the new symbols $X^{new}$ are modulated again and the PAPR of the signal $s^{new}$ is measured. This process is repeated until the desired PAPR is reached or no further reduction in the PAPR is achieved. After the iteration the signal $s^{aux}$ is used to form the analog transmission signal. The basic idea of the transmitter oriented iterative PAPR reduction for SMT with clipping is depicted in Fig. 44.

### 5.1.1 Tone reservation (TR)

As in OFDM, the TR method can be applied to reduce the PAPR of SMT signals as well. The idea is to reserve a certain number of subcarriers called Peak Reduction Tones (PRT) for PAPR-reduction, i.e.
these are not used for data transmission. The TR technique can be applied in numerous ways, as shown in [100].

In this thesis clipping-based TR is applied, in which the time domain signal $s$ is amplitude limited to get the clipped signal $s'$. The clipped signal is demodulated to retrieve the set of complex modulation symbols $X^c$ affected by clipping. The data subcarriers ($k \in N_D$) are restored to their original state, while the PRTs ($k \in N_R$) are left unchanged to form $X_{k}^{\text{new}}$. This means that $X_{k}^{\text{new}}$ will take the following values:

$$X_{k}^{\text{new}} = \begin{cases} 0 + j0, & \text{for } k \in N_Z, \\ X_k, & \text{for } k \in N_D \text{ and} \\ X_{k}^{c}, & \text{for } k \in N_R. \end{cases} \quad (5.1)$$

Finally, modulation is applied to $X_{k}^{\text{new}}$ to get the new time domain signal $s^{\text{new}}$. If this signal does not satisfy the PAPR criterion, $s = s^{\text{new}}$ is set and the next iteration is started. Basically the nonlinear noise is kept on the PRTs only, data carriers remain unchanged, which leads to a peak regrowth when $s^{\text{new}}$ is formed.

The more subcarriers are reserved, the lower PAPR can be achieved. Increasing the number of reserved subcarriers has a practical limit, as reserved tones are unavailable for data transmission. A balance is desired between data rate loss and PAPR reduction. The advantages of this method include a low complexity and an unaltered BER, as the data carriers are not affected by the method.

### 5.1.2 Active constellation extension (ACE)

ACE was proposed by Krongold and Jones [101] for PAPR-reduction. The idea behind the ACE technique is that the outer constellation points of the constellation alphabet $C$ can be dynamically extended outwards of the original constellation, such that the PAPR of the signal is reduced [79].

The clipping-based ACE method starts with the time domain signal $s$. Following clipping, the signal $s^c$ is demodulated to get $X^c$. After demodulation constraints are applied to the constellation, so that the constellation points are allowed to penetrate in to areas only which do not result in BER degradation, other demodulated symbols which do not satisfy these conditions are reset to their original value $X$.

For QPSK constellation the shaded area in Fig. 45 shows the region of the allowed extension. The figure also shows the restrictions to be applied to the constellation points regarding the maximal transmit power ($P_{\text{max}}$). With these restrictions applied to the set $X^c$, $X^{\text{new}}$ is obtained. Following modulation, the new time domain signal $s^{\text{new}}$ is formed. If the desired PAPR criterion is not met, $s = s^{\text{new}}$ is set and the next iteration is started again.
For higher order modulations the initial conditions and the extension criteria are more complex. Figure 46 shows the extension regions for 16-QAM modulation. The figure shows that only the outer constellation points can be extended without the degradation of the system performance. Also while the corner points can be extended to a larger area, the non-corner points can only be extended along a line, the symbols $X^c$ falling outside the constellation are mapped to this line. Consequently, the distorted modulation values $X^c$ which fall outside the constellation and originate from an outer point – but not from a corner point – must be rotated to fall on the given line.

The advantage of ACE is that it does not require the transmission of additional information, nor does it decrease the data rate, so the receiver can remain unchanged. As disadvantage, this method requires increased power for transmission of a data since the extended constellation points represent higher carrier amplitudes than the original ones. Another disadvantage of ACE is that soft decision based detection can not be applied due to the distorted constellation points.

5.1.3 Joint use of TR and ACE

TR and ACE can be used simultaneously, since the two methods are independent of each other. TR only uses the PRTs and leaves the data subcarriers unchanged, while ACE modifies only the data subcarriers leaving the PRTs unchanged. In the joint use of TR and ACE, a certain number of carriers are reserved as in the TR method, leading to a PAPR-reduction which features fast convergence but also exhibits the disadvantages of TR and ACE simultaneously: it leads to data rate loss, requires increased transmission power and soft decision can not be used.

5.1.4 Implementation aspects

This section discusses the implementation aspects of the presented iterative PAPR reduction schemes. The discussion starts with the computational complexity of the proposed schemes and continues with the formalization of the effects of the different parameters on PAPR reduction performance.

First, the computation requirements of the previously presented PAPR reduction techniques is discussed. As seen in [42], CF provides the best PAPR performance. If CF is used without iteration, receivers employing iterative decoding (as presented in [33, 41]) can compensate for the resulting BER degradation. Further PAPR reduction can be obtained using CF in an iterative manner, as described in [99].
In this case bit errors are introduced by the nonlinear distortion terms, which cannot be compensated for. CF only requires SMT modulation, clipping and SMT demodulation blocks at the transmitter side. Subcarriers with indices $k \in N_Z$ are reset to 0.

The complexity of the TR method is the same as that of CF, however, the performance is strongly dependent on the number of reserved tones. The subcarriers with indexes $k \in N_Z$ are reset to 0, the data subcarriers $k \in N_D$ are restored to their original values, and the value of the reserved tones $k \in N_R$ remains unaltered.

The ACE method has a slightly higher computational complexity due to the procedure of mapping of the distorted constellation point on the data subcarriers. The zero subcarriers have to be reset to 0 as well.

Most of the signal processing complexity of the iterative SMT reduction technique presented in Fig. 44 goes to the modulation and demodulation of the SMT signal. Iterative modulation, clipping and demodulation can be performed on an SMT signal burst but this is very time consuming. Due to the overlapping nature of the SMT signal, after a delay of $L_{po} = KN$ samples, the clipped symbols can be demodulated and after the frequency domain preprocessing the modulation can be restarted in a parallel manner, on a parallel thread. This implementation can be especially efficient in FPGA with a predefined number of iterations. A computationally efficient modulation with low complexity for SMT can be found in [60], for continuous and fast demodulation of the SMT symbols the recursive discrete Fourier transform can be applied as presented in [40].

The two key parameters for the PAPR reduction method are the number of iterations and the clipping ratio. During the investigations the clipping ratio is considered to be fixed during iterations. Further investigations should be performed to find the best clipping ratio profile as a function of iterations.

To characterize the gain of the PAPR reduction method a metric is defined which corresponds to the PAPR reduction performance and the invested additional signal power. The PAPR gain is defined as:

$$\Theta = \max\{\text{PAPR}(s[n]_{dB}) - \max_m\{\text{PAPR}(s^{\text{new}}[n]_{dB}) + \Delta P_s_{dB}, \}
$$

where PAPR$(s[n]_{dB}$ is the original signal’s highest PAPR, PAPR$(s^{\text{new}}[n]_{dB}$ is the highest PAPR value of the modified signal $x^{\text{new}}$ and $\Delta P_s_{dB}$ is the ratio of the average power of the original signal $s$ and the modified signal $s^{\text{new}}$ in dB. The metric also takes into account PAPR gain and increased transmit power. As a result, if the peak of the signal remains unchanged but the signal power is enlarged it has no contribution to the PAPR reduction as the PAPR reduced signal is scaled to the maximal linear range of the amplifier so it will not lead to any additional increase in the output power. For TR this can be interpreted as only the power on the reserved tones being enlarged, thus no further gain can be achieved. During the simulation this metric will also be investigated.

### 5.2 Receiver oriented iterative compensation of clipped SMT signals

Another approach is to move the majority of the signal processing efforts to the receiver side. The transmission signal is clipped and transmitted. In real-life systems not all subcarriers are used for data transmission. Usually the DC subcarrier and some carriers on the edge of the transmission band are not used due to technical difficulties and guard band purposes in the spectrum. Clipping introduces nonlinear distortions in the entire baseband, therefore the originally unused subcarriers will contain additional components. This also negatively affects the spectral behavior of the transmission signal, i.e. leakage will
appear. These additional components have to be suppressed. Digital filtering is not sufficient to suppress the clipping components on the unused subcarriers and analog filtering introduces modulation errors. Instead of filtering, the clipped transmit signal is demodulated again. The modulation values for each symbol of the used subcarriers are selected and the unused subcarriers are set to zero. After this the modulation procedure is repeated.

5.2.1 Modified BNC turbo detector for SMT systems

In case of SMT scheme, the blocks of the BNC receiver presented for OFDM in Fig. 36 must be modified and extended, both in the feedforward and in the feedback paths, due to the overlapping nature of the symbols and the absence of the CP. First, the compensation of the clipping noise is performed in time domain before demodulation. Secondly, in the presence of ISI, only a quasi Maximum Likelihood (ML) detection of the received modulation symbols \( \hat{Y} \) can be performed. Thirdly, the demapping signal processing stages have to be extended with additional signal processing blocks. First, an enlarged FFT operation is applied [102] with a length equal to the time domain impulse duration of the prototype filter. The phase compensation of the transmission channel’s response is performed for each subchannel in the frequency domain. After the phase compensation, the matched filtering is performed in the frequency domain [102]. Finally, a quasi-ML detection of the transmitted, channel distorted complex modulation values is performed similar as in case of OFDM. Equation (4-23) has to be modified taking ISI into account, yielding the following approximated probability function:

\[
p(\hat{Y}_k | a) \approx \exp \left( \frac{(\hat{Y}_k - \alpha H_k a)^2}{I + N_0 + |H_k|^2 P_{D,i}} \right),
\]

where \( I \) is the ISI term which can be calculated according to [39].

5.2.2 Practical application

In real-life systems not all subcarriers are used for data transmission. Usually the DC subcarrier and some carriers on the edge of the transmission band are not used due to technical difficulties and
guard band purposes in the spectrum. Clipping introduces nonlinear distortions in the entire baseband, therefor the originally unused subcarriers will contain components caused by clipping. This also negatively affects the spectral behavior of the transmission signal, i.e. leakage will appear. These components have to be suppressed. Digital filtering is not sufficient to suppress the clipping components on the unused subcarriers and analog filtering introduces modulation errors. Instead of filtering, the clipped transmit signal is demodulated again. The modulation values of each symbols of the used subcarriers are selected, the unused subcarriers are set to zero and repeated modulation is performed similar as described in [103, 99]. The described the transmitter modification for OFDM and SMT systems can be seen in Fig. 48. In the receiver the BNC detector of the OFDM system remains the same, as the compensation of the clipping noise is performed in the frequency domain. The BNC receiver of the SMT scheme, however, has to be modified. The same scheme as presented in Fig. 48 must be implemented in the feedback loop to reconstruct the clipping noise in the time domain.

The results of clipping and setting the unmodulated subcarriers to zero in case of OFDM and SMT can be seen in Fig. 49 and Fig. 50. In Fig. 49 the increase of the PAPR in case of both systems can be observed as a result of the filtering. This increase of the PAPR is strongly dependent on the number of the unused subcarriers. If more unmodulated subcarriers are applied, more nonlinear distortion will be removed from the signal leading to the regrowth of the peaks. Regarding the spectral characteristics the effects of clipping can be well observe in Fig. 50 as the nonlinear products appear in the baseband causing an enlarged side lobe. This leakage disappears with the filtering of the unused subcarriers. As a positive side-effect of the filtering the power of the clipping noise will also be reduced, so the receiver will operate with a reduced $P_d$.

### 5.2.3 Simulation results

Table 3 shows the summary of the simulation parameters for the two modulation schemes. The binary data are encoded with a code rate of 1/2, using a 4-state recursive systematic convolutional encoder with polynomials $(1, 5/7)_8$ in octal notation. The interleaved bits are mapped according to a 16-QAM constellation with Gray mapping. The clipping level (CR) is set to 1 dB. For the prototype filter of the SMT system the coefficients presented in [104] are applied.

To obtain comparable bit error rates, the SNR normalized to one bit energy is determined. The noise
Figure 49: CCDF of the PAPR values of the transmitted signal with clipping and additional signal processing for OFDM (a) and SMT (b).

Figure 50: Power spectrum density function of the transmitted signal with clipping and additional signal processing for OFDM (a) and SMT (b).

Table 3: Simulation parameters for SMT and OFDM system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SMT</th>
<th>OFDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>8 MHz</td>
<td></td>
</tr>
<tr>
<td>Cyclic prefix ( (P) ) length</td>
<td>0</td>
<td>128</td>
</tr>
<tr>
<td>Available subcarriers/subbands ( (N) )</td>
<td>1024</td>
<td></td>
</tr>
<tr>
<td>Modulated subcarriers/subbands ( (N_D) )</td>
<td>768</td>
<td></td>
</tr>
<tr>
<td>Overlapping factor ( (K) )</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Mapping ( (M_b) )</td>
<td>4 (16-QAM)</td>
<td></td>
</tr>
<tr>
<td>Clipping ratio</td>
<td>1 dB</td>
<td></td>
</tr>
</tbody>
</table>

The power of the AWGN channel is calculated according to the following definition:

\[
\text{SNR}_{\text{dB}} = 10 \log_{10} \left( \frac{P_b}{N_0} \right) = 10 \log_{10} \left( \frac{P_{\text{out}}(N+P)}{N_0 M_b R N_D} \right),
\]
where $P_b$ is the bit power, $N$ is the number of the subcarriers/subbands available and $N_D$ is the number of subcarriers/subbands used. $P$ is the length of the CP, $M_b$ is the number of bits transmitted by one subcarrier/subband and $R$ is the code rate. Assuming a normalized symbol duration $T$, the energy of a single bit can be expressed as $E_b = P_b T = P_b$. The parameters of the applied IEEE 802.22 channel profiles B and C can be found in Table 4. For the decoding of the received bits a BCJR decoder [90] was proposed, but due to arithmetic overflow issues the log-map decoder [92] is used.

First the simulations were performed over AWGN channel. Besides the BER performance also the Error Vector Magnitude (EVM) improvements are compared. The simulated BERs over AWGN channel can be seen in Fig. 51. Due to the absence of CP, SMT outperforms OFDM. It can be observed for both techniques that clipping severely degrades the overall system performance if it is not compensated, that is no feedback loop is active. With iterative compensation the system performance can be improved. Beyond a certain SNR limit, the BER results approach the performance of the case without clipping.

Fig. 52 shows the EVM of the constellation at the output of the receiver. EVM can be used to characterize the modulation errors, in this particular case it reflects the influence of the nonlinearity (i.e. clipping) on the signal constellation. Note that lower EVM means less distorted signal constellation and better BER performance. Let $\hat{X}_k[i]$ and $X_k[i]$ denote the constellation point at the output of the receiver on the $k^{th}$ subcarrier in the $i^{th}$ symbol and the transmitted constellation point, respectively. EVM is computed as

$$EVM = 10 \log_{10} \left( \frac{E \{ |\hat{X}_k[i] - X_k[i]|^2 \}}{P_{ref}} \right),$$

where $P_{ref}$ is the power of the outmost ideal constellation point. As it can be seen in Fig. 52, a noticeable
EVM reduction with respect to conventional receivers is achieved at low and moderate CR at an SNR value of 10 dB.

The BER simulations for Channel B can be seen in Fig. 53. In case of OFDM the CP is longer than the maximal channel delay, therefore ISI is not affecting the OFDM system, and the effects of clipping can be compensated. In case of SMT the effect of the ISI does not severely degrade the system performance, therefore it still outperforms OFDM. For both techniques the effect of clipping can be compensated and the BER after the third iteration approaches the results where no clipping was applied.

The BER simulations for Channel C are shown in Fig. 54. In this scenario the CP of OFDM is shorter than the channel impulse response so an error floor caused by the residual ISI can be observed. The presence of ISI results in an error floor also for SMT systems, but at a much lower BER.
5.3 Simulation results for the PAPR-reduction techniques

In this section simulation results are presented for the previously described PAPR-reduction methods in case of SMT signals. During the simulations $N=1024$ carriers were implemented, from which 768 were used with 16-QAM modulation. In case of TR, 10% of the subcarriers were used as PRTs. In Fig. 55 the results of TR, ACE, TR & ACE and simple clipping with filtering are presented with a target PAPR of 3 dB over the number of iterations. In Fig. 56 the same parameters are used, with the only difference that the target PAPR is 1 dB in this case. It can be observed that in both cases clipping with additional filtering provides the best performance. The price of this is the high computation complexity of the applied BNC receiver. The TR technique achieves the smallest PAPR improvement, its performance is not strongly dependent on the chosen clipping level. ACE outperforms TR in both cases, with a performance affected by the chosen clipping ratio. The joint use of TR & ACE outperforms the cases where only ACE or TR is used. The performance of clipping with filtering can reach up to 1.5 dB by the joint use of TR and ACE after the third iteration. An important measure of the performance of the PAPR reduction technique is the PAPR improvement relative to change of the required transmit power. In Fig. 57 and Fig. 58 this gain can be observed with a constant clipping level of 1 dB and 3 dB over the number of iterations. It can be observed that the best performance can be achieved by the joint use of TR and ACE and the convergence is also the fastest in this case, its maximum is reached after the fifth iteration. The TR has the slowest convergence but – as it has been discussed earlier – though it enables the use of soft detection in the receiver.

5.4 Section summary

In this section the possibilities of reducing the PAPR of SMT were studied. Three clipping based OFDM PAPR reduction method were found to be suitable for the PAPR reduction in SMT systems. The performance of TR and ACE was presented and their advantages and disadvantages were discussed. A third receiver oriented clipping mitigation method was investigated: a modified BNC receiver structure suitable for clipped SMT signal processing. Based on the EXIT chart, it was shown that the proposed iterative scheme is convergent. It was also described how the clipping technique can be applied in real-life systems for both OFDM and SMT modulation. Finally, the performance of the BNC SMT receiver was
verified and compared to OFDM systems, based on BER simulations over AWGN and Rayleigh channels. In case of both systems the clipping can be compensated and a performance of clipping-free transmission can be approached.

In this section a clipping-based PAPR-reduction schemes suitable for SMT were presented, showing that also SMT signals – similar to OFDM signal – have a large PAPR. Transmitter and receiver oriented iterative schemes have been presented which can be applied to SMT without considerable performance degradation. The PAPR-reduction schemes were compared through simulations.

None of the presented techniques degrade the ACLR of the SMT signal, as the signal processing is performed in the baseband and the subcarriers with indices \( k \in \mathbb{N}_2 \) are reset to 0 after each demodulation and re-modulation procedure. This means that the introduced PAPR reduction schemes are especially suitable for SMT in cognitive radio scenarios.

A summary and comparison of the advantages and disadvantages of the presented schemes are given in Table 5 by means of transmitter complexity and receiver requirements, data rate loss, constellation distortion and power increase. Depending on the specific application requirements, the most suitable PAPR-reduction technique can be chosen. As a future work the tradeoff between the power increase and the PAPR-reduction must be investigated. A possible dynamic setting of the clipping level should also be analyzed.

The results presented in this section were published in [41, 33, 42, 44].

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
PAPR-reduction technique & Transmitter complexity & Data rate loss & Constellation distortion & Power increase & Receiver requirements \\
\hline
TR & Moderate & Yes & No & Yes & Subcarrier indexes for TR (\( N_a \)) \\
ACE & High & No & Yes & Yes & Soft decision making is not possible \\
TR & ACE & High & Yes & Yes & Same as for TR and ACE \\
CF & Low & No & Yes & No & Receiver requires BNC [33, 46] \\
\hline
\end{tabular}
\caption{Comparison of the presented clipping-based PAPR-reduction methods for SMT.}
\end{table}
6 Recursive Discrete Fourier Transform (R-DFT) based FBMC Receivers

In this section the possible substitution of the FFT block with an R-DFT structure in cognitive radio based receivers is presented and its benefits are discussed. First, the concept of R-DFT is explained with its observer-based implementation. Then the benefits of an R-DFT receiver are given by means of signal processing complexity, spectral sensing and channel equalization applications for SMT systems.

6.1 The R-DFT

Regular DFT and FFT algorithms operate on sequences of signal samples (blocks). An element-wise evaluation of a block of samples using sliding window is possible by using recursive calculations. The DFT sum of N samples of a vector $x[n]$ can be expressed as:

$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn} = \frac{1}{N} \left\{ x[0] e^{-j \frac{2\pi}{N} k0} + x[1] e^{-j \frac{2\pi}{N} k1} + \ldots \right\}, \quad k \in 0, \ldots, N - 1. \quad (6-1)$$

In the next subsections two similar methods are described to continuously calculate the Fourier coefficient of the incoming signal $x[n]$.

6.1.1 R-DFT as a filter bank

The first solution is to realize the R-DFT procedure by a filter bank [105, 106] known as the Lagrange structure. The block diagram of this can be seen in Fig. 59. The basic idea is to use a moving average filter (realized as a comb filter and an integrator), a parallel set of demodulators $g_k[n] = \frac{1}{N} e^{-j \frac{2\pi}{N} kn}$ and modulators $c_k[n] = e^{j \frac{2\pi}{N} kn}$ to calculate the N point Fourier coefficients $\hat{X}_k[n]$ (for the samples $x[n-N], \ldots, x[n-1]$) and the signal components $\hat{x}_k[n]$.

![Block diagram of the R-DFT based on a filter bank.](image)

**Figure 59:** Block diagram of the R-DFT based on a filter bank.
6.1.2 R-DFT based on observer theory

A generalized form of a less known recursive formulation of orthogonal transforms based on observer theory was presented by Péceli [107]. The block diagram of R-DFT based on the observer theory can be seen in Fig. 60. The receiver is assumed to have enough memory to store \( N \) samples (as in regular DFT). The idea of the recursive method is to recalculate all the DFT coefficients with each new incoming sample. The calculation is not performed on a block-by-block basis, but instead using only the results of the previous DFT, taking into account the change caused by the new sample.

This idea is realized in the structure presented in Figure 60. First, the incoming sample is multiplied by the coefficients \( g_k[n] = \frac{1}{N} e^{-j \frac{2\pi}{N} kn} \), then the results are filtered. The estimated Fourier coefficient \( \tilde{X}_k[n] \) at a time instant \( n \) is remodulated using the coefficients \( c_k[n] = e^{j \frac{2\pi}{N} kn} \) and the resulting signal is subtracted from the input, realizing a circular buffer. It has been shown by Péceli [107] that the previously described observer structure is equivalent to the R-DFT and a convergence through \( N \) steps is ensured. Although the two possible implementations of the R-DFT are theoretically equivalent, in case of practical implementations the filter bank approach may lead to instability due to its finite arithmetic precision [106]. As a result, the observer-based R-DFT is more preferable due to its tolerance against such disturbances.

6.2 Applications of R-DFT

The replacement of FFT in the SMT receivers with an R-DFT leads to numerous advantages, which will be described in this section. First the signal processing complexity will be investigated, then spectral sensing is addressed. Finally, the use of R-DFT in a special channel equalization technique will be described.
6.2.1 Complexity analysis

Implemented e.g. in FPGA, while FFT requires \( \log_2 N \) butterfly stages for the calculations, R-DFT only needs parallel memory cells to realize the integrator blocks shown in Fig. 60. If only a blockwise calculation of an \( N \) point DFT is required, then conventional FFT is faster than R-DFT, because the latter requires a delay of \( N \) samples. When continuous calculation of an \( N \) point DFT in a sliding window manner over the sampled received signal is needed, R-DFT is much faster compared to rearranging the samples and reapplying an FFT for each \( N \) points of the block. The R-DFT uses the DFT result of the previous window in order to calculate the new DFT of the window shifted by one sample. The summary of the complexity requirements is given in Tab. 6.

<table>
<thead>
<tr>
<th></th>
<th>FFT</th>
<th>R-DFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFT for block of samples</td>
<td>( N \log_2 N )</td>
<td>( N^2 )</td>
</tr>
<tr>
<td>( x[n], 0 \ldots N - 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFT for a shifted window</td>
<td>( N \log_2 N )</td>
<td>( N )</td>
</tr>
<tr>
<td>( x[n], 1 \ldots N )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.2.2 Spectral sensing

In case of cognitive radio applications, it is not only the speed of the calculation what is important, but fast spectral sensing is also essential. This means that if an SMT system is implemented by FFT, a block-by-block FFT must be performed after each incoming sample to have the instantaneous spectrum available for detection. With R-DFT, this instantaneous spectral sensing capability is already implemented, as the new spectrum is obtained after each incoming sample. The implementation complexity of an R-DFT for continuous spectral sensing is smaller than in case of FFT, so it is advisable to use an R-DFT when continuous spectral monitoring is required.

6.2.3 Channel equalization

R-DFT can also be efficiently used in channel equalization as presented in Kollár et al [39]. In SMT systems no cyclic prefix is applied as in OFDM. The lack of cyclic prefix results in noticeable negative effects in case of multipath channels with longer delays on SMT transmission due to the considerable extent of inter-symbol interference. In [39] an equalization technique – called averaged MMSE (For details see Section 3.4.) – using time diversity is presented, which requires the demodulation of the time shifted versions of the transmitted symbols over a few samples. This demodulation procedure can also benefit from the R-DFT instead of using FFT for each shifted \( N \) points.

6.3 Section summary

In this section a possible use of the R-DFT in cognitive radio applications was presented. It has been stated that for multicarrier receivers using FFT for continuous spectrum sensing and demodulation of the signal the R-DFT can be computationally more efficient than the recalculation of the FFT. It can be especially useful in SMT receivers if channel equalization based on averaging technique [39] is applied.

These benefits of the observer-based R-DFT in cognitive radio are published in [40].
7 Conclusion and future work

Cognitive radio systems are a very promising candidate for the reuse of the densely occupied TV spectra especially in the WS and GS bands. The physical layer of these systems is a very important factor, where the choice of the most suitable modulation is crucial. This thesis emphasizes various aspects of the physical layer for cognitive radios.

After the introduction, first a brief description of the multicarrier modulations is given, explaining the most important analog and digital signal processing steps. OFDM, SMT and two OFDM related modulation schemes are presented. Also a computationally efficient method, the polyphase implementation of SMT is detailed.

In section 3 I give an overview and detailed comparison of possible multicarrier schemes for cognitive radio usage. It is shown that SMT technique has the most efficient spectral behavior, but from some other aspects OFDM, DFTS-OFDM or even CE-OFDM might be a better solution. Signal metrics, influence of nonlinearities on ACLR and effects of residual synchronization errors on the system performance of the four investigated schemes are shown. At the end of this section possible solutions for the channel equalization of SMT are shown.

Section 4 deals with the PAPR reduction of OFDM systems by means of clipping. Clipping is in it self a nonlinear distortion which may lead to the degradation of performance if it is not compensated in the receiver. A receiver oriented iterative method called BNC is presented. It has been shown that the receiver is divergent at low SNR values, and solutions for achieving convergence is advised. The convergence is analyzed and validated using EXIT charts, which are a well known visual representation of the iterative behavior of the turbo codes.

Due to its advantageous spectral properties, SMT modulation is an obvious solution for cognitive radio applications. In my view, this modulation scheme might be the choice also of the standardization bodies as a result of the stringent requirements set out by the communication authorities. Both OFDM and SMT are highly sensitive to the nonlinear distortions present in the transceiver chain due to the transmit signal’s large PAPR. These nonlinearities may lead to increased out-of-band radiation and degradation in BER performance. To avoid these negative effects, digital signal pre-processing is required to reduce the PAPR, if the linear range of the transmitter amplifier is lower than the crest factor of the transmitted signal. In Section 5 PAPR reduction techniques for SMT signals are investigated and their performance is evaluated through simulation and measurements. First the BNC receiver – described for OFDM – is adapted to SMT signals and its performance is validated. Then other clipping based PAPR reduction methods for SMT are described as well. For all presented methods the resulting ACLR of the transmitted signals remains unaltered due to the baseband signal processing, which makes them attractive for cognitive radio applications.

To adopt this promising development, however, some SMT specific issues must be solved. I provided solutions for two of these problems: the elimination of problems originating from nonlinear distortions as well as possible improvements of the efficiency of channel equalization techniques. Besides of all these, I recommended the application of R-DFT algorithm for spectral measurements in cognitive radios, which requires significantly less computational resources than the conventional FFT, while it is also suitable for channel equalization purposes. With this procedure the changing spectral environment can be continuously monitored, enabling the opportunistic system to react rapidly in case the primary users appear.

Besides computer simulations, I also used a hardware implementation to prove that the shoulder
attenuation necessary for cognitive radios can be achieved by SMT modulation. This USRP environment is suitable for verifying further simulation results, and, simultaneously with the birth of this thesis book, a close cooperation with National Instruments is established to make it available for the industry. In addition, it will also be deeply involved in the education of the specialists of the future in the framework of laboratory experiments, aimed to try and demonstrate wireless data transmission systems as well as state-of-the-art modulation procedures. The hardware environment and measurement results are presented in Appendix F.

Still there are several open questions related to the SMT modulation scheme and its hardware implementation, which require further research to be answered: it is important to elaborate procedures for clock-, frequency- and phase error synchronization and tracking that can be used for efficient receivers as well as to find an appropriate pilot scheme for efficient channel estimation. Furthermore, the R-DFT algorithm is likely to be efficient for synchronization purposes besides spectrum measurement and channel estimation.
A Orthogonality conditions for the filter of the SMT scheme

To achieve perfect reconstruction for both the real and the imaginary parts in SMT, the transmitted and received modulation values must be identical:

\[ \hat{a}_l[i] = a_{k=i}[m = i], \quad \forall \; l, i \in \mathbb{N}; \]  
\[ \hat{b}_l[i] = b_{k=i}[m = i], \quad \forall \; l, i \in \mathbb{N}. \]  

Evaluating the \( \Re \{ \} \) and \( \Im \{ \} \) functions and replacing \( t - iT \) with \( t \) (this is allowed as \( m \) varies between \( -\infty \) and \( \infty \)) in (2-44) and in (2-45), an extended expression for \( \hat{a}_l[i] \) and \( \hat{b}_l[i] \) can be derived:

\[ \hat{a}_l[i] = \sum_{m = -\infty}^{\infty} \sum_{k = 0}^{N-1} \int_{-\infty}^{\infty} a_k[m]p_0(t)p_0(t - mT) \cos \left( (k - l) \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) - \]
\[ b_k[m]p_0(t)p_0(t - mT - T/2) \sin \left( (k - l) \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) dt, \]  
\[ \hat{b}_l[i] = \sum_{m = -\infty}^{\infty} \sum_{k = 0}^{N-1} \int_{-\infty}^{\infty} a_k[m]p_0(t + T/2)p_0(t - mT) \cos \left( (k - l) \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) + \]
\[ b_k[m]p_0(t + T/2)p_0(t - mT - T/2) \sin \left( (k - l) \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) dt. \]

To fulfill (A-1) and (A-2) formal assumptions for (A-3) and (A-4) can be made. For (A-3) the following two equations must stand:

\[ \int_{-\infty}^{\infty} p_0(t)p_0(t - mT) \cos \left( (k - l) \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) dt = \delta_{k-l,m}, \]  
\[ \int_{-\infty}^{\infty} p_0(t)p_0(t - mT - T/2) \sin \left( (k - l) \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) dt = 0. \]

Similarly, for (A-4) the following equations must stand:

\[ \int_{-\infty}^{\infty} p_0(t + T/2)p_0(t - mT) \cos \left( (k - l) \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) dt = 0, \]  
\[ \int_{-\infty}^{\infty} p_0(t + T/2)p_0(t - mT + T/2) \sin \left( (k - l) \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) dt = \delta_{k-l,m}. \]

Taking into account that the prototype filter \( p_0(t) \) is symmetric around \( t = 0 \) it can be shown that equations (A-6) and (A-7) are antisymmetric around \( t = mT/2 + T/4 \) and \( t = mT/2 - T/4 \) respectively, so both equations are fulfilled. Concerning (A-5) and (A-8), as only neighboring subcarriers \( |k - l| > 1 \) overlap, the equations must be evaluated only at these values. First, if \( k = l \), (A-5) and (A-8) are trivially
satisfied due to the nyquist-property of the prototype filters:

\[ \int_{-\infty}^{\infty} p_0(t)p_0(t - mT) dt = \delta_{k-l,m} = \delta_m, \quad (A-9) \]

\[ \int_{-\infty}^{\infty} p_0(t + T/2)p_0(t - mT + T/2) dt = \delta_{k-l,m} = \delta_m. \quad (A-10) \]

For \( k - l = \pm 1 \) equations (A-5) and (A-8) are reduced to

\[ \int_{-\infty}^{\infty} p_0(t)p_0(t - mT) \cos \left( \frac{2\pi}{\bar{T}} t + \frac{\pi}{2} \right) dt = 0, \quad (A-11) \]

\[ \int_{-\infty}^{\infty} p_0(t + T/2)p_0(t - mT + T/2) \cos \left( \frac{2\pi}{\bar{T}} t + \frac{\pi}{2} \right) dt = 0. \quad (A-12) \]

As the prototype filters are real valued and symmetric around \( t = 0 \), both equations are satisfied.

It has been shown that with the use of a real valued evenly symmetric prototype filter satisfying the Nyquist criteria the orthogonality conditions are fulfilled. This means that perfect reconstruction of the transmitted modulation values is possible, therefore (A-1) and (A-2) are satisfied.
B Calculation of the attenuation factor for clipping

The characteristics of the soft envelope limiter can be written as

\[
A(q) = \begin{cases} 
q, & 0 \leq q < A_{\text{max}} \\
A_{\text{max}}, & A_{\text{max}} \leq q \leq \infty .
\end{cases}
\]  

(B-1)

The amplitude values of the OFDM signal follow a Rayleigh distribution given by

\[
p(q) = \frac{2q}{\sigma_s^2} e^{-\frac{q^2}{2\sigma_s^2}}.
\]  

(B-2)

The average power of the input signal is equal to \( P_s = \sigma_s^2 \). With the aid of the equation for \( \alpha \) (4-8) and the knowledge for the probability density function for the amplitude values of the OFDM signal we can write

\[
\alpha = \frac{\langle qA(q) \rangle}{\sigma_s^2} = \frac{E[rA(q)]}{\sigma_s^2} = \frac{1}{\sigma_s^2} \int_0^\infty qp(q) A(q) \, dq = \frac{1}{\sigma_s^2} \left[ \int_0^{A_{\text{max}}} qp(q) A(q) \, dq + \int_{A_{\text{max}}}^\infty qp(q) A(q) \, dq \right].
\]  

(B-3)

The main integral in (B-3) can be split into two regions: the first region \([0, A_{\text{max}}]\) where the amplitude values are not clipped, and the second region \([A_{\text{max}}, \infty]\) where the amplitudes are limited. The first integral can be expressed as

\[
\int_0^{A_{\text{max}}} r p(q) A(q) \, dq = \left. \int_0^{A_{\text{max}}} \frac{2q^3}{\sigma_s^2} e^{-\frac{q^2}{2\sigma_s^2}} \, dq \right|_{q = 0}^{q = A_{\text{max}}} = -\left( A_{\text{max}}^2 + \sigma_s^2 \right) e^{-\frac{A_{\text{max}}^2}{2\sigma_s^2}} + \sigma_s^2.
\]  

(B-4)

The second term can be expressed as

\[
\int_{A_{\text{max}}}^\infty r p(q) A(q) \, dq = \left. \int_{A_{\text{max}}}^\infty \frac{2r}{\sigma_s^2} e^{-\frac{r^2}{2\sigma_s^2}} \, dr \right|_{r = A_{\text{max}}}^{r = \infty} = \frac{A_{\text{max}}}{\sigma_s^2} \left( -\frac{A_{\text{max}}^2}{2\sigma_s^2} + \sigma_s^2 \right) = A_{\text{max}}^2 e^{-\frac{A_{\text{max}}^2}{2\sigma_s^2}} + \sigma_s \frac{\sqrt{\pi}}{2} A_{\text{max}} \text{erfc} \left( \frac{A_{\text{max}}}{\sigma_s} \right).
\]  

(B-5)

Now substituting the result of (B-5) and (B-7) into (B-4), the attenuation factor \( \alpha \) can be calculated as

\[
\alpha = \frac{1}{\sigma_s^2} \left( -\sigma_s^2 e^{-\frac{A_{\text{max}}^2}{2\sigma_s^2}} + \sigma_s^2 + \sigma_s \frac{\sqrt{\pi}}{2} A_{\text{max}} \text{erfc} \left( \frac{A_{\text{max}}}{\sigma_s} \right) \right) = \frac{1 - e^{-\frac{A_{\text{max}}^2}{2\sigma_s^2}} + \frac{\sqrt{\pi}}{2} A_{\text{max}} \text{erfc} \left( \frac{A_{\text{max}}}{\sigma_s} \right)}{\sigma_s^2} \left. \right|_{\gamma = \frac{A_{\text{max}}}{\sigma_s}} = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}}{2} \gamma \text{erfc} (\gamma),
\]  

where \( \gamma \) is defined as

\[
\gamma = \frac{A_{\text{max}}}{\sigma_s}.
\]  

(B-10)
C Calculation of the output power of the nonlinearity

With the help of (B-1) and (B-2) the output power of the nonlinearity can be expressed as

\[ P_{\text{out}} = E\{A^2(q)\} = \int_0^\infty A^2(q)p(q)dq = \int_0^{A_{\text{max}}} q^2 \frac{2q}{\sigma_s^2} e^{-\frac{q^2}{\sigma_s^2}} dq + \int_{A_{\text{max}}}^\infty A_{\text{max}}^2 \frac{2q}{\sigma_s^2} e^{-\frac{q^2}{\sigma_s^2}} dq. \quad (C-1) \]

In (C-1), the first integral can be expressed as

\[ \int_0^{A_{\text{max}}} q^2 \frac{2q}{\sigma_s^2} e^{-\frac{q^2}{\sigma_s^2}} dq = \left[ -\left( r^2 + \sigma_s^2 \right) e^{-\frac{r^2}{\sigma_s^2}} \right]_0^{A_{\text{max}}} = -\left( A_{\text{max}}^2 + \sigma_s^2 \right) e^{-\frac{A_{\text{max}}^2}{\sigma_s^2}} + \sigma_s^2. \quad (C-2) \]

The second integral is the Rayleigh CDF:

\[ \int_{A_{\text{max}}}^\infty A_{\text{max}}^2 \frac{2q}{\sigma_s^2} e^{-\frac{q^2}{\sigma_s^2}} dq = A_{\text{max}}^2 \left[ 1 - e^{-\frac{A_{\text{max}}^2}{\sigma_s^2}} \right]_{A_{\text{max}}}^{\infty} = A_{\text{max}}^2 e^{-\frac{A_{\text{max}}^2}{\sigma_s^2}}. \quad (C-3) \]

With the results of (C-2) and (C-3) we can write for (C-1):

\[ P_{\text{out}} = \sigma_s^4 - \sigma_s^4 e^{-\frac{A_{\text{max}}^2}{\sigma_s^2}} = \left( 1 - e^{-\gamma^2} \right) P_s, \quad (C-4) \]

where \( P_s \) is the average power defined as

\[ P_s = \sigma_s^4, \quad (C-5) \]

and \( \gamma \) is defined as

\[ \gamma = \frac{A_{\text{max}}}{\sigma_s}. \quad (C-6) \]
D  Modeling the a priori mutual information

The a priori information will be modeled with an equivalent Gaussian channel. The transmitted signal over an AWGN channel is given by

$$y = u + w,$$  \hspace{1cm} (D-1)

where $u \in \{-1, +1\}$ and $w$ is the AWGN with the variance $\sigma_0$. The conditional probability density function can be written as

$$p_A(y|u = U) = \frac{1}{\sqrt{2\pi} \sigma_0} e^{-\frac{1}{2} \left( y - \frac{\sigma_A^2}{2u} \right)^2}. \hspace{1cm} (D-2)$$

For the LLR values using (D-1) and (D-2) we get

$$L_A = \ln \frac{p(y|u = +1)}{p(y|u = -1)} = \frac{2}{\sigma_0^2} (u + w). \hspace{1cm} (D-3)$$

This equation can be also written, for the equivalent Gaussian channel, as

$$L_A = \mu_A u + w_A \hspace{1cm} (D-4)$$

where $w_A$ is the AWGN term of the equivalent Gaussian channel, for which the variance $\sigma_A$ can be expressed, using $\sigma_0$, as

$$\sigma_A^2 = \frac{4}{\sigma_0^4}, \hspace{1cm} (D-5)$$

and $\mu_A$ is a constant which equals to

$$\mu_A = \frac{2}{\sigma_0^2} = \frac{\sigma_A^2}{2}. \hspace{1cm} (D-6)$$

To measure the information content between the real valued random variables $A$ and $B$, the mutual information is calculated as [47]

$$I(A; B) = \int \int P(A, B) \log \frac{P(A, B)}{P(A)P(B)} \ dA dB. \hspace{1cm} (D-7)$$

The mutual information between $L_A$ and the sent symbols $u$ can be written as

$$I_A(u; L_A) = I_A$$

$$= \frac{1}{2} \sum_{u = -1, 1} \int_{-\infty}^{\infty} p_A(\xi|U = u) \log_2 \frac{2p_A(\xi|U = u)}{p_A(\xi|U = -1) + p_A(\xi|U = 1)} d\xi, \hspace{1cm} (D-8)$$

$$0 \leq I_A \leq 1,$$

Expressing the mutual information $I_A$ as a function of $\sigma_A$ we obtain the $J$-function

$$J(\sigma_A) := I_A(\sigma_A) = 1 - \int_{-\infty}^{\infty} p_A(\xi|X = u) \log_2(1 - e^{-\xi}) d\xi \hspace{1cm} (D-9)$$

for the extreme values

$$\lim_{\sigma_A \to 0} J(\sigma_A) = 0 \text{ and } \lim_{\sigma_A \to \infty} J(\sigma_A) = 1, \quad \sigma_A > 0. \hspace{1cm} (D-10)$$
With the aid of the J-function we can set the mutual information of the input $I_A$ of the SISO devices to measure the output mutual information $I_E$. The equation (D-9) cannot be expressed in a closed form, but well approximated as

$$J(\sigma) \approx \left(1 - 2^{-H_1\sigma^2H_2}\right)^{H_3},$$

\hspace*{1cm} (D-11)

where the parameteres are found by a curve-fitting method, as described in [108], and given as: $H_1=0.3073$, $H_2 = 0.8935$, $H_3=1.1064$. The approximated J-function is visualized in Fig. 61.

![J-function](image)

_Figure 61: The J-function._

To measure the mutual information content of the output LLRs from the LLR values $(L_E)$, the following expression is applied

$$I_E = 1 - E\{\log_2(1 + e^{-L})\} \approx 1 - \frac{1}{N} \sum_{n=1}^{N} \log_2(1 + e^{-L_n})^n.$$  \hspace*{1cm} (D-12)
E Soft demapping of multilevel modulations

The complex symbol $Y$, received over the AWGN channel, can be expressed as

$$Y = X + W, \quad X \in \mathcal{C}, \quad (E-1)$$

where $X$ is the originally sent constellation symbol from the constellation alphabet $\mathcal{C}$, where each symbol maps $M_b$ bits. The objective is to calculate the a posteriori values for each mapped bit. According to [89] the Log-Likelihood Ratio (LLR) value for a bit $B_0 \in \{0, 1\}$ conditioned on the a received symbol $Y$ is given by

$$L(B_0|Y) = \ln \frac{p(B_0 = 1|Y)}{p(B_0 = 0|Y)}. \quad (E-2)$$

Assuming that the bits are independent, with the use of the Bayes rule and the a priori LLR ($L_a$), for the $k^{th}$ bit of the symbol $Y$, we get

$$L(B_k|Y) = L_a(B_k) + \ln \frac{\sum_{i=0}^{2^{M-1}-1} p(Y|B_k = 1, B_{j,j=0...M-1,j\neq k} \equiv \text{bin}(i)) \exp \sum_{j=0,j \neq k}^{M-1} L_a(B_j)}{\sum_{i=0}^{2^{M-1}-1} p(Y|B_k = 0, B_{j,j=0...M-1,j\neq k} \equiv \text{bin}(i)) \exp \sum_{j=0,j \neq k}^{M-1} L_a(B_j)}, \quad (E-3)$$

where $B_k = 1, B_{j,j=0...M-1,j\neq k} \equiv \text{bin}(i)$ means the binary decomposition of $i$ where the $k^{th}$ bit is 1. The function $\text{btst}(i,j)$ is ‘1’ if the $j^{th}$ bit is set in the binary decomposition of $i$, otherwise it is ‘0’. For the calculation of the extrinsic value, the first term in (E-3) the a priori information $L_a(B_k)$, has to be ignored. For AWGN channel the conditional probability function can be written as

$$p(Y|X) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp \left( -\frac{1}{2\sigma_0^2} (Y - X)^2 \right) \quad (E-4)$$

With the help of (E-4) we can write for (E-3):

$$L(B_k|Y) = L_a(B_k) + \ln \frac{\sum_{i=0}^{2^{M-1}-1} \exp \left[ -\frac{1}{2\sigma_0^2} (Y - X_{k,(1,i)})^2 + \sum_{j=0,j \neq k}^{M-1} L_a(B_j) \right]}{\sum_{i=0}^{2^{M-1}-1} \exp \left[ -\frac{1}{2\sigma_0^2} (Y - X_{k,(0,i)})^2 + \sum_{j=0,j \neq k}^{M-1} L_a(B_j) \right]}, \quad (E-5)$$
where $X_{k,(1,i)}$ is the constellation symbol $X$ for the binary decomposition of $i$, where the $k$th bit is 1. Due to arithmetical overflow problems, the max-log approximation has to be applied, so (E-5) is simplified to

$$L(B_k|Y) = L_a(B_k) + \max_i \left[ \frac{1}{\sigma^2} \left( Y \cdot X_{k,(1,i)} - \frac{1}{2} X_{k,(1,i)}^2 \right) + \sum_{j=0, j \neq k}^{M-1} L_a(B_j) \right]$$

$$- \max_i \left[ \frac{1}{\sigma^2} \left( Y \cdot X_{k,(0,i)} - \frac{1}{2} X_{k,(0,i)}^2 \right) + \sum_{j=0, j \neq k}^{M-1} L_a(B_j) \right].$$

(E-6)

$$i = 0 \ldots 2^{M-1} - 1$$
F Measurement and implementation

Besides simulations an other important aspect is the real life measurements and hardware implementation of the presented algorithms. In this section preliminary spectral measurements are presented and hardware environment for testing is described.

![Image of a 24-hour spectral measurement in Hungary near Szolnok, indicating unused spectral band.](image)

**Figure 62**: 24 hour spectral measurements in Hungary, near Szolnok (the unused spectral band are visible).

First, in a joint cooperation with the Hungarian National Media and Communication Regulator preliminary measurements were performed in Hungary near Szolnok. A continuous 24 hour spectral monitoring was carried out in the 400-900 MHz band. A result of these spectral measurements is visible in Fig. 62. It can be observed that the 450-550 MHz band is used rather insufficiently, a large amount of spectra remains unoccupied. This "free" spectrum can be efficiently aggregated by cognitive radio application in the future.

The transmitter and receiver algorithms can be easily implemented and evaluated using a software radio device (USRP - Universal Software Radio Peripheral). Two USRP modules from Ettus Research performs the up- and down conversion, therefore the processing of the transmit and receive digital samples had to be implemented only (Fig. 63). The signals were measured using a spectrum analyzer type R&S FSH3. A measured spectra of OFDM and SMT modulations can be seen in Fig. 64. The final implementation will be performed on a USRP device manufactured by National Instruments.

An other demonstration for the comparison of the spectral behavior of SMT and OFDM transmission signal (with the same number of subcarriers) can be seen in Fig. 65 and Fig. 66. A real FM radio transmission signal – on the center frequency – was surrounded by OFDM (Fig. 65) and SMT (Fig. 66). As it can be seen the ACLR of the OFDM signal strongly disturbers the FM signal meanwhile in case of SMT the FM signal is clearly visible. This effect is not only visible, but also recognizable as the
**Figure 63:** Hardware test environment.

**Figure 64:** Measured OFDM/SMT spectra.

**Figure 65:** Baseband spectra with an FM radio surrounded by OFDM signal.

**Figure 66:** Baseband spectra with an FM radio surrounded by SMT signal.
Table 7: Simulation parameters of the SMT signal and the PAPR reduction schemes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of carriers (N)</td>
<td>512</td>
</tr>
<tr>
<td>Number of used carriers (N_D)</td>
<td>256</td>
</tr>
<tr>
<td>Number of unused carriers (N_U)</td>
<td>256</td>
</tr>
<tr>
<td>Number of reserved carriers (N_R)</td>
<td>12 (5 %)</td>
</tr>
<tr>
<td>Number of symbols</td>
<td>2000</td>
</tr>
<tr>
<td>Modulation type</td>
<td>4-QAM</td>
</tr>
<tr>
<td>Clipping ratio</td>
<td>1 dB</td>
</tr>
</tbody>
</table>

demodulated FM signal is played on a loudspeaker. The recorded sound as a wma file can be downloaded from: https://mit.bme.hu/~kollarzs/phd/fbmc_vs_ofdm_FM.wma.

Measurements were also performed in order to verify the simulation results presented in Section 5. During the measurements the parameters presented in Table 7 were used. The performance was validated with 1 and 3 iterations. The sampling frequency was set to 1 MHz and a carrier frequency of 100 MHz was applied on the USRP software radio. The signals $s^{new}$ were generated offline prior to transmission, these generated samples were used as baseband input signal for the USRPs.

The results of the measured CCDF for the various introduced techniques can be observed in Fig. 67. It can be seen that the results show the same tendencies for the various schemes as in the simulations. Besides the PAPR performance of the schemes, another important measure is the resulting PSD of the SMT transmit signal. In Fig. 68 the original SMT signal without PAPR reduction is compared to the PAPR reduced signal. For the measurements in both cases the full amplifier range was utilized. When using TR and ACE with 3 iterations, the resulting signal samples were scaled to fit the same amplifier range as in the case with no PAPR reduction. Fig. 68 shows the achieved significant power gain of approx. 5 dB, all without notable increase of ACLR, enabling the application of the proposed method in cognitive radio scenarios.

The primary goal is to implement a real time data transmission link on a USRP hardware, where the PAPR reduction procedures, channel equalization algorithms and receive-side signal processing techniques

![Figure 67: CCDF of the measured PAPR values of different PAPR reduced FBMC (SMT) signals.](image1)

![Figure 68: Measured spectrum of FBMC (SMT) signal with and without PAPR reduction.](image2)
can be tested and validated. The aim is to create a transceiver device for demonstrational purposes, enabling the presentation of SMT signals and their cognitive radio related applications. I intend to establish the SMT test environment in close co-operation with National Instruments, as well as to make it available for the industry and use it for transmission measurements in the education of students.

The radio frequency measurements were performed at the Rohde & Schwarz Reference Laboratory and the Optical Communications Systems Laboratory at HVT.
**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACE</td>
<td>Active Constellation Extension</td>
</tr>
<tr>
<td>ACLR</td>
<td>Adjacent Channel Leakage Ratio</td>
</tr>
<tr>
<td>ADSL</td>
<td>Asymmetric Digital Subscriber Line</td>
</tr>
<tr>
<td>ASCET</td>
<td>Adaptive Sine/Cosine modulated filter bank Equalizer for Transmultiplexers</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BNC</td>
<td>Bussgang Noise Cancelation</td>
</tr>
<tr>
<td>BS</td>
<td>Black Space</td>
</tr>
<tr>
<td>BST</td>
<td>Base STation</td>
</tr>
<tr>
<td>CCDF</td>
<td>Complementary Cumulative Density Function</td>
</tr>
<tr>
<td>CE-OFDM</td>
<td>Constant Envelope OFDM</td>
</tr>
<tr>
<td>CF</td>
<td>Clipping and Filtering</td>
</tr>
<tr>
<td>CM</td>
<td>Cubic Metric</td>
</tr>
<tr>
<td>CMT</td>
<td>Cosine MultiTone</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
</tr>
<tr>
<td>CPE</td>
<td>Consumer Premises Equipment</td>
</tr>
<tr>
<td>CR</td>
<td>Clipping Ratio</td>
</tr>
<tr>
<td>DAB</td>
<td>Digital Audio Broadcast</td>
</tr>
<tr>
<td>DAR</td>
<td>Decision Aided Reconstruction</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DFTS-OFDM</td>
<td>DFT-Spread OFDM</td>
</tr>
<tr>
<td>DMT</td>
<td>Discrete Multitone</td>
</tr>
<tr>
<td>DSB</td>
<td>Double SideBand</td>
</tr>
<tr>
<td>DSL</td>
<td>Digital Subscriber Line</td>
</tr>
<tr>
<td>DVB</td>
<td>Digital Video Broadcast</td>
</tr>
<tr>
<td>DWMT</td>
<td>Discrete Wavelet Multitone</td>
</tr>
<tr>
<td>EVM</td>
<td>Error Vector Magnitude</td>
</tr>
<tr>
<td>EXIT</td>
<td>EXtrinsic Information Transfer (chart)</td>
</tr>
<tr>
<td>FBMC</td>
<td>Filter Bank MultiCarrier</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>FDM</td>
<td>Frequency Division Multiplexing</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FMT</td>
<td>Filtered MultiTone</td>
</tr>
<tr>
<td>GS</td>
<td>Gray Space</td>
</tr>
<tr>
<td>HSPA</td>
<td>High Speed Packet Access</td>
</tr>
<tr>
<td>ICI</td>
<td>Inter Symbol Interference</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>IQ</td>
<td>Inphase and Quadrature</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter Symbol Interference</td>
</tr>
<tr>
<td>ISM</td>
<td>Industrial, Science and Medical (band)</td>
</tr>
</tbody>
</table>
LDPC  Low-Density-Parity-Check (code)
LLR   Log-Likelihood Ratio
LO    Local Oscillator
LPF   Low Path Filter
LTE   Long Term Evolution
MAC   Medium Access Control (layer)
ML    Maximum Likelihood
MMSE  Minimum Mean Square Error
Ofcom Office of Communications
OFDM  Orthogonal Frequency Division Multiplexing
OQAM  Offset Quadrature Amplitude Modulation
PA    Power Amplifier
PAM   Pulse Amplitude Modulation
PAPR  Peak-to-Average Power Ratio
PHY   PHYsical (layer)
PRT   Peak Reduction Tones
PSD   Power Spectral Density
PTS   Partial Transmit Sequence
QAM   Quadrature Amplitude Modulation
QoSMOS Quality of Service and MObility driven cognitive radio Systems
R-DFT  Recursive Discrete Fourier Transform
RCM   Raw Cubic Metric
SC-FDMA Single Carrier Frequency Division Multiple Access
SDR   Signal-to-Distortion Ratio
SISO  Soft-In/Soft-Out
SLM   Selected Mapping
SMT   Staggered MultiTone
SNR   Signal-to-Noise Ratio
TI    Tone Injection
TR    Tone Reservation
USRP  Universal Software Radio Peripheral
VDSL  Very high bit rate Digital Subscriber Line
VSB   Vestigial SideBand
WLAN  Wireless Local Area Network
WRAN  Wireless Regional Area Network
WS    White Space
ZF    Zero Forcing
List of symbols

\( A_{\text{max}} \)   | Clipping level          \\
\( \alpha \)          | Attenuation factor for clipping \\
\( B \)               | Binary information unit  \\
\( C \)               | Binary coded information unit \\
\( c \)               | Element of the constellation alphabet \\
\( \mathcal{C} \)      | Modulation Alphabet \\
\( d(t) \)            | Clipping noise \\
\( \delta(t) \)        | Dirac delta function \\
\( \delta_{\text{mi}} \) | Kronecker delta \\
\( F_c \)             | Carrier frequency \\
\( F_{cR} \)          | Frequency of receiver LO \\
\( F_{cT} \)          | Frequency of transmitter LO \\
\( F_s \)             | Sampling frequency \\
\( \Delta F \)         | Frequency distance between neighbouring subcarrier bands \\
\( \Delta F_c \)       | LO mismatches between the transmitter and receiver side \\
\( \phi_{\text{IQ}} \)  | Quadrature error \\
\( \gamma \)           | Clipping ratio \\
\( \gamma_1 \)         | Peak-to-average power ratio \\
\( \gamma_2 \)         | Kurtosis \\
\( h(t) \)            | Impulse response of the channel filter \\
\( h \)                | Modulation index \\
\( \Theta \)           | PAPR gain metric \\
\( \Im \)              | Imaginary value \\
\( L_{ch} \)           | Length of the channel filter in samples \\
\( L_{po} \)           | Length of the prototype filter in samples \\
\( K \)                | Overlapping factor for SMT \\
\( M_b \)              | Number of bits mapped to a constellation symbol \\
\( N \)                | Number of subcarriers/subchannels \\
\( N_D \)              | Number of data subcarriers \\
\( N_R \)              | Number of reserved subcarriers \\
\( N_Z \)              | Number of unused subcarriers \\
\( P \)                | Length of the CP \\
\( p_0(t) \)           | Continuous time impulse response of the prototype filter \\
\( P_d \)              | Average power of the clipping noise \\
\( P_s \)              | Average signal power \\
\( P_{\text{out}} \)    | Average output signal power \\
\( p_0[n] \)           | Discrete time impulse response of the prototype filter \\
\( p_R(t) \)           | Receiver matched filter \\
\( p_T(t) \)           | Transmitter pulse shaping filter \\
\( R \)                | Coding rate \\
\( r(t) \)             | Received complex baseband signal
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{R}$</td>
<td>Real value</td>
</tr>
<tr>
<td>$s(t)$</td>
<td>Transmitted complex baseband signal</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>Standard deviation of the AWGN term</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Standard deviation of the transmitted signal</td>
</tr>
<tr>
<td>$T$</td>
<td>Signalling time</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Sampling time</td>
</tr>
<tr>
<td>$v(t)$</td>
<td>Passband transmission signal</td>
</tr>
<tr>
<td>$v_s(t)$</td>
<td>Real valued passband signal</td>
</tr>
<tr>
<td>$w(t)$</td>
<td>AWGN noise term</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>Amplified passband transmission signal</td>
</tr>
<tr>
<td>$x$</td>
<td>Complex baseband modulated signal</td>
</tr>
<tr>
<td>$X_k[m]$</td>
<td>Complex modulation symbol on the $k^{th}$ subcarrier in the $m^{th}$ signalling time</td>
</tr>
<tr>
<td>$y(t)$</td>
<td>Received signal</td>
</tr>
</tbody>
</table>
References


